DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 67/38
November 1967

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Abstract:

Superconvergence relations for $\bar{N} + N \to \pi + \rho$ and $\bar{N} + N \to \pi + \omega$ are considered which follow from high energy behaviour. Saturating these relations with a number of low-lying states we obtain $\frac{g_{\phi N}}{g_{\omega N}} \simeq 0.06$, and $m_\omega = m_\rho$ and $g_{\rho\omega\pi} = 21\ (\mathrm{GeV})^{-1}$ in good agreement with experiment. Assuming ρ-meson universality and vector meson dominance we obtain $g_{\rho N} = 2.8$, $g_{\omega N} = 6.4$, $f_{\rho N} = 10.3$ in agreement, and $f_{\omega N} = 4.0$ in disagreement with the results of a least-squares fit of low-energy N-N data.

I. Introduction.

Strong interaction sum rules following from dispersion relations have recently been proposed by two groups of authors[(1)(2)]. In particular, the interest in the "superconvergent" sum rules was stimulated by the work of de Alfaro, Fubini, Furlan and Rossetti[(1)]. An amplitude which is superconvergent obeys the sum rule

$$\int_{-\infty}^{\infty} \mathrm{Im}\, T(s,t)\, ds = 0.$$

Applying this to elastic π-ρ scattering and saturating with π, ω and ϕ these authors[(1)] got $g^2_{\rho\omega\pi} = \frac{4g^2_{\rho\pi\pi}}{m^2_\rho}$ and $g^2_{\rho\phi\pi} \simeq 0$. Subsequently Frampton and Taylor[(3)] could show that the inclusion of higher states is necessary to avoid contradictions. An interesting example was given by Gilman and Harari[(4)] who considered current algebra and superconvergence sum rules together.

In the present work we apply the superconvergence sum rules to the more complicated problem $\bar{N} + N \to \pi + \rho$ and $\bar{N} + N \to \pi + \omega$. The interest in this type of processes lies in the fact that they involve pure mesonic and mesonic-baryonic coupling. Therefore, they give relations between these two types of coupling constants. In the s-channel $\rho + N \to \pi + N$, this problem was treated by Graham and Huq.[(5)]

In Section II we derive our sum rules. We start with the s-channel helicity amplitudes free of kinematical singularities and examine the high-energy behaviour. In Section III we saturate the sum rules. The intermediate states are selected according to their quantum numbers and their mass values. Conclusions of our results are briefly outlined in Section IV.

II. Derivation of the Sum Rules.

We first consider the process

(A).... $\bar{N} + N \to \pi + \rho$

It can be described in terms of six invariant amplitudes which are free of kinematical singularities[(6)]

$$(1)\quad T(t,s) = \sum_{i=1}^{6} a_i(t,s)s_i$$

$$(2)\quad \begin{aligned} s_1 &= \gamma^5(\varepsilon\cdot p_1) & s_4 &= \gamma^5(\gamma k)\ (\varepsilon\cdot p_1)\\ s_2 &= \gamma^5(\varepsilon\cdot p_2) & s_5 &= \gamma^5(\gamma k)\ (\varepsilon\cdot p_2)\\ s_3 &= \gamma^5(\varepsilon\cdot \gamma) & s_6 &= \gamma^5(\gamma k)\ (\varepsilon\cdot \gamma)\end{aligned}$$

p_1 and p_2 are the 4-momenta of the antinucleon and nucleon respectively, k and ε are the 4-momentum and polarization vector of the ρ-meson, and q is the 4-momentum of the π. Furthermore we define the scalar invariants

$$(3)\quad t = (p_1 + p_2)^2,\quad s = (p_2 - k)^2,\quad u = (p_1 - k)^2$$

In order to find out which of the invariant amplitudes are superconvergent, we first look at the kinematical singularity-free helicity amplitudes[7] in the s-channel $\rho + N \to \pi + N$. The high energy behaviour is then determined by the leading Regge trajectories. Among the six helicity amplitudes, we find the following two with additional convergence factors

