Neutrino-Nucleon Interactions in Core Collapse Supernovae

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Neutrino interactions in proto-neutron star matter are a major determinant for many key properties of core-collapse supernovae (CCSNe), such as the explosion mechanism or the nucleosynthesis of heavy elements. Recent works identify a need for increasingly precise interaction rates. This work presents an alternative way to compute the relativistic Hartree response for neutrino-nucleon interactions, improving upon previously derived expressions in several aspects. The result will then be compared to the so called elastic approximation which is widely used in supernova simulations, to asses the validity and limitations of the latter.

1 Introduction

The gravitational collapse of the inner core of a massive star at the end of its lifetime releases a huge amount of gravitational binding energy of the order of several 10^{53} erg. Eventually the collapsing core reaches supranuclear densities, where it is stopped by the strong repulsion of the nuclear interaction. At this point the binding energy is mostly converted into heat, leading to temperatures of several tens of MeV in the nascent protoneutron star (PNS). Neutrinos are emitted in large numbers both due to thermal production as well as the desire of the matter to reach weak chemical equilibrium via electron captures on protons. However for densities above $10^{11}\,\mathrm{g\,cm^{-3}}$ the neutrino mean free path (MFP) becomes so short that neutrinos are trapped in the matter, forcing them into local thermal and chemical equilibrium. Nevertheless, due to the weak coupling of neutrinos to the matter compared to all other interactions in the supernova core, the diffusion and emission of neutrinos is by far the most efficient way to transport energy into the envelope of the massive star and beyond. Consequently, it was expected early on that the vast majority of the gravitational binding energy will be released in the form of neutrinos, a prediction that was confirmed by the observation of the neutrino signal from SN 1987A.

During the first ten seconds after core bounce, the most important region for the formation of the neutrino spectra is the so called neutrinosphere, where neutrinos decouple from the matter. The first neutrino signal is an initial deleptonization burst that is emitted after the supernova shock crosses this neutrinosphere. As the shock heats the matter, the heavy nuclei of the progenitor star are dissolved into free nucleons. Chemical equilibrium at these high temperatures and densities favours then a significantly more neutron rich composition, causing rapid electron captures on free protons and subsequent emission of electron neutrinos on a timescale of only several milliseconds. After that, the neutrino spectra can be approximately understood as thermal emission from the neutrinosphere.

Once the neutrinos are outside of the neutrinosphere the probability of scattering or absorption in the envelope is rather small. However, since the neutrino luminosity is so large, on the order of a few $10^{52}\mathrm{erg\,s}^{-1}$, the small fraction of neutrinos that do interact can have an enormous impact on the envelope above the PNS. The most important effect is probably the revival of the shock that leads to the eventual supernova explosion via the so called delayed neutrino heating mechanism (for a review on CCSNe simulations and possible explosion mechanisms see e.g. [1]). After the shock looses most of its energy by photodissociating heavy nuclei in the accretion flow, it quickly stalls. In the standard core-collapse scenario it is then expected that the region below the shock (outside the neutrinosphere) is heated faster by absorbing neutrinos from the PNS than it can cool by emitting neutrinos itself. This process transfers energy and momentum from the PNS to the shock, resulting in the eventual explosion. In this context multi-dimensional hydrodynamics have been identified as a most crucial ingredient, allowing for multi-D turbulences and convection which help to increase the heating efficiency. This proved necessary to achieve successful explosion in CCSN simulations. Yet, these advances in hydrodynamics have always been accompanied by essential improvements in the neutrino transport scheme and general relativity, as well as the microphysics of nuclear matter, nuclear interaction, and weak interaction between neutrinos and matter. Since the predicted explosions still differ significantly from observed explosion energies, this process of refinement is still ongoing.

Neutrinos also play a very important role in the nucleosynthesis of heavy or rare elements in CCSNe. One such scenario is the neutrino driven wind (NDW), matter that is ejected from the PNS surface after shock revival. The idea is again that absorptions of neutrinos transfer enough energy and momentum to this matter to drive a matter outflow. Once this wind expands and cools, nuclei are formed, with the possibility of reaching very large atomic numbers. Without going into the details of nucleosynthesis, the outcome of this scenario depends strongly on the initial conditions in the NDW, in particular the expansion timescale, the entropy per baryon and the chemical composition. All of these properties are strongly affected if not fully determined both by the absolute size of the neutrino flux as well as the specific spectral characteristics and even spectral differences of the distinct neutrino flavours.

Recent studies strongly hint towards the need to improve the precision of neutrino microphysics. With respect to the neutrino heating mechanism, it was found that reaching a successful explosion of a marginal supernova in 3D might depend on 10-20% changes to the rate for neutrino-nucleon scattering[2].

