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Monoenergetic Bremsstrahlung with Linear Polarization Close to 100 % from Thin Crystal Targets

by

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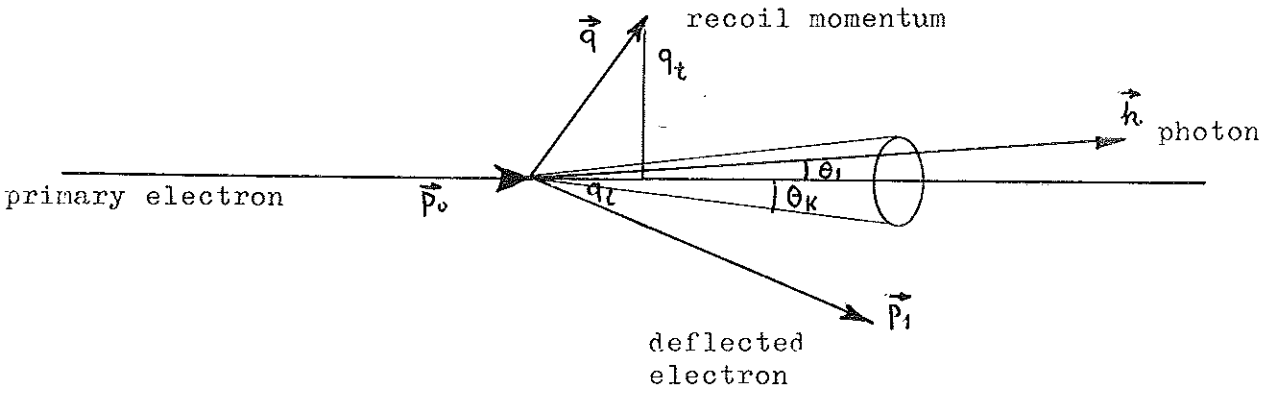
ABSTRACT

Mozley and de Wire[2] have pointed out that bremsstrahlung from thin crystals can be monochromatized if one strongly collimates the photon beam. In recent papers[3] it was shown that the linear polarization of uncollimated coherent bremsstrahlung can be raised if the crystal is orientated in such a manner that one gets the main contribution to the spectrum from one single reciprocal lattice point only. Here it will be shown that by combining this method of orientation of the crystal with strong collimation one gets not only one dominating monoenergetic spike in the spectrum due to this single reciprocal lattice point, but that, in addition, the polarization of the spike is increased to a very high value. A simple procedure is given for deriving these spectra and their polarization from some "universal" curves for silicon and diamond crystals. An estimate is given of the photon intensity which can be reached by this method.

As was pointed out by Überall[1], recoil momenta of the target nuclei are restricted to discrete values coinciding with reciprocal lattice vectors when bremsstrahlung is produced on a monocrystal. The cross section can be represented as a sum of contributions due to single reciprocal lattice points (vectors) which lie in the kinematically allowed zone of recoil momenta. In addition there is an incoherent contribution to the cross section due to thermal lattice vibrations. The region of kinematically allowed recoil momenta is given by a "pancake"-shaped volume in momentum space[1] which has a sharp lower boundary given by a plane perpendicular to the direction of the primary electron and at a distance δ to the origin. The upper limit is not sharply defined; it is given by the rapid decrease of the cross section with increasing longitudinal component of recoil momentum q_ℓ.

As one collimates the photon beam, however, one gets also a sharp upper limit of the "pancake". This is seen as follows:



momentum diagram for the bremsstrahlung process.

The photon emission angle θ_1 results from energy and momentum conservation. One gets, for small transversal recoil momentum ($q_t << 1$), by simple calculation*

$$U = E_o\theta_1 = \sqrt{\frac{q_l}{\delta} - 1} \qquad (1)$$

$$\delta = \frac{1}{2E_o}\,\frac{x}{1-x} \qquad (2)$$

E_o ... energy of the primary electron

q_l ... component of recoil momentum in direction of $\vec{p}_o$

δ ... minimal recoil momentum

$x = k/E_o$ ... relative photon energy

For photons with emission angels $U<U_k$ ($U_k=E_o\theta_k$ collimation angle) the longitudinal component of recoil momentum q_l is therefore restricted to the range

$$\delta < q_l < (1 + U_k^2) \qquad (3)$$

and one reciprocal lattice point causes bremsstrahlung intensity only in the range

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*We use units with $\hbar = c = m = 1$

$$x_\ell < x < x_u$$

$$x_u = 1 \,/\, \left(1 + \frac{1}{2E_o q_\ell}\right)$$

$$x_\ell = 1 \,/\, \left(1 + \frac{1 + U_k^2}{2E_o q_\ell}\right) \qquad (4)$$

$$\Delta x = x_u - x_\ell = x_u \,/\, \left(1 + \frac{1}{U_k^2(1-x_u)}\right)$$

For $U_k \to 0$ follows $x_\ell \to x_u$, and the contribution to the cross section due to one single reciprocal lattice point gives one monochromatic line in the bremsstrahlung spectrum.

The situation is different for the incoherent part of the cross section due to thermal lattice vibrations. The emission-angle distribution can be approximated by

$$w(U)\, dU = \frac{2U}{(1+U^2)^2}\, dU \qquad (5)$$

which is derived from the angular dependent bremsstrahlung cross section for isolated atoms. The incoherent part of the cross section then is reduced by collimation by a factor

$$V = U_k^2 \,/\, (1 + U_k^2) \qquad (6)$$

Thus collimation has two effects on monocrystal bremsstrahlung spectra as shown by Mozley and de Wire and demonstrated in Fig. 1:
1) It does not influence the contribution of one single lattice point in the range $x_\ell < x < x_u$ but it cuts the lower tails of the peaks.
2) It reduces the unpolarized incoherent part of the cross section.

Mozley and de Wire have not considered the effect of collimation on the polarization of the peaks in the bremsstrahlung spectrum. In their example it is small, however, because the peaks in the spectrum originate from a whole row of reciprocal lattice points having equal longitudinal recoil momenta q_ℓ. The contributions from different lattice points of the row show maximum polarization for different reference planes (containing the reciprocal lattice vector). So, for fixed re-

ference plane, contributions of the points cancel partially with respect to polarization. This was also shown to be the main reason for low polarization when no influence of collimation on bremsstrahlung spectra was considered[5].

