DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 69/28
~~August 1969~~

DESY-Bibliothek
9. SEP. 1969

A Consistent Approach to Exchange Currents and the Nuclear Many Body Problem

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Abstract

A consistent treatment of two-particle exchange currents in nuclei in the framework of the One-Boson-Exchange-Model for the nucleon-nucleon interaction is given. Relativistic effects are taken into account up to order $(P/M)^2$. The general formalism for the evaluation of matrix elements of two-body currents in many-nucleon systems is out-lined. Possible applications to the two-nucleon system are briefly discussed.

Introduction

In the usual quantum mechanical description of electromagnetic (e.m.) interactions of nuclei the e.m. current of the nuclear system is treated as a sum of one-body operators. Two-body operators are taken into account usually only by means of the substitution $\vec{p} \rightarrow \vec{p} - e\vec{A}$ in the momentum dependent part of the nucleon-nucleon interactions. But it has been known for a long time that such a current cannot be a conserved one, if the nucleon-nucleon interaction contains exchange forces, i.e. terms being proportional to $\vec{\tau}_i \cdot \vec{\tau}_j$. If the exchange forces are two-body operators, the one-body e.m. current must be supplemented by two-body terms $\vec{j}_{[2]}$ in such a way, that the total current $\vec{j} = \vec{j}_{[1]} + \vec{j}_{[2]}$ satisfies the continuity equation $\vec{\nabla} \cdot \vec{j} + \partial\rho/\partial t = 0$. Unfortunately, even if the N-N interaction is known, the result for $\vec{j}_{[2]}$ is not unique because to each particular solution terms like $\vec{\nabla} \times \vec{\phi}(\vec{r})$ may be added.

In a number of papers [1] the most general form of $\vec{j}_{[2]}$ and the corresponding multipole moments allowed by the usual invariance principles have been discussed. It turns out, that due to Siegert's theorem [2] electric multipole operators are completely determined in the long wave length limit by the nucleonic charge density alone [3], but the form of the magnetic multipole operators remains rather arbitrary. It is, therefore, necessary to restrict the form of $\vec{j}_{[2]}$ further by means of additional principles.

In local relativistic quantum field theory exists the principle of "minimal e.m. coupling" (i.e. the gauge invariant substitution $\partial_\mu \rightarrow \partial_\mu + ieA_\mu$) due to which the e.m. interaction of f.i. nucleons and π-mesons is uniquely determined for a given form of the πN-interaction [4]. If we would be able to solve the equations of motion in field theory, the form of NN-potentials and the corresponding e.m. currents of a nuclear system would be completely fixed. But, the only systematic treatment of field equations known at present is perturbation theory. Many authors have calculated NN-potentials in low orders of perturbation theory in the last two decades [5].

The two-body e.m. currents corresponding to the adiabatic NN-potentials in second and fourth order respectively have been derived too [6] .

But, even fourth order calculations for NN-elastic scattering, taking recoil corrections fully into account, are only in qualitative agreement with the higher partial waves and disagree with the lower ones [7] . Therefore, many of the present day NN-potentials are of a semiphenomenological character: For large distances they approach the one-pion exchange potential (OPEP) determined from second order perturbation theory, whereas the short range behaviour has to be determined by experiment [8] .

Having this in mind, some authors [9] used a mixed description of e.m. interactions of nuclei: They used one -pion exchange contributions (or somewhat refined forms) for $\vec{j}_{[2]}$ but the nuclear wave functions have been obtained by means of phenomenological forms or potentials respectively. It is obvious that such a kind of approach is highly inconsistent.

Present experimental numbers show without any doubt the non-negligible effect of exchange currents in nuclear physics: With the d-mixture in the deuteron wave function determined by NN-scattering data there is a discrepancy for the deuteron magnetic moment [10] ; the np-radiative capture process at threshold requires a 10 o/o contribution from exchange currents [11] . Recent calculations of this process by Adler, Chertok and Miller [9] based on the use of phenomenological wave functions and particular one-boson exchange contributions for $\vec{j}_{[2]}$ have only lowered the discrepancy between theory and experiment to 5 o/o. Therefore, also from the experimental point of view, a more satisfactory treatment of exchange currents is required.

From the discussion given above we conclude that a consistent approach for the construction of two-body exchange currents utilizing the principle of minimal e.m. interactions is called for.

This can be done only in the framework of a solvable dynamical model for the NN-interaction. As a dynamical model for NN-interaction we understand in this context any model based on particle exchange between the nucleons as origin of nuclear forces. Within such a model the field theoretical formulation of minimal e.m. interaction may be applied immediately. Besides the perturbation theoretical calculations mentioned above we know only one solvable dynamical model: the one-boson exchange (OBE)-model for nucleon-nucleon interactions. This model, which approximates the effect of multiple-pion exchange by means of the exchange of the known vector- and pseudoscalar mesons and two (or three) postulated scalar particles, has been worked out in great detail in the last couple of years [12] and it has been very successful in explaining experimental phase shift data by adjusting about 1o free parameters [13] .

It is the aim of this paper, to calculate exchange currents in the framework of the OBE-model and to outline the general procedure for calculating the corresponding corrections to γ-transitions and magnetic moments of complex nuclei.+)

The paper is organized as follows:
In chapter 2 we discuss more explicitly the problems arising in the construction of exchange currents for a given nuclear interaction and give a brief description of the general solution of these problems in the framework of minimal e.m. interaction.

+) Exchange current contributions derived from the OBE-model have been recently considered for the calculation of NN-Bremsstrahlung in Born-approximation [14] .

In chapter 3 a detailed discussion of exchange currents in the OBE-model is given, starting from the two-particle Dirac-equation. The explicit form of exchange currents in the non-relativistic limit, i.e. in the usual $(p/M)^2$-approximation is given by reducing the Dirac-equation to the Pauli-equation. Finally we outline possible improvements of our calculations.

In chapter 4 we discuss the treatment of two-body currents in nuclear many body systems. In section 4.1 it is shown how transition rates between low lying excited states and the groundstate of even-even nuclei can be calculated, if the excited states are dominantly linear combinations of particle-hole states. In section 4.2 we investigate the possibilities for the calculation of electromagnetic transition rates and expectation values of two-body operators for odd nuclei with the help of vertex functions.

In chapter 5 we discuss possible applications to the two nucleon system.

2. Nuclear interactions, continuity equation and exchange currents

The continuity equation for the e.m. current

$$\vec{\nabla} \cdot \vec{j}(\vec{r}) + \dot{\rho}(\vec{r}) = 0$$

takes in quantum mechanics (we use the Heisenberg picture) the form [+)]

$$\vec{\nabla}\cdot\vec{j}(\vec{r}) + i\left[H, \rho(\vec{r})\right] = 0 \tag{1}$$

If we consider a N-nucleon system in lowest order of e.m. interaction, the operators $\vec{j}$ and ρ are independent of the degrees of freedom of the e.m. fields and H is the total Hamiltonian describing strong interactions between the nucleons. If the potential term in H only consists of two-body forces (this will be assumed in the following), the total current $\vec{j}$ decomposes in a one-body and a two-body part $\vec{j}_{[1]}$ and $\vec{j}_{[2]}$ respectively with

$$\vec{\nabla}\cdot\vec{j}_{[1]}(\vec{r}) + i\left[T, \rho(\vec{r})\right] = 0 \tag{2a}$$

$$\vec{\nabla}\cdot\vec{j}_{[2]}(\vec{r}) + i\left[V, \rho(\vec{r})\right] = 0 \tag{2b}$$

where T is the operator of the kinetic energy, i.e.

$$H = T + V$$

The charge density operator $\rho(\vec{r})$ is a one-body operator as long as retardation effects in the NN-interaction are neglected (compare sections 3.1 and 3.2) :

$$\rho(\vec{r}) = e\sum_i \frac{1+\tau_{i3}}{2}\rho_i(\vec{r}) \tag{3}$$

+) In this paper we use sqare brackets $[a,b]$ for commutators and wavy brackets $\{a,b\}$ for anti- commutators.

If furthermore the extended structure of nucleons is neglected we have

$$\rho_i(\vec{r}) = \delta(\vec{r}-\vec{r}_i)$$

in case of the Dirac-equation; in case of the non-relativistic Pauli-equation with relativistic corrections taken into account up to order $(P/M)^2$, $\rho_i(\vec{r})$ contains in addition to $\delta(\vec{r}-\vec{r}_i)$ a recoil correction ρ_i^{recoil} connected with the well known "Zitterbewegung": +)

$$\rho_i^{\text{recoil}}(\vec{r}) = \frac{1}{4M^2}\left(\vec{\sigma}_i, \left(\vec{\nabla}_i\,\delta(\vec{r}-\vec{r}_i)\right) \times \vec{p}_i\right) \qquad (4)$$
$$+ \frac{1}{8M^2}\left(\Delta_i\,\delta(\vec{r}-\vec{r}_i)\right)$$

Therefore, the commutator $\left[V, \rho(\vec{r})\right]$ in eq. (2b) contains three different contributions in case of the Pauli-equation:

1. terms arising from the exchange part of V due to the non-commutativity of $\vec{\tau}_i \cdot \vec{\tau}_j$ with $\frac{1+\tau_{i3}}{2}$
2. terms arising from the momentum dependence of V
3. terms arising from the momentum- and (or) spin-dependence of ρ^{recoil}

The corresponding total $\vec{j}_{[2]}$ we call exchange current.

+) For the derivation compare section 3.2
($\vec{\nabla}_i\ \delta(\vec{r}-\vec{r}_i)$) etc. means here and in the following that the derivatives operate only on the δ-functions.

The solution of eq. (2) is not unique, because to each particular solution we can add terms like $\vec{\nabla} \times \vec{\Phi}(\vec{r})$.
The uniqueness of the solution will be achieved by utilizing the principle of " minimal e.m. interaction" for our problem.

