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DRELL EFFECT AND GAUGE INVARIANCE

by

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2 HAMBURG-GR. FLOTTBEK 1, NOTKESTIEG 1

Drell Effect and Gauge Invariance

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It is the aim of the present note to point out the necessity of a gauge invariant treatment of the Drell effect [1]. This leads for the example $\gamma+p\to N^*+\pi^{-,+}$ which we have treated in detail, to important corrections even at high energies.

The one-pion-exchange (OPE) contribution to the process

$$\gamma + N \to X + \pi^{\pm} \tag{1}$$

(X stands for anything)

has been calculated by Drell [1] according to the Feynman graph fig.1 neglecting the virtuality [2] of the pion at the vertex $\pi + N \to X$. The contribution of this diagram should dominate in the process (1) at high energies and small momentum transfer according to the usual arguments establishing the peripheral model [3].

The condition of gauge invariance for the transition amplitude $T_{\gamma+N \to X+\pi}$ may be stated in the form [4]

$$T_{\gamma+N \to X+\pi}(\epsilon) \,|_{\epsilon = k} = 0 \tag{2}$$

Only such approximations for $T_{\gamma+N \to X+\pi}$ make sense which fulfil equ.(2) because gauge invariance tells that the photon is a vector particle with zero rest mass. It is obvious that the matrix element $T^1_{\gamma+N\to X+\pi}$ corresponding to the Drell graph fig. 1 violates gauge invariance except at the nonphysical pole $t = \mu^2$ because

$$T^1_{\gamma+N \to X+\pi^{\pm}} \propto (q \cdot \epsilon) \tag{3}$$

Unfortunately it is not possible to preserve gauge invariance, at least for the cross section, by taking the polarization sum $\sum_{\mathcal{E}} |\epsilon \cdot q|^2$ at the pole because this procedure would lead to $\frac{d\sigma}{d\Omega} < 0$.

Now every non gauge invariant expression depends on a time-like gauge vector [5] a $(a \neq k)$ with $(\epsilon \cdot a) = 0$. Therefore, the Drell formula would be reliable only if it would be almost insensitive against changes of a for small physical t-values. This is not the case because for instance the choice a = q makes $T^{1}_{\gamma+N\to X+\pi^{\pm}}$ equal to zero[6]. Therefore, a gauge invariant extension of $T^{1}_{\gamma+N\to X+\pi^{\pm}}$ is called for. But this is not unique from a pure mathematical point of view [7]. To get an unique extension we are forced to look for some physical principles. We are far away from a formulation of this unique extension for the general process (1) but we have succeeded in the particular case of the reaction

$$\gamma + p \to N^{*++,0} + \pi^{-,+} \qquad (4)$$

If we treat the 3/2 3/2 -πN-resonance N^* like an ordinary elementary particle of spin 3/2 the corresponding Drell graph for reaction (4) takes the form [8] of fig. 2. The corresponding matrix element T^I has the following properties:

1. It is of lowest order in the coupling constants e and f
2. The photon interacts with the "orbital current" of a moving charged particle
3. With respect to the isospin decomposition of the total transition amplitude for reaction (4) in the t-channel T^I contains only a $T_{1,1}$ part (the total transition amplitude contains a superposition of three amplitudes T_{I,I_γ} where I is equal to the total isospin and $I_\gamma = 0,1$ is the isospin of the photon).

We now define the gauge invariant extension of T^I in such a way that the properties 1. to 3. are maintained [9]. This extension has the properties of being unique and minimal. Therefore, the extended amplitude may be called the gauge invariant OPE-contribution to reaction (4).

