DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 67/11
April 1967

Sum Rules in Perturbation Theory

by

G. Kramer
Physikalisches Staatsinstitut
II. Institut für Experimentalphysik, Hamburg

and

K. Meetz
II. Institut für Theoretische Physik,
Universität Hamburg

2 HAMBURG 52 · NOTKESTIEG 1

Sum Rules in Perturbation Theory

by

G. Kramer
Physikalisches Staatsinstitut
II. Institut für Experimentalphysik, Hamburg

and

K. Meetz
II. Institut für Theoretische Physik,
Universität Hamburg

Abstract

Sum rules derived from isospin current algebra are examined in perturbation theory for nucleons interacting with neutral pseudoscalar mesons and isovector pions. It is shown that the well-known sum rules for electromagnetic isovector form factors of nucleons and pions are correct in zero and second order of the meson-nucleon coupling constant.

I. Introduction

Sum rules have been derived by Fubini et al. [1] from the assumptions:

a. The commutation relations of the current algebra hold,
b. The dispersion relations for the retarded invariant amplitudes of the matrix element considered are unsubtracted.

It is the purpose of this paper to examine these assumptions in perturbation theory. Because a proper definition of a renormalized current operator can be given only for a conserved current, we discuss the isotopic spin current $j_{\mu,\alpha}(\alpha = 1, 2, 3)$ in meson-nucleon interaction.

We consider the matrix element of the retarded product of two isospin currents between one-particle states of equal momentum,

$$R_{\mu\nu,\alpha\beta} = i \int d^4x\, e^{iqx} (2\pi)^3 \langle p|R\left(j_{\mu,\alpha}(x) j_{\nu,\beta}(o)\right)|p\rangle \tag{1.1}$$

In order to remove vacuum contributions we define (1.1) as the limit of the corresponding non-forward matrix element. Actually it is more convenient in perturbation theory to do the calculations for the time-ordered matrix element

$$T_{\mu\nu,\alpha\beta} = i \int d^4x\, e^{iqx} (2\pi)^3 \langle p|\, T\left(j_{\mu,\alpha}(x) j_{\nu,\beta}(o)\right)|p\rangle \tag{1.2}$$

from which the retarded quantitiy can be easily obtained.

The matrix elements (1.1) and (1.2) can be related to the matrix element of the equal time commutator, if the naive definition

$$\langle p|R\left(j_{\mu,\alpha}(x) j_{\nu\beta}(o)\right)|p\rangle = \Theta(x_o) \langle p|[j_{\mu,\alpha}(x), j_{\nu\beta}(o)]|p\rangle \tag{1.3}$$

applies for the retarded product, or

$$\langle p|T\left(j_{\mu,\alpha}(x) j_{\nu,\beta}(o)\right)|p\rangle = \Theta(x_o) \langle p|j_{\mu,\alpha}(x) j_{\nu,\beta}(o)|p\rangle + \Theta(-x_o) \langle p|j_{\nu,\beta}(o) j_{\mu,\alpha}(x)|p\rangle \tag{1.4}$$

for the time-ordered product respectively.

We then find e.g.

$$-q^{\mu}T_{\mu\nu,\alpha\beta} = \int d^4x\, e^{iqx}\partial^{\mu} <p|T(j_{\mu,\alpha}(x)\, j_{\nu,\beta}(o))|p> (2\pi)^3 = \tag{1.5}$$

$$= \int d^4x\, e^{iqx}\delta(x_o)\, <p|[j_{o,\alpha}(x),\, j_{\nu\beta}(o)]|p> (2\pi)^3$$

The same relation holds for $R_{\mu\nu,\alpha\beta}$.

It is the basic assumption of current algebra that the part antisymmetric in α,β of an equal time commutator does exist and can be evaluated by application of the equal time commutation relations of free field currents. The isotopic spin currents of a free pion field and a free nucleon field ψ are:

$$\text{pions} \qquad j_{\mu,\alpha} = i : \phi t_{\alpha}\, \partial_{\mu}\phi : \tag{1.6a}$$

$$\text{nucleons} \qquad j_{\mu,\alpha} = : \tilde{\psi}\gamma_{\mu}\frac{\tau_{\alpha}}{2}\,\psi : \tag{1.6b}$$

where

$$(t_{\alpha})_{\rho\sigma} = i\varepsilon_{\rho\alpha\sigma} \qquad \rho,\sigma = 1,2,3 \tag{1.7}$$

The currents (1.6) satisfy the equal time commutation relations

pions:

$$[j_{o,\alpha}(\vec{x},o),\, j_{\nu\beta}(o)] - <o|[j_{o,\alpha}(\vec{x},o), j_{\nu,\beta}(o)]|o> = \tag{1.8a}$$

$$= i\,\varepsilon_{\alpha\beta\gamma}\, j_{\nu,\gamma}(o)\delta^{(3)}(\vec{x}) + \delta^{k}_{\nu}\partial_{k}\left(\frac{i}{2} : \phi(o)[t_{\alpha}, t_{\beta}]_{+}\phi(o) : \delta^{(3)}(\vec{x})\right)$$

(k = 1,2,3)

nucleons:

$$[j_{o,\alpha}(\vec{x},o), j_{\nu,\beta}(o)] - <o|[j_{o,\alpha}(\vec{x},o), j_{\nu,\beta}(o)]|o> = \tag{1.8b}$$

$$= i\,\varepsilon_{\alpha\beta\gamma}\, j_{\nu,\gamma}(o)\,\delta^{(3)}(\vec{x}).$$

The combination of (1.5) and (1.8) leads to the conditions for the matrix element (1.1) or (1.2) implied by assumption a. The resulting relations correspond to the well-known Ward-Takahashi identity in quantum electrodynamics [2], [3].

In Sect. II we discuss the polarization averaged matrix elements (1.1) and (1.2) between one-nucleon states of equal polarization r in second order of the meson-nucleon coupling constant g. In order to reduce calculational work we consider an interaction of nucleons with neutral pseudoscalar mesons, where only the nucleons carry isospin. Hence we obtain from (1.5) and (1.8b)

$$-q^{\mu}T_{\mu\nu,\alpha\beta} = i\,\varepsilon_{\alpha\beta\gamma}\,\frac{1}{2}\sum_{r} \langle p,r|\; j_{\nu,\gamma}(o)\,|\; p,r\rangle \quad (2\pi)^3 \qquad (1.9)$$

It is shown in Sect. II that the relation (1.9) holds in zero and second order, and there is little doubt that it holds in every order. This is by no means trivial, because it implies that the matrix elements (1.1) and (1.2) exist and that the definition (1.4) of the time-ordered product is correct for the sum of all Feynman graphs of a definite order.