The resulting unique solution $\vec{J}_{[1]}$ of eq. (2a) in case of the Dirac-equation is well known

$$\vec{J}_{[1]}(\vec{r}) = - \left. \frac{\delta T(\vec{p}_i - e_i \vec{A}(\vec{r}_i))}{\delta \vec{A}(\vec{r})} \right|_{\vec{A}=0} \tag{5}$$

leading to $\vec{J}_{[1]}(\vec{r}) = \sum_i e_i \vec{\alpha}_i \, \delta(\vec{r} - \vec{r}_i)$

The corresponding operator $\vec{j}_{[1]}$ in case of the Pauli-equation is now determined too. It contains one term constructed according to eq. (5) and an additional contribution $\vec{j}_{t[1]}$ corresponding to ρ^{recoil} +):

$$\vec{j}_{t[1]}(\vec{r}) = - \frac{e}{8M^2} \Big[T, \sum_i \frac{1+\tau_{i3}}{2} \Big(i \; \{ \vec{\sigma}_i \times \vec{p}_i, \delta(\vec{r} - \vec{r}_i) \} + \frac{1}{i} \big(\vec{\nabla}_i \, \delta(\vec{r} - \vec{r}_i) \big) \Big) \Big] \tag{6}$$

It is obvious that the expressions eq. (4) and (6) satisfy together the continuity eq. (2a).

+) For the derivation compare section 3.2

For $\vec{j}_{[\lambda]}$ the principle of minimal e.m. interaction can be utilized only within a solvable dynamical model for the NN-interaction. As a dynamical model for NN-interactions we understand in this context any model based on particle exchange between the nucleons as origin of nuclear forces. Within such a model we formulate minimal e.m. interaction as the usual minimal substitution $\partial_\mu \rightarrow \partial_\mu + ie A_\mu$ for all participating particles.

If we neglect retardation effects in the treatment of particle exchange between nucleons, and restrict ourselves to two-body forces only, the whole problem can be treated most clearly in the framework of a one-time two-particle Dirac equation. The two-particle current $\vec{J}_{[\lambda]}$ occurring in such a Dirac-equation is directly determined by the minimal e.m. interaction of the exchanged particles. The remaining part of the e.m. interaction is determined by the substitution $\partial_\mu \rightarrow \partial_\mu + ie A_\mu$ within the two-particle Dirac-equation. In this way, the exchange current $\vec{j}_{[\lambda]}$ occuring in the two-particle Pauli-equation is completely determined.

It turns out (at least in case of the OBE-model treated in this paper) that $\vec{j}_{[\lambda]}$ is not additiv with respect to contributions 1) and 2) to the commutator $[V, \rho]$ in eq. (2b) as might be expected.
But the third contribution is additiv (we call the corresponding current $\vec{j}_{t[\lambda]}$). Furthermore, if the exchange part of V is zero (isospin zero exchange between the nucleons) $\vec{j}_{[\lambda]} - \vec{j}_{t[\lambda]}$ is in general <u>not</u> given by the substitution $\vec{p} \rightarrow \vec{p} - e\vec{A}$ within the two-particle Pauli-equation, as has been assumed up to-day in the literature [15] , but additional curlterms may occur.

There exist at present two solvable dynamical models for the NN-interaction:

A) Pertubation theoretical treatment of the usual pseudoscalar meson theory with pseudoscalar coupling. Recently Wortman [7] tried to understand NN-phase shifts by means of Feynman diagrams up to the fourth order in the πN-coupling constants. In this way a qualitative understanding of the higher partial waves can be achieved, but S- and p -waves are beyond the range of the model.

B) The OBE-model, based on the exchange of vector pseudoscalar and scalar mesons between the nucleons [12] . By fitting about 1o open parameters (coupling constants, masses of scalar mesons and cut-off parameters) a reasonable agreement with experimental phase shifts can be achieved [13] .

If we take the agreement between the predictions of a model and experimental phase shifts as the criterion for the choice of a dynamical model, we have to choose the OBE-model.

3. Exchange currents within the OBE-model

In this chapter we will treat exchange currents based on the OBE-model for NN-interactions in some detail.

3.1 Two-particle Dirac-equation with OBEP and e.m. interaction.

A Lorentz covariant description of the two-nucleon problem including interaction with an external e.m. field within the framework of the OBE-model can be given by means of a Bethe-Salpeter equation (i.e. a two-time Dirac-equation) +).

In such a framework the interaction is represented by the off-shell Feynman-diagrams of fig. 1 : Diagram 1a represents the potential kernel due to the exchange of a boson B, diagram 1b represents the minimal e.m. coupling of the nucleon, diagram 1c represents the minimal e.m. coupling of the exchanged boson which has to be supplemented by diagrams of type 1d, if the $BN\bar{N}$-coupling contains derivatives. The mixed boson-exchange diagrams of fig. 1e are of a nonminimal type, but they should be considered too, if all contributions which are of a one-boson exchange type are taken into account consistently ++).
Finally, e.m. interaction terms due to the anomalous magnetic moment of the nucleon are given in the OBE-picture by the diagrams of fig. 1f.

As has been pointed out in the introduction, it is the aim of the present paper to provide the formal tools for the consistent treatment of exchange currents in systems consisting of non-relativistic nucleons.

+) A covariant formulation of the two-nucleon interaction via one-boson exchange by means of a one-time formalism has been given recently by Schierholz 16 . But the inclusion of e.m. interaction within such a framework is still an unsolved problem.

++) This has been pointed out by de Swart (private communication).

Therefore, it is not necessary to solve a BS-equation of the type described above exactly; we are only interested in the derivation of an approximate non-relativistic two-nucleon equation (Pauli-equation) which takes recoil effects into account up to the order $(P/M)^2$ as usual. +) This means in particular, that retardation effects contained in the relativistic description of one-boson exchange must only be treated approximately, i.e. up to the order $(P/M)^2$. An approximate treatment of retardation can be made by means of a one-time formalism, i.e. the usual two-particle Dirac-equation. Such an approach was first worked out by Breit for the electron interaction in atoms [17] . In connection with the OBEP most authors adopt a different point of view: By means of an unitary transformation depending on the BN$\bar{N}$-coupling constants the retardation term can be transformed into a fourth order potential term [18] . But,since fourth order potential terms are not considered within the strict OBEP, retardation terms will be neglected too. We do not agree with this point of view. Only for reasons of simplicity we neglect retardation corrections in the present paper. We will consider them in the near future.

The two-particle Dirac-equation derived from the BS-equation described above by neglecting retardation completely looks as follows

$$i\frac{\partial}{\partial t}\,\psi(\vec{r}_1,\vec{r}_2,t) = \Big(\sum_{i=1}^{2}\big(\vec{\alpha}_i\cdot\vec{\pi}_i + \beta_i M + e_i A_0(\vec{r}_i,t)\big) + V_{OBEP} + H_{e.m.[2]}\Big)\,\psi(\vec{r}_1,\vec{r}_2,t) \tag{7}$$

with $\vec{\pi}_i = \vec{p}_i - e_i\vec{A}(\vec{r}_i,t)$, $e_i = e\,\frac{1+\tau_{i3}}{2}$

+) M = nucleon mass

For OBEP one obtains from diagram 1a

$$V_{OBEP} = \sum_{B} \frac{g_B^2}{4\pi} \left(\delta_{I_B,0} + \vec{\tau}_1 \cdot \vec{\tau}_2 \, \delta_{I_B,1} \right) \times \tag{8}$$

$$\times \; \Gamma_B (1, \vec{\nabla}_1) \; \Gamma_B (2, \vec{\nabla}_2) \; J_B (r_{12})$$

with $\quad J_B(r) = \dfrac{e^{-m_B r}}{r}$

Here I_B is the isospin of the exchanged boson and Γ_B is the $BN\bar{N}$-vertex in coordiante space up to the order $(P/M)^2$. Derivatives occur only if B is a vectormeson; the time-derivative part is in this case of order $(P/M)^3$, therefore we keep only the dependence of Γ_B on $\vec{\nabla}_i$ (which operates on J_B (r_{12})).

$H_{e.m.[2]}$ in eq. (7) is that part of the e.m. interaction which corresponds to the two-particle e.m. currents, i.e.

$$H_{e.m.[2]} = \int d^3x \; J^{\mu}_{[2]}(\vec{r}) \; A_{\mu}(\vec{r}, t) \tag{9}$$

where $J^{\mu}_{[2]}$ may be decomposed according to its contributions from diagrams 1c, 1d and 1e. Due to the neglect of retardation and the fact that Γ_B contains in our approximation no time derivative we have

$$J^{0}_{[2]\,(1c+1d)} = 0 \tag{10}$$

For the space components we obtain

$$\vec{J}_{[2]\,(1c+1d)} = e \sum_{B(I_B=1)} \left\{ \frac{g_B^2}{4\pi} \frac{(\vec{\tau}_1 \times \vec{\tau}_2)_3}{4\pi} \times \right.$$

$$\times \Gamma_B(1, \vec{\nabla}_1)\, \Gamma_B(2, \vec{\nabla}_2)\, \vec{J}_B(\vec{r}, \vec{r}_1, \vec{r}_2)$$

$$- \frac{g_B^2}{4\pi} \frac{\delta}{\delta \vec{A}(\vec{r})} \Gamma_B\left(1, \vec{\nabla}_1 - (\vec{\tau}_1 \times \vec{\tau}_2)_3 \vec{A}(\vec{r}_1)\right) \Gamma_B\left(2, \vec{\nabla}_2 + (\vec{\tau}_1 \times \vec{\tau}_2)_3 \vec{A}(\vec{r}_2)\right)$$

$$\left. \times J_B(r_{12}) \right\}_{\vec{A}=0} \qquad (11)$$

with

$$\vec{J}_B(\vec{r}, \vec{r}_1, \vec{r}_2) = J_B(|\vec{r}-\vec{r}_1|) \overleftrightarrow{\nabla} J_B(|\vec{r}-\vec{r}_2|) \qquad (12)$$

$$\overleftrightarrow{\nabla} = \overrightarrow{\nabla} - \overleftarrow{\nabla}$$

We note that we get a contribution to (11) only from charged bosons, i.e. from bosons with isospin $I_B = 1$.
Using the equation

$$(\Delta - m_B^2)\, J_B(|\vec{r}-\vec{r}_i|) = -4\pi\, \delta(\vec{r}-\vec{r}_i) \qquad (13)$$

and the form of the potential of eq. (8) we see immediately that $\vec{J}_{[2]\,(1c+1d)}$ satisfies the continuity equation (2b).