To get a numerical estimate of the corrections caused by the gauge invariant extension of T^I we have to treat some of the details: In fig.3 all possible Feynman graphs for reaction (4) which are of lowest order in e and f are presented [10]. If we normalize the corresponding amplitudes $T^{I\,--IV}$ such that the photon interacts with a

- 3 -

positive charged particle of unit charge, the $T_{1,1}$ part of this class of diagrams has the form

$$T_{1,1} = T^{I} + T^{III} - \frac{1}{4} T^{II} - \frac{5}{4} T^{IV} \tag{5}$$

One gets according to the usual Feynman rules for $T^{I\,--IV}$

$$T^{I} = ef \, \frac{2(\varepsilon \cdot q)}{t - \mu^2} \, \bar{u}_\mu(p_2)\,(k-q)^\mu \, u(p_1)$$

$$T^{II} = ef \, \bar{u}_\mu(p_2)\,(-q)^\mu \, \frac{(\not{p}_1 + \not{k}) + M}{s - M^2} \, \not{\varepsilon} \, u(p_1) \tag{6}$$

$$T^{III} = ef \, \bar{u}_\mu(p_2)\,\varepsilon^\mu \, u(p_1)$$

$$T^{IV} = - f(2\pi)^3 \, \Gamma_\mu(p_2, k, \varepsilon)\,(-q)^\mu \, u(p_1)$$

with $\Gamma_\mu(p_2, k, \varepsilon) \equiv \langle N^*; p_2 | \Psi_\mu(0) | \gamma; k, \varepsilon \rangle$

where $u(p_1)$ is the Dirac spinor for the incoming proton and $u_\mu(p_2)$ resp. Ψ_μ are the momentum space wave function resp. field operator for the N^* described with the Rarita-Schwinger formalism [11].

By means of the Dirac equation $(\not{p}_1 - M)\, u(p_1) = 0$ and the commutation relations of the γ-matrices one obtains for T^{II} the decomposition

$$T^{II} = T^{II,1} + T^{II,2}$$

with

$$T^{II,1} \equiv - ef \, \bar{u}_\mu(p_2)\, q^\mu \, u(p_1) \, \frac{2(\varepsilon \cdot p_1)}{s - M^2}$$

$$T^{II,2} \equiv - \frac{ef}{s - M^2} \, \bar{u}_\mu(p_2)\, q^\mu \, \not{k} \, \not{\varepsilon} \, u(p_1) \tag{7}$$

where $T^{II,1}$ resp. $T^{II,2}$ describes the "orbital current" part resp. the contribution of the normal magnetic moment of the proton to the graph fig.3-II. For a similar decomposition of T^{IV} we use the generalized Ward identity [12] which reads for $\Gamma_\mu(p_2, k, \epsilon)$

$$\Gamma_\mu(p_2, k, \varepsilon)\Big|_{\varepsilon = k} = e(2\pi)^{-3} \, \bar{u}_\mu(p_2) \tag{8}$$

By means of Lorentz invariance and the Ward identity equ.(8) the most general form of Γ_μ is obtained as follows

$$-(2\pi)^3 \; \Gamma_\mu (p_2, k, \varepsilon) = \frac{e}{(p_2 - k)^2 - M^{*2}} \; \bar{u}^\nu(p_2)\Big[2(\varepsilon \cdot p_2)\, g_{\nu\mu} + \sum_i f_i \; C^i_{\nu\mu}(p_2, k, \varepsilon, \gamma)\Big] \qquad (9)$$

with $C^i_{\nu\mu}(p_2, k, \varepsilon, \gamma)\big|_{\varepsilon = k} = 0$

where the f_i are related to the electromagnetic moments [13] of the N^*. Therefore, the "orbital current" part $T^{IV,1}$ of T^{IV} has the form

$$T^{IV,1} = -\, e f \; \frac{2(\varepsilon \cdot p_2)}{(p_2 - k)^2 - M^{*2}} \; \bar{u}_\mu(p_2)\, q^\mu u(p_1) \qquad (10)$$

Then by means of equ. (5) to (10) the gauge invariant extension of T^I fulfilling the forementioned three properties is expressed as follows

$$T^{OPE} = T^{I} + T^{III} - \frac{1}{4} T^{II,1} - \frac{5}{4} T^{IV,1} \qquad (11)$$

According to equ. (6),(7) and (10) the gauge invariance of equ.(11) is obvious.