Let us now recall, how the sum rules follow from (1.9) ([4], [5]). If we decompose the matrix element in the isotopic spin space of the nucleon,

$$T_{\mu\nu,\alpha\beta} = {}^{(+)}T_{\mu\nu}\,\delta_{\alpha\beta} + {}^{(-)}T_{\mu\nu}\,\frac{1}{2}[\tau_\alpha,\tau_\beta] \qquad (1.10)$$

each of the coefficients is a tensor and may be written as

$${}^{(\pm)}T_{\mu\nu} = {}^{(\pm)}T_1 p_\mu p_\nu + {}^{(\pm)}T_2 q_\mu q_\nu + {}^{(\pm)}T_3 g_{\mu\nu} + {}^{(\pm)}T_4 (p_\mu q_\nu + q_\mu p_\nu) \qquad (1.11)$$

From (1.9) we obtain the following relations for ${}^{(-)}T_i$ (i = 1,2,3,4):

$$\begin{aligned} pq\,{}^{(-)}T_1 + q^{2\,(-)}T_4 &= -1 \\ q^{2\,(-)}T_2 + {}^{(-)}T_3 + pq\,{}^{(-)}T_4 &= o \end{aligned} \qquad (1.12)$$

The same relations hold of course for the amplitudes ${}^{(-)}R_i$ (i = 1,...,4). If we now assume unsubtracted dispersion relations for ${}^{(-)}R_i$ (i = 1,..,4) (assumption b),

$${}^{(-)}R_i(\nu,q^2) = \int_{-\infty}^{\infty} d\nu'\,\frac{A_i(\nu',q^2)}{\nu'-\nu-i\varepsilon}\;;\; q^2<o,\;\nu = pq \qquad i = 1,\ldots,4 \qquad (1.13)$$

and observe the relations between the A_i implied by current conservation, we arrive at the sum rules [5] :

$$\int_{-\infty}^{\infty} d\nu \, {}^{(-)}A_1(\nu,q^2) = 1$$
$$\int_{-\infty}^{\infty} d\nu \, {}^{(-)}A_4(\nu,q^2) = o \qquad \text{for } q^2 < o \qquad (1.14)$$

The latter is trivial, because it follows from crossing symmetry. The first is shown to be valid in zero and second order of g in Sect. II.

In Sect. III we turn to the more realistic model of nucleons interacting with isovector pions and consider the matrix elements (1.1) and (1.2) between one-pion states of equal momentum p. The decomposition in the isotopic spin space of the pion reads

$$T_{\mu\nu,\alpha\beta} = {}^{(o)}T_{\mu\nu} \, 2\delta_{\alpha\beta} + {}^{(+)}T_{\mu\nu} \, 2[t_\alpha,t_\beta]_+ + {}^{(-)}T_{\mu\nu} \, 2[t_\alpha,t_\beta]_- \qquad (1.15)$$

Now nucleons and pions carry isospin and the total isotopic spin current is the sum of (1.6a) and (1.6b). Combination of (1.5) and (1.8) yields the relation

$$\begin{aligned} -q^\mu T_{\mu\nu,\alpha\beta} &= i \, \varepsilon_{\alpha\beta\gamma} \langle p|j_{\nu,\gamma}(o)|p\rangle \, (2\pi)^3 \\ &+ \delta_\nu{}^k q_k \, \langle p|:\phi(o)\tfrac{1}{2}[t_\alpha,t_\beta]_+ \, \phi(o):|p\rangle \, (2\pi)^3 \end{aligned} \qquad (1.16)$$

It differs from (1.9) by the second term on the r h s that is symmetric in α, β. Thus only the antisymmetric part can be supposed to satisfy (1.9)

$$-q^\mu \, {}^{(-)}T_{\mu\nu} \, 2[t_\alpha,t_\beta]_- = i \, \varepsilon_{\alpha\beta\gamma} \langle p|j_{\nu,\gamma}(o)|p\rangle \, (2\pi)^3 \qquad (1.17)$$

We show in Sect. III that (1.17) holds in zero and second order of g. This is also true for the sum rules (1.14). However, the situation is entirely different for the symmetric parts. Here a deviation from (1.16) already occurs in zero order, due to the principal indefiniteness of the time-ordered product, i.e. the symmetric part of the zero order contribution does not agree with the definition (1.4). In second order, we encounter divergent contributions to the symmetric parts that cannot be renormalized. Therefore they do not exist and no sum rules emerge from the symmetric parts of the relation (1.16), in contrast to the model discussed in Sect. II. where the symmetric parts exist as well. But it is remarkable that the Ward-Takahashi-like identities (1.9) and (1.16) and the sum rules (1.14) are correct for that part of the matrix element, for which they have been anticipated, namely the antisymmetric.

II. Nucleons and Neutral Mesons

Our first example is a theory of interacting nucleons and neutral pseudoscalar mesons with the interaction Lagrangian

$$\mathcal{L}_I = g : \tilde{\psi}\, \gamma_5 \psi \phi : + \delta m : \tilde{\psi}\psi : + \frac{1}{2}\, \delta_\mu{}^2 : \phi^2 : \qquad (2.1)$$

where ψ, ϕ are the unrenormalized field operators of nucleons and mesons respectively. Because only the nucleons carry isospin, the isotopic spin current density is given by

$$j_{\mu,\alpha} = : \tilde{\psi}\gamma_\mu \frac{\tau_\alpha}{2} \psi : \qquad \alpha = 1,2,3 \qquad (2.2)$$

We first make some remarks on the vacuum expectation value

$$F_{\mu\nu,\alpha\beta}(q) = i \int d^4x\, e^{iqx} \langle o | T\big(j_{\mu,\alpha}(x) j_{\nu\beta}(o)\big) | o \rangle \qquad (2.3)$$

From the definition (1.4) of the time-ordered product, we obtain in zero order of g

$$F_{\mu\nu}{}^{(o)}(q) = \Big\{ (q_\mu q_\nu - q^2 g_{\mu\nu}) \Big[C_1 + q^2 \int_{4m^2}^{\infty} ds \frac{\rho(s)}{s(s - q^2 - i\varepsilon)} \Big] + g_{k\mu} g_{k\nu} C_2 \Big\} \times$$

$$\times \; Sp\Big(\frac{\tau_\alpha}{2}\frac{\tau_\beta}{2}\Big) \; ; \quad k = 1,2,3 \qquad (2.4)$$

where

$$\rho(s) = \frac{1}{12\pi^2} \Big(1 + \frac{2m^2}{s}\Big) \Big(1 - \frac{4m^2}{s}\Big)^{\frac{1}{2}} \qquad (2.5)$$

and C_1, C_2 are divergent constants,

$$C_1 = \int_{4m^2}^{\infty} ds \frac{\rho(s)}{s} \; ; \qquad C_2 = \int_{4m^2}^{\infty} ds\, \rho(s) \qquad (2.6)$$

Besides the trace factor the r h s of equ. (2.4) is identical with the lowest order of the vacuum expectation value of time-ordered electromagnetic currents in quantum electrodynamics (QED). Because of the Schwinger Term $g_{k\mu}g_{k\nu}C_2$ [6], $F^{(o)}_{\mu\nu,\alpha\beta}$ is not conserved,

$$q^\mu F^{(o)}_{\mu\nu,\alpha\beta} = q^\mu g_{k\mu}g_{k\nu}C_2 \tag{2.7}$$

This difficulty can be overcome in QED by a suitable redefinition of the electromagnetic current as the local limit of a gauge invariant non-local expression that depends on the electromagnetic potential and does not give rise to a Schwinger term ([7]). A similar argument does not apply in our case, because there is no corresponding vector field. By the same reason the constant C_1 which is equal to $(1-Z_3)/e^2$ in QED, cannot be removed by adding a counter term to the Lagrangian (2.1).

Therefore, the vacuum expectation value (2.3) cannot be given a meaning within our model besides by a redefinition of the time-ordered product that absorbs the divergent local term

$$C_1(\Box\, g_{\mu\nu} - \partial_\mu\partial_\nu)\,\delta(x) + C_2 g_{k\mu}g_{k\nu}\delta(x)$$

We shall encounter a similar situation in Sect. III.