Some remarks should be made as to the minimal e.m. interaction of charged bosons and the form of $\vec{J}_{[2]}(1c+1d)$:

1. The e.m. vertex in the case of spin-zero particles is fixed by the requirement of current conservation alone; the requirement of minimal e.m. interaction does not give any additional restriction, i.e. we have an orbital current contribution only

$$\langle p_2 |\, j_\mu(0) \,| p_1 \rangle = e(2\pi)^{-3} (p_1 + p_2)_\mu \tag{14}$$

2. In the case of vector bosons, the e.m. vertex contains, besides the orbital current term, contributions due to a magnetic dipole and an electric quadrupole moment. In contrast to the spin 1/2-case, these electromagnetic moment terms are not fixed by the usual principle of minimal e.m. interaction for particles with spin ≥ 1 [19] . We therefore apply in this case an extended principle of minimal e.m. coupling according to which only the orbital current term is taken into account. +)

With respect to the mixed current contributions $\vec{J}_{[2]}(1e)$ we have to make the following remarks:

1. If B is a scalar meson (S) and B' is a pseudoscalar meson (PS), the BB'γ -vertex vanishes due to parity conservation.

+) Such an extended principle of minimal e.m. coupling has been applied in the past with some success to the photoproduction process $\gamma + p \rightarrow \Delta(1236)^{++} + \pi^-$ [20] .

2. If B = S, B'= S' or B= PS, B' = (PS)' the BB'γ -vertex vanishes for real photons due to charge conservation.

3. Therefore, the only non-vanishing contributions come from:

a) B = V, B' = V'

b) B = PS, B' = V

c) B = S, B' = V

Only in case b) we have some experimental information on the BB'γ - coupling constant.

Because of this limited knowledge on the strength of the mixed contributions and due to the fact that they are of a non-minimal type, we postpone their detailed discussion to a separate paper.

3.2 Two particle Pauli-equation; General structure of exchange currents.

In the reduction of the Dirac-equation (7) to the two-particle Pauli equation we follow closely the procedure of Green and Sawada [12] for the OBE-potential problem: First we derive an approximate equation for the large spinor components and perform afterwards a renormalization of the wave function.

In this way we obtain a two-particle Pauli-equation

$$i \frac{\partial}{\partial t} \varphi(\vec{r}_1, \vec{r}_2, t) = H \varphi(\vec{r}_1, \vec{r}_2, t) \tag{15}$$

We split the Hamiltonian H (15) into one-and two-particle terms:

$$H = H_{[1]} + H_{[2]} \tag{16}$$

On the other hand we may decompose H with respect to strong and e.m. interaction:

$$H = T + \bar{V}_{OBEP} + H_{e.m.} \tag{17}$$

First we discuss the e.m. interaction contained in $H_{[1]}$.
We have +)

$$\begin{aligned} H_{[1]} = \; & e_1 A_0(\vec{r}_1, t) + \frac{(\vec{\sigma}_1 \cdot \vec{\pi}_1)^2}{2M} - \frac{(\vec{\sigma}_1 \cdot \vec{\pi}_1)^4}{8M^3} \\ & - \frac{i e_1}{8M^2} \left[\vec{\sigma}_1 \cdot \vec{\pi}_1 , \vec{\sigma}_1 \cdot \vec{E}(\vec{r}_1, t) \right] + (1 \leftrightarrow 2) \end{aligned} \tag{18}$$

with

$$\vec{E} = -\vec{\nabla} A_0 - \frac{\partial}{\partial t} \vec{A}$$

We treat the e.m. interaction in lowest order of perturbation theory, i.e. we consider transition matrixelements of $H_{e.m.}$ only between eigenstate of $H_{strong} = T + \bar{V}_{OBEP}$.
Keeping this in mind we may perform for the $\dot{\vec{A}}$ -terms in (18), which have the structure $\vec{O} \cdot \dot{\vec{A}}$, the substitution

$$\vec{O} \cdot \dot{\vec{A}} \longrightarrow -i \left[H_{strong} , \vec{O} \cdot \vec{A} \right] \tag{19}$$

Therefore, $H_{[1]}$ contains effectively also two-particle terms. Corresponding to (19) we have to substitute for (18) the following operators

---

+) Compare any textbook on relativistic quantum mechanics

$$H_{[1]} \longrightarrow H'_{[1]} + H_{e.m.t[1]} + H_{e.m.t[2]} \tag{20}$$

with

$$H'_{[1]} = e_1 A_0 + \frac{(\vec{\sigma}_1 \cdot \vec{\pi}_1)^2}{2M} - \frac{(\vec{\sigma}_1 \cdot \vec{\pi}_1)^4}{8M^3} + \frac{i e_1}{8M^2}\left[\vec{\sigma}_1 \cdot \vec{\pi}_1, (\vec{\sigma}_1 \cdot \vec{\nabla}_1 A_0(\vec{r}_1))\right] + (1 \leftrightarrow 2) \tag{21}$$

$$H_{e.m.t[1]} = \frac{1}{8M^2}\left[T_1\ \left(e_1\left[(\vec{\sigma}_1 \cdot \vec{\pi}_1),\ \vec{\sigma}_1 \cdot \vec{A}(\vec{r}_1)\right] + (1 \leftrightarrow 2)\right)\right] \tag{22}$$

$$H_{e.m.t[2]} = \frac{1}{8M^2}\left[\bar{V}_{OBEP},\left(e_1\left[\vec{\sigma}_1 \cdot \vec{\pi}_1, \vec{\sigma}_1 \cdot \vec{A}(\vec{r}_1)\right] + (1 \leftrightarrow 2)\right)\right] \tag{23}$$

By means of functional derivation of the expressions in eq. (21) to (23) with respect to A_0 and $\vec{A}$ respectively we obtain:

$$\left.\frac{\delta H'_{[1]}}{\delta A_0}\right|_{A_\mu = 0} = \sum_i e_i \left(\delta(\vec{r} - \vec{r}_i) + \rho_i^{recoil}\right) \tag{24}$$

where ρ_i^{recoil} is given by eq. (4) ;

$$\vec{j}_{t[1]}(\vec{r}) = - \left.\frac{\delta H_{e.m.t[1]}}{\delta \vec{A}(\vec{r})}\right|_{\vec{A}=0} \tag{25}$$

leading to the result given in eq. (6) ;

$$\vec{j}_{t[2]}(\vec{r}) = - \left. \frac{\delta H_{e.m,t[2]}}{\delta \vec{A}(\vec{r}')} \right|_{\vec{A}=0}$$

leading to

$$\vec{j}_{t[2]}(\vec{r}) = - \frac{e}{8M^2} \Big[\bar{V}_{OBEP}, \sum_i \frac{1+\tau_{i3}}{2} \times$$

$$\times \Big(i \left\{ \vec{\sigma}_i \times \vec{p}_i, \delta(\vec{r}-\vec{r}_i) \right\} + \frac{1}{i} \left(\vec{\nabla}_i \, \delta(\vec{r}-\vec{r}_i) \right) \Big) \Big] \tag{26}$$

Now we turn to the discussion of the exchange currents arising from the term $H_{[2]}$ in (16). In accordance with Green and Sawada [12] we introduce the following abreviations

$$\begin{aligned}
V_a &= - \frac{g_s^2}{4\pi} J_s + \frac{g_v^2}{4\pi} J_v \\
V_b &= \frac{g_p^2}{4\pi} J_p - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \frac{g_v^2}{4\pi} J_v \\
V_c &= \frac{g_s^2}{4\pi} J_s + \frac{g_v^2}{4\pi} J_v \\
V_d &= - \frac{g_p^2}{4\pi} J_p - \vec{\sigma}_1 \cdot \vec{\sigma}_2 \frac{g_v^2}{4\pi} J_v
\end{aligned} \tag{27}$$

$$\begin{aligned}
\vartheta(\vec{p}_i, V_K) = &- \left\{ \vec{\sigma}_1 \cdot \vec{p}_1 \; \vec{\sigma}_2 \cdot \vec{p}_2 , V_b \right\} \\
&- \vec{\sigma}_1 \cdot \vec{p}_1 \, V_c \, \vec{\sigma}_1 \cdot \vec{p}_1 - \vec{\sigma}_2 \cdot \vec{p}_2 \, V_c \, \vec{\sigma}_2 \cdot \vec{p}_2 \\
&- \vec{\sigma}_1 \cdot \vec{p}_1 \, V_d \, \vec{\sigma}_2 \cdot \vec{p}_2 - \vec{\sigma}_2 \, \vec{p}_2 \, V_d \, \vec{\sigma}_1 \cdot \vec{p}_1
\end{aligned} \tag{28}$$

For the potential term in the Pauli-equation we then obtain in the case of isoscalar boson exchange [12]

$$\overline{V}^{I=0}_{OBEP} = V_a - \frac{1}{8M^2}\sum_i \left\{ (\vec{\sigma}_i \cdot \vec{p}_i)^2, V_a \right\} - \frac{1}{4M^2}\vartheta(\vec{p}_i, V_K)$$
$$+ V_{(f^2)}(\vec{\nabla}_i) + V_{(g_v f)}(\vec{\nabla}_i, \vec{p}_K) \qquad (29)$$

where we added explicitly the terms arising from the Pauli-coupling (f-type coupling) of the vector mesons to nucleons, which have not been treated by Green and Sawada. For this terms we have (compare Bryan and Scott [12])

$$V_{(f^2)} = -\frac{1}{4\pi}\left(\frac{f}{2M}\right)^2 \left((\vec{\sigma}_1 \times \vec{\nabla}_1 \cdot \vec{\sigma}_2 \times \vec{\nabla}_2)\, J_V \right)$$

$$V_{(g_v f)} = \frac{1}{4\pi}\,\frac{g_v f}{4M^2}\left(\left\{ \vec{\sigma}_2 \cdot \vec{p}_2, (\vec{\sigma}_1 \times \vec{\sigma}_2 \cdot (\vec{\nabla}_1 J_V)) \right\} \right. \qquad (30)$$
$$\left. + i\left[\vec{\sigma}_1 \cdot \vec{p}_1, (\vec{\sigma}_1 \cdot \vec{\nabla}_1 J_V) \right] + (1 \leftrightarrow 2) \right)$$

The potential corresponding to the exchange of isovector bosons we obtain from (17) by means of the substitution $J_B \rightarrow J_B\, \vec{\tau}_1 \cdot \vec{\tau}_2$.