For the neutrino driven wind, it seems that core collapse supernova will not result in conditions that allow for a so called full r-process [3, 4], building the heaviest elements in the universe. However, a so called weak r-process is still very much possible [5] and might have contributed already to the early chemical evolution in the universe. Whether the neutrino driven wind will eventually allow for a weak r-process and to which extent depends on the detailed composition of the NDW, which depends in turn on a precise description of neutrino interactions at the level of 10%.

Eventually, it is predicted that the neutrino signal from a CCSN might undergo significant modifications due to neutrino oscillations [6]. Although this work will not look in detail on the topic of oscillations, referring instead to the various complementing contributions in this proceedings, the resulting neutrino spectra are obviously dependent on the initial spectral differences between the neutrino flavours.

All these findings lead to the conclusion, that current research on core-collapse supernova and related nuclear astrophysics requires a sufficiently accurate description of neutrino interactions in hot at dense matter. The purpose of this contribution is to discuss several problems and uncertainties at this level of precision and to propose solutions for some of them. Section 2 presents an alternative formalism for the relativistic Hartree response of neutrino-nucleon interactions, an expression that yields very precise values for the neutrino MFP at various conditions in CCSNe. In Section 3 the relativistic Hartree response is used to assess the limitations of the so called elastic approximation[7], an analytic expression for the MFP that is widely used in supernova simulations. Finally, the work is concluded in Section 4.

2 Hartree Response for Neutrino-Nucleon Interaction

The general derivation of the Hartree response for neutrino-nucleon interactions is the same for charged and neutral-current reactions. This work studies reactions of the type

$$\nu_1 + N_2 \rightarrow l_3 + N_4.$$

The particle ν_1 can represent either a neutrino or an antineutrino, while the final state lepton l_3 will be a neutrino of the same type (neutral current) or the corresponding charged lepton (charged current). For charged leptons the finite mass is taken into account, so that the description is applicable to charged-current interactions of muon neutrinos. In free space and at the energies encountered in CCSNe, these weak interactions can be described by a current-current Lagrangian

$$\mathcal{L} = \frac{G}{\sqrt{2}} l_\mu j^\mu,$$

where $G = G_F V_{ud}$ for charged current and $G = G_F$ for neutral current, with G_F the Fermi coupling constant and V_{ud} the up-down element of the CKM-matrix. For reactions with a neutrino in the initial state the lepton current reads

$$l_\mu = \bar{\psi}_3 \gamma_\mu \left(1 - \gamma_5\right) \psi_1,$$

where the ψ_i are Dirac spinors. Note that for an antineutrino in the initial state, indices 1 and 3 have to be exchanged. The nucleon current is likewise given by

$$\psi_4 \left[c_V \gamma^\mu + \frac{iF_2}{2M_N} \sigma^{\mu\nu} q_\nu - c_A \gamma^\mu \gamma_5 \right] \psi_2$$

The value of the coupling constants c_V, c_A, and F_2 depends on the specific reaction (see e.g. [15]) For a noninteracting Fermi gas, the Lagrangian $\mathcal{L}$ is equivalent to the matrix element. Hence, using Fermi's Golden Rule the mean free path λ, considering final state Pauli blocking, can be computed by

$$\frac{1}{\lambda(E_1)} = 2 \int \frac{d^3\boldsymbol{p}_2}{(2\pi)^3} \int \frac{d^3\boldsymbol{p}_3}{(2\pi)^3} \int \frac{d^3\boldsymbol{p}_4}{(2\pi)^3} \frac{\left\langle |\mathcal{M}|^2 \right\rangle}{16 E_1 E_2 E_3 E_4} \, \delta^4(P_1 + P_2 - P_3 - P_4) \, f_2 \left(1 - f_3\right) \left(1 - f_4\right),$$

with the particle four momenta P_i. The f_i are standard Fermi distributions.

When studying nucleonic matter at densities above $10^{12}\,\mathrm{g\,cm^{-3}}$ the assumption of noninteracting nucleons is not justified. Both the strong and the electromagnetic interaction affect the spectra of nucleons and/or the correlations between them. These effects have to be taken into account for neutrino interactions. One approach is to compute the nucleon response via the medium polarization tensor in the so called Hartree approximation [8, 9], which takes into account how the nucleon spectrum is altered by the mean effect of the interaction. Instead, this work computes the Hartree response by replacing the free space nucleon current with an effective nucleon current in medium, treating the nucleons as quasi free Fermions. However, the formalism of the medium polarization allows to go beyond the mean field by computing the nucleon response via the random phase approximation (RPA) [8, 10, 11, 12, 13]. The RPA includes correlations among the nucleons, stemming from the residual interaction. This is particularly relevant for the neutral-current response, where the correlations may be more important than the mean field, especially for subsaturation densities.

