DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 70/2
January 1970

The Total Pair Production Cross-Section in Hydrogen and Helium

Part I - The Integration of the Jost, Luttinger and Slotnick Formula for σ_T

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The Total Pair Production Cross-Section in Hydrogen and Helium

Part I - The Integration of the Jost, Luttinger and Slotnick Formula for σ_T .

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Abstract

The total pair production cross-section is evaluated using the formula of Jost, Luttinger, and Slotnick, for the elements Hydrogen and Helium. The accuracy of the work is 0.1%.

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Introduction

The absorption of photons by pair production has been treated by several authors.[1] Regrettably the theoretically most accurate work, that of Jost, Luttinger, and Slotnick (JLS) has up to now, not been evaluated. The JLS calculation involves no approximations, and is good for all photon energies. The formula of Bethe and Heitler, neglects electron screening of the nucleus, and is therefore good only at small photon energies (below 50 MeV). The formula of Bethe has approximations of the order 1/k and is thus only good at high energies (greater than 10 GeV).

In this paper we present numerical evaluations of the JLS formula, for the cases of Hydrogen and Helium. The precision of this work is 1 part in 1000. In both the high energy, and low energy limits the values obtained agree with the Bethe and Bethe-Heitler results respectively providing a valuable check on the work. In the intermediate region of photon energies the JLS formula should provide the most accurate values for the total pair cross-sections currently available. This formula has been recently verified to ± 0.3% precision in the region 1 to 4 GeV photon energy.[2]

EVALUATION OF THE JOST, LUTTINGER AND SLOTNICK FORMULA

The Basic Formula

JLS, by a covariant calculation, utilizing the unitarity of the S matrix, obtain for the total pair production cross-section

$$\sigma(k_o) = \int_{K-(K^2-1)^{1/2}}^{K+(K^2-1)^{1/2}} dQ\ P(Q,K) \qquad (1)$$

with $K = k_o/2m$

k_o = incident γ energy in MeV

$m = m_e c^2$, the electron rest mass.

The integrand is

$$P(Q,\ K) = 2\ \frac{Z^2\ r_o^2}{137\ K^2}\ \left(\{1 - F(Q)\}^2\ \frac{I(Q,\ K)}{Q^3}\right) \tag{2}$$

where

Z = atomic charge

r_o = classical electron radius = $e^2/m_e c^2$

$F(Q)$ = coherent atomic scattering function

Q = momentum transfer in units of $2m_e c$

K = energy, in units of $2m_e c^2$

and finally, $I(Q,\ K)$ is given by JLS as:

$$I(Q,\ K) = (1 - 2Q^2)\ J_1 + (1 - 4Q^2 - 8QK + \frac{4Q^2 - 1}{3KQ})\ \times$$

$$\ell n\left[(y)^{1/2} + (y - 1)^{1/2}\right] + (3 + \frac{2K}{3Q} + \frac{2Q^2 - 1}{3KQ})\left(y(y - 1)\right)^{1/2}\ \times$$

$$+ \{-2(1 + Q^2) + \frac{2K^2}{3}\ (-4 + \frac{1}{Q^2})\} \times \left(\frac{1}{1 + \frac{1}{Q^2}}\right)^{1/2} \times$$

$$\ell n\ \frac{(1 + 1/Q^2)^{1/2} - (1 - 1/y)^{1/2}}{(1 + 1/Q^2)^{1/2} + (1 - 1/y)^{1/2}}\ . \tag{3}$$

With

$$J_1 = -R(1/Z\lambda) - R(\lambda/Z) + \frac{\pi^2}{6} + \frac{1}{2}[\ell n\ (\lambda)]^2$$

$$+ \frac{1}{2}(\ell n\ Z)^2 - (\ell n\ Z)\ (\ell n\ 8KQ)$$

$$Z = [(y - 1)^{1/2} + y^{1/2}]^2$$

$$\lambda = [Q + (Q^2 + 1)^{1/2}]^2$$

$$y = 2KQ - Q^2$$

$$R = R(t) = \int_o^t \ell n(1 + x)\ \frac{dx}{x}$$

The Approximate Expression for R(t)