Exchange currents in case of isoscalar boson exchange:

We note, that in deriving the potential term (29) we did not yet use the commutation relations of the Pauli matrices. This enables us to obtain the exchange current contribution, arising in case of isoscalar boson exchange only from the momentum dependence of the potential term, immediately according to eq. (7), by means of the substitution $\vec{p}_i \longrightarrow \vec{\pi}_i$ in eq. (29).

$$\vec{j}^{\,I=0}_{[2]}(\vec{r}) = \frac{\delta}{\delta \vec{A}(\vec{r})} \Big(- V_a + \Big\{ \frac{1}{8M^2} \sum_i (\vec{\sigma}_i \cdot \vec{\pi}_i)^2, V_a \Big\}$$
$$+ \frac{1}{4M^2}\, \vartheta(\vec{\pi}_i, V_k) - V_{(g_v f)}(\vec{\nabla}_i, \vec{\pi}_k) \Big)_{\vec{A}=0} \qquad (31)$$
$$+ \vec{j}^{\,I=0}_{t[2]}(\vec{r})$$

where, according to eqs. (26) and (29) $\vec{j}^{\,I=0}_{t[2]}$ is given by

$$\vec{j}^{\,I=0}_{t[2]}(\vec{r}) = - \frac{e}{8M^2} \Big[V_a, \sum_i \frac{1+\tau_{i3}}{2} \Big(i \{ \vec{\sigma}_i \times \vec{p}_i, \delta(\vec{r}-\vec{r}_i) \}$$
$$+ \frac{1}{i} (\vec{\nabla}_i\, \delta(\vec{r}-\vec{r}_i)) \Big) \Big] \qquad (32)$$

With V_a from eq. (27) we obtain finally

$$\vec{j}^{\,I=0}_{t[2]}(\vec{r}) = \frac{e}{4M^2} \sum_i \frac{1+\tau_{i3}}{2}\, \delta(\vec{r}-\vec{r}_i) (\vec{\sigma}_i \times (\vec{\nabla}_i V_a)) \qquad (33)$$

The occurence ot terms like $(\vec{\sigma}_i \cdot \vec{\pi}_i)^2$ in (31) leads, in general, to additional curl-terms in $\vec{j}_{[2]}$ compared to the result obtained from the substitution $\vec{p}_i \rightarrow \vec{\pi}_i$ in the final expression for the potential. It turns out, that these curl-terms cancel each other (add) in case of vector meson (scalar meson) exchange.

<u>Exchange currents in case of isovector boson exchange</u>

First we define quantities $\vec{J}_{[2]K}$ by means of eq. (27) with the r.h.s. changed by the substitution $J_B\ (r_{12}) \rightarrow e\frac{(\vec{\tau}_1 \times \vec{\tau}_2)_3}{4\pi}\ \vec{J}_B(\vec{r}, \vec{r}_1, \vec{r}_2)$

Then according to eqs. (7), (8) and (11) we obtain $\vec{j}^{\,I=0}_{[2]}$ by means of the substitutions

$$\text{a)} \quad V_K \longrightarrow \vec{\tau}_1 \cdot \vec{\tau}_2\, V_K - \int d^3x\, \vec{J}_{[2]K} \cdot \vec{A}(\vec{r})$$

in the first term of eq. (31) and

$$\text{b)} \quad V_a \longrightarrow \vec{\tau}_1 \cdot \vec{\tau}_2\, V_a \qquad \text{in eq. (32).}$$

To this we have to add the contributions form diagram 1 d (last term in eq. (11)).

In this way we obtain

$$\vec{J}^{\,I=1}_{[2]}(\vec{r}) = \vec{J}_{[2]a} - \frac{1}{8M^2}\sum_i \{\vec{p}_i^{\,2}, \vec{J}_{[2]a}\}$$
$$- \frac{1}{4M^2}\vartheta(\vec{p}_i, \vec{J}_{[2]K}) + \frac{\delta}{\delta\vec{A}(\vec{r})}\Big(\frac{1}{8M^2}\sum_i \{(\vec{\sigma}_i\cdot\vec{\pi}_i)^2, \vec{\tau}_1\cdot\vec{\tau}_2 V_a\}$$
$$+ \frac{1}{4M^2}\vartheta(\vec{\pi}_i, \vec{\tau}_1\cdot\vec{\tau}_2 V_K) - \tilde{V}_{(f^2+g_V f)}(\vec{\nabla}_i - e(\vec{\tau}_i\times\vec{\tau}_{i+1})_3\vec{A}(\vec{r}_i),$$
$$,\vec{\pi}_K)\Big)_{\vec{A}=0} - \frac{e}{8M^2}\Big[\vec{\tau}_1\cdot\vec{\tau}_2 V_a, \sum_i \frac{1+\tau_{i3}}{2}\Big(i\{\vec{\sigma}_i\times\vec{p}_i, \delta(\vec{r}-\vec{r}_i)\}$$
$$+ \frac{1}{i}(\vec{\nabla}_i\,\delta(\vec{r}-\vec{r}_i))\Big)\Big] \qquad (34)$$

$$\text{with} \quad \tilde{V}_{(f^2+g_V f)} = V_{(f^2+g_V f)}\Big(J_V \rightarrow \vec{\tau}_1\cdot\vec{\tau}_2 J_V - e\frac{(\vec{\tau}_1\times\vec{\tau}_2)_3}{4\pi}\int d^3x\,\vec{J}_V\cdot\vec{A}\Big)$$

3.3 Explicit form of exchange currents

In this section we collect the explicit expressions for exchange currents arising from the exchange of scalar, pseudoscalar and vector mesons respectively with either isospin zero or one.

A) Isoscalar boson exchange

The formulas for exchange currents in case of isoscalar boson exchange are obtained by means of an explicit evaluation of eq. (31).

a)

Scalar mesons

$$\vec{j}^{\,I=0}_{[2]}(\vec{r}) = \frac{e}{2M^2}\,\frac{1+\tau_{13}}{2}\,\frac{g_s^2}{4\pi}\Big(2\,J_s\,\delta(\vec{r}-\vec{r}_1)\,\vec{p}_1 + J_s\Big(\frac{\vec{\nabla}_1}{i}\,\delta(\vec{r}-\vec{r}_1)\Big) + \frac{1}{i}\,J_{1s}\,\vec{r}_{12}\,\delta(\vec{r}-\vec{r}_1) + J_s\Big(\vec{\sigma}_1 \times \big(\vec{\nabla}_1\,\delta(\vec{r}-\vec{r}_1)\big)\Big) + (1 \leftrightarrow 2) \qquad (35)$$

where we introduced the abreviation

$$J_{1B}(r) = \frac{1}{r}\,\frac{\partial}{\partial r}\,J_B(r)$$

We note, that the current generated by means of the substitution $\vec{p}_i \longrightarrow \vec{\pi}_i$ from the spin-orbit term in the potential and $\vec{j}_{t[2]}$ cancel each other. Furthermore, there occurs in (35) an additional curl term.

b) Pseudoscalar mesons

$$\vec{j}^{\,I=0}_{[2]}(\vec{r}) \equiv 0 \qquad (36)$$

The result (36) is an immediate consequence of the following facts:

1. The corresponding potential has no momentum dependence

2. No curl-terms can occur, because $(\vec{\sigma}_i \cdot \vec{\pi}_i)^2$-terms in (31) are absent in that case

3. $\vec{j}_{t[2]} = 0$ due to $V_a = 0$

c) Vector mesons

We decompose the current with respect to its contributions from g-and f-type couplings of vector bosons with nucleons:

$$\vec{j}^{\,I=0}_{[2](g_V^2)}(\vec{r}) = -\frac{e}{2M^2}\left(\frac{1+\tau_{13}}{2}\right)\frac{g_V^2}{4\pi}\,\delta(\vec{r}-\vec{r}_1)\Big(2\,J_V\,\vec{p}_2 + i\,\vec{r}_{12}\,J_{1V} + 2\,(\vec{r}_{12}\times\vec{S})\,J_{1V}\Big) + (1\leftrightarrow 2) \tag{37}$$

with

$$\vec{S} = \frac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$$

In contrast to the scalar case, the spin orbit generated current terms and $\vec{j}_{t[2]}$ add but the curl-terms cancel each other. $\vec{j}^{\,I=0}_{[2](f^2)}$ vanishes for the same reasons as in the pseudo-scalar case.

$$\vec{j}^{\,I=0}_{[2](g_V f)}(\vec{r}) = \frac{e}{M^2}\left(\frac{1+\tau_{13}}{2}\right)\frac{g_V f}{4\pi}\,\delta(\vec{r}-\vec{r}_1)\,J_{1V}\,(\vec{S}\times\vec{r}_{12}) + (1\leftrightarrow 2) \tag{38}$$

B) Isovector boson exchange

The following formulas are obtained by means of an explicit evaluation of eq. (34).