By standard methods we then obtain from equ. (11) for the cross section of the reaction $\gamma + p \to \pi + N + \pi^\pm$ in the isobar approximation for the πN-scattering in the high energy limit [14]

$$\frac{d^3\sigma}{dq_0\, d\Omega} \underset{s \to \infty}{\simeq} e^2 (2\pi)^{-3} \, \frac{|\underset{\sim}{p}|\sqrt{s'}}{\sqrt{s'}} \, \sigma_{33}(s') \left\{ \frac{-t}{(t-\mu^2)^2} + \frac{1}{2} |\underset{\sim}{p}|^{-2} + \frac{5}{64} |\underset{\sim}{p}|^{-2} \, \frac{3\mu^2 + 5t + 8s' - 8M^2}{s'} \, \frac{(-t)}{t-\mu^2} \right\} \qquad (12)$$

with

$$s' \equiv (k + p_1 - q)^2 \;, \qquad |\underset{\sim}{p}|^2 \equiv \frac{(s' - M^2 + \mu^2)^2}{4s'} - \mu^2$$

i.e. S' is the total energy squared in the rest system of the N^*, $|\underset{\sim}{p}|$ the corresponding momentum and σ_{33} the isobar contribution to the total πN-cross section.

Now the first term in equ. (12) corresponds to the Drell formula [1] while the second and third terms are corrections. These corrections do not vanish at high energies contrary to the supposition of Drell [15]. They are at low momentum transfer of an order of magnitude comparable to the original Drell prediction [1] as can be read off from fig. 4. The general tendency of this corrected cross section for momentum transfer $-t \lesssim \mu^2$ agrees with some recent measurements in the BeV-region at CEA [16].

We further note:

a) In the static limit ($M \to \infty$) our results correspond to the results obtained by F. Hadjioannou [17] and K. Itabashi [18] within the static Chew-Low model because $T^{II,1} = 0$ for the coulomb gauge in the CMS and $T^{IV,1} \underset{M \to \infty}{\to} 0$. Therefore, our model is a simple relativistic generalisation of the static theory. The forementioned static results agree remarkable with the Caltech measurements [19] at about 1,2 BeV.

b) In the cross section corresponding to the single diagram fig.3-I some kinematical "off shell" effects are included automatically. Their importance has already been stressed [20].

c) The proportionality of our results equ.(12) to the πN total cross section depends very strongly on the used isobar model for the πN-scattering. In a more general situation of reaction (1) this may be false.

References and footnotes

1) S.D. Drell : Phys.Rev.Lett. 5 (1961) 278

2) Off shell corrections have been considered in a recent paper by M.L. Thiebaux,Jr. : Phys. Rev.Lett. 13 (1964) 29

3) Compare:

S.D. Drell: Phys.Rev.Lett 5 (1960) 342

F. Salzman and G. Salzman: Phys.Rev. 120 (1960) 599

" " " Phys.Rev.Lett. 5 (1960) 377

" " " Phys. Rev. 125 (1962) 1703

E. Ferrari and F. Selleri: Nuovo Cimento Suppl. 24 (1962) 453

4) We denote the polarization resp. momentum vector of the incoming photon by ϵ resp. k and the momenta of the incoming nucleon, outgoing X resp. outgoing pion by p_1, p_2 resp. q. Further we introduce the usual invariants $s = (k+p_1)^2$ and $t = (k-q)^2$. We use the metric $g_{oo} = - g_{11} = -g_{22} = -g_{33} = 1$.

5) Drell has calculated the cross section corresponding to the graph fig. 1 with the coulomb gauge in the overall CMS (i.e. $a = p_1$).

6) This is no contradiction to the gauge invariance at the pole because the limits $a \to q$, $t \to \mu^2$ do not commute for the polarization sum $\sum_\epsilon | q.\epsilon |^2$.

7) For instance an extension of equ.(3) in the form

$$(\varepsilon \cdot q) \longrightarrow (\varepsilon \cdot q) - \frac{(\varepsilon \cdot b)(k \cdot q)}{(k \cdot b)}$$

with an arbitrary four vector b would fulfil the requirement of gauge invariance.