We see that the expansion of the integrand of R(t) for small x and integration yield the formula

$$R(t) = t - t^2/4 + t^3/9 - t^4/16 + \text{etc.} \qquad (4)$$

By evaluating Eq.(1) for 2, 3 and 4 terms in R(t), it was possible to determine the error introduced by the approximate R equation. These results are presented in Table I. The error in the use of R(t) will be the dominant error in the value of σ_T at low photon energies, where a value of as much as 5% error could be obtained at 2 MeV. However, above 4 MeV the error introduced is less than 1%, and above 20 MeV less than 0.1%. Thus we are justified in the use of a 4-term equation for R(t) in the subsequent work; bearing in mind the precision quoted above.

The Approximate Expression for I (Q, K) when Q << K

In the region of $Q \ll K$, JLS give an approximate formula:

$$I\,(Q,\,K) = (1 - 2Q^2)\,J_1 + \frac{1}{2}\,(1 - 4y - \frac{2}{3y})\,\ell n\,Z\, +$$

$$+\,\frac{y^2(1 - 1/y)^{1/2}}{3}\,\left[11 - \frac{13}{3}\,(1 - \frac{1}{y}) - 2(1 - \frac{1}{y})^2\right] \qquad (5)$$

This is of value in checking any numerical evalutation of I(Q, K) as formula (3) is difficult to compute accurately for $Q \ll K$ due to the cancellation of terms. By evaluation of both Eq.(3) and (5) over the entire Q range using 16 significant figures in each calculation, it was determined that for Q of the order of 4/K that Eq.(3) and (5) agreed to at least 5 significant figures. Thus Eq.(3) only was used in a numerical integration of Eq.(1) with full confidence it accurately represents the JLS recoil momentum distribution.

Tables of σ_T from the JLS Formula

The end result of this work is to prepare tables of the cross-section predicted by the JLS formula for various energies, along with an indication of the calculational error. In order to make a realistic comparison with other formulae and with experimental data we include the screening effect, in particular we shall use the exact Hydrogen atom screening function, and two different screening functions for Helium. These are the radially correlated and uncorrelated wave functions explained in detail in Reference (3).

The Equation (1) was integrated numerically by applying the definition of the integral as a summation, and increasing the number of terms in the summation until the precision was at least 1 part in 1000 for the value σ_t . An equal number of steps were taken in variable Q for Q values below 1 MeV/c, and above this point. This is because the integrand is roughly constant up to Q = 1 MeV/c, but falls approximately as $1/Q^2$ beyond 1 MeV/c. Figure 1 presents the error with step no. for various cases. Table II presents the values of σ_T at various energies, for atomic Hydrogen and Helium. It is interesting to note that at high energies the JLS evolution agrees with a value of σ_T obtained for the case of coherent and incoherent atomic Hydrogen screening, respectively obtained from the integrated Bethe formula.[2] The precision of the comparison is 1 part per 1000.

The results of the Bethe formula, which is exact in the high energy limit, were obtained in two different ways, once by exact evaluation of the Bethe formula at 10^9 MeV by computer calculation, and once by an analytical integration of the Bethe formula in the high energy limit. Both methods agreed to much better than one part per thousand. At lower energies the JLS and Bethe formulae give different results. As the Bethe formula is not expected to be highly accurate at lower energies we attribute this error solely to the Bethe formula. In fact the error goes approximately as 1/k as mentioned by Bethe. At very low energies the screening terms are entirely negligible and comparison of the JLS with the integrated Bethe-Heitler formula without screening can be done. To within about .1% at

3 MeV these formulae do agree. We thus conclude that the JLS formula provides a method of obtaining reliable total cross section values. The Table II is useful, however, only to compare theoretical predictions. Certain small corrections have yet to be discussed which are necessary for comparison with experimental data. Part II of this paper will be devoted to the discussion of these points and will allow total cross sections to be computed to approximately 0.3% precision for comparison with experimental data. However above 1 GeV photon energy, the given σ_T values are precise to 1% if the radiation correction (computed by Mork and Olsen[4]) of + 0.9% of σ_T is added to the values of Table II.