It has been reported[5] that the polarization of (uncollimated) bremsstrahlung can be raised if one orientates the crystal in such a manner that one obtains one dominating peak in the bremsstrahlung spectrum which is due to one single lattice point only. By collimating the photon beam very strongly the unpolarized incoherent contribution as well as contributions from other lattice points are suppressed at the dominating peak of the spectrum. Therefore only the contribution from one single lattice point will remain at the peak, and - contrary to the previous case - no contributions of other lattice points cancel its high polarization.

Bremsstrahlung spectra and linear photon polarization can be calculated from the cross sections $\sigma_\perp(\sigma_\parallel)$ for bremsstrahlung polarized normal (parallel) to a reference plane given in Ref. 4 by Formulas 4, 7, 8, 9, 10 and 11*:

$$\frac{x}{\bar{\sigma}N}\,\frac{d\sigma_\perp + d\sigma_\parallel}{dx} = \left[1 + (1-x)^2\right]\left[\psi_1 + \psi_1^i\right] - \frac{2}{3}(1-x)\left[\psi_2 + \psi_2^i\right]$$

$$\frac{x}{\bar{\sigma}N}\,\frac{d\sigma_\perp - d\sigma_\parallel}{dx} = 2(1-x)\,\psi_3$$

$$\psi_1 = \frac{N_o}{N}\,\frac{(2\pi)^2}{a^3}\,4\delta \sum_{\vec{g}} |S|^2\, e^{-Ag^2}\,\frac{(1-F(g))^2}{g^4}\,\frac{q_t^2}{q_\ell^2} \qquad (7)$$

$$\psi_2 = \frac{N_o}{N}\,\frac{(2\pi)^2}{a^3}\,24\delta^2 \sum_{\vec{g}} |S|^2\, e^{-Ag^2}\,\frac{(1-F(g))^2}{g^4}\,\frac{q_t^2(q_\ell-\delta)}{q_\ell^4}$$

$$\psi_3 = -\frac{N_o}{N}\,\frac{(2\pi)^2}{a^3}\,4\delta^2 \sum_{\vec{g}} |S|^2\, e^{-Ag^2}\,\frac{(1-F(g))^2}{g^4}\,\frac{q_t^2\cos 2\Phi^o}{q_\ell^4}$$

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*The formulas were given earlier for a special reference plane of polarization in Ref. 3

N ..... number of atoms of the crystal

N_o ..... number of atoms of the fundamental cell

a ..... lattice constant in units of $\lambda\!\!\!^{-}_c$ ($\lambda_c = 2\pi\lambda\!\!\!^{-}_c$ Compton wave length)

$\sigma = Z^2 \, (e^2/\hbar c) \, (e^2/mc^2)^2 = Z^2 \; 5 \; . \; 795 \; . \; 10^{-28} \; cm^2$

q_t ..... component of reciprocal lattice vector $\perp \vec{p}_o$

q_ℓ ..... component of reciprocal lattice vector $\parallel \vec{p}_o$

$\phi_o = \phi - \arctan \frac{g_3}{g_2}$ angle between the reference plane for polarisation and the plane containing $\vec{g}$ and $\vec{p}_o$

In this formulation the explicit introduction of crystal angles (θ, α, ϕ) is suppressed.

The forumulas give the bremsstrahlung cross section integrated over angles. Collimation restricts the contribution of each reciprocal lattice point to the range $x_\ell < x < x_u$, as was demonstrated earlier (4). For including the influence of collimation for each peak due to one single lattice point, only the corresponding terms of the sums in formulas (7) must be taken. In addition to this the incoherent part of the cross section (represented by ψ_1^i and ψ_2^i) has to be multiplied by the factor $V = U_k^2/(1+U_k^2)$ (6).

The linear polarization of bremsstrahlung defined by

$$P = \frac{\sigma_\perp - \sigma_\parallel}{\sigma_\perp + \sigma_\parallel} \qquad (8)$$

is easily calculated with these modifications of Eqs. 7.
Its maximum value is reached for a reference plane containing the reciprocal lattice vector and the primary electron direction ($\phi^o = 0$), as seen from the expression for ψ_3.

In the limiting case of very sharp collimation ($U_k \to 0$) the spectrum consists of monoenergetic spikes at positions x_u, with x_u given by Formula 4 for each lattice point. At x_u one has

$$\psi_1 = \psi_3 \qquad \psi_2 = 0$$

The incoherent contribution to bremsstrahlung is suppressed by colli-

mation, and the polarization for this limiting case is given by

$$P_M = \frac{2(1-x)}{1+(1-x)^2} \qquad (9)$$

which is only dependent on the photon and primary electron energies. This is the upper limit for the polarization which can be reached.

Two spectra obtained with electrons of 6 GeV on a silicon crystal are shown in Fig. 1. The collimation angles are $U_k = 0.3$ and 0.5 respectively. In both cases the photon energy at the peak is 1.8 GeV.

Graphical Representation for Cross Section and Polarization

In Ref. 4 numerical information concerning cross section for bremsstrahlung on diamond and its polarization dependence was collected. It is possible to determine bremsstrahlung spectra and their polarization for different crystal orientations from some "universal graphs". For the monoenergetic spectra considered a very similar representation of numerical data is possible. Its derivation from Eqs. 7 is analogous to that given in Rev. 4. Here only the procedure of deriving the spectra and the polarization from the "universal curves" is given. The same notation is used as in Ref. 4. Data are given for diamond and silicon crystals.

It is assumed that the electron beam ($\vec{p}_o$) hits the diamond at a relatively small angle $\theta \leq 0.1$ against the crystal axis $\vec{b}_1 = (110)$. The plane $\vec{p}_o$, (110) is inclined by a small angle $\alpha \leq 5^\circ$ against the plane (001), (110). Then the dominating peak in the spectrum comes from the most suitable reciprocal lattice point $(2\bar{2}0)$. With the aid of Fig. 2 (diamond) or Fig. 5 (silicon) the crystal angles θ, α can be chosen for the desired electron and photon energies. These figures give also the positions x_u of the other important spikes of the spectrum due to the lattice points $(4\bar{4}0)$, $(6\bar{6}0)$. The width of the spikes is calculated from the third equation of (3). The bremsstrah-

lung intensity at x_u is given by

$$I = \frac{x}{N\bar{\sigma}} \frac{d\sigma}{dx} = \frac{U_k^2}{1+U_k^2} I^i(x) + E_o(\text{GeV})\, I^c \qquad (10)$$

