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Low Momentum Transfer Neutron Form Factors  
from Analysis of Deuteron Electrodisintegration.

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## A b s t r a c t

Electromagnetic form factors of the neutron are calculated from recent inelastic e-d-scattering measurements which were performed at the quasi-elastic peak and at momentum transfers below  $5 \text{ f}^{-2}$ . The theory used in the analysis includes a realistic description of the deuteron structure and of the effect of interactions between the outgoing nucleons. We find that the rescattering corrections are very important in this region of momentum transfer. Concerning the charge form factor  $G_{2E}$  of the neutron it is concluded that meaningful results cannot be given at the moment. The magnetic form factor of the neutron  $G_{2M}/\mu_2$  comes out still larger than but closer to the magnetic form factor  $G_{1M}/\mu_1$  of the proton compared to former analyses. More measurements around  $q^2 = 1 \text{ f}^{-2}$  seem to be very interesting in this respect.

## I. Introduction.

Recently E.B. Hughes et al.<sup>1)</sup> measured very precisely the ratio of the elastic electron-proton cross section to the quasi-elastic electron-deuteron cross section over a range of momentum transfer  $q^2$  from 0.5 to  $35 \text{ f}^{-2}$ . From their data they deduced the form factors of the neutron on the basis of a theory developed by Durand<sup>2)</sup> which was supplemented by final-state interaction effects taken from the theoretical work of Nuttall and Whippmann<sup>3)</sup>. It was found that for  $q^2$  values larger than about  $6 \text{ f}^{-2}$  the square of the neutron's charge form factor  $(G_{2E})^2$  is consistent with zero, and that the magnetic form factor  $G_{2M}$  follows rather well the law

$$G_{2M} = \mu_2 \frac{G_{1M}}{\mu_1} = \mu_2 G_{1E}; \quad \text{where } G_{1E} \text{ and } G_{1M} \text{ are}$$

the form factors of the proton and  $\mu_1$  and  $\mu_2$  the magnetic moments of the proton and neutron respectively. On the other hand, for  $q^2$  below  $6 \text{ f}^{-2}$  imaginary values for the neutron's charge form factor  $G_{2E}$  were found even when the experimental errors had been properly taken into account. The magnetic form factor  $G_{2M}$  came out appreciably larger than one would have expected according to the rule  $G_{2M} = \frac{\mu_2}{\mu_1} G_{1M}$  which is fulfilled for the higher  $q^2$ .

Unfortunately, in the low  $q^2$  range the corrections to Durand's Born approximation theory<sup>2)</sup> applied by the Stanford group are large; they were not based on a real evaluation of the theory but had to be extrapolated from the results of Nuttall and Whippman<sup>3)</sup> for  $q^2$  above  $4 \text{ f}^{-2}$ . This procedure cannot be regarded as reliable as has already been pointed out by Hughes et al.<sup>1)</sup>. Therefore, further theoretical analysis of the inelastic electron-deuteron scattering for small  $q^2 < 5 \text{ f}^{-2}$  is very desirable.

Last year two of us presented results on a theoretical study of the inelastic electron-deuteron scattering in the  $q^2$  range from 2.5 to  $40 \text{ f}^{-2}$ <sup>4)</sup>. Only a few examples concerning  $q^2$  and electron-scattering angle  $\theta$  could be investigated because of the rather lengthy numerical calculations necessary to evaluate the cross section with final state corrections. Furthermore, our study was limited to scattering at the quasi-elastic peak. We put our emphasis on a realistic description of the d-n-p vertex

function and on the corrections caused by the final state interaction. Concerning the last point it was found that the rescattering corrections are not very important for momentum transfers  $q^2$  above  $6 \text{ f}^{-2}$  but that for  $q^2$  below this value they lead to an appreciable reduction of the theoretical cross section as compared to the Born approximation. In this paper we present a more complete analysis of the theoretical cross section for  $q^2$  less than  $5 \text{ f}^{-2}$ . Apart from some minor modifications which will be discussed later, we base our analysis on the theory as outlined in (I). In (I) we followed the procedure that we calculated the cross section under the assumption that the neutron form factors are also given, and we then compared the values of the complete theoretical cross section with approximate evaluations. Here we try to determine the form factors of the neutron directly from the measured ratio  $R_0$  of the elastic electron-proton cross section to the quasi-elastic electron-deuteron cross section as well as the empirical form factors of the proton. For  $R_0$  we take only the results of ref. 1 which supersede the older data of de Vries et al. <sup>5)</sup>. Besides of these data there exist some recent measurements of the Orsay group which will not be included in our analysis <sup>6)</sup>.