An advantage of the new formalism is that the relativistic Hartree response is analytically more closed, allowing for a simpler and faster numerical computation. Also, for the first time a finite lepton mass is taken into account to describe charged current reactions of ν_μ.

In astrophysical simulations the nucleonic matter is often described by a relativistic mean field (RMF) equation of state (EOS) [14], in particular by a $\sigma\omega\rho$-model. Here, the free nucleon spectrum is replaced by a quasi-particle dispersion relation, given by

$$E_{n,p} = \sqrt{m^{*2}_{n,p} + \boldsymbol{p}^2} + U_{n,p},$$

where $m^*_{n,p}$ and $U_{n,p}$ are the nucleon effective masses and relativistic mean field potentials, respectively, of neutrons and protons. They are directly related to the mean meson fields.

To compute the Hartree response, one replaces the free nucleon spinors ψ_i in the nucleon current by effective nucleon spinors ψ_i^*,

$$\psi_i^* = \frac{1}{\sqrt{E_i^{*2} + m_i^{*2}}} \begin{pmatrix} \chi_s \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{E_i^* + m_i^*}\chi_s \end{pmatrix}.$$

Here χ_s are normalized Pauli spinors so that $\chi_s^\dagger \chi_{s'} = \delta_{ss'}$, $\boldsymbol{\sigma}$ is the vector of Pauli matrices, and E_i^* is given by

$$E_i^* = E_i - U_i = \sqrt{m_i^{*2} + \boldsymbol{p}_i^2}.$$

Note that ψ_i^* becomes the free nucleon spinor for vanishing meson fields. Plugging ψ_* in the nucleon current one can derive the squared matrix element in medium $\left\langle |\mathcal{M}_{HRT}|^2 \right\rangle$. To compute then the MFP one only needs to adjust the normalization of the phase space integrals to conserve Lorentz invariance,

$$\frac{1}{\lambda(E_1)} = 2\int \frac{d^3\boldsymbol{p}_2}{(2\pi)^3} \int \frac{d^3\boldsymbol{p}_3}{(2\pi)^3} \int \frac{d^3\boldsymbol{p}_4}{(2\pi)^3} \frac{\left\langle |\mathcal{M}_{HRT}|^2 \right\rangle}{16 E_1 E_2^* E_3 E_4^*} \delta^4(P_1 + P_2 - P_3 - P_4)\, f_2\,(1-f_3)\,(1-f_4)\,,$$

where the Fermi distributions for the nucleons describe the quasiparticle distributions,

$$f_{2,4}(E_{2,4}) = \left[1 + \exp\left(\left[\sqrt{m^{*2}_{2,4} + \boldsymbol{p}^2} + U_{2,4} - \mu_{2,4}\right]/T\right)\right]^{-1}.$$

The MFP cannot be computed fully analytically. However, it can be closed analytically up to a 2-dimensional numerical integral,

$$\frac{1}{\lambda} = \frac{G^2}{4\pi^3}\frac{1}{E_1^2}\int_{E_{2-}}^{\infty} dE_2 \int_{m_3}^{E_{3+}} dE_3 f_2 \left[1 - f_3\right]\left[1 - f_4\right] I_{tot}.$$

The integration limits E_{2-} and E_{3+} are constraints that arise from four-momentum conservation. The term I_{tot} is the fully analytical solution of the relation

$$I_{tot} = \frac{p_1 p_2 p_3 p_4}{4\pi^2}\int d\Omega_2 d\Omega_3 d\Omega_4 dE_4 \frac{\left\langle \left|\mathcal{M}_{HRT}\right|^2 \right\rangle}{16G^2}\delta^4(P_1 + P_2 - P_3 - P_4)$$

with $p_i = |\boldsymbol{p}_i|$. The detailed formalism and derivation will be presented in a future publication.

Previous solutions for the relativistic Hartree response were closed only up to three numerical integrals [12]. Yet, for neutral-current interactions the above approach can be applied only when the final state neutrino is either distributed isotropically or when final state neutrino blocking can be neglected. Otherwise the factor $(1 - f_3)$ cannot be factored out of the angular integrals. In that case the relativistic Hartree response as given in the literature cannot be further closed. The following section will therefore focus on charged-current interactions.

