DESY 65/8

September 1965
Experimente

INTENSITY AND POLARISATION OF COHERENT BREMSSTRAHLUNG SPECTRA FROM A DIAMOND CRYSTAL IN UNIVERSAL REPRESENTATION

by

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Summary

Plots of intensity and polarisation functions are represented and their use is explained in obtaining the main features of coherent spectra produced from a diamond radiator for any desired electron energy and crystal orientation. The polarisation to be expected may be obtained from the curves for the first peak of the spectrum, which gives the only significant contribution. In addition the contribution of a single lattice point is represented.

Experimental situation

An electron beam with momentum $\vec{p}_o$, energy E_o, strikes the diamond radiator at the small angle Θ_o with respect to the [110]-axis of the crystal. α is the angle between the planes $\vec{p}_o$, [110] and [001], [110] in the reciprocal lattice space, see Fig.1.

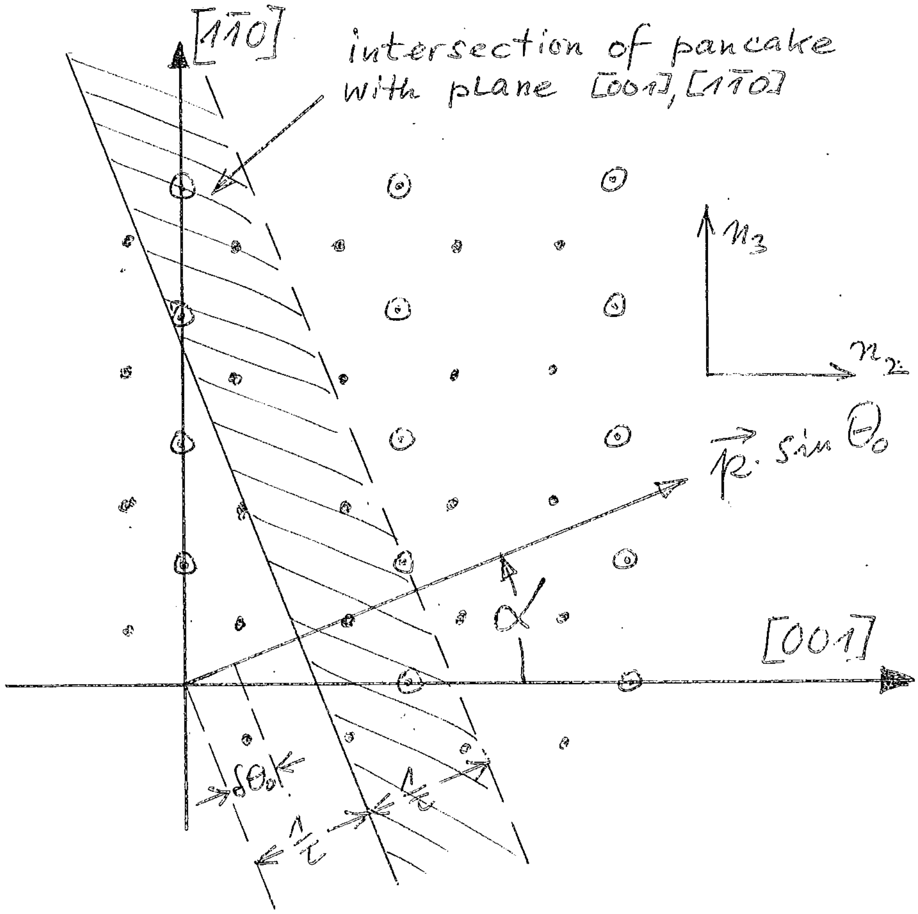


Fig.1 Reciprocal lattice plane, view ⊥ [110].

Coherent spectra for $\alpha = 0, 90^{\circ}$

The resulting coherent spectrum, i.e. the dependence of the normalized intensity

$$(1) \qquad I = \frac{x \, d\sigma/dx}{N \bar{\sigma}} \qquad \text{(compare DESY 64/9, equ. 39)}$$

on the relative quantum energy

$$(2) \qquad x = k/E_o$$

may be represented as a function of the product $\Theta_o \cdot E_o$ and is in its coherent part proportional to E_o. Also, the position of the discontinuities x_d as well as the polarisation P are functions of $\Theta_o \cdot E_o$, which have the following form:

$$(3) \quad x_d = 1/\left(1 + \frac{37{,}6}{\Theta_o E_o n_2}\right), \quad \alpha = 0^{\circ}, \quad n_2 \text{ integer, but } n_2 \neq 2, 6, 10, \ldots$$

$$(4) \quad x_d = 1/\left(1 + \frac{26{,}7}{\Theta_o E_o n_3}\right), \quad \alpha = 90^{\circ}, \quad n_3 \text{ integer}$$

$$(5) \quad I = E_o \cdot I_k(x, \Theta_o E_o) + I_i(x), \qquad E_o \text{ in GeV}$$

$$(6) \quad P = E_o \cdot p_o(x, \Theta_o E_o)/I$$

I_k and I_i are the coherent and incoherent parts of the intensity. (5) and (6) are valid for any value of α. The reciprocal lattice points in the [001], [110] plane are numbered by indices n_2, n_3 as shown in Fig.1. The polarisation is defined such that the electrical vector is parallel to the plane $\vec{p}_o$, [110].

Given Θ_o, E_o and $\alpha = 0^{\circ}$ or 90°, one can construct a spectrum by the aid of the plots 1 A to 6 A, 6 AB ($\alpha = 0$), or 1 B to 5 B, 6 AB ($\alpha = 90^{\circ}$), e.g.:

1 A gives the position of discontinuities x_d as function of $\Theta_o \cdot E_o$ for a number of lattice rows n_2. The dashed curve in addition gives the x-value at which the maximum intensity of the first peak

has its half value. With x_d fixed, I_k becomes a function of $\Theta_o E_o$ only. 2 A to 6 A give the upper and lower intensities, $\hat{I}_k$ and $\check{I}_k$ for the value $\Theta_o . E_o$ choosen. These have to be multiplied according to (5) and then $I_i(x_d)$ must be added (plot 6AB). In this way the spectrum is defined by 2 intensity points for each step and by a half-intensity point on the slope of the first peak. At the high energy end the spectrum approaches the incoherent part 6AB, which is not dependent from E_o or Θ_o.

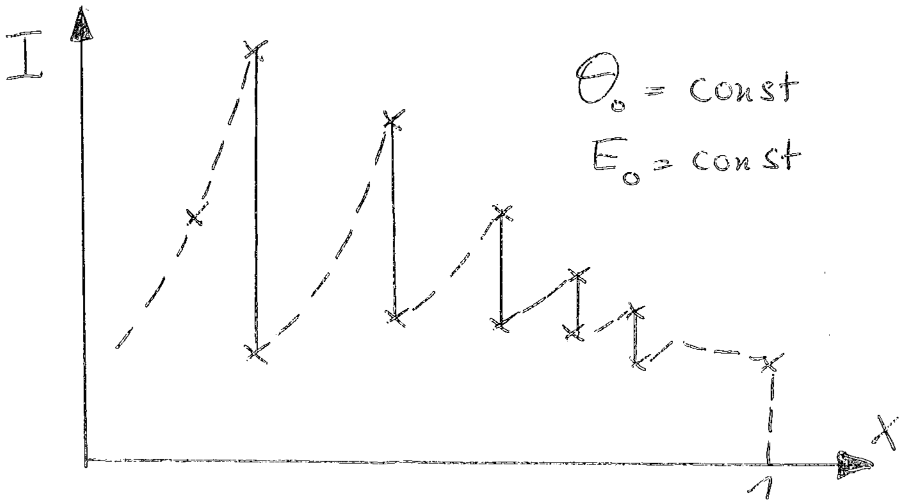


Fig.2 Spectrum constructed from the plots 1A,B-6A,B

The polarisation function p $(\Theta_o E_o)$ is given in 7 AB. With the intensity of the first peak now being known, it serves to calculate the polarisation of the photons in the first peak.

Θ_o-dependence of intensity

For the alignment of the crystal, it is useful to know the intensity as a function of Θ_o, with $x = x_o$ and E_o fixed. Again the plots 1 A, 1 B give the values $\Theta_d = \frac{\Theta_o \cdot E_o}{E_o}$ for $x = x_o$ at which the discontinuities appear. The intensity steps are taken from curves 2 A to 6 A, 2 B to 5 A, whichever case applies, as in the case of an energy spectrum. In addition one now has to use the plot 8 AB, which represents the total intensity, integrated over x, as function of $\Theta_o E_o$:

$$(7)\quad I_T = \int_0^1 I(x,\Theta_0 E_0)\,dx = E_0 \int_0^1 I_k(x,\Theta_0 E_0)\,dx + \int_0^1 I_i(x)\,dx$$

$$= E_0 \cdot I_{kT}(\Theta_0 E_0) + I_{iT}$$

This has the follwoing reason: Experimentally, the intensity distribution of the coherent photon beam is measured with a pair spectrometer, taking the total intensity of the beam as monitor. Taking an energy spectrum, $\Theta_0 E_0$ = const, the total intensity is constant. However it changes with Θ_0 if a Θ-dependence is measured, such that the intensity distribution actually observed is:

$$(8)\quad \frac{I(\Theta_0)}{I_T} = \frac{E_0\, I_k(x_0,\Theta_0 E_0) + I_i(x_0)}{E_0\, I_{kT}(\Theta_0 E_0) + I_{iT}}$$

The Θ-distribution is symmetrical to $\Theta_0 = 0$.