We discuss the theoretical results in Section II and use them in Section III to calculate the form factors of the neutron. In Section IV the results are briefly discussed and compared with other data.

## II. Theoretical Results.

Following (I) we write for the inelastic electron-deuteron cross section

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{\text{Mott}} \frac{m^2 p}{\pi \sqrt{m^2 + p^2}} J(\theta, E') \quad (1)$$

where  $\sigma_{\text{Mott}}$  is the Mott cross section

$$\sigma_{\text{Mott}} = \frac{\alpha^2}{4 E^2} \frac{\cos^2 \theta/2}{\sin^4 \theta/2} \quad (2)$$

The notation is the same as in (I). The angular distribution function  $J(\theta, E')$  is subdivided into its Born approximation part  $J_B(\theta, E')$  and the rest

$$\Delta(\theta, E') = J(\theta, E') - J_B(\theta, E') \quad (3)$$

For  $\Delta(\theta, E')$  a multipole expansion was derived in (I). There it was found that it is sufficient to take into account the partial waves up to  $J = 5$ . Since the cross section is derived in the one-photon exchange approximation,  $J(\theta, E')$  is a linear function of  $\tan^2 \theta/2$ . We prefer to write  $J(\theta, E')$  in the following way:

$$J(\theta, E') = J_\ell(E') + J_t(E') \frac{q^2}{4m^2} \left[ \frac{1}{1 + q^2/4m^2} + \tan^2 \theta/2 \right] \quad (4)$$

Then  $J_\ell$  and  $J_t$  are proportional to the longitudinal and the transversal parts of the cross section. By comparison of this definition for  $J_\ell$  and  $J_t$  with Eq.(I,32) or Eq.(I,33) the Born approximation parts to  $J_\ell$  and  $J_t$  are defined, whereas from Eq.(I,44) the rescattering contributions to  $J_\ell$  and  $J_t$  can be read off immediately. In (I) the matrix element  $M_{el}^{\text{conv}}$  of the transversal electric part of the convection current was

set equal to zero. This was justified because in the Born approximation the transversal convection current gave a completely negligible contribution of the order of 0.3 per cent. It was concluded that this matrix element should be retained in the analysis reported here, because for small  $q^2$  the final state interaction is important as it could substantially enhance this special matrix element. In the notation of (I) this matrix element is

$$M_{el}^{conv}(L, J, \lambda) = (G_{1E} + (-)^L G_{2E}) \sqrt{3} \sqrt{\frac{2L+1}{2L(L+1)}} \quad (5)$$

$$\times \sum_{\ell, \ell_i} \sqrt{2\ell_i+1} C(L, \ell_i, \ell; 00) W(11\ell L; \ell_i J) K_{conv}(\ell J \lambda; L \ell_i)$$

where the radial matrix element is

$$K_{conv}(\ell J \lambda; L \ell_i) = \int_0^{\infty} v_{\ell J \lambda}(p, r) \left\{ \left( \frac{\bar{q}_0}{q} - \frac{q}{4m} \right) \frac{d}{dr} r j_L\left(\frac{qr}{2}\right) - \frac{q}{2m} j_L\left(\frac{qr}{2}\right) r \frac{d}{dr} \right\} u_{\ell_i}(r) dr \quad (6)$$

and where  $\bar{q}_0$  is defined by

$$\bar{q}_0 = 2\sqrt{p^2 + m^2} - m_d \quad (7)$$

This formula for  $M_{el}^{conv}(L, J, \lambda)$  is derived by applying Siegert's theorem which has been successfully employed in the theory of deuteron photodisintegration<sup>7)</sup>. Therefore the formulation given here reduces in the limit  $q^2 = 0$  to the nonrelativistic theory of photodisintegration.