We now turn to the matrix-element $T_{\mu\nu,\alpha\beta}$(1.2). The Feynman graphs of Fig. 1 give the contributions in zero order

$$T^{(o)}_{\mu\nu,\alpha\beta} = -\frac{1}{2}\sum_r \tilde{u}^{(r)}(p)\left\{\gamma_\mu\frac{\tau_\alpha}{2}\frac{1}{\gamma(p+q)-m}\gamma_\nu\frac{\tau_\beta}{2} + \gamma_\nu\frac{\tau_\beta}{2}\frac{1}{\gamma(p-q)-m}\gamma_\mu\frac{\tau_\alpha}{2}\right\}u^{(r)}(p)$$

$$= \frac{1}{2}\,Sp\left\{\gamma_\mu\big(\gamma(p+q)+m\big)\gamma_\nu(\gamma p+m)\right\}\frac{1}{m^2-(p+q)^2-i\varepsilon}\frac{\tau_\alpha}{2}\frac{\tau_\beta}{2}$$

$$+\ \frac{1}{2}\,Sp\left\{\gamma_\nu\big(\gamma(p-q)+m\big)\gamma_\mu(\gamma p+m)\right\}\frac{1}{m^2-(p-q)^2-i\varepsilon}\frac{\tau_\beta}{2}\frac{\tau_\alpha}{2} \tag{2.8}$$

(we use the normalization $\tilde{u}u = 2m$).

The identity (1.9) is satisfied by (2.8) in zero order

$$-q^\mu T^{(o)}_{\mu\nu,\alpha\beta} = i\,\varepsilon_{\alpha\beta\gamma}\frac{1}{2}\sum_r \tilde{u}^{(r)}\gamma_\nu\frac{\tau_\gamma}{2}u^{(r)} \tag{2.9}$$

On the other hand, we obtain from (2.8) the invariant amplitudes

$$^{(\pm)}T_1^{(o)} = \frac{1}{m^2 - (p+q)^2 - i\varepsilon} \pm \frac{1}{m^2 - (p-q)^2 - i\varepsilon} \qquad (2.10)$$

and similar expressions for $^{(\pm)}T_i^{(o)}$ (i = 2,3,4). For $q^2<o$ the corresponding retarded amplitudes, e.g.

$$^{(\pm)}R_1^{(o)} = \frac{1}{m^2 - (p+q)^2 - i\varepsilon(p+q)} \pm \frac{}{m^2 - (p-q)^2 + i\varepsilon(p-q)} \qquad (2.11)$$

have the analytic properties required in I and the sum rule (1.14) is valid in zero order

$$\int_{-\infty}^{\infty} d\nu \; ^{(-)}A_1^{(o)}(\nu,q^2) = 1 \qquad (2.12)$$

It follows that we must have in all higher orders

$$\int_{-\infty}^{\infty} d\nu \; ^{(-)}A_1^{(n)}(\nu,q^2) = 0 \qquad n = 2,4,\ldots. \qquad (2.13)$$

if the sum rule (1.14) holds in perturbation theory.

The derivation of the sum rule in Sect. I was based on the assumptions (1.9) and (1.13) which we now discuss in second order of g.

The contributions to the r h s of (1.9) are given by the Feynman graphs of Fig. 2 and can be written as

$$(2\pi)^3 \langle p,r|j_{\nu,\gamma}^{(2)}(o)|p,r\rangle = \tilde{u}^{(r)}(p)\{\Lambda_\nu^{(2)}(p,p) + $$

$$+ \frac{1}{2}\left(\frac{i}{(2\pi)^4}\Sigma^{(2)}(p) - \delta m\right)\frac{1}{\gamma p - m}\gamma_\nu + \frac{1}{2}\gamma_\nu\frac{1}{\gamma p - m}\left(\frac{i}{(2\pi)^4}\Sigma^{(2)}(p) - \delta m\right)\} \times$$

$$\times \frac{\tau_\gamma}{2}u^{(r)}(p) \qquad (2.14)$$

Here we have defined

$$\Sigma^{(2)}(p) = g^2\int d^4k\, \gamma_5 \frac{\gamma(p+q-k) + m}{(p+q-k)^2 - m^2}\gamma_5 \frac{1}{k^2 - \mu^2} \qquad (2.15)$$

$$\Lambda_\nu^{(2)}(p+q,p) =$$

$$\frac{ig^2}{(2\pi)^4}\int d^4k\,\gamma_5\,\frac{\gamma(p+q-k)+m}{(p+q-k)^2-m^2}\,\gamma_\nu\,\frac{\gamma(p-k)+m}{(p-k)^2-m^2}\,\gamma_5\,\frac{1}{k^2-\mu^2} \qquad (2.16)$$

$\sum^{(2)}$ and $\Lambda_\nu^{(2)}$ are linked by the Ward-Takahashi identity ([2])

$$(2\pi)^4\, i\, \Lambda_\nu^{(2)}(p+q,\,p)q^\nu = \sum^{(2)}(p+q) - \sum^{(2)}(p) \qquad (2.17)$$

as in quantum electrodynamics. The relation (2.17) is maintained, if the meson propagator is regularized and leads as usual to the equality of vertex and self-energy renormalization constants,

$$Z = Z_2 \qquad (2.18)$$

We find in second order

$$Z - 1 \cong -\frac{g^2}{16\pi^2}\left\{\frac{1}{2}\ln\frac{M^2}{m^2} - \frac{3}{4} + \int_0^1 dx\; x\ln\frac{m^2}{x^2m^2+(1-x)\mu^2} + \int_0^1 dx\; x\,\frac{x^2m^2}{x^2m^2+(1-x)\mu^2}\right\} \qquad (2.19)$$

for large values of the regulator mass M. While Z_2 describes the well-known renormalization of the nucleon field by the interaction with the meson field, Z is a additional renormalization constant that renormalizes the isotopic spin current (2.2). It is defined by

$$\Lambda_{\nu,\gamma}(p,p) = -(Z-1)\gamma_\nu\frac{\tau_\gamma}{2} \qquad (2.20)$$

The renormalized current is

$$j'_{\mu,\alpha} = \frac{Z}{Z_2} : \tilde{\psi}'\gamma_\mu\frac{\tau_\alpha}{2}\psi' : = j_{\mu,\alpha} \qquad (2.21)$$

where

$$\psi' = \frac{1}{\sqrt{Z_2}}\,\psi \qquad (2.22)$$

is the renormalized nucleon field. Due to (2.20) the unrenormalized and the renormalized currents are the same. Hence it follows from (2.14) that

$$(2\pi)^3 \langle p,r|j^{(2)}_{\nu,\gamma}(o)|p,r\rangle = o \tag{2.23}$$

The tensor $T^{(2)}_{\mu\nu,\alpha\beta}$ receives contributions from the Feynman graphs of Fig. 3 and the corresponding crossed graphs. (We have not pictured the mass renormalization counter terms):

$$T^{(2)}_{\mu\nu,\alpha\beta} = -\frac{1}{2}\sum_r \tilde{u}^{(r)}(p) \tag{2.24}$$

$$\left\{\gamma_\mu \frac{\tau_\alpha}{2} \frac{1}{\gamma(p+q) - m} \left(\frac{i}{(2\pi)^4} \Sigma^{(2)}(p+q) - \delta m\right) \frac{1}{\gamma(p+q) - m} \gamma_\nu \frac{\tau_\beta}{2} + \right. \quad (3a)$$

$$+ \Lambda^{(2)}_\mu(p,p+q) \frac{\tau_\alpha}{2} \frac{1}{\gamma(p+q) - m} \gamma_\nu \frac{\tau_\beta}{2} + \quad (3b)$$

$$+ (Z_2-1)\gamma_\mu \frac{\tau_\alpha}{2} \frac{1}{\gamma(p+q) - m} \gamma_\nu \frac{\tau_\beta}{2} + \quad (3c) + (3d)$$

$$+ \gamma_\mu \frac{\tau_\alpha}{2} \frac{1}{\gamma(p+q) - m} \Lambda_\nu(p+q,p) \frac{\tau_\beta}{2} + \quad (3e)$$

$$\left. + \frac{\tau_\alpha}{2} M^{(2)}_{\mu\nu}(p,q) \frac{\tau_\beta}{2}\right\} u^{(r)}(p) + \{\alpha,\mu,p+q \leftrightarrow \beta,\nu,p-q\} \quad (3f)$$

where

$$M^{(2)}_{\mu\nu}(p,q) =$$
$$\frac{ig^2}{(2\pi)^4}\int d^4k \gamma_5 \frac{\gamma(p-k) + m}{(p-k)^2 - m^2} \gamma_\mu \frac{\gamma(p+q-k) + m}{(p+q-k)^2 - m^2} \gamma_\nu \frac{\gamma(p-k) + m}{(p-k)^2 - m^2} \gamma_5 \frac{1}{k^2-\mu^2} \tag{2.25}$$