a) Scalar mesons

$$\vec{j}^{\,I=1}_{[2]}(\vec{r}) = \vec{\tau}_1\cdot\vec{\tau}_2\;\vec{j}^{\,I=0}_{[2]}(\vec{r}) + e\,\frac{(\vec{\tau}_1\times\vec{\tau}_2)_3}{4\pi}\,\frac{g_s^2}{4\pi}\,\vec{J}_s$$

$$+ \frac{e}{2M^2}\,\frac{g_s^2}{4\pi}\,(\vec{\tau}_1\times\vec{\tau}_2)_3\Big(\delta(\vec{r}-\vec{r}_1)\Big(\frac{1}{i}\,J_s\,\vec{p}_1 + J_s(\vec{p}_1\times\vec{\sigma}_1)$$

$$+ \frac{1}{2}\,i\,J_s(\vec{\sigma}_1\times\vec{r}_{12})\Big) - (1\leftrightarrow 2)\Big)$$

$$+ \frac{e}{4M^2}\,\frac{(\vec{\tau}_1\times\vec{\tau}_2)_3}{4\pi}\,\frac{g_s^2}{4\pi}\Big(2\,\vec{J}_s\,\vec{p}_1^{\,2} - \frac{1}{2}\,m_s^2\,\vec{J}_s +$$

$$+ \frac{2}{i}\,J_{1s}(|\vec{r}-\vec{r}_1|)\,(\vec{r}_1-\vec{r},\vec{p}_1)\,\overleftrightarrow{\nabla}\,J_s(|\vec{r}-\vec{r}_2|) +$$

$$+ J_{1s}(|\vec{r}-\vec{r}_1|)\big(\vec{\sigma}_1,(\vec{r}_1-\vec{r})\times\vec{p}_1\big)\overleftrightarrow{\nabla}\,J_s(|\vec{r}-\vec{r}_2|) - (1\leftrightarrow 2)\Big)$$

$$- \frac{e}{8M^2}\,(\vec{\tau}_1\times\vec{\tau}_2)_3\,\frac{g_s^2}{4\pi}\,J_s\Big(\vec{\nabla}\,\big(\delta(\vec{r}-\vec{r}_1) - \delta(\vec{r}-\vec{r}_2)\big)\Big) \tag{39}$$

b) Pseudoscalar mesons

$$\vec{j}^{\,I=1}_{[2]}(\vec{r}) = \frac{e}{4M^2}\,(\vec{\tau}_1\times\vec{\tau}_2)_3\,\frac{g_P^2}{4\pi}\,\delta(\vec{r}-\vec{r}_1)\,\vec{\sigma}_1\,(\vec{\sigma}_2\cdot\vec{r}_{12})\,J_{1P}$$

$$+ (1\leftrightarrow 2)$$

$$- \frac{e}{4M^2}\,\frac{g_P^2}{4\pi}\,\frac{(\vec{\tau}_1\times\vec{\tau}_2)_3}{4\pi}\Big((\vec{\sigma}_1\cdot\vec{\nabla}_1)(\vec{\sigma}_2\cdot\vec{\nabla}_2)\,\vec{J}_P\Big) \tag{40}$$

c) Vector mesons

$$\vec{j}^{\,I=1}_{[2](g_V^2)}(\vec{r}) = \vec{\tau}_1\cdot\vec{\tau}_2\ \vec{j}^{\,I=0}_{[2](g_V^2)}(\vec{r}) +$$

$$+ \frac{e}{8M^2}(\vec{\tau}_1\times\vec{\tau}_2)_3\ J_V\ \frac{g_V^2}{4\pi}\left(\vec{\nabla}\left(\delta(\vec{r}-\vec{r}_1)-\delta(\vec{r}-\vec{r}_2)\right)\right)$$

$$+ e\frac{(\vec{\tau}_1\times\vec{\tau}_2)_3}{4\pi}\ \frac{g_V^2}{4\pi}\ \vec{J}_V + \frac{e}{4M^2}\ \frac{g_V^2}{4\pi}(\vec{\tau}_1\times\vec{\tau}_2)_3\Big(\delta(\vec{r}-\vec{r}_1)\,\cdot$$

$$\times\Big(-J_{1V}\left(i\,\vec{r}_{12}\times(\vec{\sigma}_1-\vec{\sigma}_2)-(\vec{\sigma}_1\cdot\vec{\sigma}_2)\,\vec{r}_{12}+(\vec{\sigma}_1\cdot\vec{r}_{12})\,\vec{\sigma}_2+\vec{r}_{12}\right)$$

$$+ 2J_V\left(i\vec{p}_2-\vec{p}_2\times\vec{\sigma}_1\right)\Big) - (1\leftrightarrow 2)\Big)$$

$$- \frac{e}{4M^2}\frac{(\vec{\tau}_1\times\vec{\tau}_2)_3}{4\pi}\ \frac{g_V^2}{4\pi}\Big(-\frac{1}{2}m_V^2\ \vec{J}_V + 2\ \vec{J}_V(\vec{p}_1\cdot\vec{p}_2)$$

$$- J_{1V}\left(|\vec{r}-\vec{r}_1|\right)\left(\vec{\sigma}_1\,,(\vec{r}_1-\vec{r})\times\vec{p}_1\right)\overleftrightarrow{\nabla}\ J_V\left(|\vec{r}-\vec{r}_2|\right)$$

$$+ \frac{2}{i}\ J_{1V}\left(|\vec{r}-\vec{r}_1|\right)\left(\vec{\sigma}_1\,,\vec{r}_1-\vec{r}\right)\overleftrightarrow{\nabla}\,J_V\left(|\vec{r}-\vec{r}_2|\right)(\vec{\sigma}_1\cdot\vec{p}_2)$$

$$- \frac{1}{2}\ J_{1V}\left(|\vec{r}-\vec{r}_1|\right)\left(\vec{\sigma}_1\cdot\vec{r}_1-\vec{r}\right)\overleftrightarrow{\nabla}\left(\vec{\sigma}_2\cdot\vec{r}_2-\vec{r}\right)J_{1V}\left(|\vec{r}-\vec{r}_2|\right)\ \vec{\sigma}_1\cdot\vec{\sigma}_2$$

$$- (1\leftrightarrow 2)\Big)$$

(41)

$$\vec{j}^{\,I=1}_{[2](f^2)}(\vec{r}) = e\left(\frac{f}{2M}\right)^2\frac{(\vec{\tau}_1\times\vec{\tau}_2)_3}{4\pi}\ \delta(\vec{r}-\vec{r}_1)\left(\vec{\sigma}_1\times(\vec{r}_{12}\times\vec{\sigma}_2)\right)J_{1V}$$

$$+ (1\leftrightarrow 2)$$

$$- \frac{e}{4\pi}\left(\frac{f}{2M}\right)^2\frac{(\vec{\tau}_1\times\vec{\tau}_2)_3}{4\pi}\left((\vec{\nabla}_1\times\vec{\sigma}_1)\cdot(\vec{\nabla}_2\times\vec{\sigma}_2)\,\vec{J}_V\right)$$

(42)

$$\vec{j}^{\,I=1}_{[2](g_V f)}(\vec{r}) = \vec{\tau}_1 \cdot \vec{\tau}_2 \; \vec{j}^{\,I=0}_{[2](g_V f)}(\vec{r})$$

$$+ \frac{e}{4\pi} \frac{g_V f}{4M^2} (\vec{\tau}_1 \times \vec{\tau}_2)_3 \Big(\delta(\vec{r}-\vec{r}_1) \Big(i\,(\vec{\sigma}_1 - \vec{\sigma}_2) \times \vec{r}_{12} \; J_{1V}$$

$$+ \; 2\, \vec{\sigma}_1 \times (\vec{r}_{12} \times \vec{\sigma}_2)\, J_{1V} - 4\, J_V \; \vec{\sigma}_1 \times \frac{(\vec{p}_1 - \vec{p}_2)}{2} \Big) - (1 \leftrightarrow 2) \Big)$$

$$+ \frac{e}{4\pi} \frac{g_V f}{4M^2} \frac{(\vec{\tau}_1 \times \vec{\tau}_2)_3}{4\pi} \Big(\frac{1}{i} \Big((\vec{\nabla}_2, \vec{\nabla}_1 \times (\vec{\sigma}_1 - \vec{\sigma}_2))\, \vec{J}_V \Big)$$

$$- 2 \Big((\vec{\nabla}_1 \times \vec{\sigma}_1) \cdot (\vec{\nabla}_2 \times \vec{\sigma}_2)\, \vec{J}_V \Big) + 4 \Big(\vec{\sigma}_1 , (\vec{\nabla}_1 \; \vec{J}_V) \times \frac{(\vec{p}_1 - \vec{p}_2)}{2} \Big)$$

$$+ m_V^2 \, \vec{J}_V - (1 \leftrightarrow 2) \Big)$$

$$+ \frac{ie}{4\pi} \frac{g_V f}{4M^2} (\vec{\tau}_1 \times \vec{\tau}_2)_3 \Big(\Big(\vec{\nabla} \times \vec{\sigma}_1 \, \delta(\vec{r}-\vec{r}_1) \Big) J_V - (1 \leftrightarrow 2) \Big)$$

(43)

3.4 Form factor effects

In order to achieve agreement between the predictions of the OBEP-model and experimental s-wave phase shifts one must introduce a high momentum cut-off in the OBE-potentials [13]. This cut-off serves as an approximation of vertex and propagator corrections to the potential diagram 1a, represented by the diagram fig. 2.

The transition from diagram 1a to 2 in terms of formulas is given by the substitution

$$\overline{V}_{OBEP}(B) \longrightarrow \int dm_B \, \rho(m_B) \, \overline{V}_{OBEP}(B) \tag{44}$$

The corresponding change in the formulas for our exchange current contribution, i.e. the interaction of a photon with the different bubbles in the diagram fig. 2 depends on the unknown dynamical structure of these vertex and propagator terms. On the other hand if we change the potential according to eq. (44) additional terms must be considered also in the current in order to maintain current conservation.

From a pure phenomenological point of view we propose a change of our currents in analogy to the change in the potentials

$$\vec{j}^{\,I}_{[2]}(B) \longrightarrow \int dm_B \, \rho(m_B) \, \vec{j}^{\,I}_{[2]}(B) \tag{45}$$

3.5 Outline of possible improvements

There are some points where our general scheme for the calculation of exchange currents can be improved:

1. As discussed already in section 3.1 retardation effects can be taken into account approximately.

2. In its present form our formalism is not adequate for the description of processes initiated by virtual photons (i.e. by electrons) because we did neither consider the extended structure of nucleons nor that of the exchanged bosons. The best way to take such electromagnetic form factor effects into account consistently is given by means of the vector dominance model (VDM) [21]. In the VDM the photon couples directly to a vector boson which then interacts universally (i.e. by means of a conserved current) to all hadrons. This means, that in our diagrams 1b-1c we have to change the e.m. vertices according to the prescription given in fig. 3.