For the analysis to be described in the next section, we have to know the dependence of the cross section on the form factors of the proton  $G_{1E}$  and  $G_{1M}$  and on the form factors of the neutron  $G_{2E}$  and  $G_{2M}$ . From the appropriate equation in (I) together with the addition Eq.(5) above it is obvious that  $J_\ell$  and  $J_t$  depend on the form factors in the following way:

$$\begin{aligned}
 J_{\lambda} &= (G_{1E}^2 + G_{2E}^2) A_{\lambda} + 2 G_{1E} G_{2E} B_{\lambda} \\
 J_t &= (G_{1M}^2 + G_{2M}^2) A_t + 2 G_{1M} G_{2M} B_t \\
 &+ (G_{1E}^2 + G_{2E}^2) C_t + 2 G_{1E} G_{2E} D_t \\
 &+ 2(G_{1E}^2 G_{1M} + G_{2E}^2 G_{2M}) E_t + 2(G_{1E} G_{2M} + G_{2E} G_{1M}) F_t
 \end{aligned}
 \tag{8}$$

The coefficients  $A_{\lambda}$ ,  $B_{\lambda}$ ,  $A_t$ ,  $B_t$ , etc. are determined by the theory. The structure of the deuteron and the final scattering state has been described by the Hamada-Johnston potential as was already discussed in (I). The results for these coefficients are given in Table 1 for the  $q^2$  we are interested in for analysing the experimental data of ref. 1. We remark that the values of the momentum transfer  $q^2$  for inelastic deuteron scattering at the quasi-elastic peak given in the first row of Table 1 are slightly different from the elastic electron-proton scattering values  $q^2 = 0.5, 1.0, 2.5, \text{ and } 4.6 \text{ f}^{-2}$ . The results in Table 1 show that there is an appreciable interference in  $J_{\lambda}$  and  $J_t$  between the proton and neutron contributions. The amount of interference decreases with increasing  $q^2$ . In (I) we found that the contribution of the proton-neutron interference is less than 1 per cent even at  $q^2 = 2.5 \text{ f}^{-2}$ , if it is evaluated in the Born approximation. So the proton-neutron interference is very sensitive to final-state interaction effects. For some values of  $q^2$  we investigated this effect further and found that  $B_t$  and  $B_{\lambda}$  are completely altered by rescattering contributions. The change of  $A_{\lambda}$  and  $A_t$  is much less. In Table 1 we have written down the corresponding changes in per cent in parentheses behind the values of  $A_{\lambda}$  and  $A_t$  respectively. We see that  $A_{\lambda}$  and  $A_t$  are modified differently for  $q^2$  below  $1.5 \text{ f}^{-2}$  but that they are always reduced as compared to the Born-approximation values.

The fact that for small  $q^2$  the interference terms are not negligible is interesting and has been found before in connection with the application of dispersion theoretical methods<sup>8)</sup>. In particular, it will be shown in the next section that the interference term proportional to  $B_{\lambda}$  is very important for the analysis towards the charge form factor of the neutron. As functions of  $q^2$  the coefficients  $B_{\lambda}$  and  $B_t$  decrease rather strongly. For  $q^2 = 4.6 \text{ f}^{-2}$  the ratios  $B_{\lambda}/A_{\lambda}$  and  $B_t/A_t$  which are negative are only of the order of  $-0.05$ , whereas for  $q^2 = 0.5 \text{ f}^{-2}$  these ratios are ten times larger.

It is clear that the simple formula of Durand (see for instance Eq(I,50))



which has been used extensively for the analysis of electrodisintegration experiments is not valid in the considered  $q^2$  range.

### III. Analysis of the Experimental Data.

The experimental values for  $J_\ell$  and  $J_t$  are obtained by a least-squares fit of the relation to the data of Hughes et al. <sup>1)</sup>:

$$J = J_\ell + J_t y \quad (9)$$

where

$$y = \frac{q^2}{4m^2} \left( \frac{1}{1 + q^2/4m^2} + 2 \tan^2 \theta/2 \right) \quad (10)$$

In this paper were reported results for  $R_o$ , which is the ratio of the elastic electron-proton cross section to the quasi-elastic electron-deuteron cross section.  $R_o$  is connected with  $J$  by the following relation:

$$J = \pi \frac{\sqrt{p^2 + m^2}}{m^2 p} \frac{1}{1 + \frac{2E}{m} \sin^2 \theta/2} G_p(\theta, q^2) \frac{1}{R_o} \quad (11)$$

Here  $G_p(\theta, q^2)$  is the elastic electron-proton cross section up to the nuclear cross section which is factored out:

$$G_p(\theta, q^2) = \frac{1}{1 + q^2/4m^2} G_{1E}^2 + \frac{q^2}{4m^2} \left( 2 \tan^2 \theta/2 + \frac{1}{1 + q^2/4m^2} \right) G_{1M}^2. \quad (12)$$