$$U_k = \frac{E_o(\text{GeV})}{0.511.10^{-3}} \theta_k$$

This formula looks like Eq. 16 of Ref. 4, but with the incoherent contribution I^i having been reduced by the factor $U_k^2/(1+U_k^2)$ due to collimation. The meaning of the coherent contribution I^c is changed inasmuch as only the contribution of one single reciprocal lattice point is included. I^i is represented in Fig. 3 (Fig. 6) as a function of $x_u = k/E_o$. I^c is given in Fig. 4 (Fig. 7).
For the reference plane for polarization containing the important reciprocal lattice points given above (the axis b_3=(110) respectively; ϕ=0) the linear polarization is easily calculated with Eq. 9 because there is only a coherent contribution to the bremsstrahlung spectrum from one reciprocal lattice point in one spike. One obtains with Eqs. 8 and 10

$$P = \frac{2(1-x_u)}{1+(1-x_u)^2} \quad \frac{E_o(\text{GeV})\, I^c}{I} \qquad (11)$$

Bremsstrahlung Intensities

What is the bremsstrahlung intensity which can be reached by this method? Sharp collimation of the photon beam is needed. Evidently the method only works if the primary electron direction is determined to at least the same order of magnitude as the collimation angle. This means that the mean angle of multiple scattering and hence the target thickness must be small. At first glance it would seem that in consequence of these two conditions only very small bremsstrahlung intensities can be reached with respect to amorphous target bremsstrahlung. This is not so, however. The reason is, that the angular distribution for the coherent part of the cross section is quite different from that for the incoherent contribution. In addition, interference effects raise the bremsstrahlung cross section very much above the Bethe-Heitler value.

The calculation of the influence of the target thickness and the collimation on bremsstrahlung intensity for arbitrary conditions is quite involved, but two limiting cases can be treated easily:

1) $\theta_v^2 \ll \theta_k^2$

2) $\theta_k^2 \ll \theta_v^2$

θ_k is the angle of collimation, θ_v is the angle of multiple scattering. It is obvious that for getting monochromatic spikes in the bremsstrahlung spectrum both angles must be smaller than the natural photon emission angle. In the following only the coherent contribution to the bremsstrahlung spectrum is considered.

1) $\theta_v^2 \ll \theta_k^2$

The mean square angle of multiple scattering at half target thickness is given by [6]

$$\theta_v = \frac{21.10^{-3}}{E_o(\mathrm{GeV})} \sqrt{\frac{T}{2}} \qquad (12)$$

T is the target thickness in radiation length X_o. Therefore one can write the condition $\theta_v^2 \ll \theta_k^2$ also in the form

$$T \ll T_k$$

$$T_k = 2\left(\frac{0.511}{21}\right)^2 U_k^2 = 1.184.10^{-3}\, U_k^2 \qquad (13)$$

The number of atoms per cm^2 target area is given by

$$N(cm^{-2}) = T\, X_o\, (g/cm^2)\, \frac{L}{A(g/Mol)} \qquad (14)$$

(L Avogadro's number, A atomic weight)

The bremsstrahlung intensity (coherent part) is calculated from Eq. 10.

$$\frac{Q_{eff}'}{n_e} = k \frac{dn(k)}{dk} \Big/ n_e = T X_o \frac{L}{A} \bar{\sigma} E_o(GeV) I^c \qquad (15)$$

Here the "effective number of quanta" Q_{eff}' is introduced. Q_{eff}' is the effective number of quanta in a (1/k) - photon spectrum having the same number of photons at the position of the peak as the coherent spectrum. n_e is the number of electrons.
For a diamond one has

$$\frac{Q_{eff}'}{n_e} = k \frac{dn(k)}{dk} \Big/ n_e = 0.0444 \; T \; E_o(GeV) \; I^c \qquad (16)$$

with $X_o = 42.4$ g/cm^2, $A = 12.001$ g/Mol

and for a silicon crystal

$$\frac{Q_{eff}'}{n_e} = 0.0531 \; T \; E_o(GeV) \; I^c \qquad (17)$$

with $X_o = 21.8$ g/cm^2, $A = 28.09$ g/Mol

Before discussing these results the second limiting case will be treated:

2) $\quad \theta_k^2 << \theta_v^2$ or $T >> T_k$

For this case the situation is a little bit more complicated. The angular distribution of electrons at half target thickness is

$$W_e(\theta) \, d\Omega = \frac{1}{\pi \theta_v^2} \; e^{-\left(\frac{\theta}{\theta_v}\right)^2} \; d\Omega, \qquad (18)$$

θ_v given by Eq.12.

It is suitable first to consider infinitely small collimation and to calculate which electrons can emit radiation in the direction of the collimator axis. At x_u given by Eq. 4, the photon emission angle $U = E_o \theta_1$ is zero, and the direction of the electron coincides with the direction of the photon. For a certain range below x_u the photon emission angle θ_1 is small with respect to the multiple-scattering angle θ_v. The (coherent) bremsstrahlung intensity in this range

therefore is given by Eqs. 18 and 10

$$\frac{dQ_{eff}'}{n_e} = N\bar{\sigma}\ \frac{1}{\pi\theta_v^{\,2}}\ E_o(\mathrm{GeV})\ I^c\ d\Omega_k \qquad (19)$$

Ω_k... solid angle of collimation in sterad

With Eqs. 12 and 14 one gets

$$\frac{dQ_{eff}'}{n_e} = X_o\ \frac{L}{A}\ \bar{\sigma}\ \frac{1}{\pi}\ \frac{2E_o^{\,3}\ (\mathrm{GeV})}{(21.10^{-3})^2}\ I^c\ d\Omega_k \qquad (20)$$

independent of target thickness.