$$(4)\quad \begin{aligned} \phi_1 &= \frac{\langle 0,\frac{1}{2}|T|1,-\frac{1}{2}\rangle}{(\sin\frac{\Theta}{2})^2\cdot\cos\frac{\Theta}{2}} = \frac{1}{k(\sin\frac{\Theta}{2})^2}\sum_\ell \phi_1^\ell\left[-\sqrt{\frac{\ell}{\ell+2}}\,P'_{\ell+1}(\cos\Theta) + \sqrt{\frac{\ell+2}{\ell}}\,P'_\ell(\cos\Theta)\right]\\ \phi_2 &= \frac{\langle 0,-\frac{1}{2}|T|1,-\frac{1}{2}\rangle}{\sin\frac{\Theta}{2}\cdot(\cos\frac{\Theta}{2})^2} = \frac{1}{k(\cos\frac{\Theta}{2})^2}\sum_\ell \phi_2^\ell\left[\sqrt{\frac{\ell}{\ell+2}}\,P'_{\ell+1}(\cos\Theta) + \sqrt{\frac{\ell+2}{\ell}}\,P'_\ell(\cos\Theta)\right]\end{aligned}$$

with Θ the scattering angle in the s-channel and $k = |\vec{k}|$. For $t \to \infty$ ϕ_1 and ϕ_2 behave like $t^{\alpha-1/2-1}$. Only Regge poles with parity $P = -(-1)^{J-1/2}$ can contribute to the combination $\phi_1 + \phi_2$, whereas only those with $P = (-1)^{J-1/2}$ contribute to $\phi_1 - \phi_2$. Thus for isospin $I = 1/2$ $\phi_1 + \phi_2$ is controlled by the N_γ trajectory with $\alpha(0) \simeq -.8$, and for $I = 3/2$ by the Δ_δ trajectory with $\alpha(0) \simeq 0$;for $I = 1/2$ $\phi_1 - \phi_2$ is controlled by N_α trajectory with $\alpha(0) \simeq -.4$, while for $I = 3/2$ no trajectory is known experimentally. Therefore we conclude that $\phi_1 - \phi_2$ and $\phi_1 + \phi_2$ are superconvergent for $t \to \infty$.

Next we need the relation between the helicity amplitudes and the invariant amplitudes. After some algebra we obtain

(5)
$$\phi_1 + \phi_2 = \sqrt{2}\ \frac{|\vec{p}_1|}{M} \sqrt{(E_1 - M)(E_2 + M)} \left[a_1 + (\sqrt{s} - M) a_4 \right]$$

$$\phi_1 - \phi_2 = \sqrt{2}\ \frac{|\vec{p}_1|}{M} \sqrt{(E_1 + M)(E_2 - M)} \left[a_1 - (\sqrt{s} + M) a_4 \right]$$

As a_1 and a_4 are superconvergent also in the u-channel, we can write the superconvergence sum rules

(6)
$$\int_{-\infty}^{\infty} \mathrm{Im} \left[a_1\ (t,s) + (\sqrt{s} - M) a_4 (t,s) \right] dt = 0$$

$$\int_{-\infty}^{\infty} \mathrm{Im} \left[a_1\ (t,s) - (\sqrt{s} + M) a_4 (t,s) \right] dt = 0$$

In the analyticity domain these relations should hold for all s, and that allows us to write down sum rules separately for a_1 and a_4 at $s = 0$

(7a) $$\int_{-\infty}^{\infty} \mathrm{Im}\ a_1 (t, s=0) dt = 0$$

(7b) $$\int_{-\infty}^{\infty} \mathrm{Im}\ a_4 (t, s=0) dt = 0$$

for the two isospins $I = 1/2$ and $I = 3/2$ which can be exchanged in the s-channel.

III. Saturation of the Sum Rules

In order to exploit the sum rules in Equs. (7a) and (7b) we will try to saturate them by number of low-lying intermediate states in the t and u-channels. Candidates among the known particles and resonances are the π, ω, Φ, A_1, N and N^*. The following couplings shall be used:

$\pi\bar{N}N$: $g \cdot \bar{v}\ (p_1) \gamma^5\ u(p_2)$

$\rho\bar{N}N$: $g_{\rho N}\ \bar{v}\ (p_1) \gamma_\mu\ u(p_2) \varepsilon^\mu + i\ \frac{f_{\rho N}}{2M}\ \bar{v}\ (p_1) \sigma_{\mu\nu} u(p_2) k^\nu \varepsilon^\mu$

$\pi\bar{N}N^*$: $\frac{g_{\pi NN^*}}{M^*}\ \bar{v}\ (p_1) \psi_\nu q^\nu$

$\rho\bar{N}N^*$: $G\ \bar{v}\ (p_1) \gamma^5 \psi_\nu \varepsilon^\nu + \frac{G}{M^* + M}\ \bar{v}\ (p_1) \gamma^5 \gamma_\mu \psi_\nu k^\nu \varepsilon^\mu$