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TABLE I

CROSS SECTION EVALUATED FOR THE CASE OF COHERENT PAIR PRODUCTION IN HYDROGEN

(Exact atomic wave function used in screening correction.) The Jost et al formula with 2, 3 and 4-term expansion of R(t) are compared. $\Delta(ij) = |\sigma(i \text{ term } R) - \sigma(j \text{ term } R)|$.

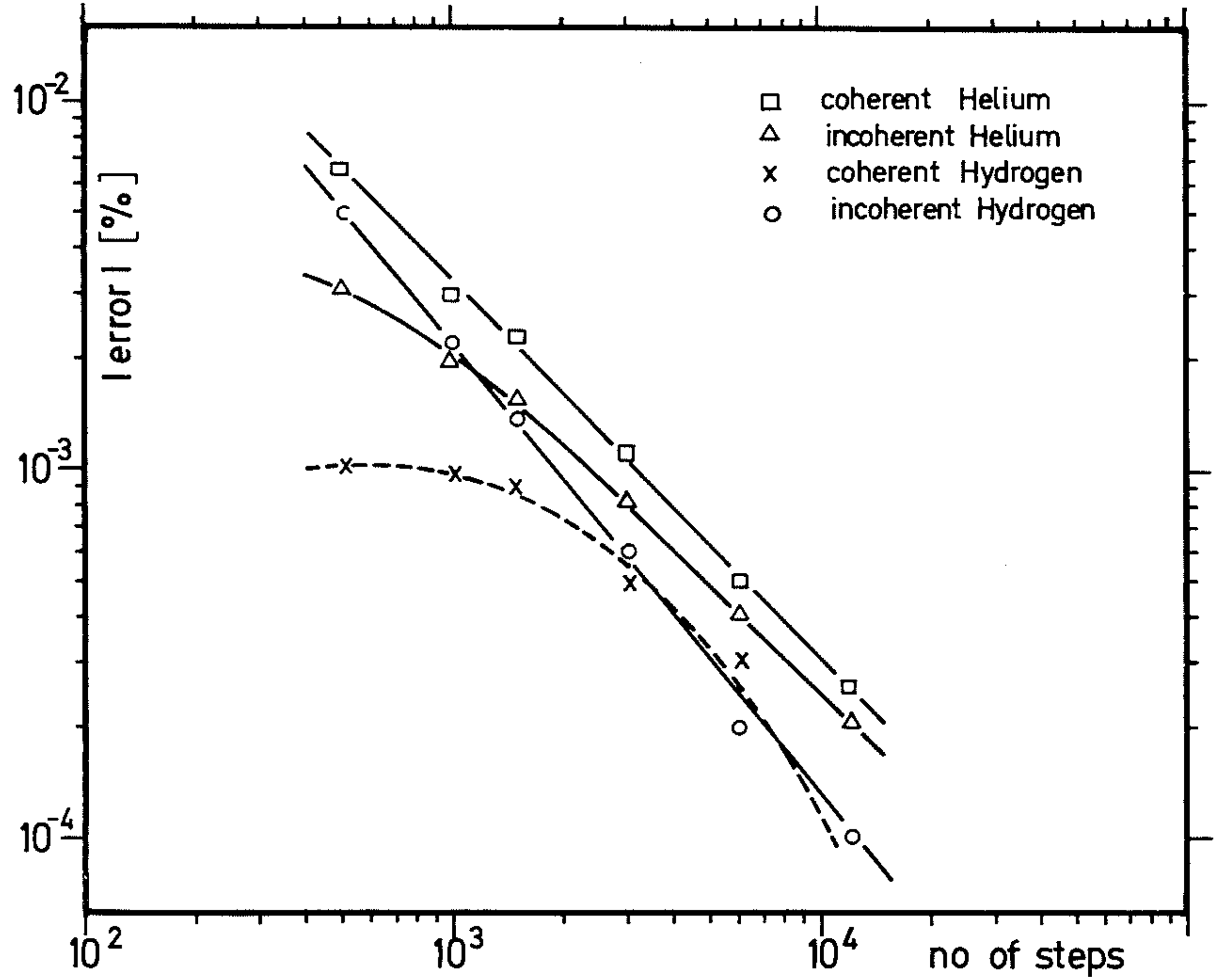
Energy MeV	$\Delta(32)/\sigma(3)$	$\Delta(43)/\sigma(4)$
2	.08	.05
4	.02	.007
6	.01	.004
10	.005	.002
20	.003	.001
40	.002	.0008
100	.0006	.0002
1000	$<10^{-6}$	$<10^{-6}$