8) The coupling constant f at the $\pi N N^*$- vertex is related to the width of the N^*. Compare:

A.W. Hendry: Nucl. Phys. 37 (1962) 283

9) In a similar way the gauge invariant extension of the OPE-amplitude for the process $\gamma + N \to N + \pi^\pm$ has been dicussed by G. Kramer and P. Stichel: Zeitschr. f.Physik 178 (1964) 519

10) The so called contact graph fig.3-III comes about by the derivative in the effective $\pi N\ N^*$-coupling $f\ \bar{\Psi}_\mu(\partial^\mu\ \varphi_\pi)\Psi$ and the gauge invariant substitution $\partial_\mu \rightarrow \partial_\mu - ie\ A_\mu$ in the presence of electromagnetic interactions.

11) W. Rarita and J. Schwinger :Phys.Rev. 60 (1941) 61

12) Y. Takahashi: Nuovo Cimento 6 (1957) 371

13) One could think of an inclusion of the terms being proportional to the f_i into our gauge invariant extension, i.e. to drop requirement 2 . But thereby one would run into some difficulties connected with the high-energy behavior of these terms and the non-uniqueness of the "normal electrodynamic moments" for particles with spin greater or equal one. For the latter compare:

J.A. Young and S.A. Bludman: Phys.Rev. 131 (1963) 2326

14) We remember that $\underset{\sim}{q}^2 \sin^2\theta \underset{s\rightarrow\infty}{\simeq} -t$

15) S.D. Drell: Rev.Mod.Phys. 33 (1961) 458

16) R.B. Blumentahl, W.L. Faissler, P.M. Joseph, L.J. Lanzerotti, F.M. Pipkin, D.G. Stairs, J. Ballam, H. DeStaebler, Jr., and A. Odian: Phys.Rev.Lett. 11 (1963) 496
But these results do not agree with the measurements by W.A. Blanpied, J.S. Greenberg, V.W. Hughes, D.C. Lu, and R.C. Minehart: Phys.Rev.Lett. 11 (1963) 477

17) F. Hadjioannou: CERN-Report 1962

18) K. Itabashi: Phys.Rev. 123 (1961) 2157

19) J.R. Kilner, R.E. Diebold and R.L. Walker: Phys.Rev.Lett. 5 (1960) 518

2P) L.V. Laperashvili and S.G. Matinyan: Soviet Physics JETP 14 (1962) 195

Figure captions:

fig. 1 Drell graph for reaction $\gamma + p \rightarrow X + \pi^{-,+}$

fig. 2 Drell graph for reaction $\gamma + p \rightarrow N^{*++,o} + \pi^{-,+}$

fig- 3 Lowest order Feynman diagrams for reaction $\gamma + p \rightarrow N^{*++,o} + \pi^{-,+}$

fig. 4 $\frac{d\sigma}{d|t|}$ in arbitrary units at high energies for $\gamma + p \rightarrow N^{*++,o} + \pi^{-,+}$ according to the Drell formula [1] (lower curve) resp. equ.(12) at $s' = M^{*2}$ (upper curve).