3 Comparison to Elastic Approximation

A simplification of the mean free path for neutrino-nucleon interactions that is often used in CCSN simulations is the so called *elastic approximation* [7]. Its advantage is that it delivers an analytic expression for λ. Starting from the noninteracting Fermi gas approach, the elastic approximation is inherently nonrelativistic. Expanding $\left\langle |\mathcal{M}|^2 \right\rangle$ in powers of (E_1/m_2) and dropping all terms of order 1 or higher, the squared matrix element simplifies to

$$\frac{\left\langle |\mathcal{M}|^2 \right\rangle}{16 E_1 E_2 E_3 E_4} = c_A^2\left(3 - x\right) + c_V^2\left(1 + x\right),$$

where x is the angle cosine between initial and final lepton momenta $\boldsymbol{p}_1$ and $\boldsymbol{p}_3$. Also, one assumes that the absolute momentum of the nucleons stays constant, $p_2 = p_4$. The final state lepton energy equals then the initial neutrino energy plus the mass difference between nucleons, and the mean free path can be derived analytically. It is straightforward to expand this approximation to consider the quasiparticle dispersion relation for nucleons in the mean field theory [3],

$$\frac{1}{\lambda_{elst}} = \frac{G^2}{\pi}\left(c_V^2 + 3c_A^2\right) E_3 p_3 \left(1 - f_3\right)\frac{n_2 - n_4}{1 - \exp\left[\left(\eta_4 - \eta_2\right)/T\right]}\Theta(E_3 - m_3),$$

with

$$E_3 = E_1 + m_2^* - m_4^* + U_2 - U_4 \qquad \text{and} \qquad \eta_i = m_i^* + U_i - \mu_i.$$

As an improvement over the elastic approximation, analytic correction factors for finite nucleon recoil and subleading order terms in the matrix element (often called *weak magnetism*

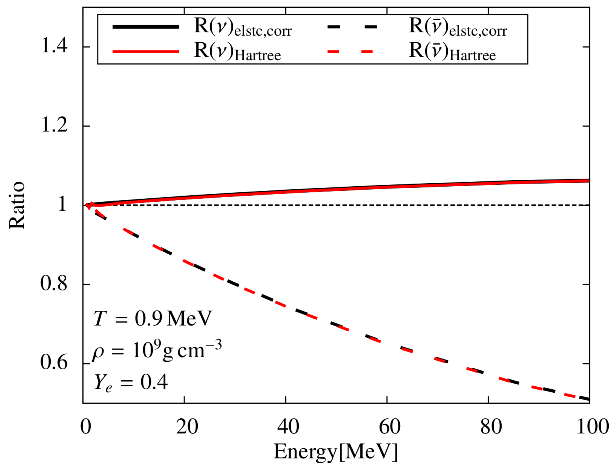

Figure 1: Ratio of mean free path from relativistic Hartree response to elastic approximation (bright/red) at low densities and temperatures, compared to nucleon recoil and weak magnetism correction factor(black). Solid and dashed lines correspond to neutrino and antineutrino absorption, respectively.

corrections) were suggested [15]. Essentially, these correction factors are the ratio of the exact free space cross section and its nonrelativistic approximation. Multiplying with this ratio shall correct the simplifications that were made in the matrix element and the final lepton phase space factor. This approach is nearly exact for nucleons at low densities and temperatures. However, it can not correct the approximations that were made in the Pauli blocking factors.

For charged-current reactions, the appropriate correction factor is

$$R(E_1) = \left\{ c_V^2 \left(1 + 4e + \frac{16}{3}e^2\right) + 3c_A^2 \left(1 + \frac{4}{3}e\right)^2 \pm 4\left(c_V + F_2\right) c_A e \left(1 + \frac{4}{3}e\right) + \frac{8}{3} c_V F_2 e^2 + \frac{5}{3}e^2 \left(1 + \frac{2}{5}e\right) F_2^2 \right\} / \left[\left(c_V^2 + 3c_A^2\right)(1 + 2e)^3\right].$$

Looking at the elastic approximation with nucleon recoil and weak magnetism corrections, one can compare it to the exact mean free path (bar correlations) from the Hartree response, in order to assess the precision of the former for various conditions in a CCSN. The situation at lower densities and temperatures, corresponding e.g. to the region in the neutrino driven wind, is investigated in Fig.1. Here the ratio between the Hartree response and the elastic approximation is shown along the analytic correction factor. Note that in both Fig.1 and Fig.2 the underlying composition and interaction of the nucleons is described by the Hempel-EOS [14] in the DD2-parametrization [16]. However, all findings presented here are practically independent of the chosen RMF-parametrization. For a temperature of $T = 0.9\,\mathrm{MeV}$ and density $\rho = 10^9\,\mathrm{g\,cm^{-3}}$ one can see a nearly perfect agreement between the two. This seems reasonable, as final state blocking is probably negligible under these conditions and the mean free path is almost proportional to the cross section. Also the thermal energy of the nucleons is not very large, justifying the assumption of an initial nucleon at rest. It seems safe to use the corrected elastic approximation e.g. to determine the equilibrium electron fraction in the

NDW when the neutrino spectrum from the proto-neutron star is known.