Table Captions

Table II: Total cross section in millibarns for Hydrogen (H) [T = total cross section = coherent (C) + incoherent(I)] and Helium (He1) correlated wave function, and (He2) uncorrelated wave function. K is photon energy.

Figure Caption

% error on step size for integration of JLS formula for various cases.



10^-2
10^-3
10^-4
| error | [%]
10 2
10^3
10^4
no of steps
coherent Helium
incoherent Helium
coherent Hydrogen
incoherent Hydrogen

TABLE 2

units=millibarns/atom

k MeV	σ_T H	σ_T He(1)	σ_I He(2)	σ_C H	σ_I H	σ_C He(1)	σ_I He(1)	σ_C He(2)	σ_I He(1)
2.0	0.394	1.183	1.183	0.197	0.197	0.789	0.394	0.789	0.394
3.0	1.011	3.034	3.034	0.506	0.506	2.023	1.011	2.023	1.011
4.0	1.630	4.890	4.890	0.815	0.815	3.260	1.630	3.260	1.630
5.0	2.189	6.566	6.566	1.094	1.094	4.377	2.189	4.377	2.189
6.0	2.685	8.056	8.056	1.343	1.343	5.371	2.685	5.371	2.685
7.0	3.129	9.386	9.386	1.564	1.564	6.257	3.129	6.257	3.129
8.0	3.527	10.582	10.582	1.764	1.764	7.055	3.527	7.055	3.527
9.0	3.889	11.666	11.666	1.944	1.944	7.777	3.889	7.777	3.889
10.0	4.219	12.656	12.656	2.109	2.109	8.437	4.219	8.437	4.219
20.0	6.509	19.521	19.522	3.254	3.254	13.012	6.509	13.013	6.509
30.0	7.909	23.707	23.711	3.954	3.955	15.798	7.910	15.801	7.910
40.0	8.917	26.703	26.710	4.458	4.460	17.784	8.918	17.792	8.919
50.0	9.704	29.018	29.030	4.850	4.854	19.312	9.706	19.323	9.707
60.0	10.348	30.889	30.907	5.170	5.178	20.539	10.350	20.555	10.352
70.0	10.892	32.449	32.470	5.440	5.452	21.554	10.894	21.574	10.897
80.0	11.361	33.775	33.801	5.671	5.690	22.412	11.364	22.434	11.367
90.0	11.774	34.923	34.952	5.873	5.900	23.148	11.775	23.172	11.780
100.0	12.140	35.928	35.960	6.052	6.088	23.788	12.141	23.814	12.146
200.0	14.447	41.894	41.928	7.142	7.305	27.468	14.426	27.494	14.433
300.0	15.651	44.744	44.764	7.677	7.974	29.140	15.604	29.158	15.606
400.0	16.415	46.459	46.465	8.001	8.414	30.114	16.344	30.125	16.340
500.0	16.949	47.619	47.613	8.220	8.729	30.759	16.860	30.765	16.848
600.0	17.347	48.464	48.447	8.380	8.967	31.221	17.242	31.222	17.225
700.0	17.657	49.109	49.084	8.502	9.156	31.570	17.539	31.567	17.517
800.0	17.907	49.621	49.588	8.598	9.309	31.844	17.777	31.837	17.750
900.0	18.112	50.037	49.997	8.676	9.436	32.065	17.973	32.056	17.942
1000.0	18.284	50.384	50.338	8.741	9.543	32.247	18.137	32.236	18.102
2000.0	19.181	52.138	52.058	9.068	10.113	33.152	18.985	33.128	18.929
3000.0	19.545	52.827	52.732	9.195	10.350	33.499	19.328	33.470	19.262
4000.0	19.747	53.204	53.100	9.264	10.483	33.686	19.518	33.654	19.446
5000.0	19.878	53.444	53.335	9.308	10.570	33.804	19.640	33.770	19.564
6000.0	19.970	53.612	53.498	9.339	10.631	33.886	19.726	33.851	19.647
7000.0	20.038	53.737	53.620	9.362	10.676	33.947	19.790	33.911	19.709
8000.0	20.091	53.833	53.713	9.379	10.712	33.994	19.840	33.957	19.757
9000.0	20.134	53.910	53.788	9.393	10.741	34.031	19.879	33.993	19.795
100000.0	20.503	54.565	54.425	9.511	10.992	34.342	20.223	34.299	20.126
1000000.0	20.552	54.649	54.506	9.526	11.026	34.380	20.268	34.337	20.169
10000000.0	20.559	54.660	54.516	9.528	11.030	34.385	20.274	34.341	20.175
100000000.0	20.559	54.661	54.517	9.528	11.031	34.386	20.275	34.342	20.175
1000000000.0	20.559	54.661	54.518	9.529	11.031	34.386	20.275	34.342	20.176