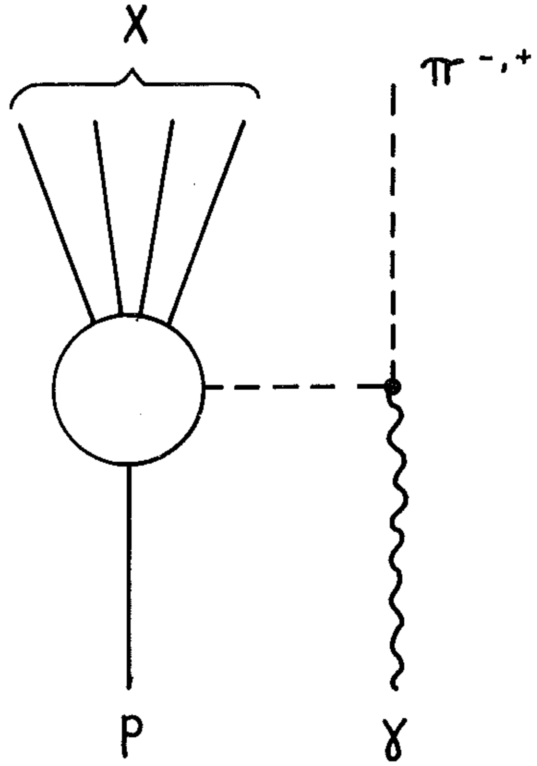

fig. 1

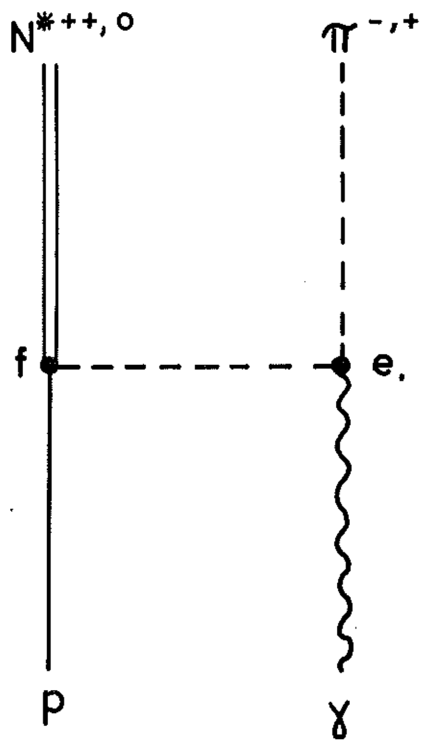

fig. 2