(8)

$$A_1\bar{N}N: \quad i\, g_{A_1N}\, \bar{v}(p_1)\gamma^5\gamma_\mu\, u(p_2)\, e^\mu$$

$$\pi\pi\rho: \quad g_{\rho\pi\pi}\, \varepsilon^\mu\, (q_n + q)_\mu$$

$$\pi\omega\rho: \quad g_{\rho\omega\pi}\, \varepsilon_{\alpha\beta\gamma\delta}\, (p_1 + p_2)^\alpha\, e_\omega^\beta\, k^\gamma\, \varepsilon^\delta$$

$$\pi A_1\rho: \quad g_L \left[q_n^\mu - \frac{(q_n\cdot k)k^\mu}{k^2} \right]\cdot \left[k^\nu - \frac{(q_n\cdot k)q_n^\nu}{q_n^2} \right] e_\nu \varepsilon_\mu + \frac{g_T}{m_{A_1}^2}\, \varepsilon^{\lambda\alpha\beta\gamma}\varepsilon_{\mu\nu\rho\gamma} q_{n\alpha} k_\beta q_n{}^\mu k^\nu e_\lambda \varepsilon^\mu .$$

Here M is the mass of the nucleon, M^* that of the N^*, e^μ and e_ω^μ are the polarization vectors of the A_1 and ω respectively. q_n is always used for the 4-momentum of the exchanged particle. The couplings $\omega\bar{N}N$ and $\Phi\bar{N}N$ are defined analogously to the coupling $\rho\bar{N}N$, and $\pi\Phi\rho$ is defined analogously to the coupling $\pi\omega\rho$. The $\rho\bar{N}N^*$ coupling should be described rigorously by three coupling constants, but we will here confine ourselves to the so called "magnetic dipole coupling," since we shall include the contribution of the N^* merely as a correction.

First we try to saturate Equs. (7a) and (7b) for the process (A) with π,ω,Φ and N as intermediate states. We find contradiction between the equations for I = 1/2 and I = 3/2, so that we have to take into account A_1 as an intermediate state. Furthermore, to obtain more independent equations, we consider not only process (A) but also

(B) $\bar{N} + N \rightarrow \pi + \omega$

(C) $\bar{N} + N \rightarrow \pi + \Phi$.

These processes differ only in isospin structure from process (A). For process (B) Equs. (7a) and (7b), saturated with ρ and N, give

$$(9) \quad - f_{\rho N}\cdot g_{\rho\omega\pi}\, \frac{M^2+m_\omega^2}{2M} + 2g\cdot g_{\omega N} = 0$$

$$(10) \quad g_{\rho N}\cdot g_{\rho\omega\pi} + \frac{g f_{\omega N}}{M} = 0$$

Combined with the analogous equations for the Φ they yield

$$(11) \quad \frac{2g_{\Phi N} - f_{\Phi N}}{2g_{\omega N} - f_{\omega N}} = \frac{g_{\rho\Phi\pi}}{g_{\rho\omega\pi}} \cdot \frac{f_{\rho N}(M^2 + m_\Phi^2) + 2M^2 g_{\rho N}}{f_{\rho N}(M^2 + m_\omega^2) + 2M^2 g_{\rho N}}$$

We apply the Gell-Mann-Sharp-Wagner model[9] to the decays $\omega \to 3\pi$ and $\Phi \to 3\pi$; with the masses and branching ratios from the Rosenfeld Tables of September 1967 [8] we have $g_{\rho\omega\pi} = 19(\text{GeV})^{-1}$ and $g_{\rho\Phi\pi} = 1.3(\text{GeV})^{-1}$. We note that

$$(12) \quad \frac{f_{\Phi N}}{g_{\Phi N}} = \frac{f_{\omega N}}{g_{\omega N}} = \mu_p + \mu_n = -0.12$$

should hold approximately, μ_p and μ_n being the anomalous magnetic moments of proton and neutron, respectively. Inserting this in Equ. (11) gives

$$(13) \quad \frac{g_{\Phi N}}{g_{\omega N}} \simeq 0.06$$

One would also expect such a small ratio in the quark model, where in the ideal mixing the Φ is build up only from λ-quarks. It is also in agreement with the result obtained from a least-squares fit of low-energy N-N data[10].