It can easily be checked that the contributions corresponding to the diagrams (3a) - (3b) and (3c)-(3d) separately satisfy the relation

$$q^\mu T^{(2)}_{\mu\nu,\alpha\beta} = o \tag{2.26}$$

This follows from the Ward-Takahashi identity (2.17) and its generalization to $M_{\mu\nu}$.

$$q^\mu M^{(2)}_{\mu\nu}(p,q) = \Lambda^{(2)}_\nu(p,p) - \Lambda^{(2)}_\nu(p+q,p) \tag{2.27}$$

Thus the relation (1.9), which combines the conditions on $T_{\mu\nu,\alpha\beta}$ implied by current conservation and equal time commutation relations, is valid in second order of g. A similar proof applies in all higher orders.

We now turn to the analytic properties required in Sect. I. It is clear that all the contributions to the invariant amplitudes ${}^{(\pm)}T_i$ $(i = 1,\ldots,4)$ of (2.24) can be cast for fixed $q^2 < o$ into the dispersive form

$$ {}^{(\pm)}T_i\left(p^2,(p+q)^2\right) = \int_{m^2}^{\infty} ds \, \frac{\rho_i(s,q^2)}{s-(p+q)^2-i\varepsilon} \qquad i = 1,\ldots,4 \tag{2.28} $$

if we use the well-known machinery for the analysis of analytic properties of Feynman graphs. Similarly we obtain for the crossed graphs

$$ {}^{(\pm)}T_i\left(q^2,(p-q)^2\right) = \int_{m^2}^{\infty} ds \, \frac{\rho_i(s,q^2)}{s-(p-q)^2-i\varepsilon} \qquad i = 1,\ldots,4 \tag{2.29} $$

Thus the corresponding retarded amplitudes ${}^{(\pm)}R_i(q^2,\nu)$ satisfy dispersion relations in ν of the usual variety. The remaining question is: are all these dispersion relations unsubtracted? If so, (2.13) must hold, or in other words ${}^{(-)}R_1(q^2,\nu)$ mus decrease faster than $|\nu|^{-1}$ as $\nu \to \infty$ for fixed $q^2 < o$.

We shall now prove this assertion in second order. One may expect that the relation (2.26) provides for a cancellation of the leading asymptotic terms for $|(p \pm q)^2| \to \infty$. We therefore restrict ourselves to the contributions of (2.26) and check whether a cancellation takes place in the two subgroups of terms that separately satisfy (2.26). We only list the results of the laborious but straightforward calculations. The contributions to ${}^{(\pm)}T_1^{(2)}$ from the graphs of Fig. 3a and 3b have the asymptotic form

$$ {}^{(\pm)}T_1(2) \cong -\frac{g^2}{4\pi^2}\left\{\frac{1}{2}\ln\frac{M^2}{m^2}+\frac{3}{4}+\frac{1}{2}\ln\left|\frac{m^2}{(p+q)^2}\right|\right\}\frac{1}{|p+q|^2} + o\left(\frac{1}{|(p+q)^2|}\right) \quad (3a) $$

(2.30)

$$ {}^{(\pm)}T_1(2) \cong \frac{g^2}{4\pi^2}\left\{\frac{1}{2}\ln\frac{M^2}{m^2}+\frac{3}{4}+\frac{1}{2}\ln\left|\frac{m^2}{(p+q)}\right|\right\}\frac{1}{|(p+q)^2|} + o\left(\frac{1}{|(p+q)^2|}\right) \quad (3b) $$

for $|(p+q)^2| \to \infty$

The sum of the two terms decreases in fact faster that $|(p+q)^2|^{-1}$ as $|(p+q)^2| \to \infty$. In our simple model this holds equally for the symmetric and antisymmetric amplitude ${}^{(\pm)}T_1$. In more complex cases only the antisymmetric part ${}^{(-)}T_1^{(2)}$ has this property as will be seen in Sect. III. Finally we note the asymptotic expressions from the graphs 3c - 3f:

$$^{(\pm)}T_1^{(2)} \cong 4\left(Z_2-1\right) \frac{1}{|(p+q)|^2} \qquad (3c) + (3d)$$

$$^{(\pm)}T_1^{(2)} \cong \frac{g^2}{4\pi^2} \left\{\frac{1}{2} \ln \frac{M^2}{m^2} + \frac{3}{4} + \frac{1}{2} \ln \frac{m^2}{|(p+q)^2|}\right\} \frac{1}{|(p+q)^2|} + o \qquad (3e)$$

$$^{(\pm)}T_1^{(2)} \cong \frac{g^2}{4\pi^2} \left\{- \frac{1}{2} \ln \left|\frac{m^2}{(p+q)^2}\right| - \frac{3}{2} + \int_0^1 dx\, x(1-x) \ln \frac{m^2}{(1-x)m^2+x\mu^2} \right. \qquad (2.31)$$

$$\left. + \int_0^1 dx\, x \frac{x^2m^2}{x^2m^2+(1-x)\mu^2} \right\} \frac{1}{|(p+q)|^2} + o \quad \text{for } |(p+q)^2| \to \infty. \qquad (3f)$$

Again the sum decreases faster than $|(p+q)^2|^{-1}$ as can be seen with (2.19). The situation is completely the same for the crossed diagrams. Thus the retarded amplitude $^{(-)}R_1$ has the required asymptotic behaviour and (2.13) holds in second order of g.

The preceding discussion makes clear that there is an intimate link between the asymptotic behaviour for large $|\nu|$ and the Ward-Takahashi-like identity (1.9). Because the latter holds in every order there is little doubt that this is also true for (2.13). We shall employ a different method in Sect. III which makes these things more transparent. The sum rule (1.14) is, therefore, in fact equivalent with current conservation and equal time commutation relations.

III. Nucleons and Isovector Mesons.

Our second example is a theory of interacting pseudoscalar mesons and nucleons with the interaction Lagrangian

$$\mathcal{L}_I = g \sum_{\alpha=1}^{3} : \tilde{\psi}\, \gamma_5 \tau_\alpha \psi \phi_\alpha : + \delta m : \tilde{\psi}\psi : + \frac{1}{2} \delta\mu^2 \sum_{\alpha=1}^{3} :\phi_\alpha \phi_\alpha: \qquad (3.1)$$

where ϕ_α and ψ are the unrenormalized field operators of the mesons and nucleons respectively. The isotopic spin current consists of a meson part and a nucleon part (see (1.6a) and (1.6b))

$$\begin{aligned} j_{\mu,\alpha} &= \varepsilon_{\alpha\beta\gamma} : \phi_\beta \partial_\mu \phi_\gamma : + : \tilde{\psi}\gamma_\mu \frac{\tau_\alpha}{2} \psi : \\ &= i : \phi\, t_\alpha \partial_\mu \phi : + : \tilde{\psi}\gamma_\mu \frac{\tau_\alpha}{2} \psi : \quad , \alpha = 1,2,3 \end{aligned} \qquad (3.2)$$

For the model defined by the interaction Lagrangian (3.1) we investigate only one-meson matrix elements.