$G_p(\theta, q^2)$  can be calculated from the measured form factors of the proton. Since the electron-proton measurements have not been made at exactly the same  $q^2$  for which we need the form factors  $G_{1E}$  and  $G_{1M}$  we calculated them from three parameter interpolation formulas of the following form:

$$\begin{aligned} G_{1E} &= 1 + a_E q^2 + b_E q^4 + c_E q^6 \\ G_{1M} &= \mu_1 (1 + a_M q^2 + b_M q^4 + c_M q^6) ; \quad \mu_1 = 2.793 \end{aligned} \quad (13)$$

The coefficients  $a_E$  to  $c_M$  as well as their error matrices were determined by least-squares fits to the data of several laboratories which are published in papers collected in ref. 9.

The ratio  $R_0$  in Eq.(11) may differ from the measured ratio  $R_{exp}$  by radiative corrections to the proton and deuteron cross sections, but these corrections have been applied in ref. 1.

The results for  $J_\ell$  and  $J_t$  are shown in Table 2<sup>†</sup>). The errors of these two quantities come mostly from the error of  $R_0$ , but the errors of the form factors of the proton are included as well. The values for  $q^2$  in the first column of Table 2 are the momentum transfers for elastic e-p scattering. They differ by a small amount from the corresponding momentum transfer for which the authors of ref.1 did the quasi-elastic e-d scattering measurements.

It is rather clear that it is not possible to obtain significant results for the neutron's charge form factor  $G_{2E}$  from these data since the experimental accuracy of  $J_\ell$  is much too small. Nevertheless we have done the few computations which lead to  $G_{2E}$ . From an analysis of these calculations, one could see the accuracy of  $J_\ell$  needed to come to useful results for  $G_{2E}$ . Furthermore, we might expect from this analysis a hint as to the direction in which the theory must be modified when complex values for  $G_{2E}$  should be encountered as was the case in the analysis done by Hughes et al. 1).

$G_{2E}$  is calculated from the following formula:

$$G_{2E} = G_{1E} \left( -\frac{B_\ell}{A_\ell} + \sqrt{\frac{J_\ell}{G_{1E}^2 A_\ell} + \left(\frac{B_\ell}{A_\ell}\right)^2 - 1} \right) \quad (14)$$

In this formula  $B_\ell$  and  $A_\ell$  are known from our theory (see Table 1).  $J_\ell/G_{1E}^2$  is obtained in the least-squares fit procedure described above and is tabulated with its error which is very important for the following analysis in Table 2. Instead of using  $J_\ell/G_{1E}^2$  we could have calculated  $G_{2E}$  from  $J_\ell$  and  $G_{1E}^2$ , but the method suggested by Eq.(14) is more advantageous. For  $J_\ell/G_{1E}^2$  the influence of the error of the proton form factors is minimized since  $J_\ell$  is roughly proportional to  $G_{1E}^2$ . Furthermore, the complicated procedure of error propagation calculations had to be done only for the quantity  $J_\ell/G_{1E}^2$ . Let us denote the radicand

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<sup>†</sup> The data for  $E = 240.6$  and  $\theta = 60^\circ$  are omitted because the corresponding  $q^2$  at the peak comes out as  $q^2 = 1.306 \text{ f}^{-2}$ , which is too far from the  $q^2$  of the other measurements around  $q^2 = 1.5 \text{ f}^{-2}$ .

of the square root in Eq.(14) by  $r$ . Now the numerical computations show that  $J_\ell / (G_{1E}^2 A_\ell)$  is roughly equal to 1 whereas  $(B_\ell / A_\ell)^2$  is very small and less than 0.04 for the four  $q^2 > 1 \text{ f}^{-2}$ . The numbers of these quantities are collected in Table 3. Therefore a rather strong cancellation takes place between  $J_\ell / G_{1E}^2 A_\ell$  and  $1 - (B_\ell / A_\ell)^2$ . Of course this cancellation has its origin in the subtraction of the proton contribution to the cross section. Although some of the cross section measurements are so precise as to lead to a statistical accuracy for  $J_\ell$  or  $J_\ell / G_{1E}^2$  of the order of 2 per cent, the quantity  $\sqrt{r}$  is barely determined at all. We obtain  $\sqrt{r} = 0.05 \begin{smallmatrix} + 0.10 \\ - 0.05 \end{smallmatrix}$  for  $q^2 = 1.5 \text{ f}^{-2}$ , which is the most favourite case. So only a change of  $r$  by systematic errors in the experimental data, that is in  $J_\ell$ , or by a change of the theoretical value for  $A_\ell$  of the order of 1 per cent or less is needed to obtain a value for  $G_{2E}$  for  $q^2 = 1.5 \text{ f}^{-2}$  and also for  $q^2 = 1.0 \text{ f}^{-2}$  which is in agreement with the number one calculates from the slope of  $G_{2E}$ :