For a finite collimator one has to integrate Eq. 19 up to the collimation angle. There the difficulty arises that the projection of the reciprocal lattice vector onto the photon direction - which coincides with c_ℓ at x_u - is not constant. Hence also x_u is not constant. This difficulty is not a serious one, however, because of the restriction $\theta_k^{\,2} \ll \theta_v^{\,2}$ and the fact that Eq. 20 is approximately true for a certain range below x_u. For this range one can integrate Eq. 20:

$$\frac{Q_{eff}'}{n_e} = X_o\ \frac{L}{A}\ \bar{\sigma}\ \frac{1}{\pi}\ \frac{2E_o(\mathrm{GeV})}{(21.10^{-3})^2}\ I^c\ E_o^{\,2}(\mathrm{GeV})\Omega_k \qquad (21)$$

For circular collimation with half collimation angle U_k one has

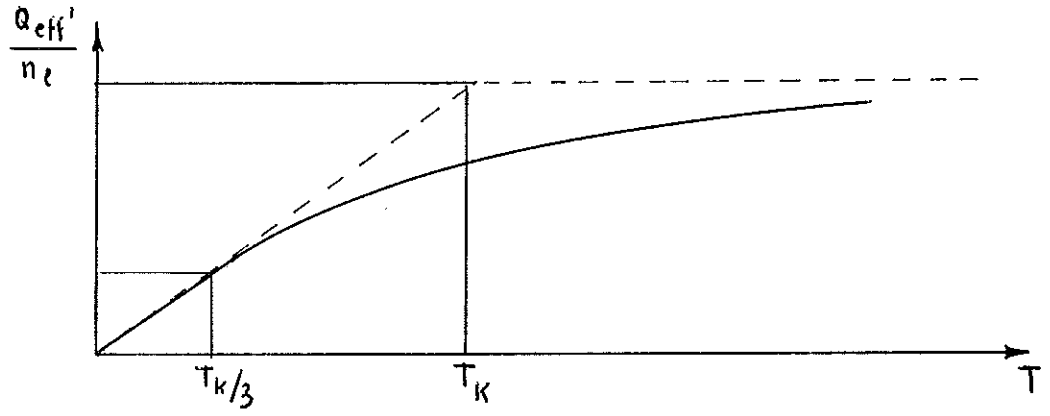
$$\frac{Q_{eff}'}{n_e} = 0.526.10^{-4}\ E_o(\mathrm{GeV})\ I_c U_k^{\,2} \qquad (22)$$

for a diamond, and

$$\frac{Q_{eff}'}{n_e} = 0.629.10^{-4}\ E_o(\mathrm{GeV})\ I_c U_k^{\,2} \qquad (23)$$

for a silicon crystal.

The situation for the two limiting cases is qualitatively demonstrated in the figure below.



bremsstrahlung intensity as a function of target thickness.

If one assumes the value for $T_k/3$ given by Eq. 17 as the attainable photon intensity one has, in the two cases demonstrated in Fig. 1, $Q_{eff}'/n_e = 0.11.10^{-3}$ and $0.31.10^{-3}$ respectively.

Now bremsstrahlung intensities should be compared with these obtained from an amorphous target. Only the limiting case of large target thickness will be considered ($\lambda_o^2 >> 1$, $\lambda_o^2 >> U_k^2$).
The probability that bremsstrahlung passes the collimator opening is given, with Eqs. 18 and 12, by

$$\frac{1}{\pi\theta_v^2}\,\Omega_k = \frac{1}{\pi}\frac{2}{T}\left(\frac{0.511}{21}\right)^2 \pi U_k^2 = 1.18 \cdot 10^{-3}\,\frac{U_k^2}{T} \qquad (24)$$

The number of effective quanta emitted in the target is by definition of radiation length

$$\frac{Q_{eff}}{n_e} = T$$

The number of effective quanta which pass the collimator then is

$$\frac{Q_{eff}}{n_e} = 1.18 \cdot 10^{-3}\, U_k^2 \qquad (25)$$

What would be the bremsstrahlung intensity if a thick amorphous target would be used, all the other conditions being the same as in the two examples demonstrated in Fig. 1? For a thin silicon crystal target the bremsstrahlung intensity Q_{eff}'/n_e is about that obtainable from thick amorphous target (Eq. 24). The situation is improved when a diamond target is used instead of a silicon crystal. Then the bremsstrahlung intensity is four times higher for the same photon and primary electron energies.

ACKNOWLEDGEMENT

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FIGURES

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2) R.F. Mozley, J. de Wire, Nuovo Cim. 27, 1281 (1963)

3) G. Barbiellini, G. Bologna, G. Diambrini, G.P. Murtas Phys. Rev. Lett. 8, 454 (1962); Nuovo Cim. 28, 435 (1963)

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6) B. Rossi, High Energy Particles, Prentice-Hall, Inc. Englewood Cliffs, N.J. (1956)