The outline of the proof for the second model will be similar to the presentation in Section II. But instead of investigating the asymptotic properties of separate graphs we first shall sum up several diagrams and then study the behaviour of these sums for $\nu \to \infty$. For the summation we shall use extensively Ward-Takahashi-like indentities.

The matrix element $T_{\mu\nu,\alpha\beta}$ is defined as in Section I (1.2). The zero order contributions exhibited by the Feynman graphs of Fig. 4 are as follows:

$$T^{(o)}_{\mu\nu,\alpha\beta} = t_\alpha t_\beta \; (2p+q)_\mu \frac{1}{\mu^2 - (p+q)^2 - i\varepsilon} (2p+q)_\nu + t_\beta t_\alpha \; (2p-q)_\nu \frac{1}{\mu^2 - (p-q)^2 - i\varepsilon} (2p-q)_\mu \qquad (3.3)$$

where μ is the renormalized meson mass. If the naive definition of the time ordered product equ (1.4) is used we encounter in $T^{(o)}_{\mu\nu,\alpha\beta}$ an additional noncovariant term equal to

$$[t_\alpha, t_\beta]_+ \; g_{\mu o} \; g_{\nu o} \qquad (3.4)$$

The divergence of the zero order contribution equ (3.3) is:

$$-q^\mu T^{(o)}_{\mu\nu,\alpha\beta} = [t_\alpha, t_\beta]_+ \, q_\nu + i\varepsilon_{\alpha\beta\gamma} \, t_\gamma \, 2p_\nu \qquad (3.5)$$

The identity (1.16) is satisfied for the isotopic spin antisymmetric part. But the right hand side of equ (3.5) yields a different isotopic symmetric part than the relation (1.16). The difference is accounted for by the additional term (3.4). Now the invariant amplitudes ${}^{(\pm)}T_i^{(o)}$ $(i = 1,\ldots,4)$ can be immediately read off from equ (3.3), for example

$${}^{(\pm)}T_1^{(o)} = \frac{1}{\mu^2 - (p+q)^2 - i\varepsilon} \pm \frac{1}{\mu^2 - (p-q)^2 - i\varepsilon} \qquad (3.6)$$

and similar expressions for ${}^{(\pm)}T_i^{(o)}$ $(i = 2,3,4)$. All ${}^{(o)}T_i^{(o)}$ are zero in this approximation.

As in Section II the corresponding retarded amplitudes have the analytic properties required in I and the sum rule (1.14) is valid in zero order

$$\int_{-\infty}^{\infty} d\nu^{(-)} \; A_1^{(o)}(\nu,q^2) \; = \; 1 \tag{3.7}$$

Therefore we must have in all higher orders

$$\int_{-\infty}^{\infty} d\nu^{(-)} \; A_1^{(n)}(\nu,q^2) \; = \; o \qquad n = 2,4,\ldots \tag{3.8}$$

The next step is to show that the isotopic spin antisymmetric part of $T_{\mu\nu,\alpha\beta}$ fulfills the relation (1.17) and that the retarded amplitudes ${}^{(-)}R_i(q^2,\nu)$ satisfy unsubtracted dispersion relations in $pq = \nu$ for fixed $q^2 < o$. As in Section II we prove these properties in second order of g.

The right hand side of (1.17) is given by the Feynman diagrams of Fig. 5 and is equal to the following expression:

$$(2\pi)^3 <p|j_{\nu,\gamma}^{(2)}(o)|p> \; = \; t_\gamma \left(\Lambda_\nu^{(2)}(p,p) + 2\, p_\nu \frac{i}{(2\pi)^4} \left. \frac{\partial \sum^{(2)}}{\partial p^2} \right|_{p^2=\mu^2} \right) \tag{3.9}$$

where $\sum^{(2)}$ is the meson self-energy in second order

$$\sum^{(2)}(p) \; = \; - g^2 \int d^4k \; Sp\left(\gamma_5 \frac{1}{\gamma(p+k) - m} \; \gamma_5 \frac{1}{\gamma k - m} \right) \tag{3.10}$$

and $\Lambda_\nu^{(2)}$ the vertex function

$$\Lambda_\nu^{(2)}(p+q,p) \; = \; - \frac{ig^2}{(2\pi)^4} \int d^4k \; Sp\left(\gamma_5 \frac{1}{\gamma(p+q+k) - m} \gamma_\mu \frac{1}{\gamma(p+k) - m} \gamma_5 \frac{1}{\gamma k - m} \right) \tag{3.11}$$

As in Section II the functions $\sum^{(2)}$ and $\Lambda^{(2)}$ obey the Ward-Takahashi itentity:

$$(2\pi)^4 i \; \Lambda_\nu^{(2)} \; (p+q,p) q^\nu \; = \; \sum^{(2)}(p+q) - \sum^{(2)}(p) \tag{3.12}$$

As usual we get the Ward identity from equ (3.12) by differentiation:

$$(2\pi)^4\, i\, \Lambda_\nu^{(2)}(p,p) = \frac{\partial}{\partial p^\nu} \sum^{(2)}(p) = 2p_\nu \frac{\partial}{\partial p^2} \sum^{(2)} \tag{3.13}$$

and therefore equ (3.9) says that

$$(2\pi)^3 \langle p | j_{\nu,\gamma}^{(2)} | p \rangle = 0 \tag{3.14}$$

just as in the first model. Equ (3.13) leads to the equality of vertex and meson self-energy renormalization constants

$$Z = Z_3 \tag{3.15}$$

if the functions $\sum^{(2)}$ and $\Lambda_\nu^{(2)}$ are regularized in such a way that equ (3.12) is maintained. Then as in Section II due to the relation (3.15) the unrenormalized and the renormalized currents are the same.

The Feynman graphs in Fig. 6 which give the contributions to $T_{\mu\nu,\alpha\beta}^{(2)}$ in second order will be considered in two groups. The first group consists of the diagrams a) and b) and the second group is the rest of the graphs in Fig. 6. The diagram a) and the corresponding crossed one are:

$$T_{\mu\nu,\alpha\beta}^{(2a)} =$$

$$- t_\alpha t_\beta (2p+q)_\mu \frac{1}{(p+q)^2 - \mu^2} \frac{i}{(2\pi)^4} \left(\sum^{(2)}(p+q) - \sum^{(2)}(p)\right) \frac{1}{(p+q)^2 - \mu^2} (2p+q)_\nu$$

$$- t_\beta t_\alpha (2p-q)_\nu \frac{1}{(p-q)^2 - \mu^2} \frac{i}{(2\pi)^4} \left(\sum^{(2)}(p-q) - \sum^{(2)}(p)\right) \frac{1}{(p-q)^2 - \mu^2} (2p-q)_\mu \tag{3.16a}$$

whereas diagram b) and the corresponding crossed one are equal to:

$$T_{\mu\nu,\alpha\beta}^{(2b)} = - t_\alpha t_\beta\, \Lambda_\mu^{(2)}(p,p+q) \frac{1}{(p+q)^2 - \mu^2} (2p+q)_\nu$$

$$- t_\beta t_\alpha (2p-q)_\nu \frac{1}{(p-q)^2 - \mu^2} \Lambda_\mu^{(2)}(p-q,p) \tag{3.16b}$$