Now we examine the sum rules from process (A). Since $g_{\Phi N}$ and $g_{\rho\Phi\pi}$ are small compared with the other couplings we will drop the contribution of the Φ and keep only π, ω, A_1 and N. This gives us the equations

(14)

$$-2gg_{\rho\pi\pi} + f_{\omega N}g_{\rho\omega\pi}\frac{M^2 + m_\rho^2}{2M} - g_{A_1N}\, g_L \frac{M(m_\pi^2 - m_\rho^2 - m_{A_1}^2)}{m_{A_1}^2} - g_{A_1N}\, g_T \frac{2Mm_\rho^2}{m_{A_1}^2} + 4gg_{\rho N} = 0$$

(15)

$$4gg_{\rho\pi\pi} + f_{\omega N}g_{\rho\omega\pi}\frac{M^2 + m_\rho^2}{2M} + 2g_{A_1N}\, g_L \frac{M(m_\pi^2 - m_\rho^2 - m_{A_1}^2)}{m_{A_1}^2} + 4g_{A_1N}\, g_T \frac{Mm_\rho^2}{m_{A_1}^2} - 2gg_{\rho N} = 0$$

(16)

$$g_{\omega N}\, g_{\rho\omega\pi} - g_{A_1N}\, g_L - g_{A_1N}\, g_T \frac{m_\pi^2 - m_\rho^2 - m_{A_1}^2}{2m_{A_1}^2} + 2\,\frac{g \cdot f_{\rho N}}{M} = 0$$

(17)

$$g_{\omega N}\, g_{\rho\omega\pi} + 2g_{A_1N}\, g_L + g_{A_1N}\, g_T \frac{m_\pi^2 - m_\rho^2 - m_{A_1}^2}{m_{A_1}^2} - \frac{g \cdot f_{\rho N}}{M} = 0$$

From these expressions, together with Equs. (9) and (10), we obtain after some slight rearrangements

$$(18)\quad \frac{g_{\rho\omega\pi}\cdot M}{g} = -\frac{f_{\omega N}}{g_{\rho N}} = \frac{4gM}{g_{\rho\omega\pi}(M^2+m_\rho^2)} = -\frac{f_{\rho N}}{g_{\omega N}} = \frac{4gM}{g_{\rho\omega\pi}(M^2+m_\omega^2)}$$

The unique solution of Equ. (18) is

$$(19)\quad m_\omega = m_\rho$$

$$(20)\quad g^2_{\rho\omega\pi} = \frac{4g^2}{M^2+m_\rho^2}$$

The first of these two predictions is satisfied experimentally with an error of about 1% [8]. It is also obtained by exact SU(6)-symmetry. With $(\frac{g^2}{4\pi}) = 14.4$, Equ. (20) gives $g_{\rho\omega\pi} = \pm 21\ (\mathrm{GeV})^{-1}$, in remarkable agreement with the experimental value $g_{\rho\omega\pi} = \pm(18 \pm 3)(\mathrm{GeV})^{-1}$. Additional information is needed for predictions of meson-nucleon couplings. We take the relation

$$(21)\quad \frac{f_{\rho N}}{g_{\rho N}} = \mu_p - \mu_n = 3.7$$

which should approximately hold in the vector dominance model[11]. Furthermore we assume ρ-meson universality[12]

$$(22)\quad g_{\rho N} = \frac{1}{2}\, g_{\rho\pi\pi}$$

With these assumptions and with $(\frac{g^2_{\rho\pi\pi}}{4\pi}) = 2.5$ which has been calculated from the width of the ρ-meson[8], we obtain from Equ.(18) (without regard to an overall sign)

$$(23a)\quad g_{\rho N} = 2.8, \quad g_{\omega N} = 6.4, \quad f_{\rho N} = 10.3$$

$$(23b)\quad f_{\omega N} = 4.0$$

We want to compare this with the results obtained from a least-squares fit of low-energy N-N data, made by Köpp and Söding:[10]

$g_{\rho N} = 3.9 \pm 0.4, \quad g_{\omega N} = 7.7 \pm 1.9, \quad f_{\rho N} = 13.1 \pm 1.3, \quad f_{\omega N} = -0.8 \pm 0.2.$

We see that Equ. (23a) is in qualitative agreement with Ref. 10) but that Equ. (23b) is in strong disagreement. Equ. (23b) disagrees also with the assumption in Equ. (12). The origin of this discrepancy can be traced to Equ. (10). If we would forget Equ. (10) and would use Equ. (12) instead, we would have

$$\text{(23c)} \quad f_{\omega N} = -0.77$$

instead of Equ. (23b), and this would in a trivial way establish agreement with Ref. 10).