$$C = \left[ \frac{d}{dq^2} G_{2E}(q^2) \right]_{q^2=0} = (0.021 \pm 0.001) \text{f}^2 \quad 11)$$

or (15)

$$C = \left[ \frac{d}{dq^2} G_{2E}(q^2) \right]_{q^2=0} = (0.0178 \pm 0.0009) \text{f}^2 \quad 12)$$

known from n-e scattering experiments. In the first place we cannot expect our theory to be so complete that this change of  $A_\ell$  by 1 per cent must be excluded. Therefore, a discussion of other possibilities is pointless. Contrary to the analysis of Hughes et al.<sup>1)</sup> we obtain positive numbers for  $r$  within the statistical errors of  $J_\ell / G_{1E}^2$  for the two cases  $q^2 = 1.0 \text{ f}^{-2}$  and  $q^2 = 1.5 \text{ f}^{-2}$ , which makes the final result for  $G_{2E}$  real.

Unfortunately, the situation is different for the two larger momentum transfers  $q^2 = 2.5 \text{ f}^{-2}$  and  $q^2 = 4.6 \text{ f}^{-2}$ . Here the radicand  $r$  is negative; thus real values for  $G_{2E}$  cannot be derived from the data with our theory. Hughes et al.<sup>1)</sup> observed a similar discrepancy for the data with  $q^2$  below  $5 \text{ f}^{-2}$ , although they used a different formula for their analysis. At the moment we have no remedy for this discouraging result. A change of either the theory or the experimental data by roughly 15 per cent is necessary to get a positive  $r$ . According to our analysis there seems to be no need for a modification of the theory of this order of magnitude for  $q^2 = 1.0 \text{ f}^{-2}$  and  $q^2 = 1.5 \text{ f}^{-2}$ , and apparently

also not for the larger  $q^2$  above  $6.0 f^{-2}$ . Therefore the additional terms in the theory which might account for the discrepancies at  $q^2 = 2.5 f^{-2}$  and  $q^2 = 4.6 f^{-2}$  had to depend rather strongly on  $q^2$ , which seems unlikely to us. Our analysis makes it clear that only very crude results for  $G_{2E}$  can be obtained because of the strong cancellation taking place in  $r$ . This will hold even when the empirical data would come out slightly different but with the same accuracy.

To complete our survey, we have also collected in Table 3 the results for  $B_\ell/A_\ell$ , which gives a large contribution, and  $G_{2E}/G_{1E}$ . For the calculation of  $G_{2E}$  we have chosen the minus sign before the square root in Eq. (14) since we expect that  $G_{2E}$  is roughly equal to  $cq^2$  with the  $c$  mentioned above. For  $q^2 = 2.5 f^{-2}$  and  $q^2 = 4.6 f^{-2}$  we have arbitrarily set  $r$  equal to zero. Therefore only upper limits for  $G_{2E}$  are obtained. The same procedure was gone through in connection with the results for the lower limits of  $J_\ell$  at  $q^2 = 1.0 f^{-2}$  and  $q^2 = 1.5 f^{-2}$ , and for  $I_\ell$  itself with  $q^2 = 1.0 f^{-2}$ .

The computation of  $G_{2M}$  is straightforward. No appreciable cancellations take place. In the representation of  $J_t$  the coefficients  $C_t$ ,  $D_t$ ,  $E_t$ , and  $F_t$  are so small that they can be neglected; they would change the final result by less than 0.5 per cent. Then  $G_{2M}$  is calculated from

$$G_{2M} = G_{1M} \left( -\frac{B_t}{A_t} - \sqrt{\frac{J_t}{G_{1M}^2 A_t} - 1 + \left(\frac{B_t}{A_t}\right)^2} \right) \quad (16)$$