The Ward-Takahashi identity eq (3.12) allows us to express $\sum^{(2)}(p\pm q) - \sum^{(2)}(p)$ by the divergence of the corresponding vertex function $\Lambda^{(2)}$. Then in both contributions, a) and b), the same vertex functions appear.
The sum of a) and b) is:

$$T^{(2a+ab)}_{\mu\nu,\alpha\beta} =$$

$$t_\alpha t_\beta \{\Lambda^{(2)}_\mu(p,p+q) - (2p+q)_\mu \frac{q^\mu \Lambda^{(2)}_\mu(p,p+q)}{(p+q)^2 - \mu^2}\} \frac{1}{\mu^2 - (p+q)^2} (2p+q)_\nu$$

$$+ \quad t_\beta t_\alpha (2p-q)_\nu \frac{1}{\mu^2 - (p-q)^2} \{\Lambda^{(2)}_\mu(p-q,p) + (2p-q)_\mu \frac{q^\mu \Lambda^{(2)}_\mu(p-q,p)}{(p-q)^2 - \mu^2}\} \qquad (3.17)$$

Equ.(3.17) can be simplified by introducing the following decomposition of $\Lambda^{(2)}_\mu(p',p)$ into invariant functions $\Lambda^{(2)}_\pm(p',p)$:

$$\Lambda^{(2)}_\mu(p',p) = (p'+p)_\mu \Lambda^{(2)}_+(p',p) + (p'-p)_\mu \Lambda^{(2)}_-(p',p) \qquad (3.18)$$

It is easily shown that the experission on the right hand side of equ (3.17) is identical to:

$$T^{(2a+2b)}_{\mu\nu,\alpha\beta} =$$

$$t_\alpha t_\beta \Lambda^{(2)}_-(p,p+q) \left[(2p+q)_\mu \frac{q^2}{(p+q)^2 - \mu^2} - q_\mu\right] \frac{1}{\mu^2 - (p+q)^2} (2p+q)_\nu$$

$$- \quad t_\beta t_\alpha (2p-q)_\nu \frac{1}{\mu^2 - (p-q)^2} \left[(2p-q)_\mu \frac{1}{(p-q)^2 - \mu^2} + q_\mu\right] \Lambda_-^{(2)}(p-q,p) \qquad (3.19)$$

Since $p^2 = \mu^2$ the sum of the contributions a) and b) has zero divergence

$$q^\mu T^{(2a+2b)}_{\mu\nu,\alpha\beta} = o \qquad (3.20)$$

$\Lambda_-^{(2)}$, a function of q^2 and pq, has been calculated in terms of an integral representation whose asymptotic properties for $pq = \nu \to \infty$ were studied. It is found that $\Lambda^{(-)}$ behaves for $\nu \to \infty$ like ln ν. Then it follows from equ (3.19) that the invariant function $T_1^{(2)}(\nu,q^2)$ in $T^{(2a+2b)}_{\mu\nu,\alpha\beta}$ decreases faster than $|\nu|^{-1}$ for $|\nu| \to \infty$. We remark that the diagrams a) and b) seperately are logarithmic divergent. It is well known that the divergent contributions cancel each other in the sum of vertex and self-energy corrected graphs due to the Ward identity. Our derivation shows that the same mechanism leads to a better asymptotic behaviour of the invariant functions than would be expected from the convergent contributions of the graphs a) and b) seperately.

In $T_{\mu\nu,\alpha\beta}^{(2a+ab)}$ the isotopic spin symmetric and antisymmetric part of $T_1^{(2)}$ have the nice asymptotic behaviour $\ln\nu/\nu^2$ for $\nu \to \infty$. Unfortunately in the sum of the contributions from graphs c), d), e) and f) only the antisymmetric part ${}^{(-)}T_1^{(2)}$ has this asymptotic property whereas the symmetric parts ${}^{(o)}T_1^{(2)}$ and ${}^{(+)}T_1^{(2)}$ still diverge.

The contributions of these diagrams and their corresponding crossed ones are:

$$T_{\mu\nu,\alpha\beta}^{(2c+2d)} = \frac{1}{2}[t_\alpha,t_\beta]_+ \{(2p+q)_\mu \frac{1}{\mu^2-(p+q)^2}(2p+q)_\nu \frac{i}{(2\pi)^4}\frac{\partial}{\partial p^2}\sum^{(2)} +$$

$$+ (2p-q)_\nu \frac{1}{\mu^2-(p-q)^2}(2p-q)_\mu \frac{i}{(2\pi)^4}\frac{\partial}{\partial p^2}\sum^{(2)}\}$$

$$+\frac{1}{2}[t_\alpha,t_\beta]\{(2p+q)_\mu \frac{1}{\mu^2-(p+q)^2}(2p+q)_\nu \frac{i}{(2\pi)^4}\frac{\partial}{\partial p^2}\sum^{(2)} \times$$

$$\times -(2p-q)_\nu \frac{1}{\mu^2-(p-q)^2}(2p-q)_\mu \frac{i}{(2\pi)^4}\frac{\partial}{\partial p^2}\sum^{(2)}\}$$

(3.21a)

$$T_{\mu\nu,\alpha\beta}^{(2e)} =$$

$$\frac{1}{2}[t_\alpha,t_\beta]_+ \{(2p+q)_\mu \frac{1}{\mu^2-(p+q)^2}\Lambda_\nu^{(2)}(p+q,p) + \Lambda_\nu^{(2)}(p,p-q)\frac{1}{\mu^2-(p-q)^2}(2p-q)_\mu\}$$

$$+\frac{1}{2}[t_\alpha,t_\beta]\{(2p+q)_\mu \frac{1}{\mu^2-(p+q)^2}\Lambda_\nu^{(2)}(p+q,p) - \Lambda_\nu^{(2)}(p,p-q)\frac{1}{\mu^2-(p-q)^2}(2p-q)_\mu\}$$

(3.21b)

$$T_{\mu\nu,\alpha\beta}^{(2f)} =$$

$$-\frac{1}{2}\delta_{\alpha\beta}\{M_{\mu\nu}^{(2)}(p,q) + M_{\nu\mu}^{(2)}(p,-q)\} - \frac{1}{2}[t_\alpha,t_\beta]\{M_{\mu\nu}^{(2)}(p,q) - M_{\nu\mu}^{(2)}(p,-q)\} \quad (3.21c)$$

where

$$M_{\mu\nu}^{(2)}(p,q) =$$

$$-\frac{ig^2}{(2\pi)^4}\int d^4k\ Sp(\gamma_5\frac{1}{\gamma(p+k)-m}\gamma_\mu\frac{1}{\gamma(p+q+k)-m}\gamma_\nu\frac{1}{\gamma(p+k)-m}\gamma_5\frac{1}{\gamma k-m}) \quad (3.22)$$

As in Section II $M_{\mu\nu}^{(2)}(p,q)$ obeys also a Ward-Takahashi identity:

$$q^{\mu}M_{\mu\nu}^{(2)}(p,q) = \Lambda_{\nu}^{(2)}(p,p) - \Lambda_{\nu}^{(2)}(p+q,p)$$

and

$$M_{\nu\mu}(p,q)q^{\mu} = \Lambda_{\nu}^{(2)}(p,p) - \Lambda_{\nu}^{(2)}(p,p+q) \tag{3.23}$$