4. Treatment of two-particle currents in the nuclear many-body system.

The exchange current operator $\vec{J}ex\,(\vec{r})$ is of the general form

$$\vec{J}_{ex}(\vec{r}) = \iint d^3r_1\, d^3r_2\, \psi^\dagger(\vec{r}_1)\, \psi^\dagger(\vec{r}_2)\, \vec{j}_{[2]}(\vec{r}, \vec{r}_1, \vec{r}_2)\, \psi(\vec{r}_2)\, \psi(\vec{r}_1)$$

where $\vec{j}_{[2]}$ is composed of various terms according to eqs. (35) to (43). The interaction of $\vec{J}ex$ with an external electromagnetic field represented by the vector potential $\vec{A}\,(\vec{r}, t\,)$ is given by

$$\begin{aligned} V_{ex}(t) &= -\int d^3r\, \vec{J}_{ex}(\vec{r})\, \vec{A}(\vec{r}, t) \\ &= -\int d^3r\, \vec{A}(\vec{r},t) \iint d^3r_1\, d^3r_2\, \psi^\dagger(\vec{r}_1)\, \psi^\dagger(\vec{r}_2)\, \vec{j}_{[2]}(\vec{r}, \vec{r}_1, \vec{r}_2)\, \psi(\vec{r}_2)\, \psi(\vec{r}_1) \end{aligned} \tag{46}$$

Expanding

$$\psi^\dagger(\vec{r}) = \sum_n a_n^\dagger\, \varphi_n^*(\vec{r})$$

$$\psi(\vec{r}) = \sum_n a_n\, \varphi_n(\vec{r})$$

where $a_n^\dagger$, a_n are creation- and destruction operators for the single particle states $\varphi_n(\vec{r})$, we obtain

$$V_{ex}(t) = -\sum_{n_1, n_2, n_3, n_4} \int d^3r\, \vec{A}(\vec{r}, t)\, \vec{j}_{[2]\, n_1 n_2 n_3 n_4}(\vec{r})\, a_{n_1}^\dagger\, a_{n_2}^\dagger\, a_{n_3}\, a_{n_4}$$

with

$$\vec{j}_{[2]\, n_1 n_2 n_3 n_4}(\vec{r}) = \iint d^3r_1\, d^3r_2\, \varphi_{n_1}^*(\vec{r}_1)\, \varphi_{n_2}^*(\vec{r}_2)\, \vec{j}_{[2]}(\vec{r}, \vec{r}_1, \vec{r}_2) \times$$
$$\times\, \varphi_{n_3}(r_2)\, \varphi_{n_4}(\vec{r}_1)$$

Carrying out the integration over $\vec{r}$, V appears as a two-particle operator

$$V_{ex}(t) = \sum_{n_1 n_2 n_3 n_4} V_{n_1 n_2 n_3 n_4}(t)\, a^{+}_{n_1} a^{+}_{n_2} a_{n_3} a_{n_4} \tag{47}$$

where

$$V_{n_1 n_2 n_3 n_4}(t) = - \int d^3r \, \vec{A}(\vec{r},t)\, \vec{j}_{[2]\, n_1 n_2 n_3 n_4}(\vec{r})$$

Quite generally expectation values and transition matrix elements of the exchange current operator can be calculated with the help of vertex functions. In the case of electromagnetic groundstate transitions in even-even nuclei the use of vertex function can be avoided, if the particle-hole (ph) amplitudes and/ or two particle- two hole (2p - 2h) amplitudes of an excited state are known. We will treat this case first.

4.1 Groundstate transitions in even-even nuclei.

We consider a transition between the groundstate $|0\rangle$ and an excited state $|S\rangle$ of an even-even nucleus. The contribution of the exchange currents to the transition matrix element is given by (see eq. (47))

$$\langle S | V_{ex} | 0 \rangle = \sum_{n_1 n_2 n_3 n_4} V_{n_1 n_2 n_3 n_4} \langle S | a^{+}_{n_1} a^{+}_{n_2} a_{n_3} a_{n_4} | 0 \rangle \tag{48}$$

The problem is therefore reduced to the calculation of the 2p - 2h amplitudes $\langle s | a^{+}_{n_1} a^{+}_{n_2} a_{n_3} a_{n_4} | 0 \rangle$. In principle they can be extracted from the spectral representation of the eight-point function. However no reliable methods are known for the present time which would permit to obtain the necessary information on the eight-point function for nuclei. On the other hand, well founded methods exist for the calculation of p-h amplitudes of low lying excited states. If it is known that a given excited state is primarily a linear combination of p-h states then the following procedure would be applicable: As a first approximation the excited state in question is considered as a linear combination of p-h excitations: $| s^{(0)} \rangle = \sum_{n,m} d^{(0)}_{n,m} a^{+}_{n} a^{+}_{m} | 0 \rangle$. Then the 2p-2h amplitudes can be determined by perturbation theory. The wave function is then of the form $|s\rangle = A | s_{ph} \rangle + B | s_{2p,2h} \rangle$,

with
$$|A|^2 \langle s_{ph} | s_{ph} \rangle + |B|^2 \langle s_{2p2h} | s_{2p2h} \rangle = 1$$

$$|B| \ll |A|$$

The contribution of $| s_{2p,2h} \rangle$ can be taken directly from eq. (48):

$$\langle s_{2p2h} | V_{ex} | 0 \rangle = \sum_{n_1 n_2 n_3 n_4} V_{n_1 n_2 n_3 n_4} \langle s_{2p2h} | a^{+}_{n_1} a^{+}_{n_2} a_{n_3} a_{n_4} | 0 \rangle$$

$| s_{ph} \rangle$ gives also a contribution to the transition matrix, coming from those summation terms where indices of creation and destruction operators are equal:

$$\langle s_{ph} | V_{ex} | 0 \rangle = \sum_{n_1 n_2 n_3 n_4} V_{n_1 n_2 n_3 n_4} \langle s_{ph} | a^{+}_{n_1} a^{+}_{n_2} a_{n_3} a_{n_4} | 0 \rangle$$
$$\times \left(\delta_{n_1 n_3} + \delta_{n_1 n_4} + \delta_{n_2 n_3} + \delta_{n_2 n_4} \right)$$

Making use of the anticommutation relations we obtain:

$$\langle s_{ph}| V_{ex} |0\rangle = \sum_{n_1 n_2 n_4} V_{n_1 n_2 n_1 n_4} \langle s_{ph}| a^{+}_{n_2} a^{+}_{n_1} a_{n_4} a_{n_1} |0\rangle$$

$$- \sum_{n_1 n_2 n_3} V_{n_1 n_2 n_3 n_1} \langle s_{ph}| a^{+}_{n_2} a^{+}_{n_1} a_{n_3} a_{n_1} |0\rangle$$

$$- \sum_{n_1 n_2 n_4} V_{n_1 n_2 n_2 n_4} \langle s_{ph}| a^{+}_{n_1} a^{+}_{n_2} a_{n_4} a_{n_2} |0\rangle$$

$$+ \sum_{n_1 n_2 n_3} V_{n_1 n_2 n_3 n_2} \langle s_{ph}| a^{+}_{n_1} a^{+}_{n_2} a_{n_3} a_{n_2} |0\rangle$$

$$= \sum_{n_1 n_2 n_4} V_{n_1 n_2 n_1 n_4} \langle s_{ph}| a^{+}_{n_2} (\delta_{n_1 n_4} - a_{n_4} a^{+}_{n_1}) a_{n_1} |0\rangle$$

$$- \sum_{n_1 n_2 n_3} V_{n_1 n_2 n_3 n_1} \langle s_{ph}| a^{+}_{n_2} (\delta_{n_1 n_3} - a_{n_3} a^{+}_{n_1}) a_{n_1} |0\rangle$$

$$- \sum_{n_1 n_2 n_4} V_{n_1 n_2 n_2 n_4} \langle s_{ph}| a^{+}_{n_1} (\delta_{n_2 n_4} - a_{n_4} a^{+}_{n_2}) a_{n_2} |0\rangle$$

$$+ \sum_{n_1 n_2 n_3} V_{n_1 n_2 n_3 n_2} \langle s_{ph}| a^{+}_{n_1} (\delta_{n_2 n_3} - a_{n_3} a^{+}_{n_2}) a_{n_2} |0\rangle$$

The contributions of the terms containing the Kronecker symbol cancel pairwise and we obtain:

$$\langle S_{ph} | V_{ex} | 0 \rangle = - \sum_{n_1 n_2 n_4} V_{n_1 n_2 n_1 n_4} \langle S_{ph} | a^+_{n_2} a_{n_4} a^+_{n_1} a_{n_1} | 0 \rangle$$

$$+ \sum_{n_1 n_2 n_3} V_{n_1 n_2 n_3 n_1} \langle S_{ph} | a^+_{n_2} a_{n_3} a^+_{n_1} a_{n_1} | 0 \rangle$$

$$- \sum_{n_1 n_2 n_3} V_{n_1 n_2 n_3 n_2} \langle S_{ph} | a^+_{n_1} a_{n_3} a^+_{n_2} a_{n_2} | 0 \rangle$$

$$+ \sum_{n_1 n_2 n_4} V_{n_1 n_2 n_2 n_4} \langle S_{ph} | a^+_{n_1} a_{n_4} a^+_{n_2} a_{n_2} | 0 \rangle$$

Introducing the occupation number N_n of the single particle state n in the groundstate, defined by $a^+_n a_n |0\rangle = N_n |0\rangle$, interchanging the indices 1 and 2 in the third and fourth sum and relabeling $n_4 \longleftrightarrow n_3$ we get finally:

$$\langle S_{ph} | V_{ex} | 0 \rangle = \sum_{n_2, n_3} \langle S_{ph} | a^+_{n_2} a_{n_3} | 0 \rangle \times$$

$$\times \sum_{n_1} N_{n_1} \left[V_{n_2 n_1 n_1 n_3} - V_{n_1 n_2 n_1 n_3} + V_{n_1 n_2 n_3 n_1} - V_{n_2 n_1 n_3 n_1} \right]$$

The preceeding expression corresponds to an effective two-particle vertex, defined by the graphical equation of fig.4. (For the meaning of symbols, see the explanation given in connection with fig. 5).

The corresponding form factor diagram for the free nucleon gives zero contribution, because the occupation numbers are identically zero in that case.

The preceeding discussion has shown that for lowlying p-h states with small admixtures of 2p-2h states the exchange current contribution to groundstate transition rates can be calculated with reasonable accuracy.