The results are tabulated in Table 4, together with  $G_{1M}$ . Finally we tested whether the form factors  $G_{1E}$ ,  $G_{1M}$ , and  $G_{2M}$  show some common dependence on  $q^2$  (besides normalization) in the region of the analysis, as it was observed for higher  $q^2$  (1). In between we obtained  $G_{2M}/G_{1M}$ , for which we expect  $G_{2M}/G_{1M} \approx \mu_2/\mu_1 = -0.685$  and independent of  $q^2$  as has been found for the higher  $q^2$  (1). We see that we come rather close to this number except for  $q^2 = 1.0 f^{-2}$ . The column  $-G_{2M}/\mu_2$  ( $\mu_2 = -1.91$ ) can be directly compared with the results of the analysis in ref. 1. We see that  $-G_{2M}/\mu_2$  is now smaller. This is the effect of the interference contribution  $-B_t/A_t$  which lowers the absolute value of  $G_{2M}$  compared to Durand's formula without interference. The column for  $(-G_{2M}/\mu_2)$  should be compared with the column for  $G_{1E}$  in Table 3 as a test of the relation  $G_{1E} \approx -G_{2M}/\mu_2$  which is also rather well satisfied for the higher momentum transfer. For the sake of completeness the

quantity  $G_{1M}/\mu_1$  ( $\mu_1 = 2.793$ ) is tabulated in the last column of Table 4. It agrees almost exactly with  $G_{1E}$  for the fit we have made. The results obtained for  $G_{2M}$  can be considered as encouraging. More experimental data on inelastic electron-deuteron scattering are needed, in particular around  $q^2 = 1 \text{ f}^{-2}$ , before more accurate magnetic form factors for the neutron can be deduced in the low momentum transfer range.

Finally we remark that the results for the magnetic form factor  $G_{2M}$  depend strongly on the measurements for the small electron scattering angles  $\theta$  which are more precise than the measurements in the backward hemisphere. It is interesting to note that  $G_{2M}$  would have smaller values than we obtained when the measurements in the forward hemisphere would be consistent with charge form factors  $G_{2E}$  of the order of 0.02 for  $q^2 = 4.6 \text{ f}^{-2}$ . For such  $G_{2E}$  the quantity  $J_\lambda$  has the following values: 6.10, 4.15, 2.35, 0.89, and that for  $q^2 = 1.0, 1.5, 2.5, \text{ and } 4.5 \text{ f}^{-2}$ . With these values for  $J_\lambda$  we obtain by a least squares fit to all measurements of the particular  $q^2$  for  $J_\lambda: J_{\lambda 2} = 112.9 \pm 10.5, 61.2 \pm 1.6, 30.14 \pm 1.8, 10.09 \pm 0.15$  again for  $q^2 = 1.0, 1.5, 2.5, \text{ and } 4.6 \text{ f}^{-2}$ . These numbers have to be compared with the results of  $J_\lambda$  in Table 2. They are smaller than the values of  $J_\lambda$  in this table which we obtained from the straight line fit to the data with  $J_\lambda$  not fixed.

#### IV. Comparison with other Data.

A second source for neutron form factors are elastic electron-deuteron scattering experiments <sup>13)</sup>. At small  $q^2$  they give very small values of  $G_{2E}$  but seem to deviate from the extrapolated neutron-electron interaction. This deviation, if it is real, is perhaps due to theoretical uncertainties in the deuteron form factors, but when nonrelativistic theory is accepted as a basis for analysis the form factors  $G_{2E}$  obtained from elastic e-d scattering have much higher accuracy than the values we arrived at in Table 3; these latter are, of course, con-

sistent with the data from elastic scattering. The upper limit for  $G_{2E}$  at  $q^2 = 4.6 \text{ f}^{-2}$  can also be compared with the results of P. Stein et al. <sup>14)</sup> who obtained  $G_{2E} = 0.041 \pm 0.052$  at  $q^2 = 5.5 \text{ f}^{-2}$  from analysis of electron-neutron coincidence measurements.

As to the magnetic form factor of the neutron, measurements of the elastic electron-deuteron scattering give results which obey rather closely the rule  $G_{2M}/G_{1M} = \mu_2/\mu_1$ . We have collected these data in Fig. 1 and compared them with our results from Table 4 in the form that we plot the ratio  $G_{2M}/\mu_2 / G_{1M}/\mu_1$ . The elastic data give a ratio systematically smaller than 1, whereas our results lead to a ratio larger than 1. We see from Fig. 1 that measurements with  $q^2 \leq 1 \text{ f}^{-2}$  seem to be particularly interesting. Unfortunately, we are not able to include in our present analysis any recent high-accuracy measurements <sup>6)</sup> of inelastic e-d scattering cross sections around  $q^2 = 3.5 \text{ f}^{-2}$ , but we hope to present such an analysis in a later publication.