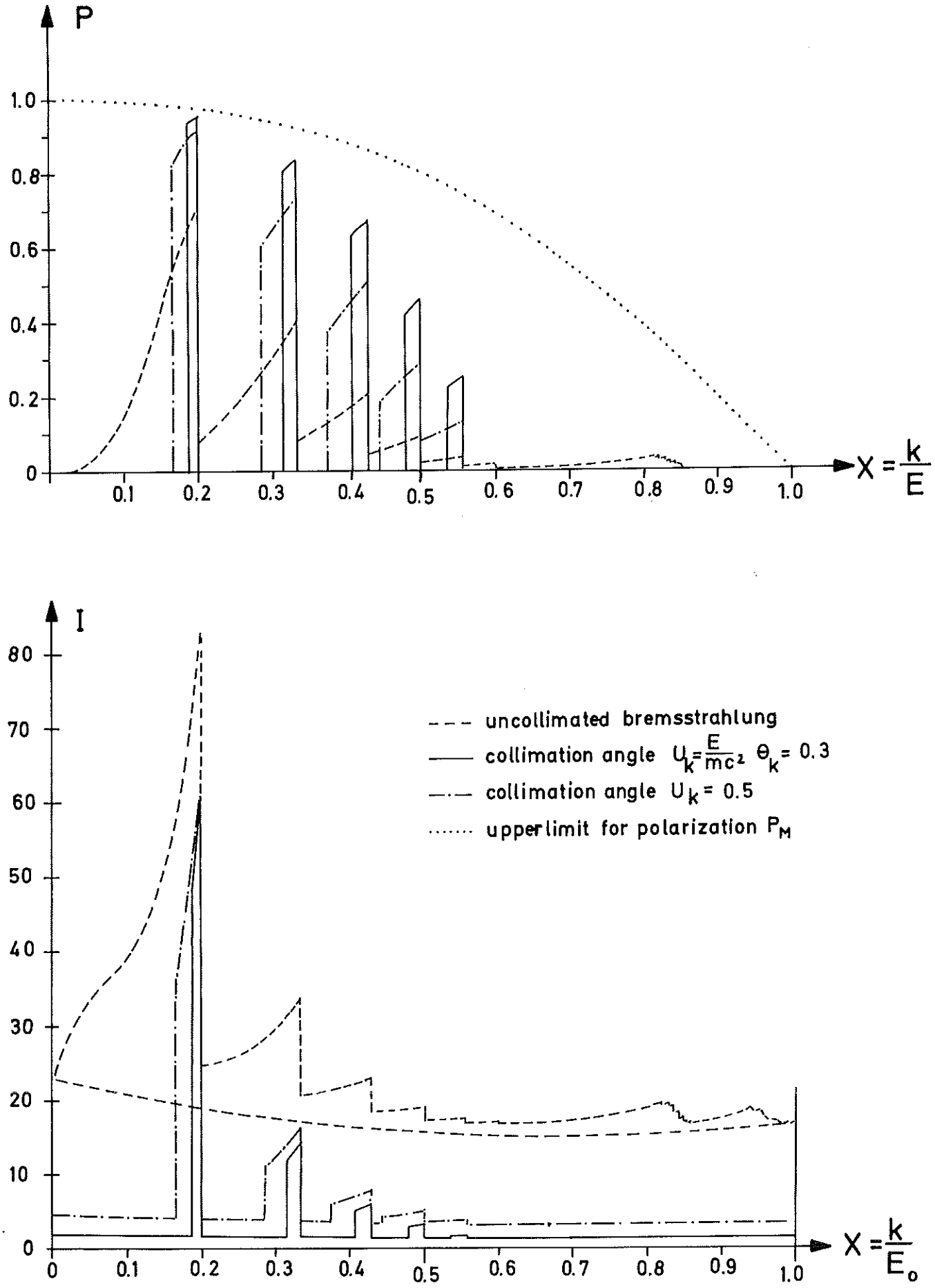

Fig. 1 Bremsstrahlung spectra and photon polarization obtained from silicon crystal $E = 6$ BeV; $\Theta = 50$ mrad; $\alpha = 0.962$
collimation angle $U_k = \frac{E_o}{mc^2}\Theta_k = 0.3$ and 0.5

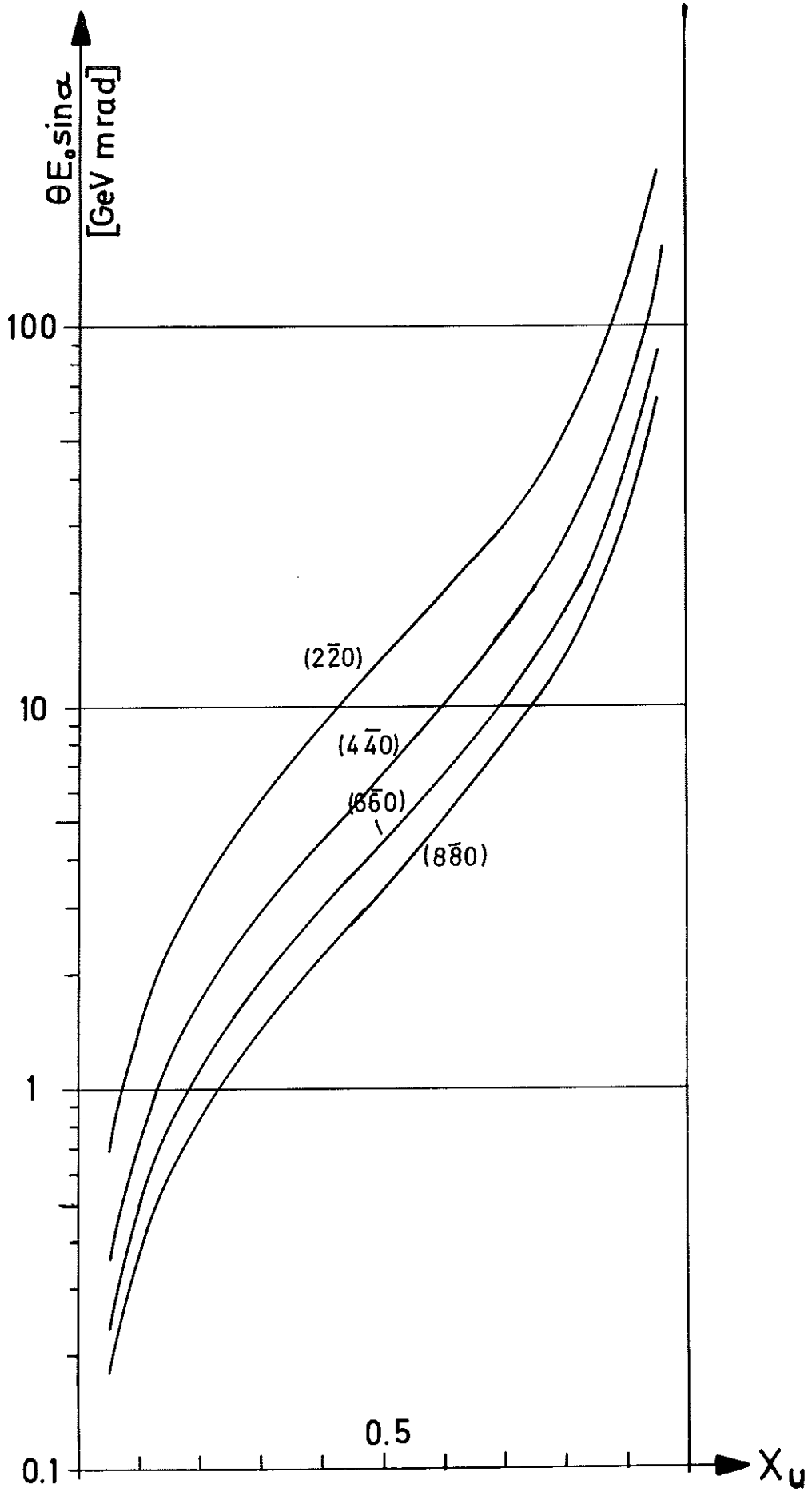

Fig. 2 Bremsstrahlung peak position as function of $\theta E_0 \sin\alpha$ for diamond