Now we sum up all contributions of c), d), e) and f) proportional to $[t_{\alpha},t_{\beta}]$. For this purpose we express $2p_{\nu} \frac{\partial}{\partial p^2} \sum^{(2)}$ by $\Lambda_{\nu}^{(2)}(p,p)$ according to equ (3.13). Then in the sum of c), d) and e) the vertex functions $\Lambda^{(2)}$ can be replaced by the divergence of $M_{\mu\nu}^{(2)}$ by means of equ (3.23). The last step is to add the contribution of (3.21c) and to introduce the invariant functions $M_i^{(2)}(p,q)$ and $\tilde{M}_i^{(2)}(p,q) = M_i^{(2)}(p,-q)$ respectively which represent the decompositon of $M_{\mu\nu}^{(2)}(p,q)$:

$$M_{\mu\nu}^{(2)}(p,q) =$$

$$M_1^{(2)}\, p_{\mu}p_{\nu} + M_2^{(2)}\, q_{\mu}q_{\nu} + M_3^{(2)}\, g_{\mu\nu} + M_4^{(2)}(p_{\mu}q_{\nu} + q_{\mu}p_{\nu}) \tag{3.24}$$

Then the result for ${}^{(-)}T_{\mu\nu}^{(2c+2d+2e+2f)}$ is:

$${}^{(-)}T^{(2c+2d+2e+2f)} =$$

$$\begin{aligned}
&\tfrac{1}{4}\Big\{\Big(q_{\mu} + (2p+q)_{\mu}\frac{q^2}{\mu^2-(p+q)^2}\Big)\Big((\tfrac{1}{2}M_1 - M_4)p_{\nu} + \tfrac{1}{2}(M_4 - M_2)q_{\nu}\Big)\\
&- M_3 g_{\mu\nu} + (2p+q)_{\mu}q_{\nu}\frac{1}{\mu^2-(p+q)^2}\Big(\frac{\partial}{\partial p^2}\textstyle\sum^{(2)} - M_3\Big)\\
&- \Big(q_{\mu} - (2p-q)_{\mu}\frac{q^2}{\mu^2-(p-q)^2}\Big)\Big((\tfrac{1}{2}\tilde{M}_1 - \tilde{M}_4)p_{\nu} - \tfrac{1}{2}(\tilde{M}_4 - \tilde{M}_2)q_{\nu}\Big)\\
&+ \tilde{M}_3 g_{\mu\nu} + (2p-q)_{\mu}q_{\nu}\frac{1}{\mu^2-(p-q)^2}\Big(\frac{\partial}{\partial p^2}\textstyle\sum^{(2)} - \tilde{M}_3\Big)\Big\}
\end{aligned} \tag{3.25}$$

One verifies easily that

$$q^{\mu}\,{}^{(-)}T_{\mu}^{(2c+2d+2e+2f)} = 0 \tag{3.26}$$

so that together with equ (3.20)

$$q^{\mu}\,{}^{(-)}T_{\mu\nu}^{(2)} = 0 \tag{3.27}$$

On the other hand the divergences of the tensor-amplitudes ${}^{(+)}T^{(2)}_{\mu\nu}$ and ${}^{(o)}T^{(2)}_{\mu\nu}$ do not vanish. We remark that the right hand side of equ (3.25) is a finite quantity. The divergent term proportional to $\frac{\partial}{\partial p^2}\Sigma^{(2)}$ is cancelled by an identical term contained in M_3.
The asymptotic behaviour or ${}^{(-)}T^{(2)}$ for $\nu \to \infty$ now depends on the asymptotic property of the amplitude $\frac{1}{2} M_1 - M_4$. This function behaves for $\nu \to \infty$ like:

$$\left(\frac{1}{2} M_1 - M_4\right) \;\tilde{=}\; c_1 + c_2 \frac{\ln\nu}{\nu} \tag{3.28}$$

where c_1 and c_2 depend not on ν. The amplitude $(\frac{1}{2} M_1 - M_4)$ has the same asymptotic behaviour with a different constant c_2 but the same constant c_1. The terms in the right hand side of equ (3.25) proportional to c_1 cancel each other. Therefore ${}^{(-)}T_1^{(2)}$ decreases for $\nu \to \infty$ at least like $\ln\nu/\nu^2$.

Concerning the analytic properties of the invariant amplitudes ${}^{(-)}R_i$ $(i = 1,\ldots,4)$ we refer to the discussion in Section II which is valid also for the invariant amplitudes of the second model. The unsubtractedness of the dispersion relations for ${}^{(-)}R_i^{(2)}(q^2,\nu)$ $(i = 1,\ldots,4)$ then follows from the asymptotic property of ${}^{(-)}T_1^{(2)}$ as explained in Section II.

References.

[1] S. Fubini, G. Furlan and C. Rossetti, Nuovo Cim. 40, A1171 (1965)

[2] Y. Takahashi, Nuovo Cim. 6, 370 (1957)

[3] E. Kazes, Nuovo Cim. 13, 1226 (1959)

[4] S. Fubini, Nuovo Cim. 43, A475 (1966)

[5] G. Kramer and K. Meetz, DESY 66/8 (1966)

[6] J. Schwinger, Phys. Rev. Letters 3, 296 (1959)

[7] K. Johnson, Nucl. Phys. 25, 431 (1961).

Figure Captions.

Fig. 1.: Feynman Graphs for the Nucleon Matrix Element in zero order

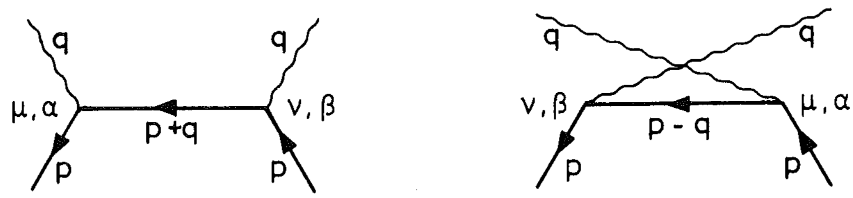
Fig. 2.: Feynman Graphs for the Nucleon Vertex in second order

Fig. 3.: Feynman Graphs for the Nucleon Matrix Element in second order

Fig. 4.: Feynman Graphs for the Meson Matrix Element in zero order

Fig. 5.: Feynman Graphs for the Meson Vertex in second order

Fig. 6.: Feynman Graphs for the Meson Matrix Element in second order



a) Fig.1 b)