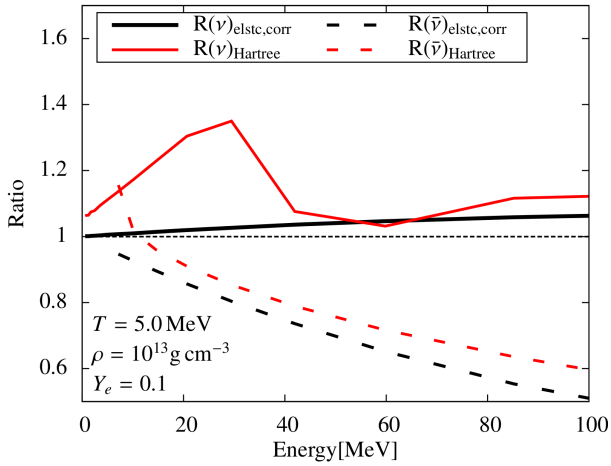

Figure 2: Ratio of mean free path from relativistic Hartree response to elastic approximation (bright/red) at low densities and temperatures, compared to nucleon recoil and weak magnetism correction factor(black). Solid and dashed lines correspond to neutrino and antineutrino absorption, respectively.

Higher densities and temperatures are studied in Fig.2. At a temperature of $T = 5\,\text{MeV}$ and a density of $\rho = 10^{13}\,\text{g cm}^{-3}$ one finds that the corrected elastic approximation underestimates the mean free path for almost all energies, both for neutrinos and antineutrinos. The deviation is on the order of 10%, and it can reach more than 30% for neutrinos in the energy range around 20 MeV, which is particularly important for neutrino transport and emission in CCSN. Therefore it is likely that this might affect the neutrino spectrum in the early cooling and NDW phase, starting several hundred seconds post bounce. During the accretion and explosion phase, the electron type neutrinos decouple mostly from lower densities, where this deviation is smaller. However, one can find deviations on the order of 10% already above densities $\rho = 10^{12}\,\text{g cm}^{-3}$. Also, for temperatures above 5 MeV an increase in the Hartree response compared to the corrected elastic approximation by several percent is observed, even for the lower densities, reaching $\sim$ 10% for $T = 10\,\text{MeV}$. Such temperatures can be realized at the neutrinospheres in the early postbounce phase, potentially affecting the accretion and explosion phase.

The general understanding behind the suppression of the elastic approximation compared to the Hartree response is that the restrictions on momentum and energy exchange force the final state particles into configurations with too large Pauli blocking. The correction factor cannot improve this situation since it does not address the statistical factors. Ultimately, further work has to be done to understand this observation in more detail. Yet one can conclude that the elastic approximation, despite adding nucleon recoil and weak magnetism corrections, does not reach a precision beyond the 10% level for charged current reactions of electron type neutrinos at high densities (relevant only later times) and/or high temperatures (relevant at early times postbounce). However, the approximation is well suited for conditions such as in the NDW, where it reproduces the Hartree response with great accuracy.

4 Conclusion

The relativistic Hartree response for weak neutrino-nucleon interactions was derived different from the standard formalism in the literature. For charged-current reactions, the resulting new expression is analytically more closed and allows to consider finite lepton masses like in charged-current ν_μ-absorption. The improvements can also be applied to neutral-current scatterings, if the neutrino distribution is isotropic or final state lepton blocking is negligible. The Hartree response was compared to the elastic approximation with nucleon recoil and weak magnetism corrections to assess the precision of the latter in CCSNe simulations. It was found that the accuracy will be less than 10% for temperature above 5 to 10 MeV and densities higher than $10^{12}\,\mathrm{g\,cm^{-3}}$. The likely reason is an artificially strong final state blocking enforced by the rigid kinematic constraints of the elastic approximation. Hence, it must be used with caution both during the early accretion and explosion phase as well as the initial cooling phase. Nevertheless it is well suited to describe neutrino-nucleon interactions in the neutrino driven wind, which takes place at lower densities and temperatures. Future studies should try to understand the limitations of the elastic approximation better. Also it would be interesting to compare both the Hartree response and the elastic approximation in the framework of a dynamical CCSN simulation to quantify the impact of different rates both on the explosion mechanics and the subsequent neutrino spectrum in the initial cooling phase.

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