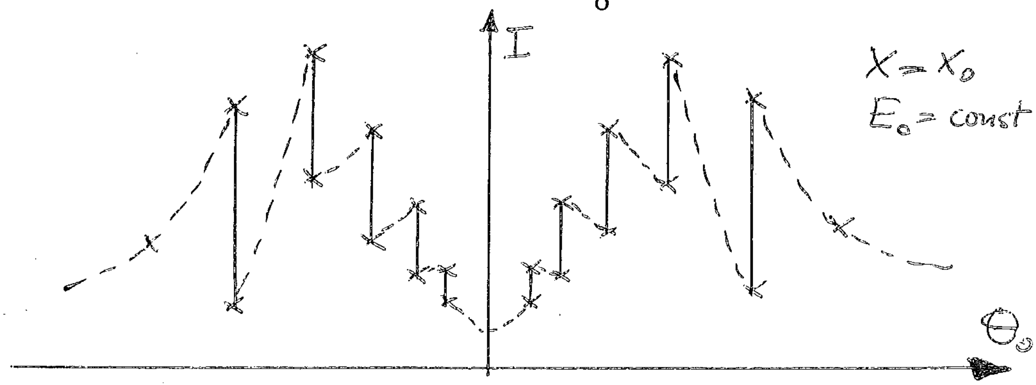


Fig.3 Θ-Dependence of intensity from plots 1A,B-8A,B.

Contribution of the single lattice point (0,2)

In the general case $0 \leq \alpha \leq 90^\circ$ the following relation gives the position of all possible discontinuities in the spectrum:

$$(9)\quad x_d = 1\Big/\left[1 + \frac{10^3}{\Theta_0 E_0 (26{,}6\, n_2 \cos\alpha + 37{,}7\, n_3 \sin\alpha)}\right], \quad 0 \leq \alpha \leq 90^\circ$$

The intensity now does not allow a simple representation as in the cases $\alpha = 0,\ 90^\circ$, but it has been shown experimentally by

Bologna, Lutz, Schulz, Timm, Zimmermann (to be published) that orientations with small α and large Θ_o are possible to allow the separation of certain single lattice points such that the intensity from this point dominates the spectrum, contributions from all other points being small.

The most favorable lattice point with respect to intensity and polarisation is $(n_2, n_3) = (0,2)$; this contribution is therefore also represented here. The position of this, single, discontinuity, the intensity and polarisation are given by the following formulas:

$$(10) \quad x_d = 1 \Big/ \left[1 + \frac{13{,}27}{\Theta_o E_o \sin\alpha}\right], \quad 0 \leq \alpha \leq 90^\circ \quad n_2 = 0\ ;\ n_3 = 2$$

$$(11) \quad I = E_o \cdot I_k(x_d) + I_i$$

$$(11') \quad I_k = \frac{1 - x_d}{x_d}\left[1 + (1 - x_d)^2\right] \cdot 7{,}36 \quad [GeV^{-1}] \quad n_2 = 0\ ;\ n_3 = 2$$

$$(12) \quad P = E_o \cdot p(x_d) \cdot \cos\alpha \,/\, I$$

$$(12') \quad p = \frac{(1 - x_d)^2}{x_d} \cdot 14{,}72 \quad [GeV^{-1}], \quad n_2 = 0\ ;\ n_3 = 2$$

The functions x_d $(\Theta_o \cdot E_o \cdot \sin\alpha)$, I_k (x_d), and $p(x_d)$ for the single point (0,2) are plotted on pages 1 C, 2 C. $I_k(x_d) = 0$ in this case because contributions from other lattice points are not included.

Given Θ_o, E_o and α one can find the position of the discontinuity x_d from 1 C. The curves 2 C then give the necessary information to calculate intensity and polarisation according to (11) and (12). The polarisation obtained from this single lattice point is higher than in the case $\alpha = 0, 90^\circ$. For small x_d the polarisation approaches

$$(13) \quad P \rightarrow \frac{1}{1 + \frac{I_i}{E_o} \frac{x_d}{(1 - x_d)^2}} \rightarrow 1 \quad ; \quad x_d \ll 1$$

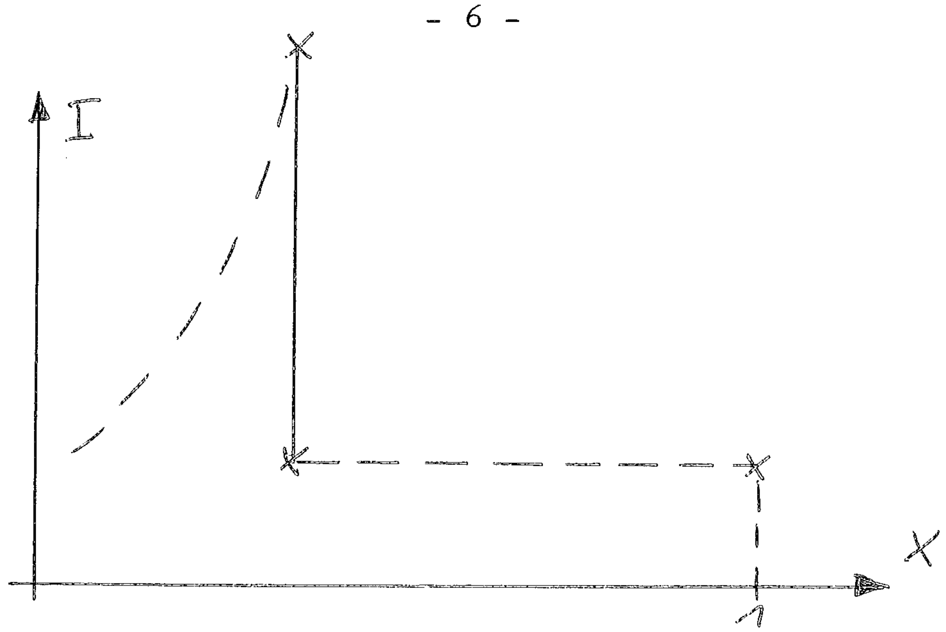


Fig.4 Spectrum from a single lattice point as constructed from 1 C, 2 C, 6 AB.

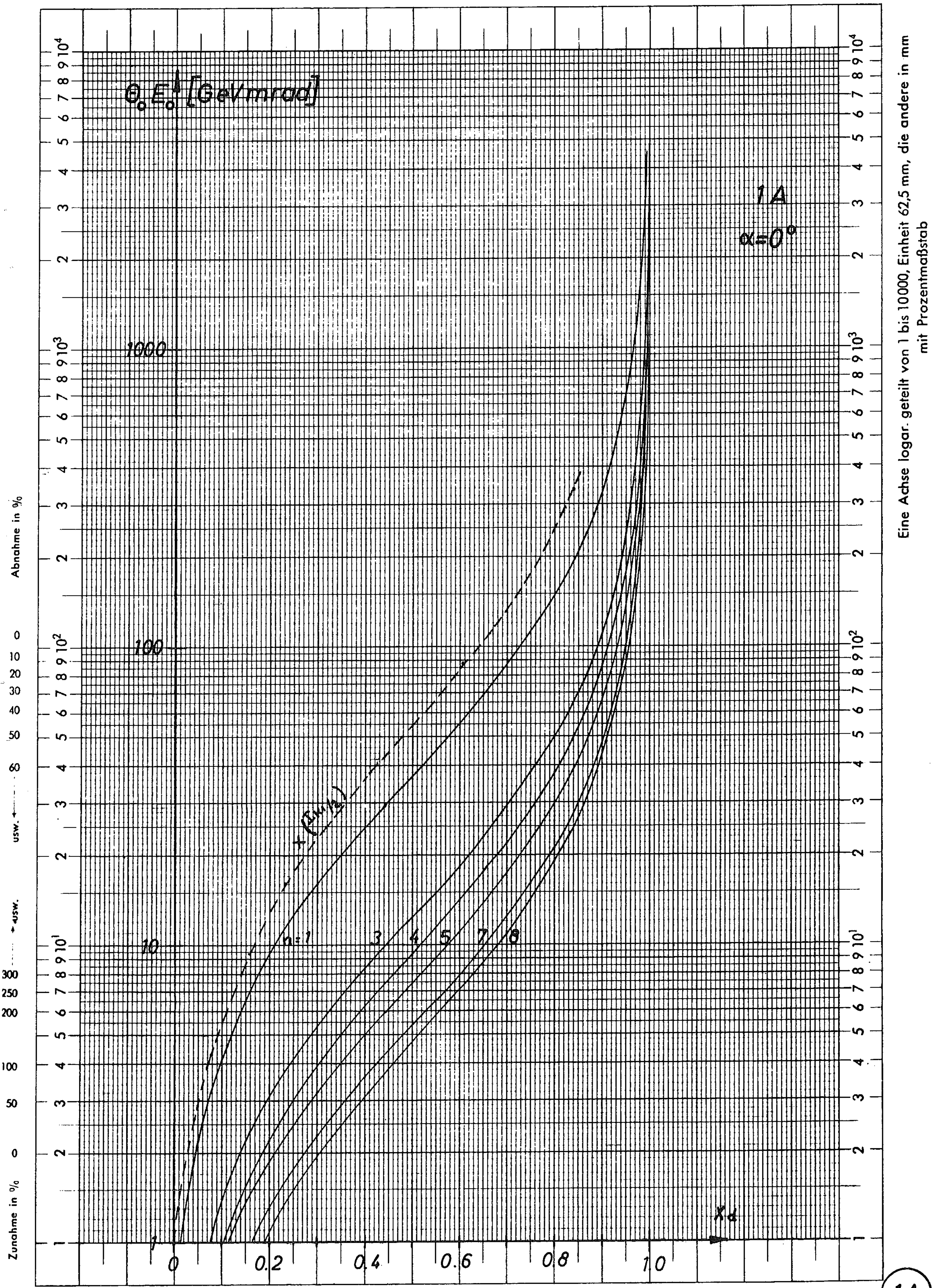
Final remarks

The spectral intensities obtained in this way do not, of course, take into account all the experimental effects which tend to smooth out the discontinuities such as primary divergence, multiple scattering, mosaic spread of the crystal and apparatus resolution.

The universal character of the intensity- and polarisation functions which are given here can be recognised in the formulas for the cross section of coherent Bremsstrahlung production which have been given by Barbiellini, Bologna and Murtas*).