By the way we remark that by including the Φ-meson we would not get Equ. (19) but

$$\text{(24)} \quad g^2_{\rho\Phi\pi} = \frac{4g^2}{M^2 + m^2_\rho} \cdot \frac{m^2_\omega - m^2_\rho}{m^2_\Phi - m^2_\rho}$$

This shows a direct connection between the small fraction of $\Phi \to 3\pi$ and the ω-ρ mass difference. Instead of Equ. (20) we would have

$$\text{(25)} \quad g^2_{\rho\omega\pi} = \frac{4g^2}{M^2 + m^2_\rho} \cdot \frac{m^2_\Phi - m^2_\omega}{m^2_\Phi - m^2_\rho}$$

and this does not change significantly the numerical value of $g_{\rho\omega\pi}$ as compared with Equ. (20).

Gilman and Harari[4] considered superconvergence and current algebra relations for the process $\pi + \rho \to \pi + \rho$ and derived $g_T = 0$. Using this result in Equs. (14) - (17) we would be led to a contradiction with our assumed ρ-meson universality and would obtain results in disagreement with Ref. 10). To solve this contradiction we would need a transverse admixture of about

$$\frac{g_T}{g_L} \simeq 0.2.$$

Finally we shall also take into account N* as an intermediate state. The exact expressions would become very clumsy, however, and some couplings are not known very well. Therefore we will treat the contribution of the N* merely as a correction to see whether our previous results are changed significantly or not. The righthand sides of Equs. (14)-(17) are now changed from 0 to $0.17 g_{\pi NN^*} G$, to $-0.68 g_{\pi NN^*} G$, to $-0.23 \frac{g_{\pi NN^*} G}{M^*}$ and to $0.92 \frac{g_{\pi NN^*} G}{M^*}$ respectively. From these new equations we obtain an additional constraint

$$(26) \quad \frac{f_{\rho N}}{g_{\rho N}} \simeq 3.76$$

which supports our assumptions Equ. (21). Instead of Equ. (20) we get a more complicated relation, but if we take $g_{\pi NN^*} = 17$ from the width of the N*[8] and G = 15 from Ref. 13) the numerical value of $g_{\rho\omega\pi}$ is altered only about 10% to $g_{\rho\omega\pi} = \pm 19\,(\mathrm{GeV})^{-1}$. Therefore, the values in Equs. (23a) and (23b) will also change only by about 10%.

The inclusion of the N* also enables us to estimate the coupling of the A_1-particle, because now there is no longer a contradiction to the result of Gilman and Harari[4]. We can take $g_T = 0$ and with $g_L = 57\ (\mathrm{GeV})^{-1}$ we obtain

$$(27) \quad g_{A_1 N} = 1.2$$

IV. Conclusions.

Two of the six invariant amplitudes for the processes $\bar{N} + N \to \pi + \rho$ and $\bar{N} + N \to \pi + \omega$ were found to obey superconvergence relations. Saturating these relations for the first process with π, ω, Φ and N, we were led to contradictions. Saturation with π, ω, A_1, N for the first process, and saturation with ρ and N for the second process gave $m_\omega = m_\rho$ and $g_{\rho\omega\pi} = 21\,(\mathrm{GeV})^{-1}$, in remarkable agreement with experiment. Assuming vector dominance and ρ-meson universality we obtained $g_{\rho N} = 2.8$, $g_{\rho N} = 6.4$, $f_{\rho N} = 10.3$, in agreement and $f_{\omega N} = 4.0$ in strong disagreement with a least-squares fit of N-N data. This disagreement was due to an equation for the second process. It might possibly be improved by inclusion of an additional higher resonance. Furthermore it was seen that inclusion of the N* changed the results only by about 10%.

Acknowledgements:

The authors thank Professor P. Urban for his encouragement and his interest in this work. A part of this work was done at DESY, Hamburg. One of the authors (A.B) wants to thank Professor W. Jentschke for the hospitality at DESY extended to him during the summer of 1967. He is grateful to Professor H. Joos and Drs. F. Gutbrod, M. Krammer and D. Schildknecht for many helpful discussions.

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