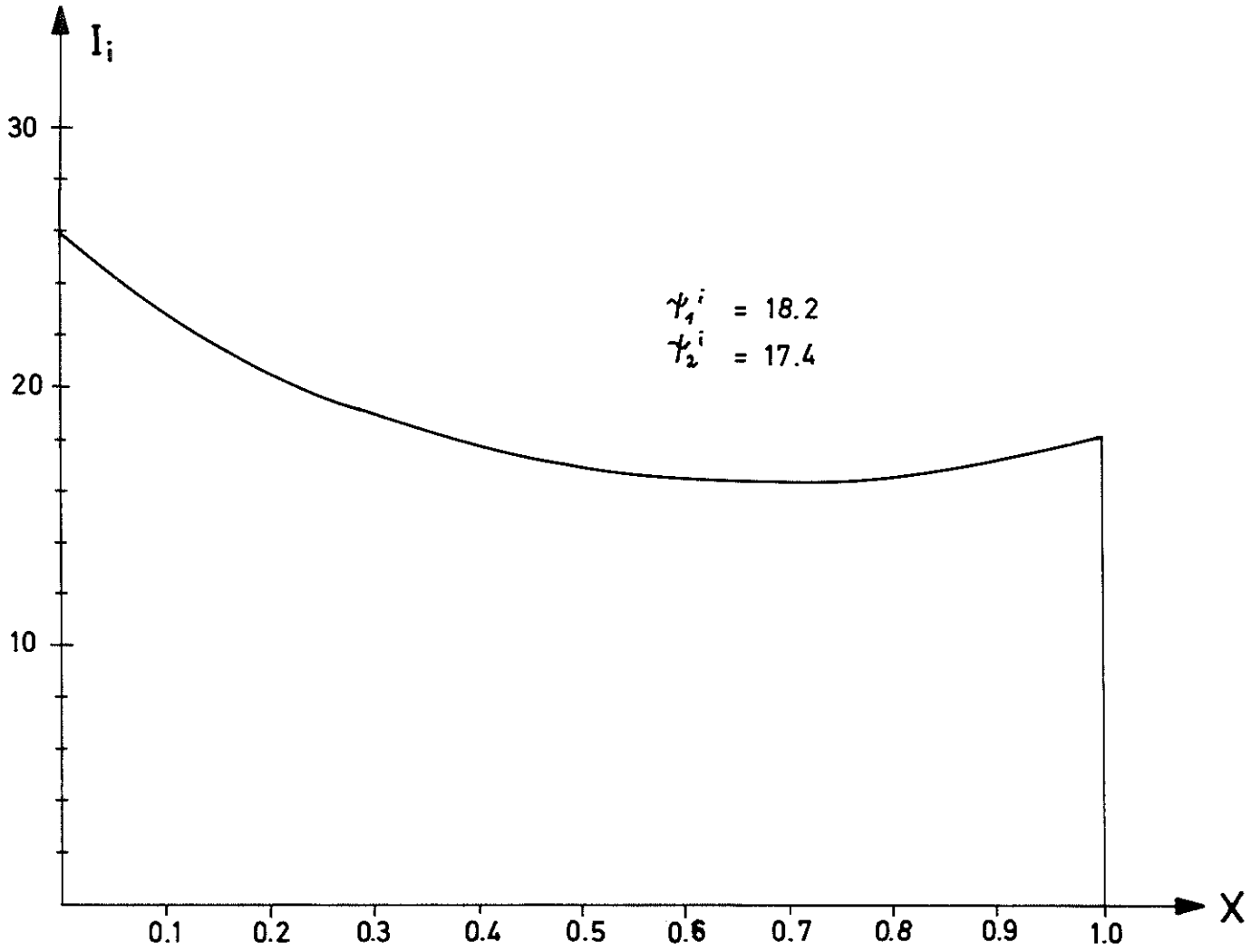

Fig. 3 Incoherent contribution to bremsstrahlung cross section for diamond

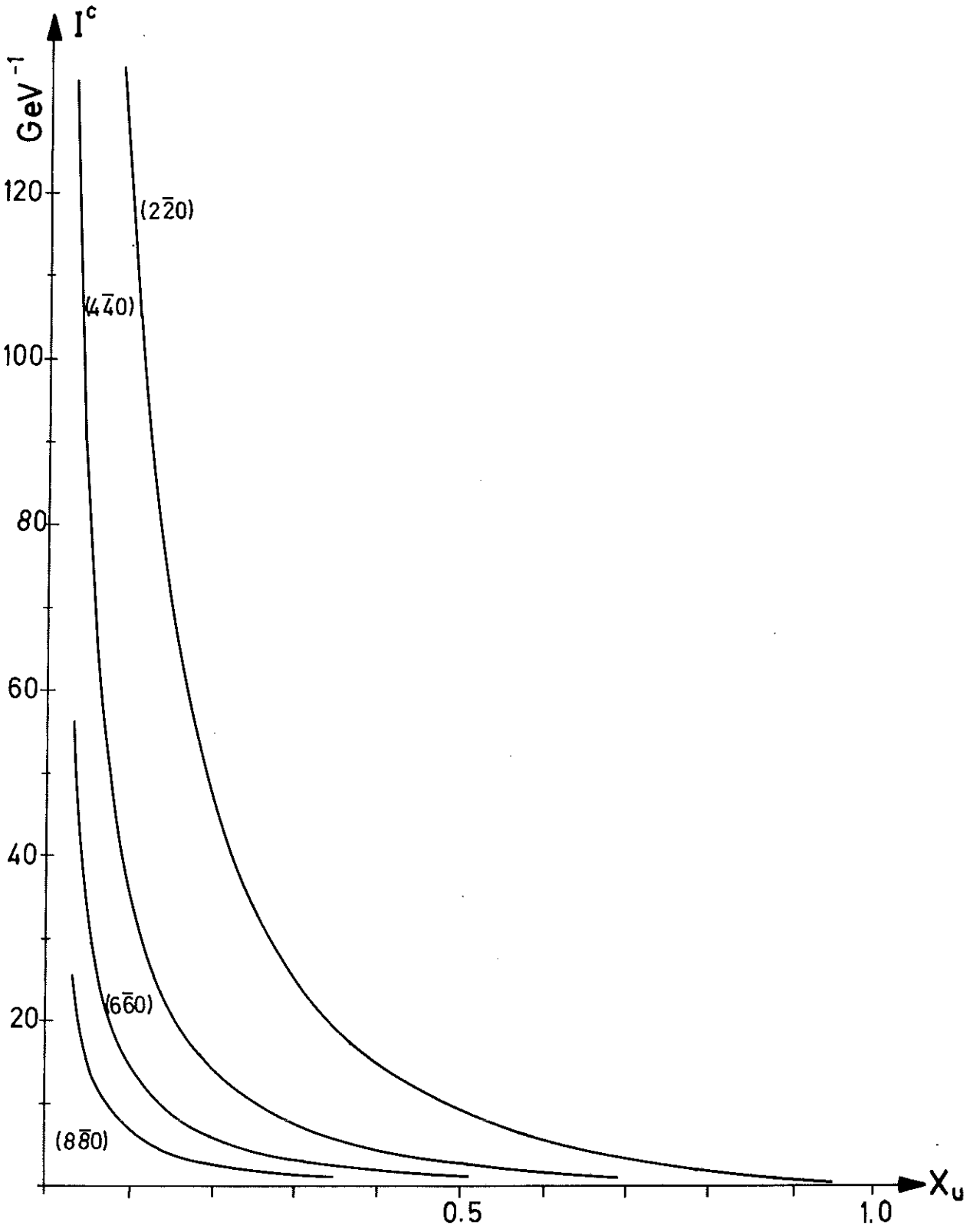

Fig. 4 Contribution of one single lattice point I^c as function of the peak position X_u for diamond crystal