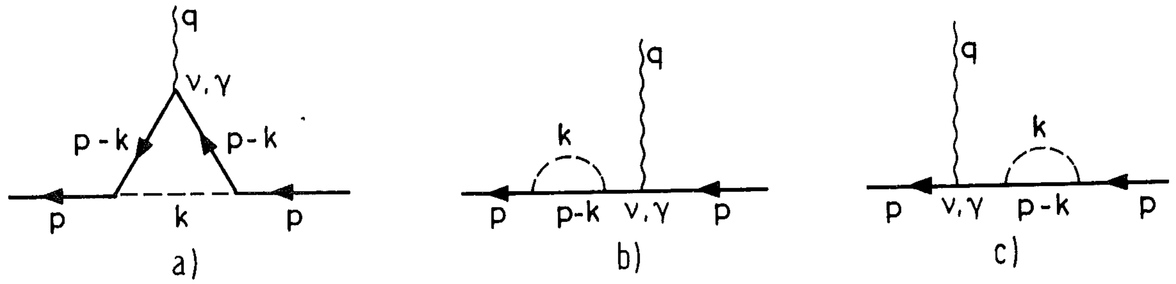


Fig.2

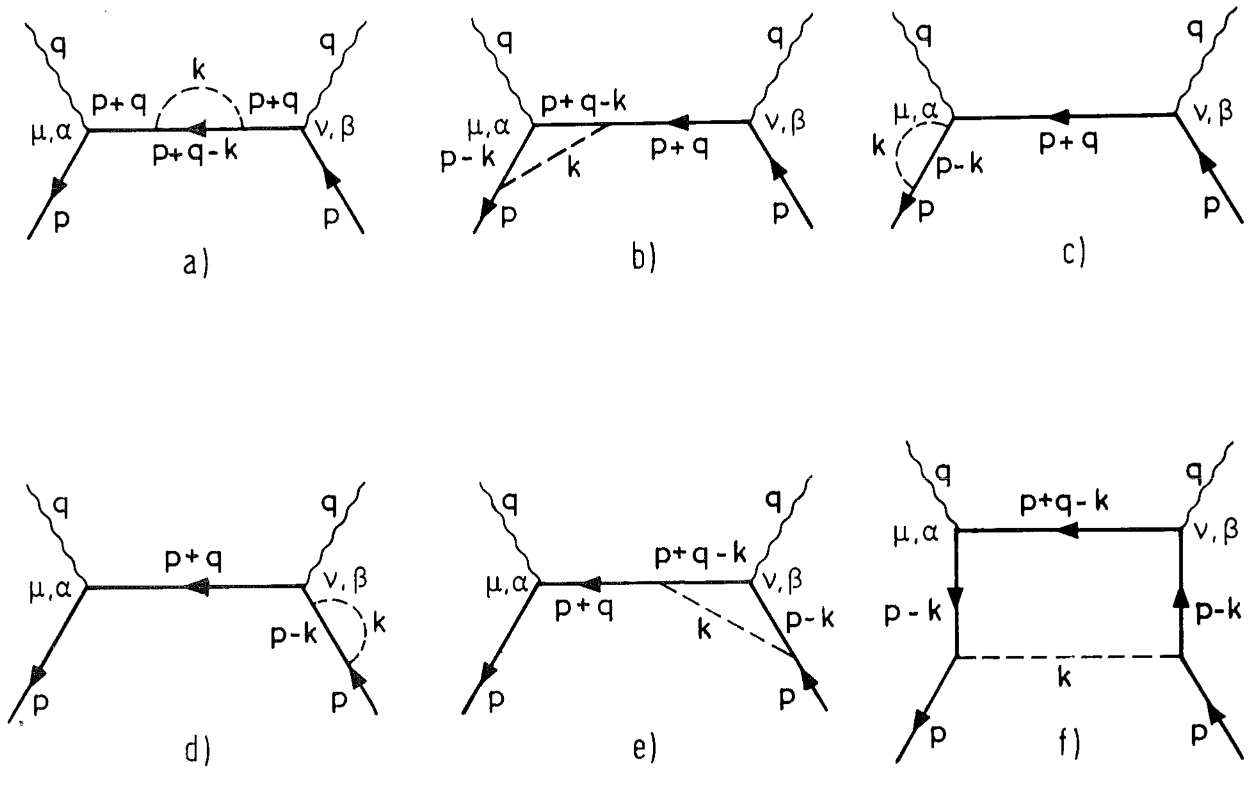


Fig.3

Fig. 4

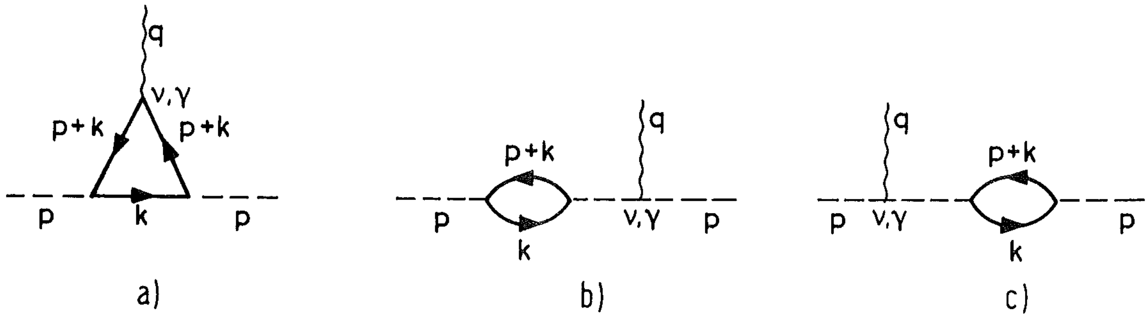


Fig. 5

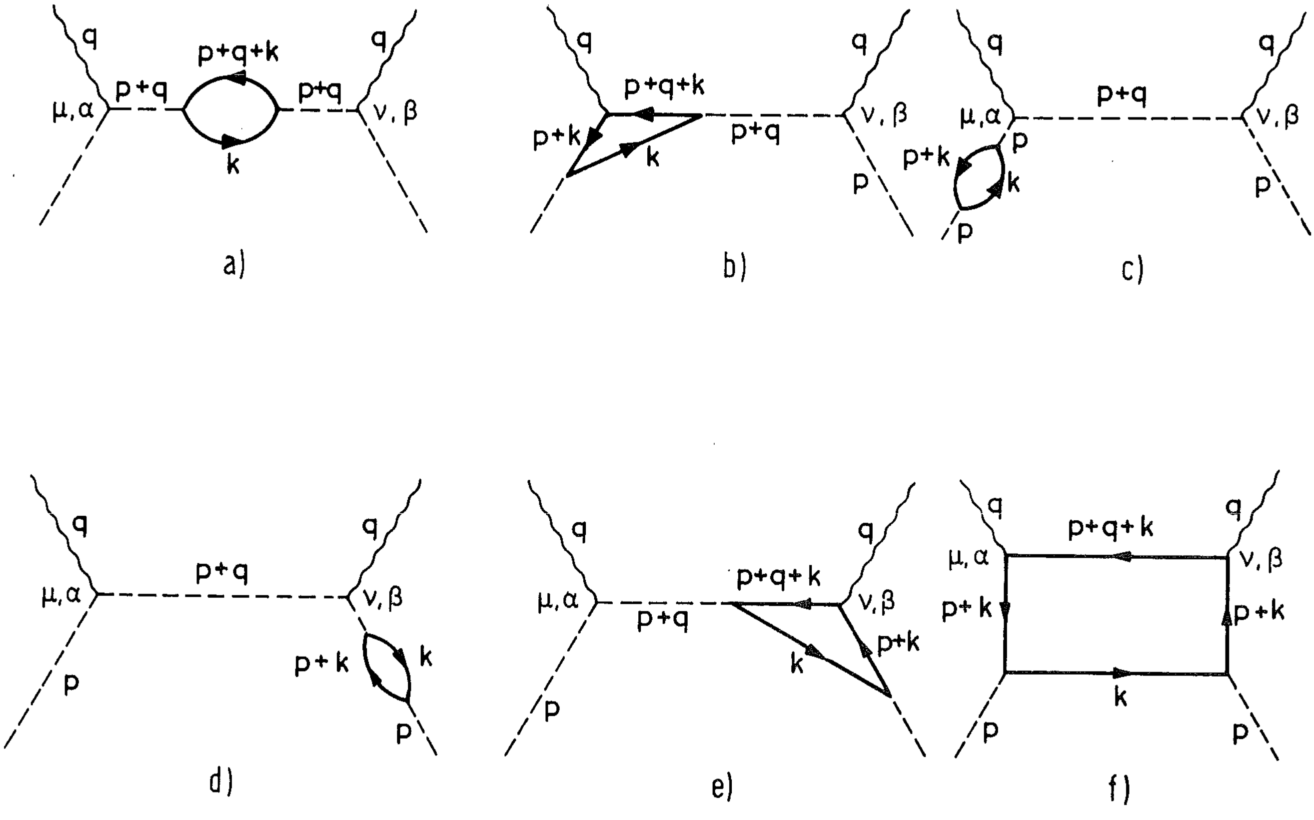


Fig. 6