4.2 Transitions in nuclei of odd mass number.

We consider a transition between two states $n_1^{(i)}$ and $n_4^{(j)}$, being characterised by the i-th and j-th pole of the propagators $G_{n_1}(\omega_1)$ and $G_{n_4}(\omega_4)$ respectively. The residues of the poles we call $Z_{n_1}^{(i)}$ and $Z_{n_2}^{(j)}$; the corresponding energies are called $\varepsilon_{n_1}^{(i)}$ and $\varepsilon_{n_4}^{(j)}$. The electromagnetic transition probability $W_{n_4 \to n_1}$ is given by (see for example ref. 22)

$$W_{n_4 \to n_1} = \frac{2\pi}{\hbar} \left| \sum_{K=1}^{2} T^{[K]}_{n_1^{(i)} n_4^{(j)}} \left(\frac{\varepsilon_{n_1}^{(i)} + \varepsilon_{n_4}^{(j)}}{2}, \varepsilon_{n_1}^{(i)} - \varepsilon_{n_4}^{(j)} \right) \right|^2$$

$$\times \; Z_{n_1}^{(i)} Z_{n_4}^{(j)}$$

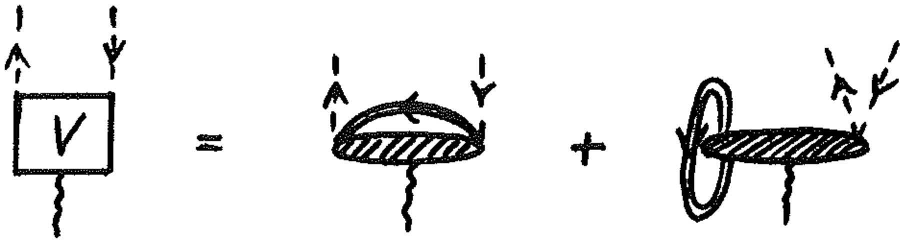
Here $\mathcal{T}^{[1]}$ is the vertex, due to the usual one body current; $\mathcal{T}^{[2]}$ is a sum of effective vertices, due to two body currents. To each meson exchange diagram corresponds a vertex, defined by a diagram of the type shown in fig. 5. Here the double arrows are renormalized single particle (or hole) propagators; the dashed lines indicate ingoing and outgoing states; $\gamma_{2,2}$ and $\gamma_{4,2}$ are the complete scattering amplitudes for the relevant number of ingoing and outgoing propagators.+) The calculation of expectation values of a two-body operator Q leads to a pictorially identical vertex diagram. One only has to replace the elementary electromagnetic vertex by a matrix element of the operator Q. In the arguments of the effective field $\mathcal{T}^{[2]}$ one has put $\varepsilon_{n_1}^{(i)} = \varepsilon_{n_4}^{(i)}$ which gives for the expectation value of Q in the state $n_1^{(i)}$ (see ref. 22) :

$$\langle Q \rangle_{n_1^{(i)}} = \left(Z_{n_1}^{(i)} \right)^2 \sum_{K=1}^{2} \mathcal{T}^{[K]}_{n_1^{(i)} n_1^{(i)}} \left(\varepsilon_{n_1}^{(i)}, 0 \right)$$

If virtual 2p-2h excitations are important, the evaluation of $\mathcal{T}^{[2]}$ is hopeless, because in that case a detailed knowledge of the amplitude $\gamma_{4,2}$ would be required. However, if the coupling between virtual p-h and 2p-2h excitations is weak (this corresponds to the case treated in detail in section 4.1 for the even-even nuclei), the main contribution to the effective vertex comes from diagrams of the type shown in fig. 6a and 6b.

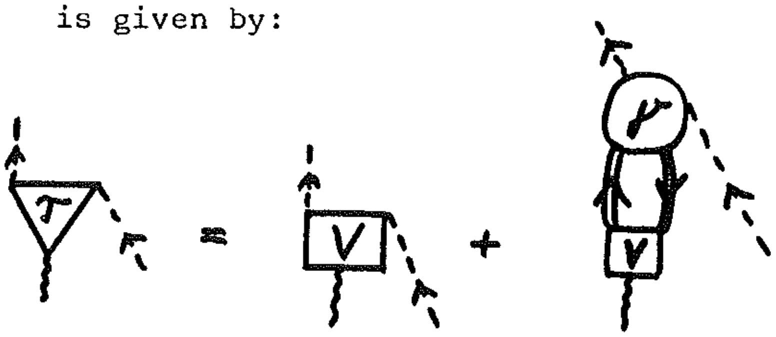
+) For brevity the bubbles $\gamma_{2,2}$ and $\gamma_{4,2}$ contain also the case of no interaction.

If we introduce an elementary interaction vertex through the definition



then the vertex $\mathcal{T}^{[2]}$, defined graphically by the symbol

is given by:



Here, in contrast to $\gamma_{2,2}$, γ contains only genuine interaction diagrams, the free propagation contribution being already included in the first term on the right hand side. For low lying states the equation for the effective field can be solved by known techniques [22] , just as the corresponding equation for $\mathcal{T}^{[1]}$.

We close this chapter in summarizing: If in the respective nuclear states the coupling between p-h and 2p-2h components is small, then the two body current contribution to electromagnetic ground state transition rates in even-even nuclei as well as to expectation values and electromagnetic transition rates in odd nuclei can be calculated with the same degree of accuracy as the usual one body current contributions.

5. Applications

In this section we discuss some examples for possible applications of our present approach to e.m. interactions of nuclear systems. In particular we concentrate ourselves on those cases, where we hope to resolve present discrepancies between experimental data and recent theoretical treatments.

1. Deuteron magnetic moment

Only the isoscalar part of the exchange current can contribute to the magnetic moment of the deuteron. The difference $\Delta\mu$ between $\mu_{exp.}$ and the theoretical value obtained from calculations without using any exchange current contribution (ECC) and a d -state probability $P_D \simeq$ 6,5 o/o determined from NN-scattering data +) amounts to be about 0.014 nuclear magnetons [23] . Different authors have discussed the effect of particular ECC's:

a) Adler and Drell [9], [1o] considered the ρ-π mixed ECC. With a partial width for the decay $\rho \rightarrow \pi\gamma$ of 0.5 MeV they could explain $\Delta\mu$. On the other hand an OPE-analysis of recent experimental data for the process $\gamma p \rightarrow \Delta^{++}\rho^-$ leads to $\Gamma(\rho^- \rightarrow \pi^-\gamma) <$ 0.24 MeV [24] . Therefore, we agree with Kisslinger [23] , that the ρ-π exchange current seems not to be the dominant effect.

b) Gersten and Green [25] considered the ECC arising by means of the substitution $\vec{p}_i \rightarrow \vec{\pi}_i$ from the momentum dependence of the OBE-potential. They conclude that sign and

---

+) P_D is not a direct measurable quantity, but it can be infered from the 3S effective range parameters by means of phenomenological potentials [8] .

magnitude of this contribution to $(\Delta\mu)_{tt}$. depend critically on the particular version of the used OBEP-model.

In a consistent approach the effects discussed in a) and b), the additional curl-terms one obtains from scalar boson exchange and the contribution of $\vec{j}^{\,I=0}_{t[2]}$ have to be treated additively. But, because of the smallness of $\Delta\mu$, it may happen that small relativistic corrections and (or) contributions of virtual N^{+}-excitations [23] which one neglects otherwise, are important in this case.

2. Elastic and inelastic electron-deuteron scattering at low q^2.

Any comparison of theoretical predictions with experimental data on elastic and inelastic e-d scattering depend on the values of the e.m. formfactors of nucleons. As neutron e.m. formfactors are just determined by means of e-d scattering experiments, only a comparative theoretical analysis of both elastic and inelastic e-d scattering will give reliable information on the importance of ECC's. A rough analysis along this line has been done by Dietz and Month [26] by using neutron formfactors from inelastic e-d scattering (analysed without ECC's) and a simple parametrization of ECC's in an analysis of elastic e-d scattering. They conclude, that there is an appreciable ECC to the charge and quadrupole form factors of the deuteron. A small ECC to the magnetic form factor cannot be excluded (compare [10]).

A discrepancy up to a factor of two has been reported by Adler [27] for e-d inelastic scattering. He uses a dipole formula for the nucleon form factors as input and considers also particular ECC's +).

+) Some criticisms on the treatment of ECC's by Adler we bring in connection with the deuteron photodisintegration.

3. Photo disintegration of the deuteron at low energies

Noyes [11] concluded from a careful analysis of the inverse reaction np- radiative capture at threshold that the experimental cross section data require a 1o% contribution from exchange currents. Recent calculations on this process by Adler, Chertok and Miller [9] based on the use of phenomenological wave functions and particular one-boson exchange contributions for $\vec{j}[2]$ have only lowered the discrepancy between theory and experiment to 5 % . In particular the following important contributions to $\vec{j}[2]^{I=1}$ +) have been neglected by these authors:

a) the $\sigma_1 - \sigma_1$ -exchange current (compare eq. (39)

b) the $\rho - \rho$ -exchange current (compare eq. (41 - 43))

c) $\vec{j}_t[2]^{I=1}$

In particular Adler claims [27] that the spin-independent term in the ρ $N\bar{N}$-coupling has no effect on the exchange current. Our result eq. (41) disagrees with this statement.

4. N-N- Bremsstrahlung

Baier and Kühnelt [14] considered recently ECC's to N-N-Bremsstrahlung in Born-approximation. They negelcted ECC's of the mixed type. Their theoretical predictions for the cross section are systematical too low at low energies.

+) The transition is mainly a $M1, |\Delta I| = 1$ -transition (compare the discussion and the references given by Adler et al. [9]) .

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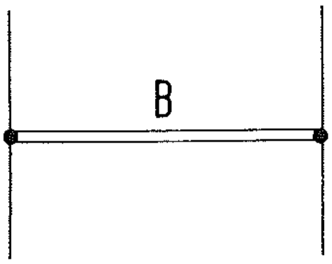
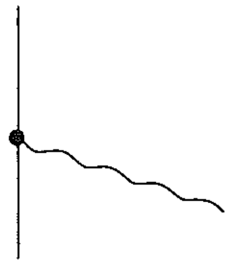
Figure captions

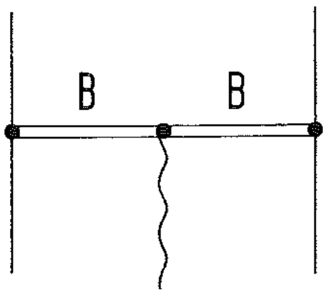
1a - 1f:	Interaction diagrams representing strong and electromagnetic terms in the two-particle Dirac equation.
2 :	Vertex and propagator corrections to the simple OBEP.
3a,3b :	Electromagnetic form factor effects due to the VDM.
4 :	Effective electromagnetic two-particle current vertex for p-h states in even-even nuclei.
5 :	General form of the effective two-particle current vertex for odd nuclei.
6a, 6b :	Effective two-particle current vertices for odd nuclei in the absence of (p,h)- (2p,2h) correlations.