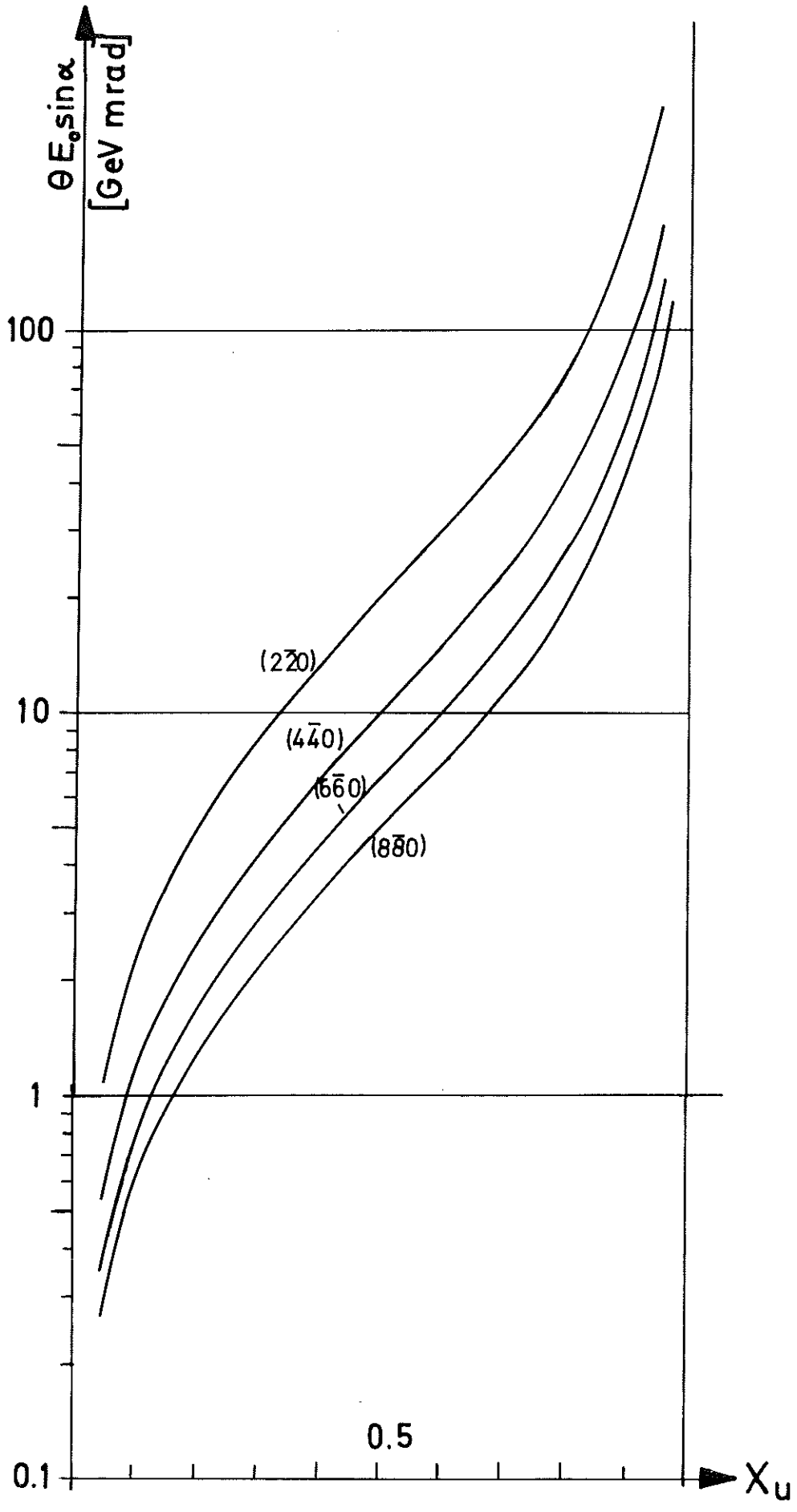

Fig. 5 Bremsstrahlung peak position as function of $\theta E_0 \sin\alpha$ for silicon crystal