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Table Captions.

- Table 1: Coefficients of longitudinal and transversal cross sections  $J_l(\theta, E')$  and  $J_t(\theta, E')$  in  $f$  as a function of  $q^2$ .
- Table 2: Experimental longitudinal and transversal cross section in  $f$  from ref. 1.
- Table 3: Analysis towards the charge form factor  $G_{2E}$  of the neutron.
- Table 4: Results of the calculation of the magnetic form factor of the neutron and comparison with the magnetic proton form factor.

Figure Caption.

- Fig. 1 Comparison of the magnetic form factors of the neutron and proton for  $q^2 \leq 5.0 \text{ f}^{-2}$ , including results from elastic electron-deuteron scattering. 13)

Table 1

$q^2 f^{-2}$	$A_{\ell}$	$B_{\ell}$	$A_t$	$B_t$	$C_t 10^{+2}$	$D_t^{+2}$	$E_t 10^{+2}$	$F_t 1P^{+2}$
0.492	14.06 (-37.0)	-6.357	19.92 (-10.7)	-11.20	4.567	-1.798	25.26	-4.562
0.990	7.893 (-31.7)	-1.588	8.580 (-25.8)	-1.924	2.324	2.092	1.028	10.58
1.488	5.978 (-22.4)	-0.8851	6.161 (-20.0)	-0.9242	1.634	0.9654	-0.3612	7.497
2.486	4.040 (-10.7)	-0.4110	4.104 (-10.4)	-0.4792	0.9959	0.4682	-0.5182	4.087
4.600	2.297 (-5.3)	-0.09564	2.261 (- 6.8)	-0.1175	0.4893	0.4889	-0.2607	1.734

Table 2

$[f^{-2}]^q$	$J_\ell$	$J_t$	$J_\ell/G_{1E}^2$	$J_t/G_{1M}^2$
1.0	$5.80 \pm 0.51$	$126.9 \pm 26.8$	$7.33 \pm 0.64$	$20.50 \pm 4.42$
1.5	$4.15 \pm .10$	$61.0 \pm 2.2$	$5.85 \pm 0.14$	$10.95 \pm 0.40$
2.5	$2.055 \pm 0.52$	$32.93 \pm 0.66$	$3.514 \pm 0.087$	$7.34 \pm 0.14$
4.6	$0.774 \pm 0.035$	$11.00 \pm 0.31$	$1.946 \pm 0.089$	$3.68 \pm 0.09$

Table 3

$q^2$	$\frac{J_l}{G_{1E} A_l}$	$r$	$-\frac{B_l}{A_l}$	$G_{2E}/G_{1E}$	$G_{1E}$	$G_{2E}$
1.0	1.0098	0.0503		-0.0231		-0.025
	0.9287	-0.0208	0.2012	<0.2012	0.8888	<0.1788
	0.8476	-0.1019		<0.2012		<0.1788
1.5	1.0020	-0.0239	0.1481	-0.0065	0.8427	-0.0055
	0.9786	0.0025		0.0981		0.0827
	0.9552	-0.0209		<0.1481		<0.1248
2.5	0.8913	-0.0984				
	0.8698	-0.1199	0.1017	<0.1017	0.7646	<0.0778
	0.8483	-0.1414				
4.6	0.8910	-0.1072				
	0.8520	-0.1462	0.04202	<0.04202	0.6321	<0.0266
	0.8130	-0.1852				

Table 4

$q^2$	$G_{1M}$	$-(G_{2M}/G_{1M})$	$-G_{2M}$	$G_{2M}/\mu_2$	$G_{1M}/\mu_1$
1.0	2.4978	+0.960 +0.098 -0.238	2.40 +0.49 -0.60	1.25 +0.26 -0.31	0.8943
1.5	2.3735	+0.727 $\pm$ 0.037	1.72 $\pm$ 0.08	0.90 $\pm$ 0.04	0.8498
2.5	2.1179	0.763 $\pm$ 0.019	1.62 $\pm$ 0.04	0.85 $\pm$ 0.02	0.7583
4.6	1.7322	0.730 $\pm$ 0.025	1.26 $\pm$ 0.04	0.66 $\pm$ 0.02	0.6202

Fig. 1