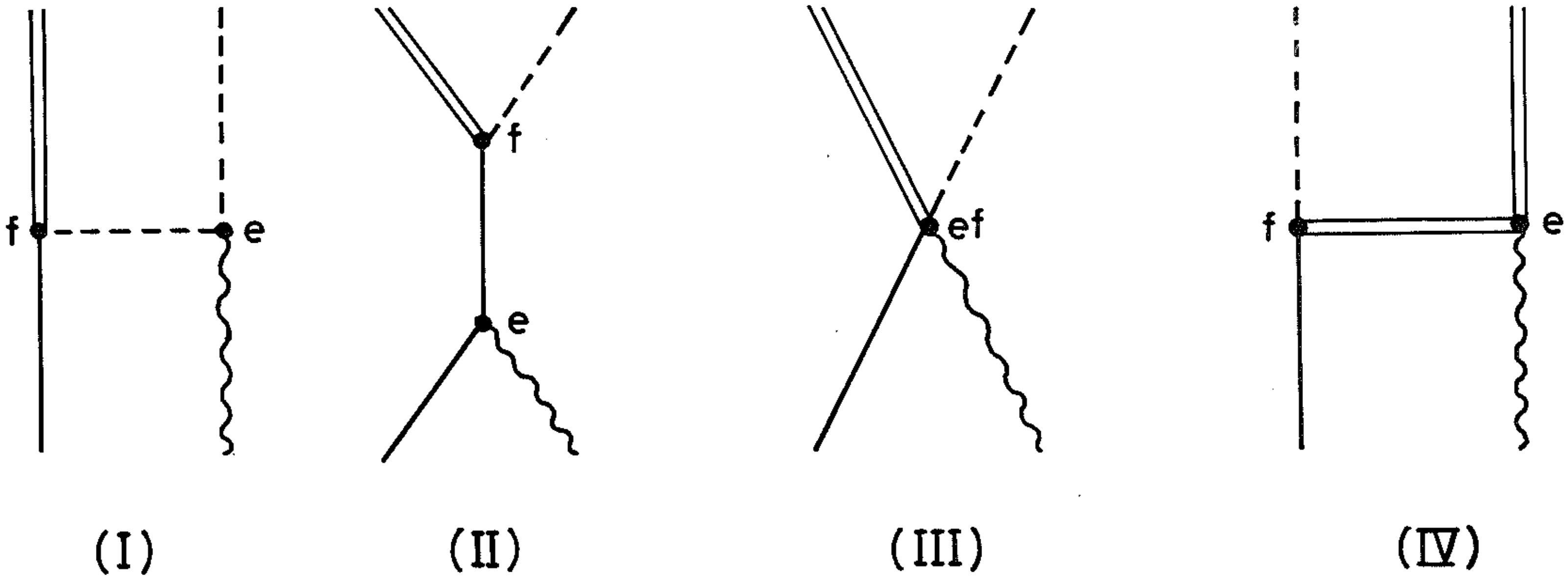

fig. 3

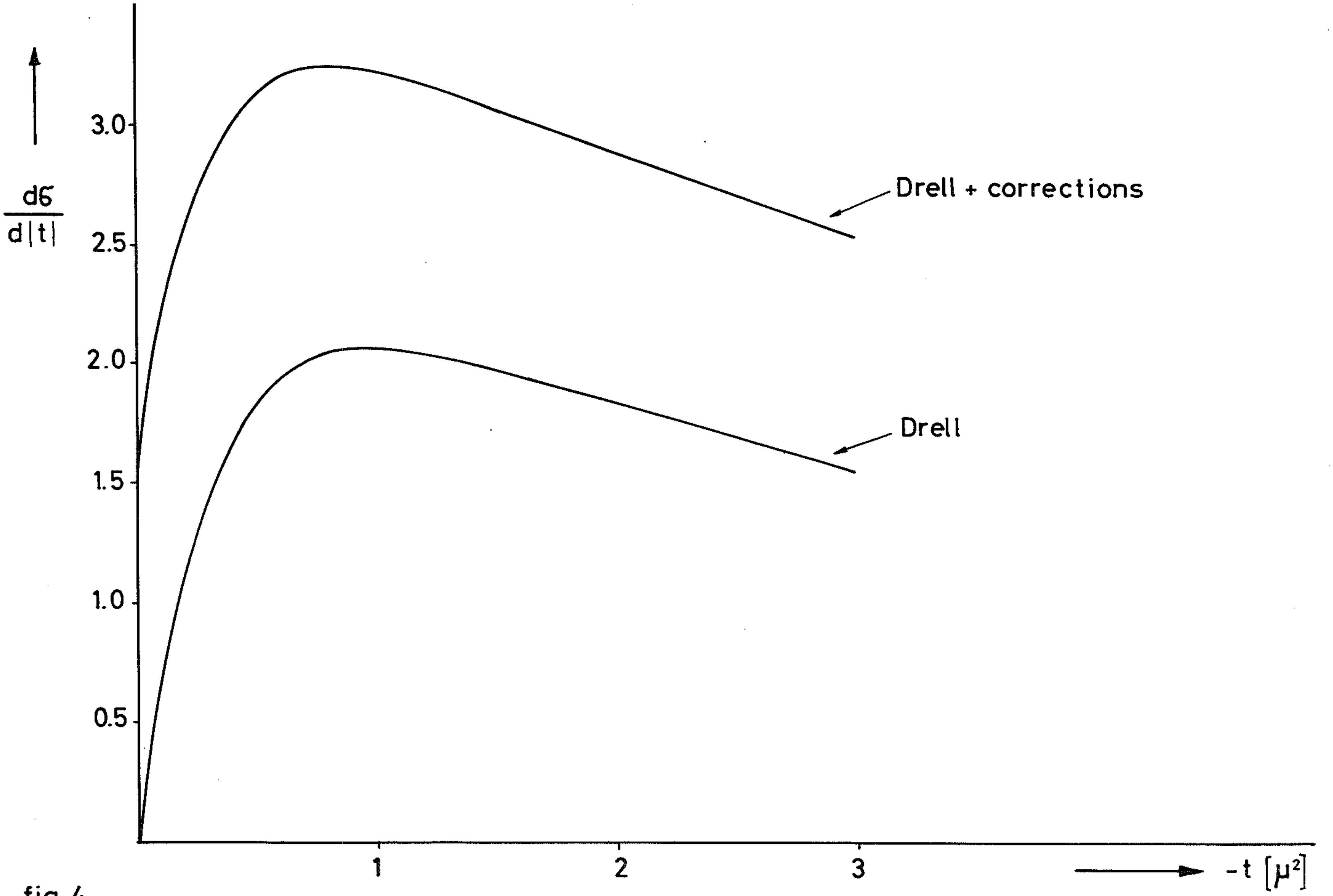

fig. 4