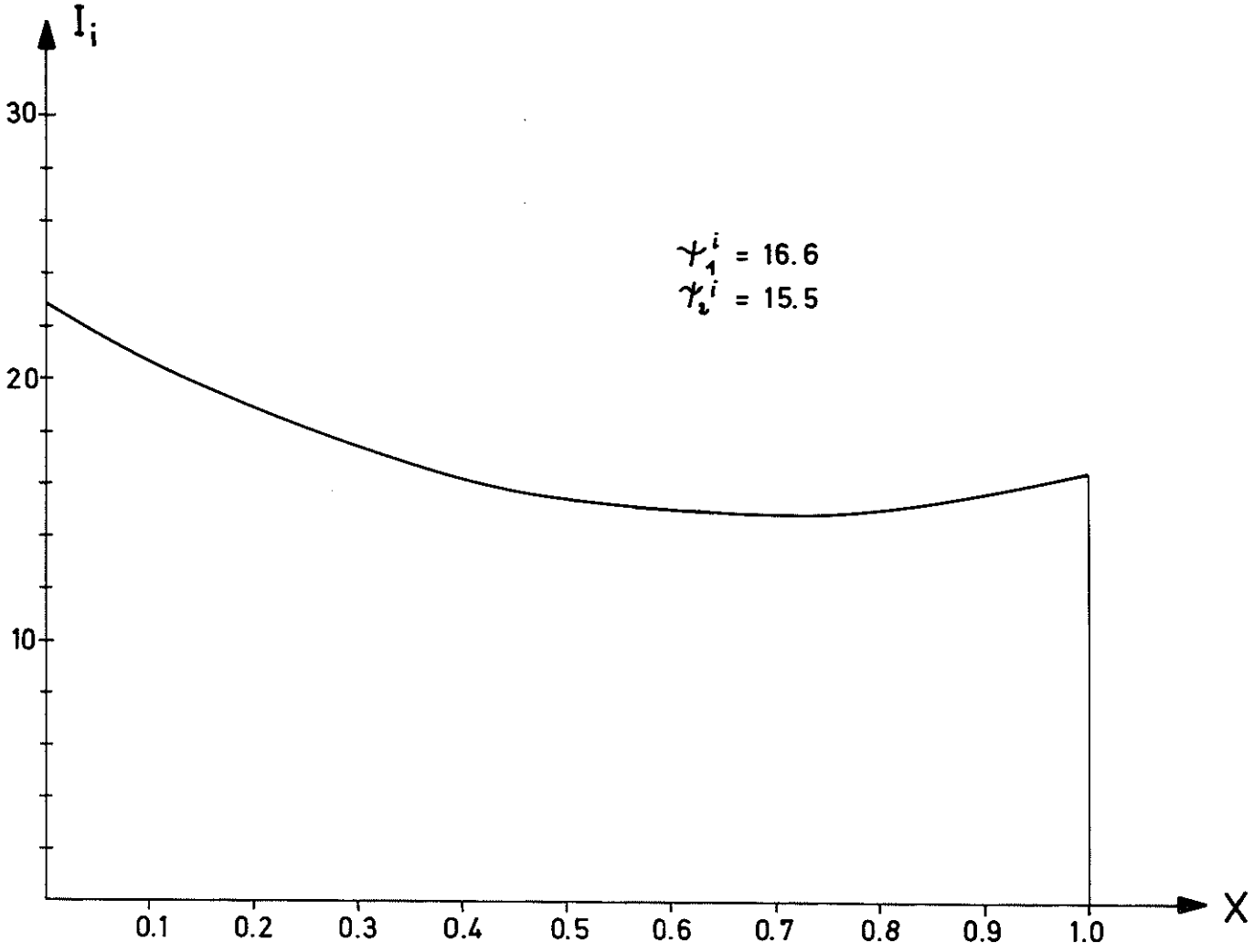

Fig. 6 Incoherent contribution to bremsstrahlung cross section for silicon crystal

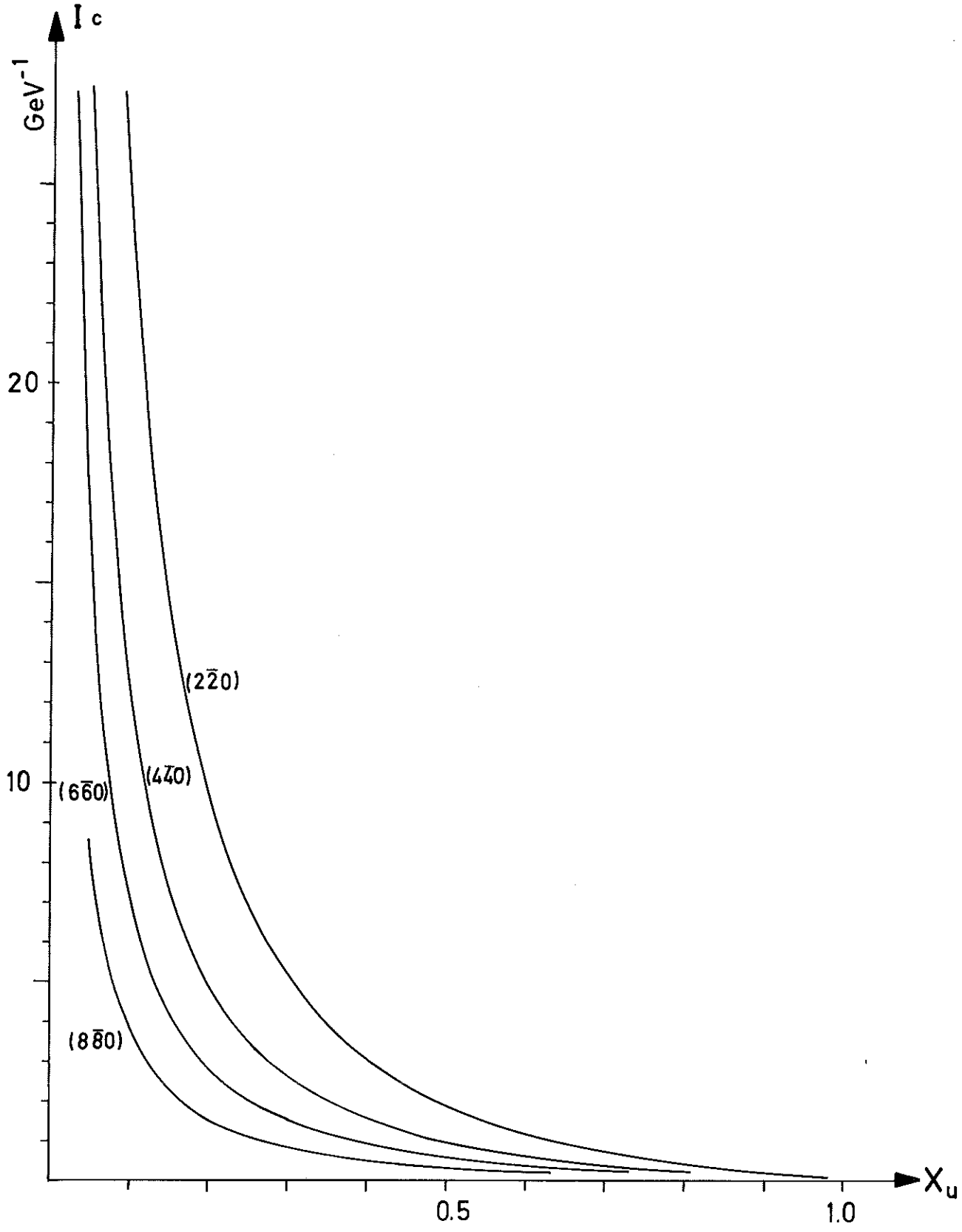

Fig. 7 Contribution of one single lattice point I^c as function of the peak position X_u for silicon crystal