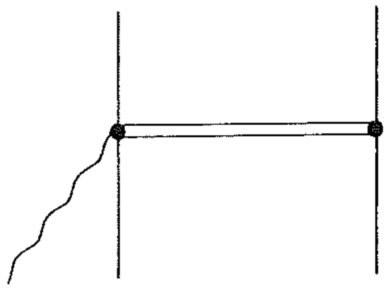
(1a)



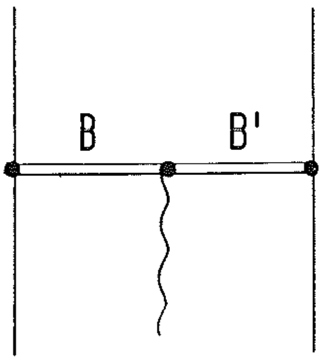
(1b)



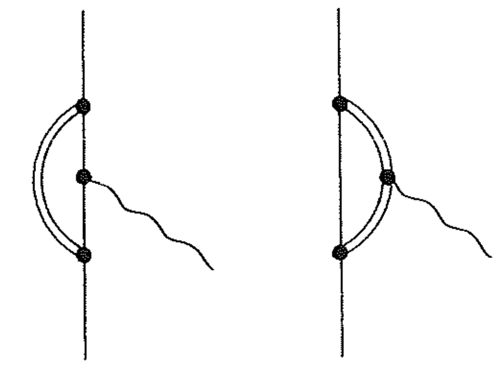
(1c)



(1d)



(1e)



(1f)

Fig.1

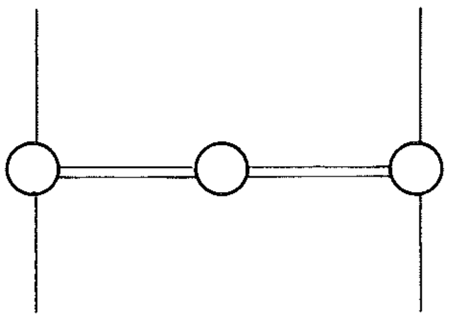
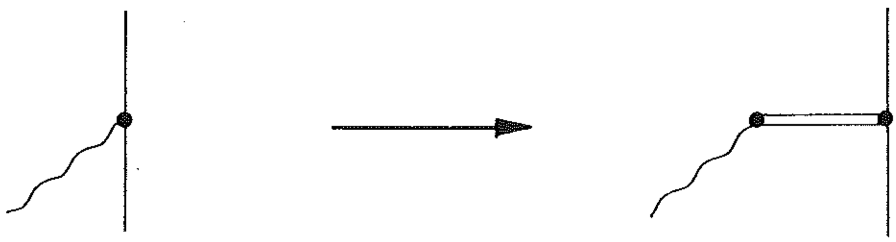
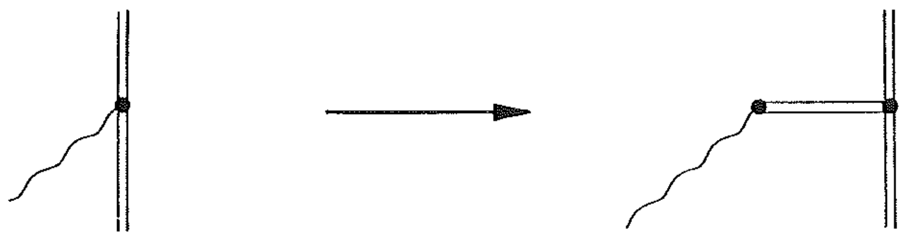


Fig. 2



(3a)



(3b)

Fig. 3

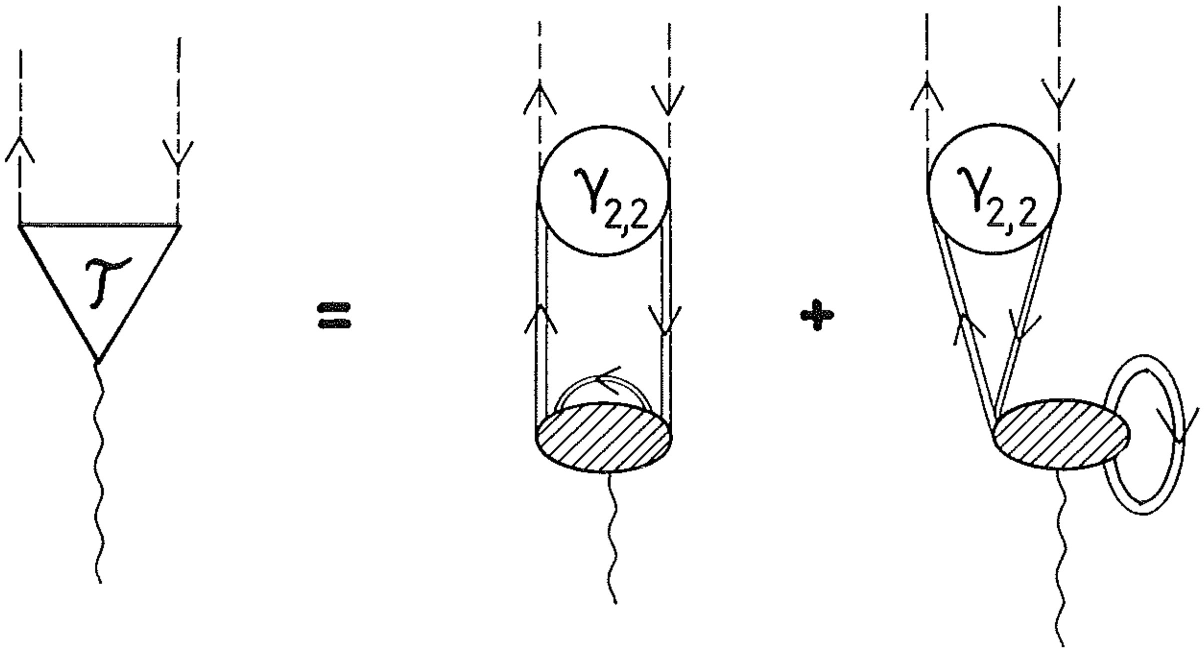


Fig. 4

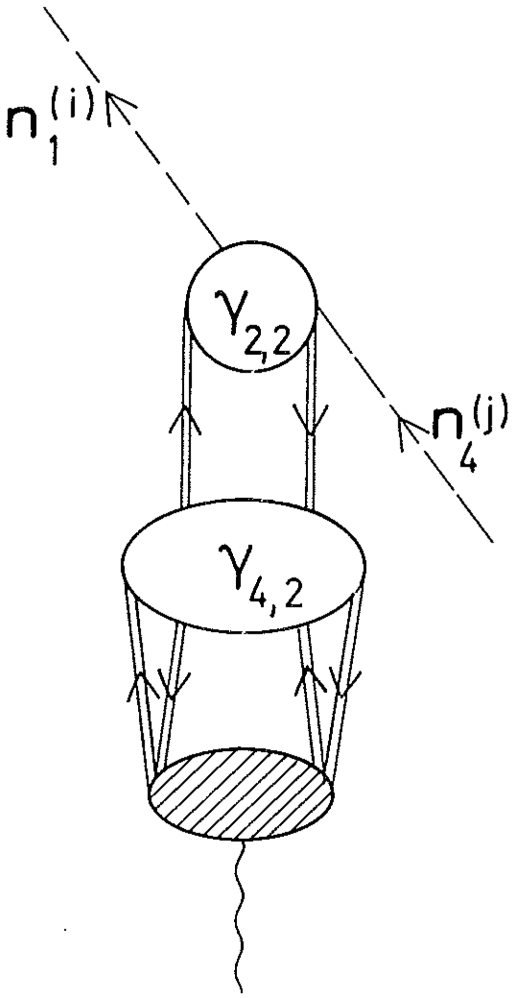
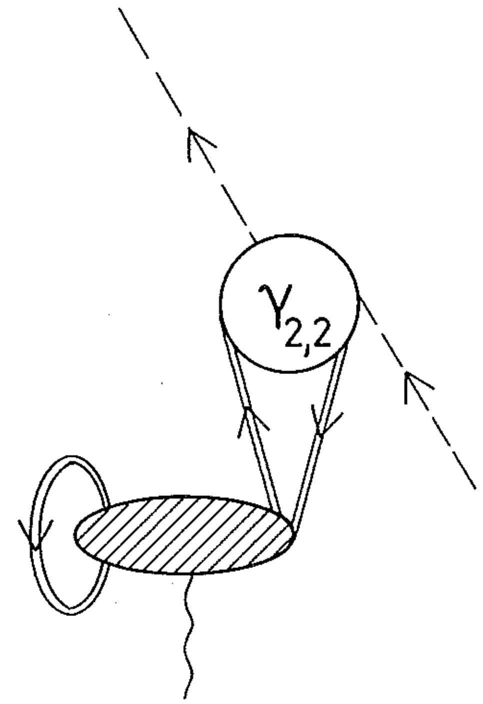
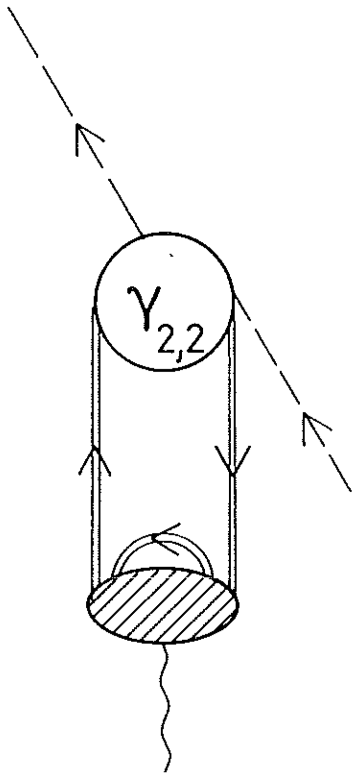


Fig. 5



(6a)



(6b)

Fig. 6