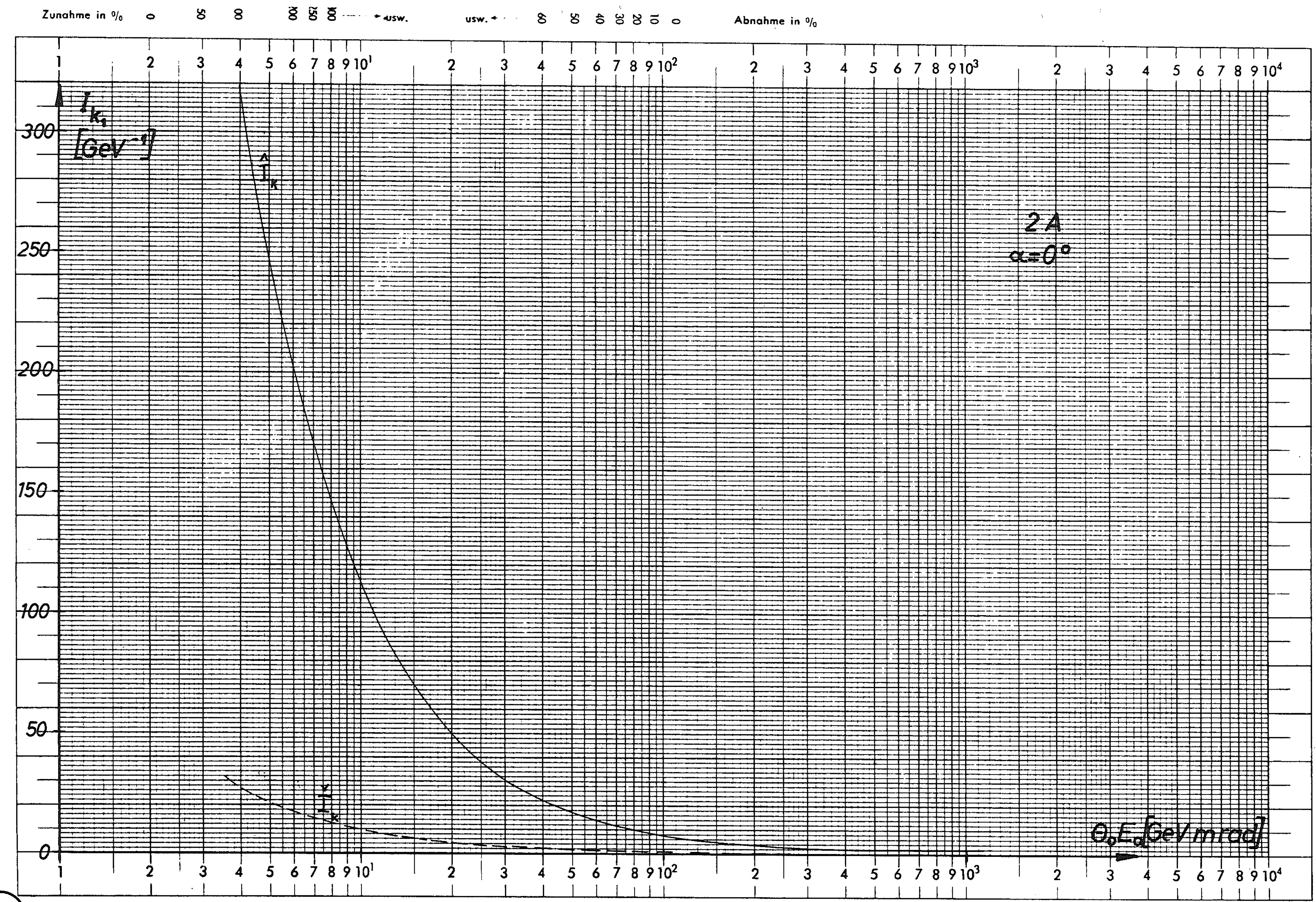
I thank Dr.G.Bologna und Mr.G.Lutz for discussions and Miss W.Kuffner for help in evaluating the data and for preparing the drawings.

*) Phys.Rev.Lett. 8, 454, (1962), eqns. 3, 4 for $\alpha = 0, 90^{\circ}$.
Nuovo Cim. 28, 435, (1963), eqn.3 for any α.



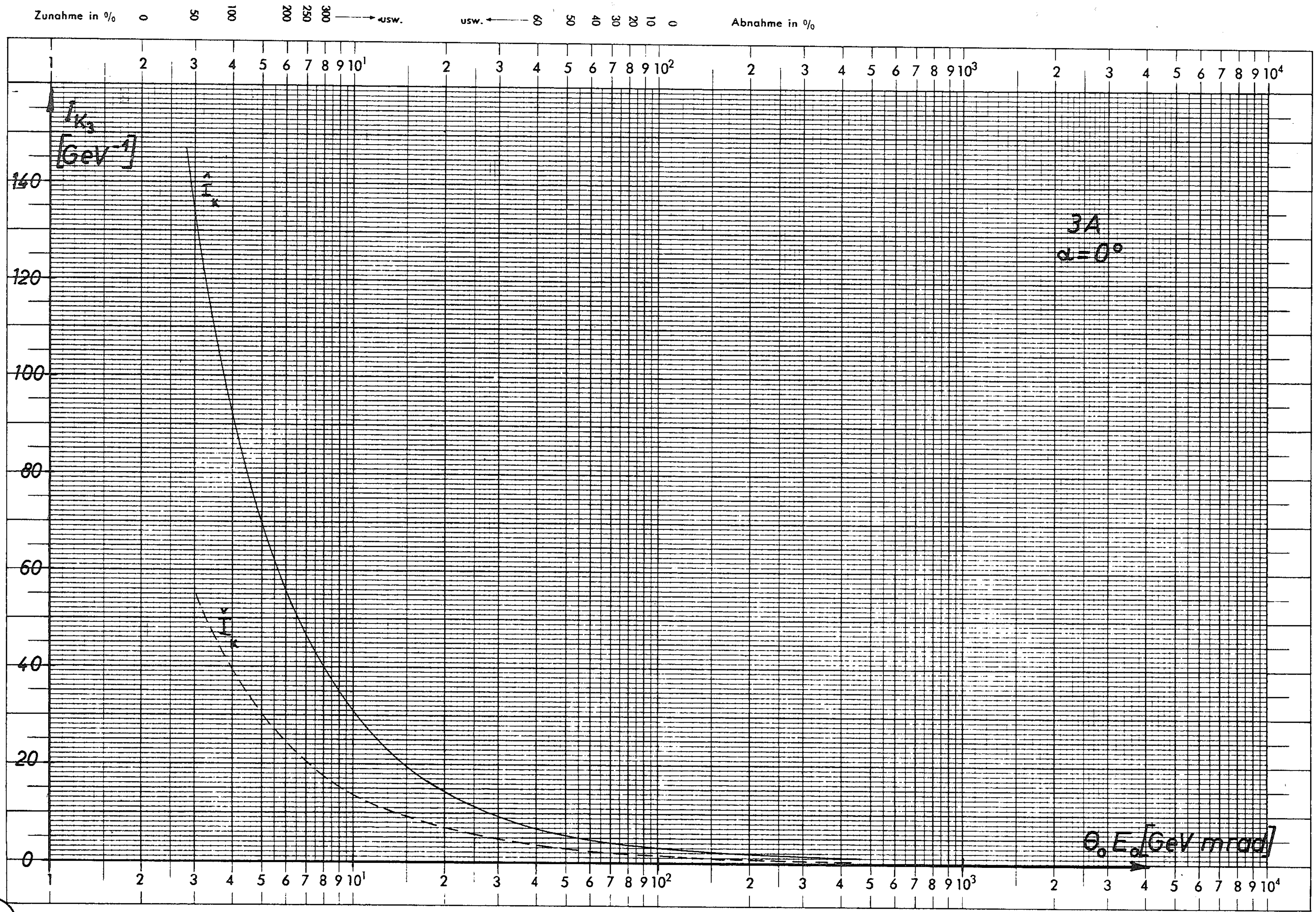
$\Theta_0 E_0^{\frac{1}{2}}$ [GeVmrad]
1A
α=0°
1000
100
10
1
n=1
3
4
5
7
8
$X(E_{th}/E_0)$
x_d
0
0.2
0.4
0.6
0.8
1.0
Abnahme in %
Zunahme in %
usw.
Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab

1A



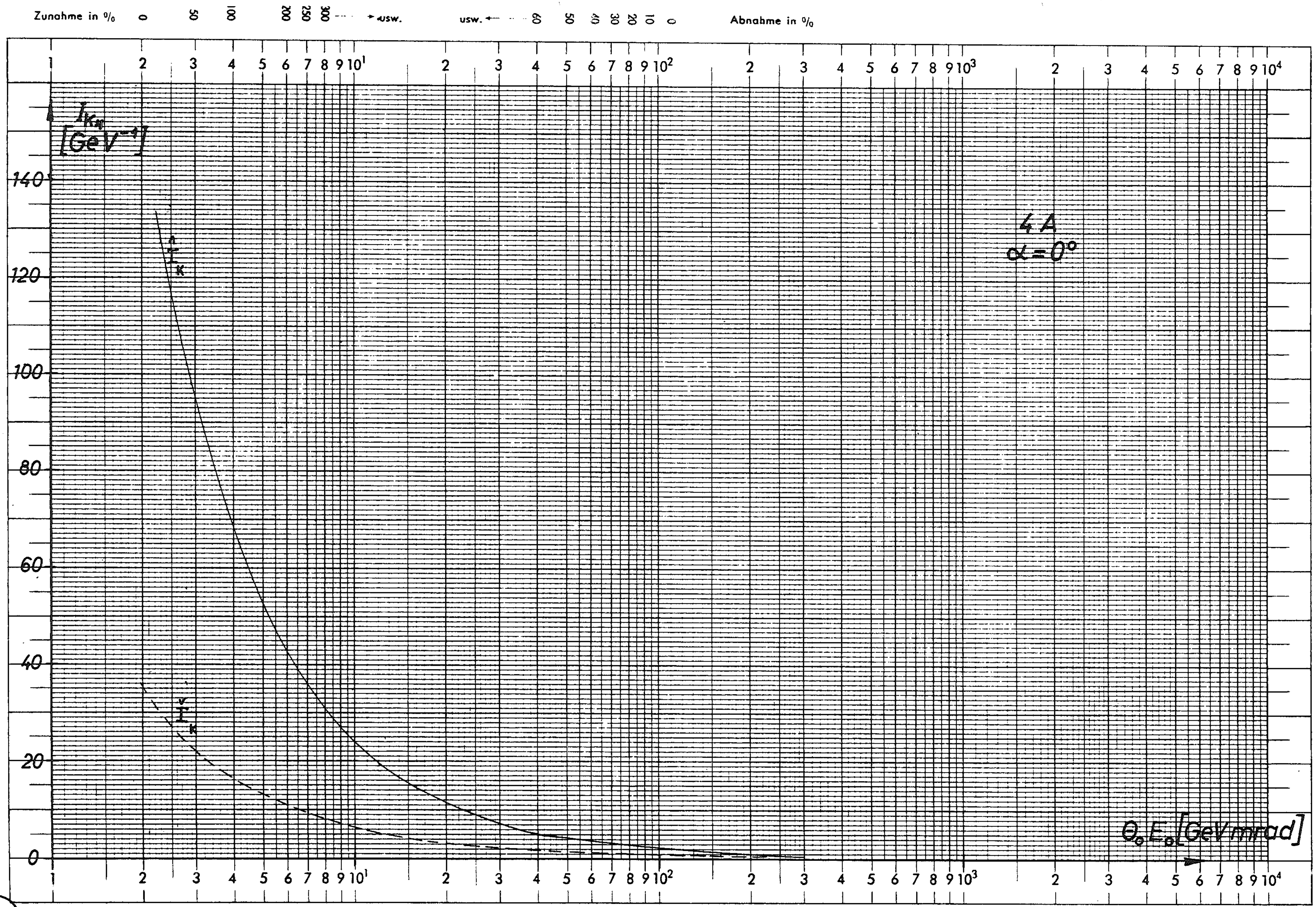
Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab

2A

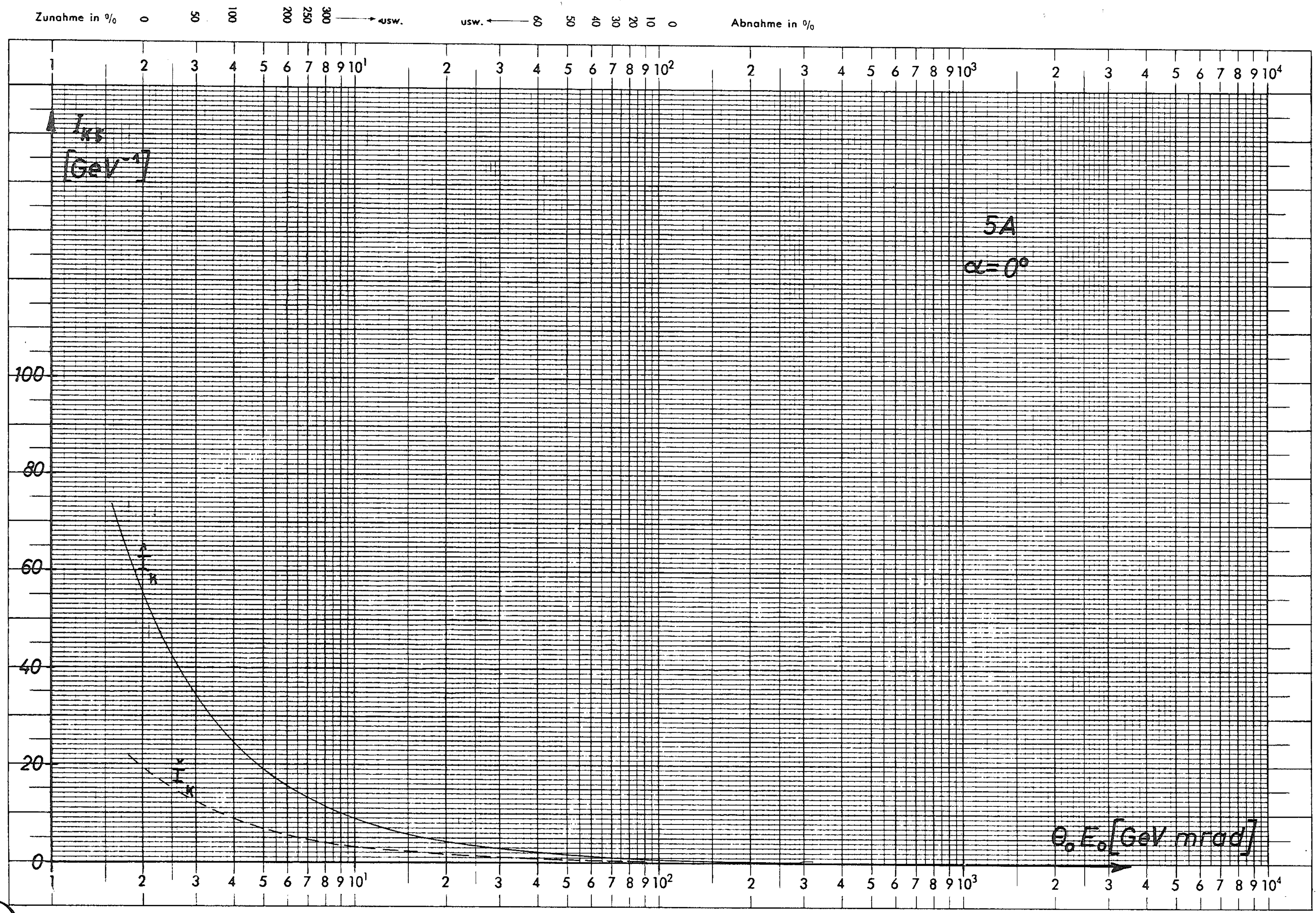


Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab

3A

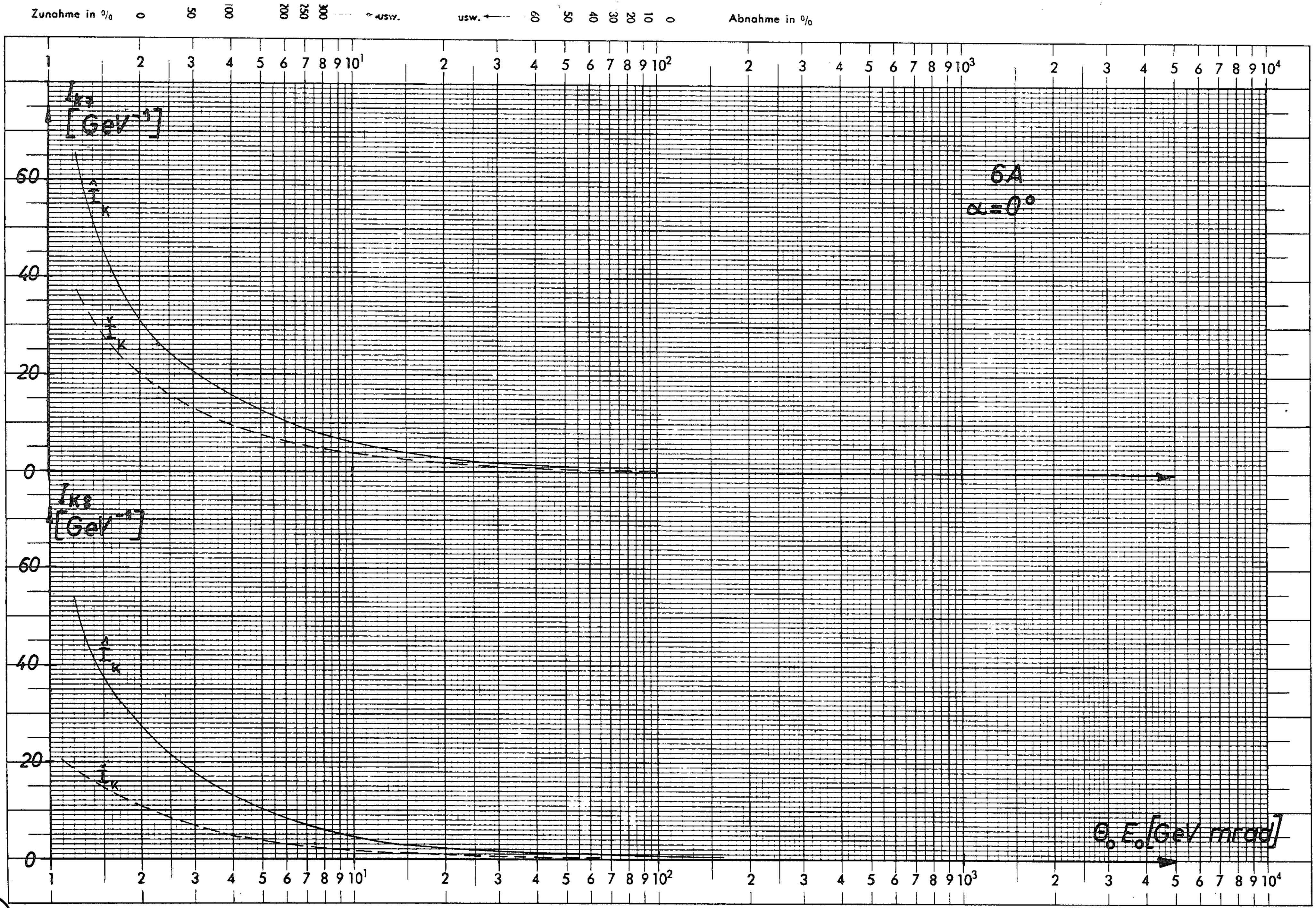


Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab



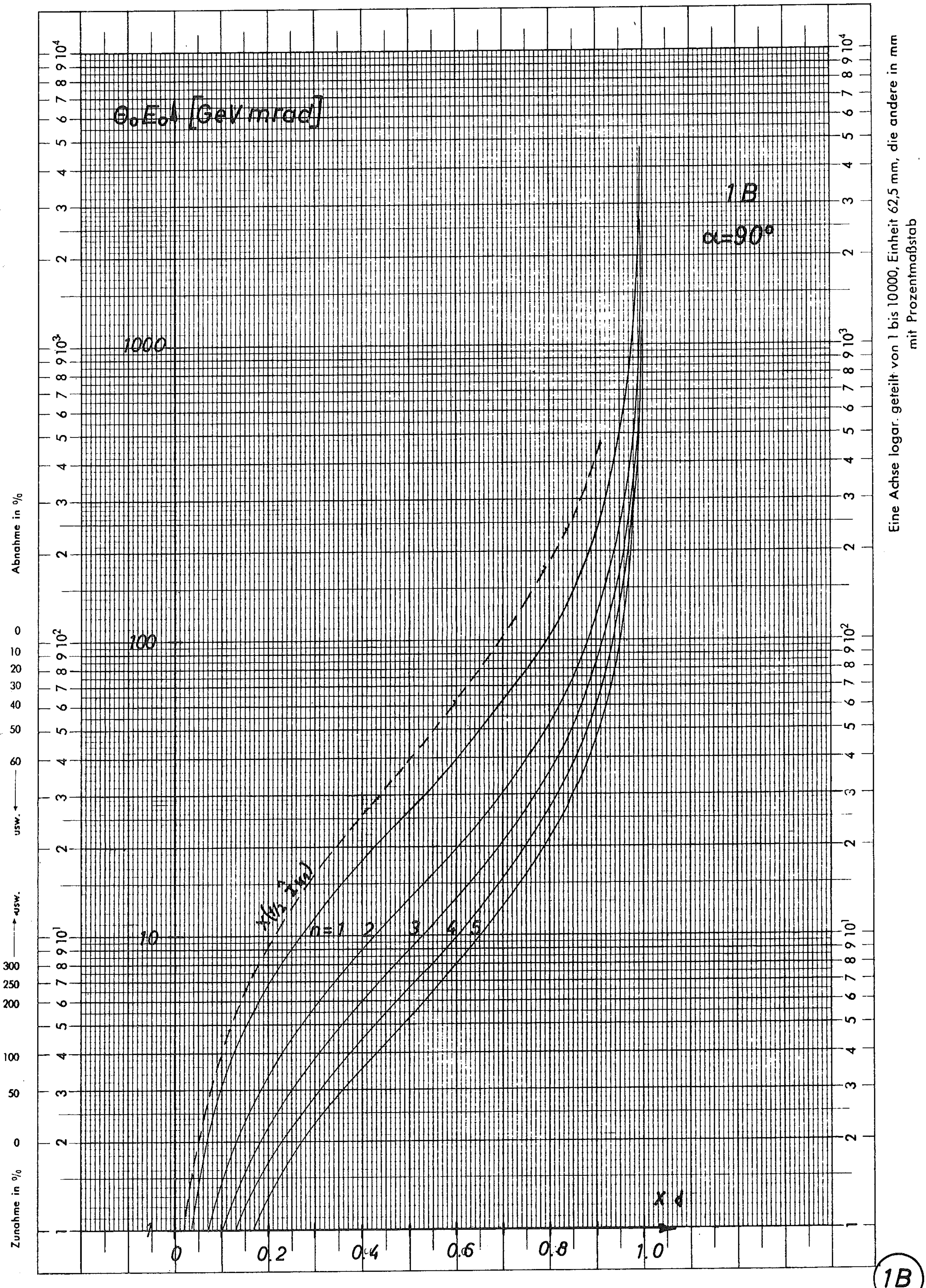
Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab

5A



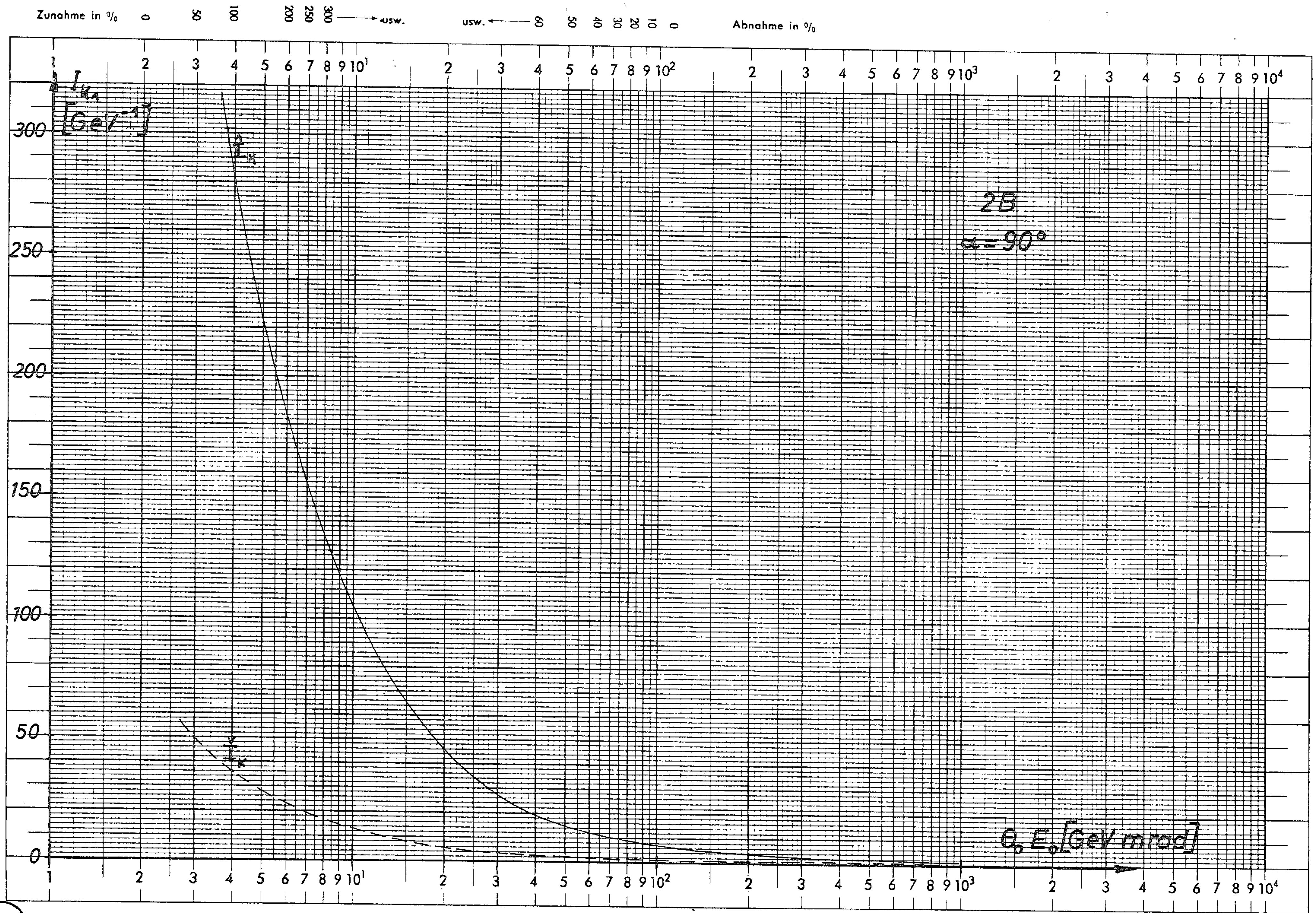
Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab

6A



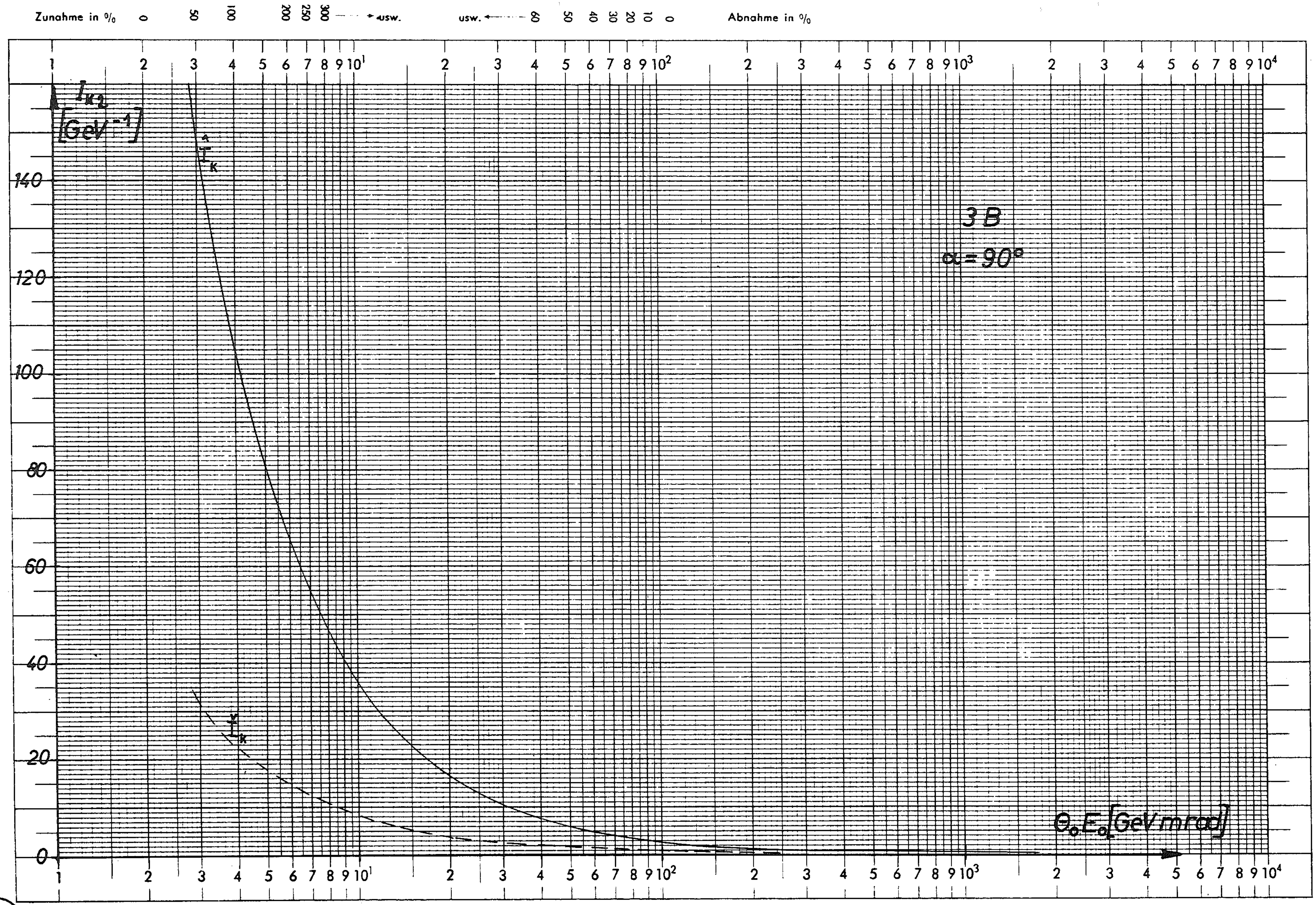
$\Theta_0 E_0 A$ [GeV mrad]
1 B
$\alpha = 90°$
1000
100
10
1
$x(\eta_1, \lambda \eta_2)$
n=1
2
3
4
5
0
0.2
0.4
0.6
0.8
1.0
x
Abnahme in %
Zunahme in %
usw.
Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab

1B



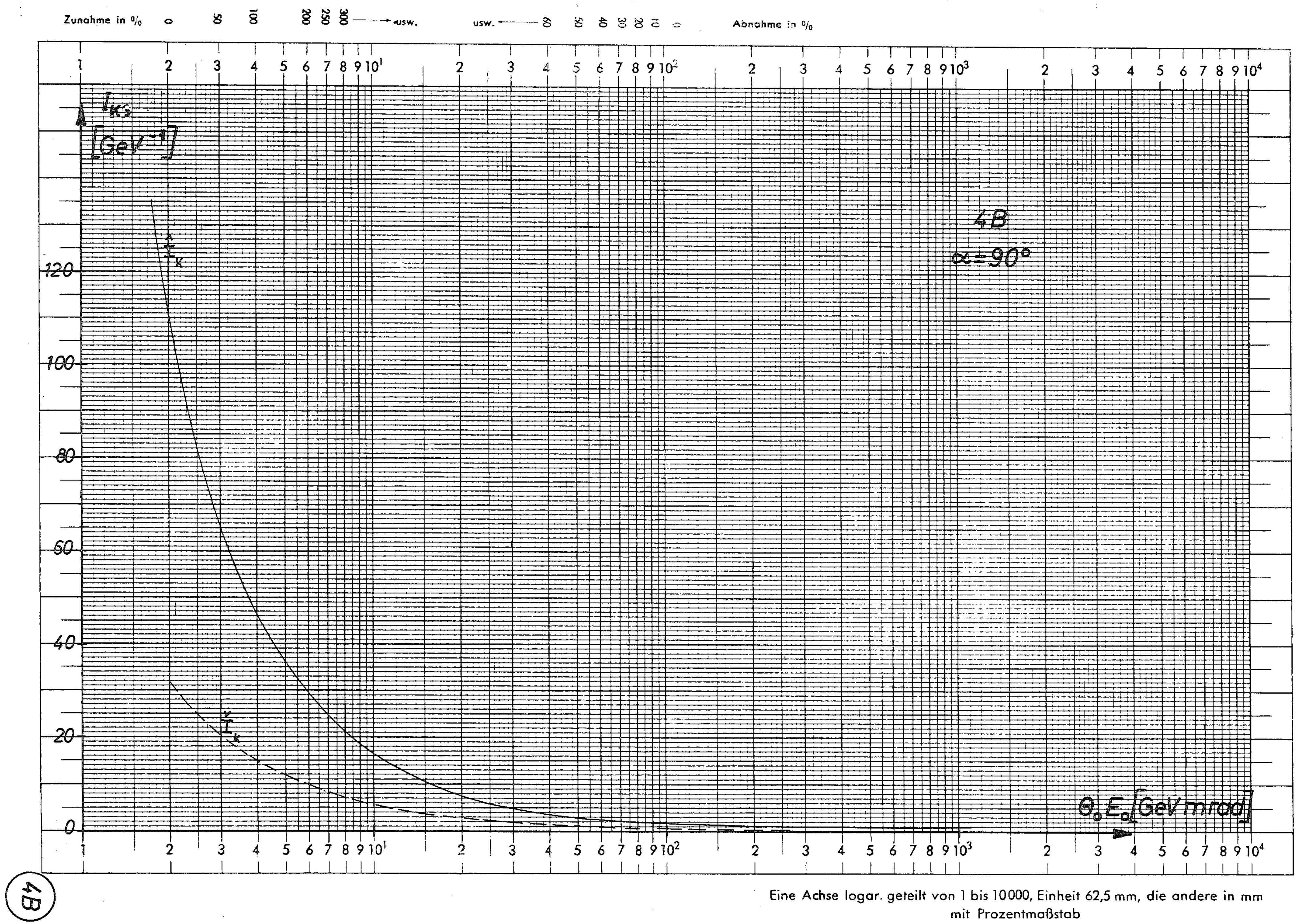
Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab

2B

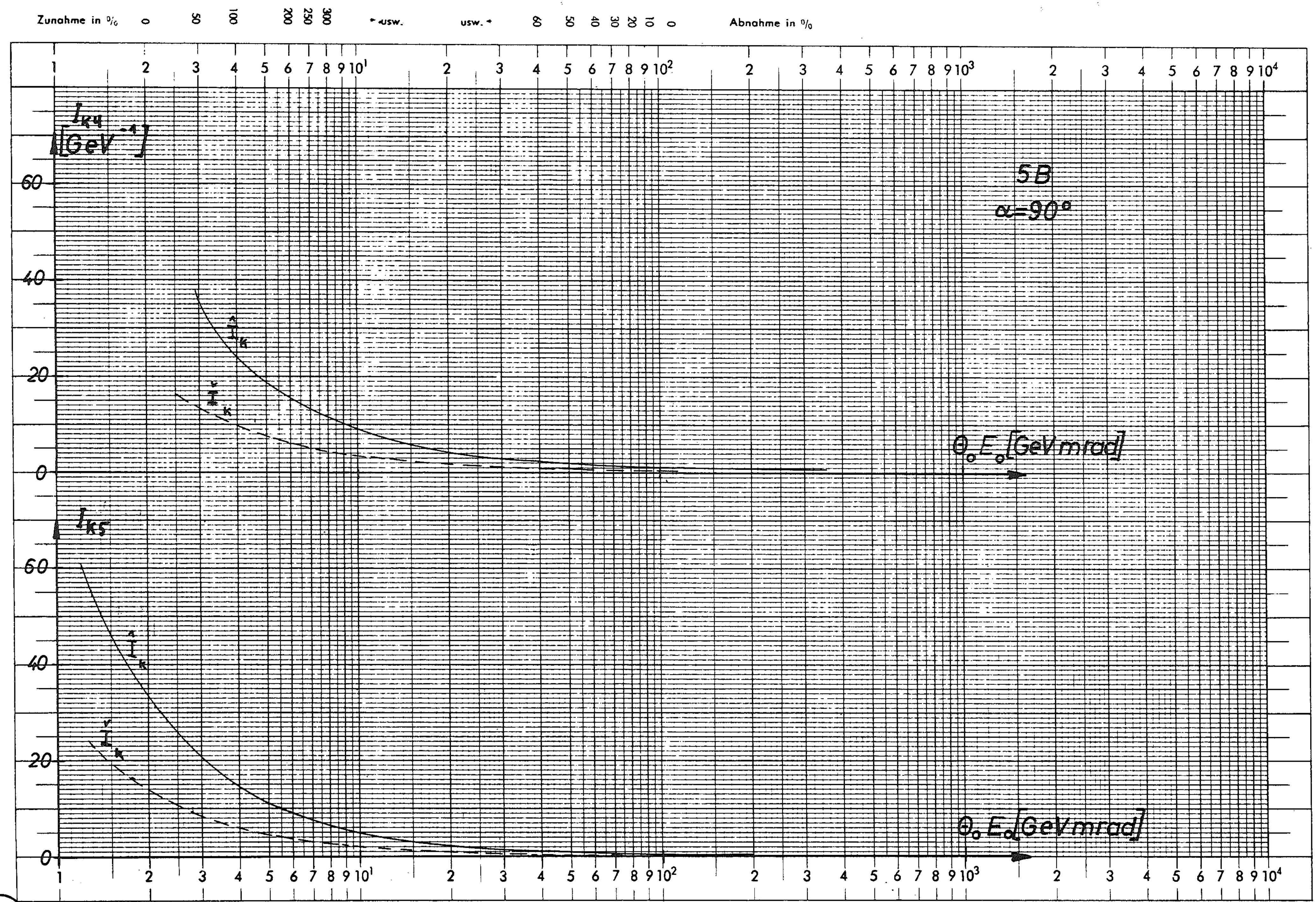


Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab

3B

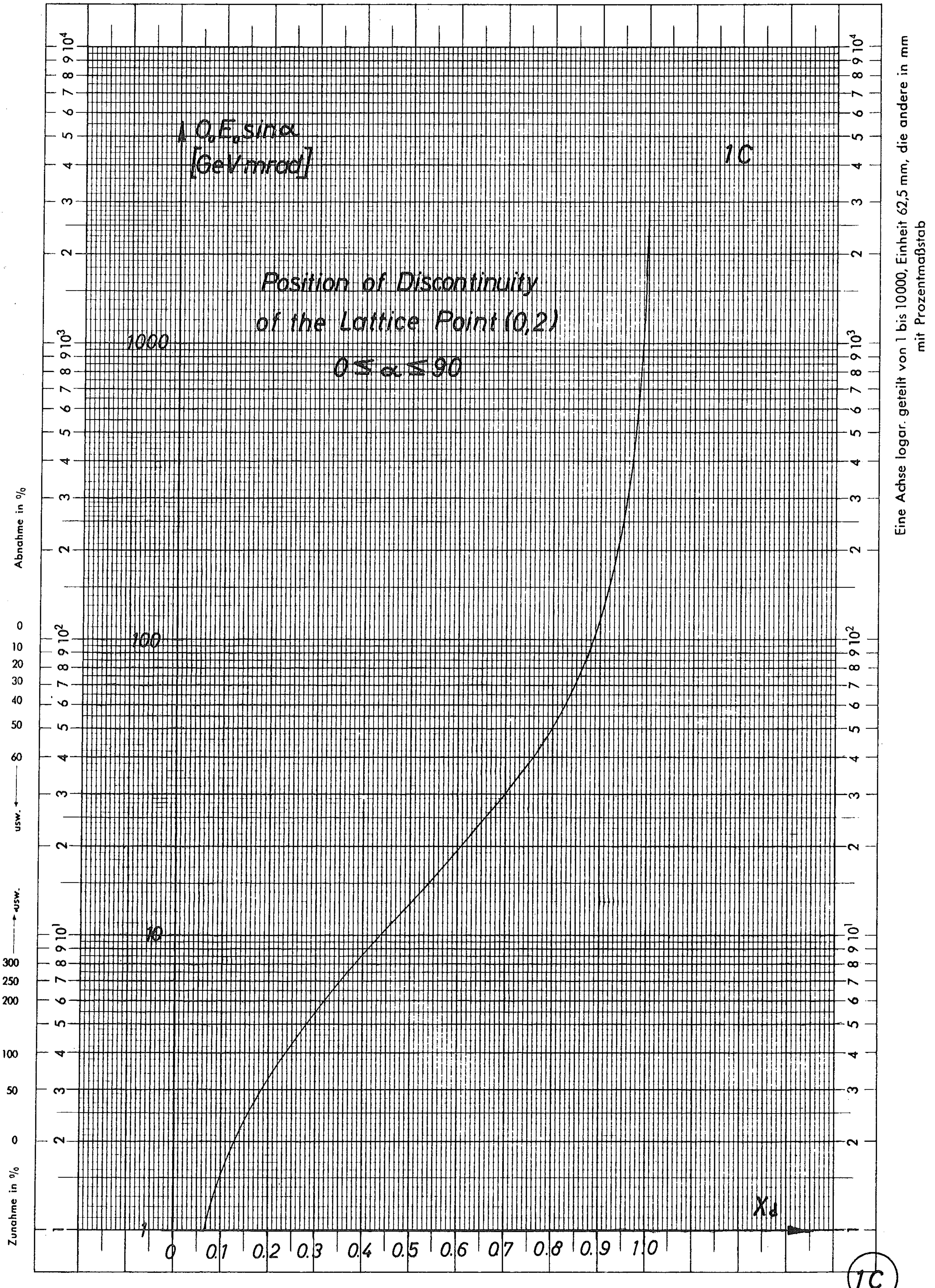


Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab



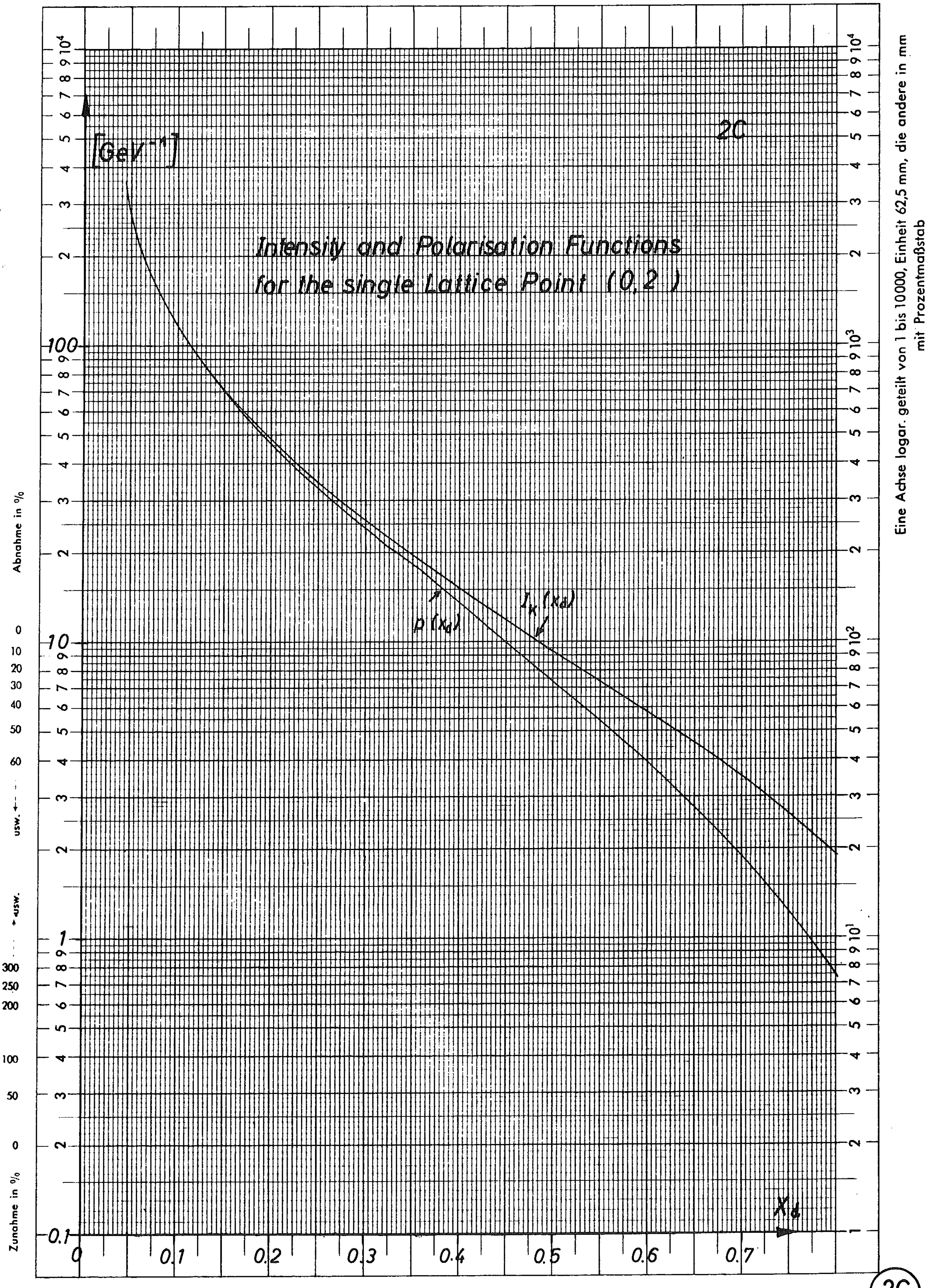
Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab

(5B)



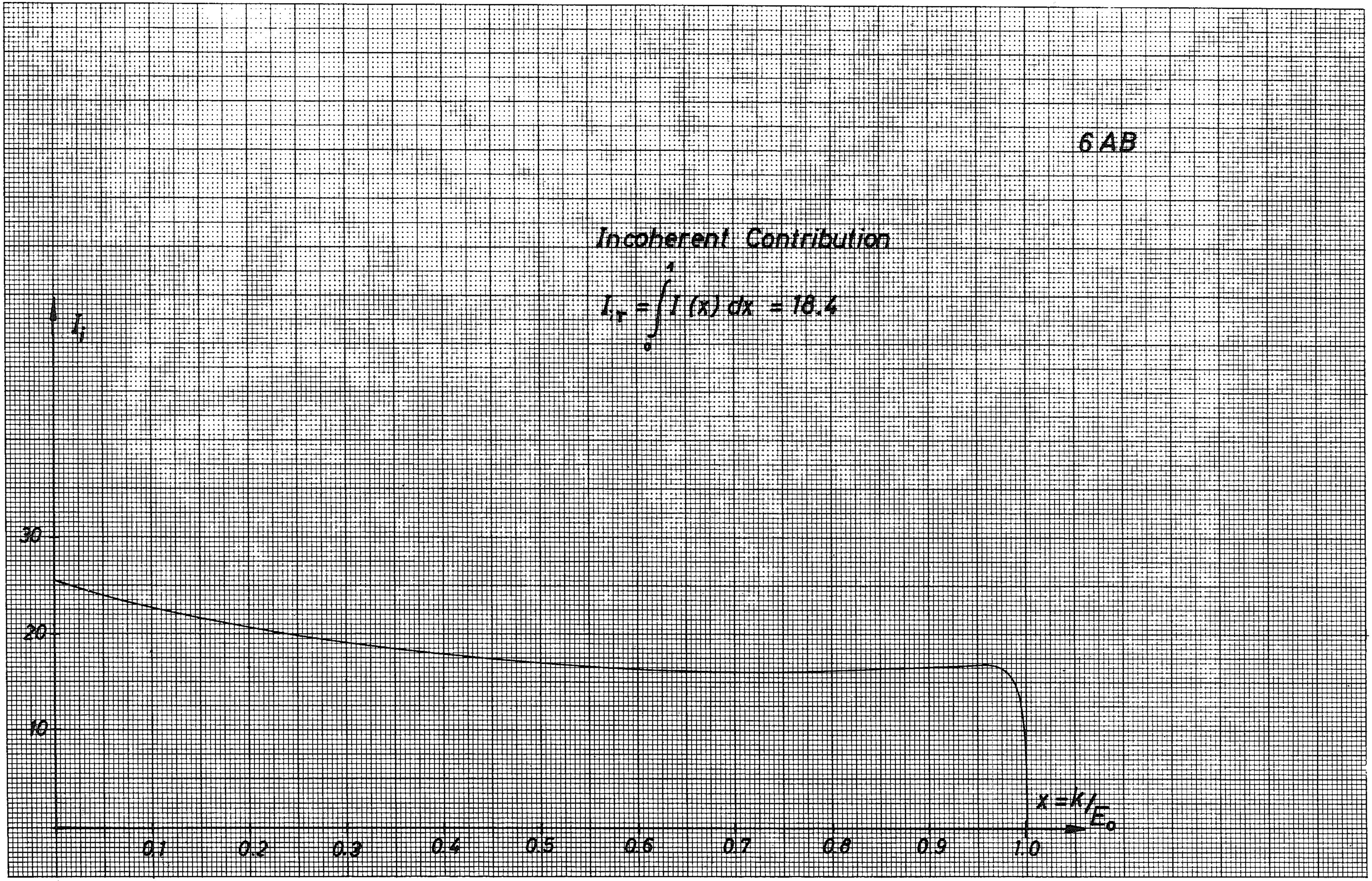
$Q_0E_0\sin\alpha$
[GeV mrad]
1C
Position of Discontinuity
of the Lattice Point (0,2)
$0 \leq \alpha \leq 90$
1000
100
10
1
X_d
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Abnahme in %
Zunahme in %
usw.
Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab

1C

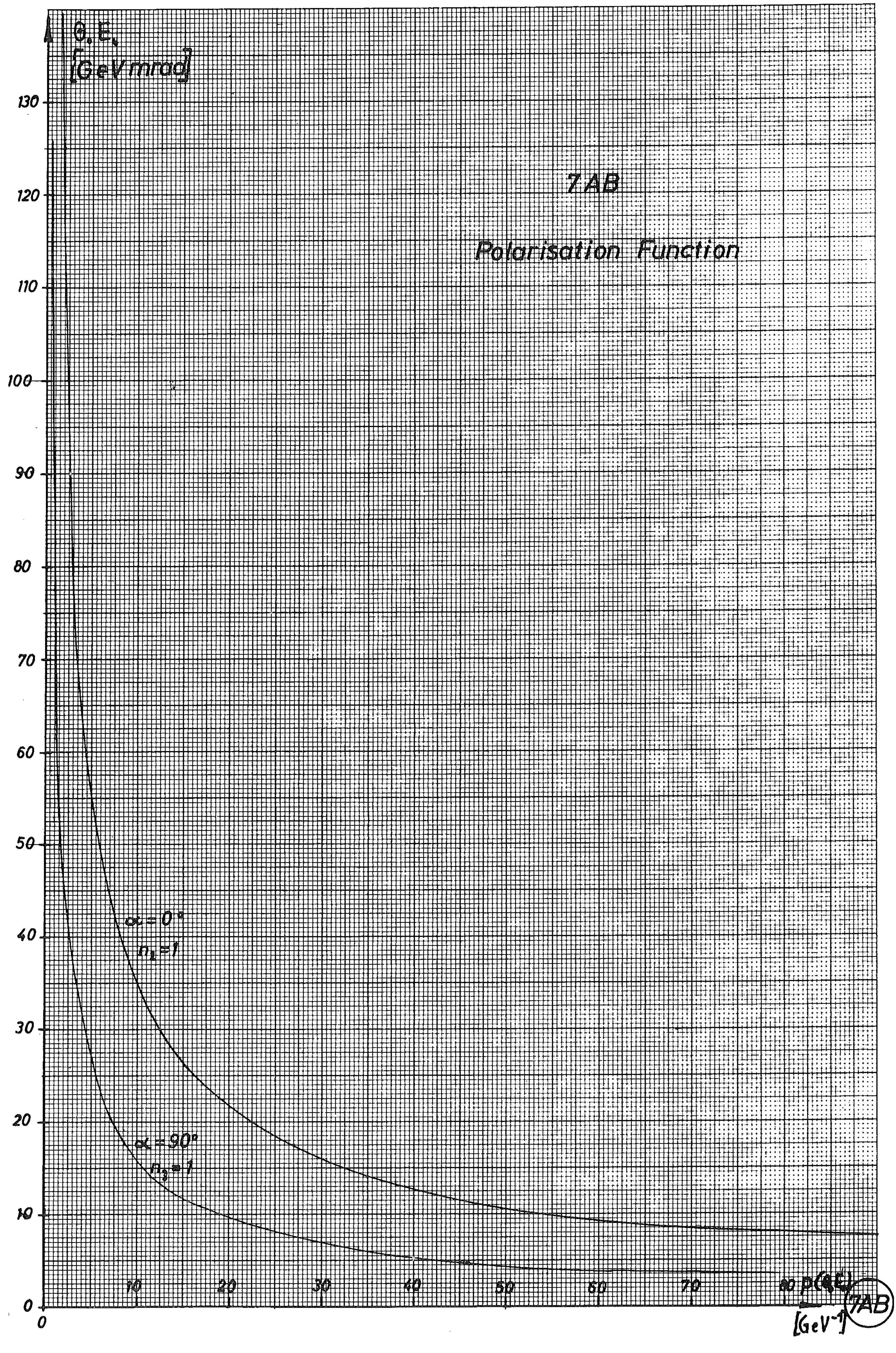


Intensity and Polarisation Functions
for the single Lattice Point (0,2)
[GeV⁻¹]
2C
p (xd)
Ik (xd)
Xd
Abnahme in %
Zunahme in %
Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab

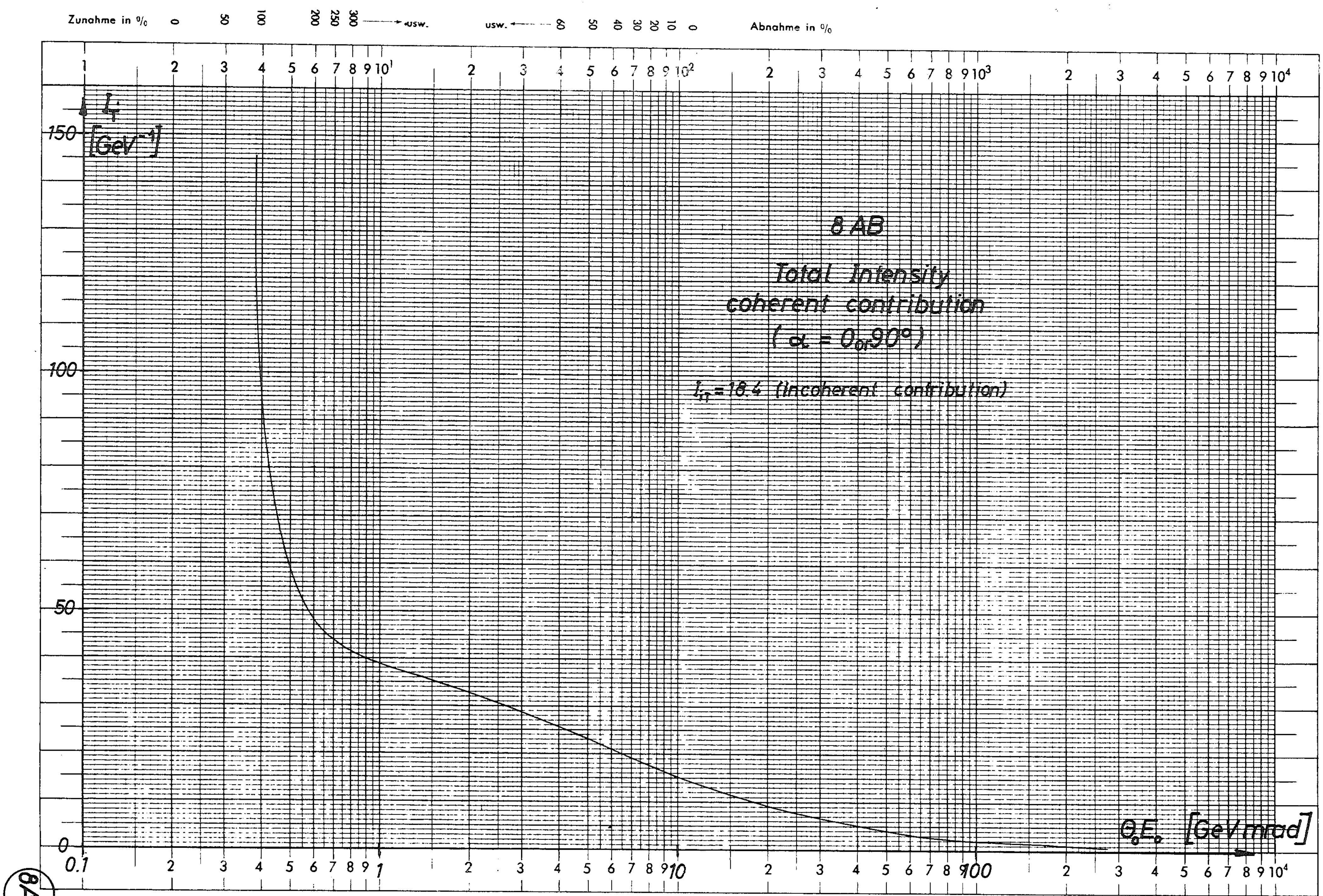
2C


6 AB
Incoherent Contribution
$I_i = \int_0^1 I(x)\,dx = 18.4$
I_i
30
20
10
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
$x = k/E_0$

6AB



$\Theta_o E_o$
[GeV mrad]
7 AB
Polarisation Function
$\alpha = 0°$
$n_2 = 1$
$\alpha = 90°$
$n_3 = 1$
130
120
110
100
90
80
70
60
50
40
30
20
10
0
0
10
20
30
40
50
60
70
80
[GeV^{-1}]
7AB



8AB

Eine Achse logar. geteilt von 1 bis 10000, Einheit 62,5 mm, die andere in mm mit Prozentmaßstab