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DESY 70/3
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Part II - Correction to the JLS value for σ_T

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Abstract

Comparison of the Jost, Luttinger and Slotnick formula for the total pair production cross section to more recent theoretical work by Maximon, (for total coherent pair production), and by Mork (for total incoherent pair production) is presented. If one neglects the effect of atomic screening then the coherent and incoherent cross sections will be identical except for small corrections important only at low photon energies. The result of the comparison shows that the Jost, Luttinger and Slotnick formula (for coherent production) agrees with the formula of Maximon to better than 1 part per 1000 above 5 MeV. A very simple approximate formula for the total coherent (unscreened) pair production cross section is given. Finally a comparison of the difference between the Mork calculation for the unscreened incoherent cross section with the JLS formula gives the correction term that must be applied to the JLS formula in the incoherent case (called the retardation correction). The addition of screening correction and the retardation correction to the JLS formula then gives the most precise value for the total pair production . Comparison with recent experimental data shows good agreement.

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Introduction

In part I of this paper the formula of Jost, Luttinger, and Slotnick[1] for the total cross-section for pair production was evaluated for the first time. It was shown that the values obtained were in good agreement with previous calculations of Bethe and of Bethe and Heitler in restricted energy regions. It is also possible to make a comparison with a recently derived formula of Maximon[2] (which does not include screening) and this will be done in the first section of this paper. Our second task will be to describe the various corrections that are necessary to be added to the JLS cross section to make this cross section directly comparable with experimental data. In particular at low energies a formula for the correction due to the retardation of the electron's field in incoherent pair production will be given.

1. Comparison of the JLS Formula to the Maximon Formula. (Without Screening)

Recently Maximon has computed the total pair production cross section in the Born approximation in a way involving no approximation of high energy behaviour.[2] His work however does not include screening (an effect due to the atomic electrons) a correction that will be fairly important as the photon energy increases above about 100 MeV. In principle his work and that of JLS should agree for all energies - if screening is neglected in the JLS case. As we have mentioned in Part I, up to now no evaluations of the JLS formula were available for such a check. In order to ascertain the correctness of these formulae, the JLS prescription was integrated with no screening correction over a large range of photon energies. The integration was carried out in a similar way to that mentioned in Part I. Table I presents the results of the JLS evaluation with no screening. The values up to 100 MeV are precise to .1%, and values up to 1000 MeV are precise to 1%. Also shown in Table I are the values of Maximon. The following comments can be made about the comparison.

a) For 3 MeV and above the JLS and Maximon values are in excellent agreement, consistent with the error in the numerical evaluation.

b) Below 3 MeV the JLS and Maximon formulae differ - this may be entirely attributable to the use of an approximate formula for R(t) in the JLS formula. As was discussed in Part I this introduces an error at very low photon energies.

Thus we can conclude that the evaluations of the two prescriptions for pair production give identical results. This confirmation of the JLS formula is important as it involves a comparison of two independently derived formulae, both without energy dependent approximations. In Part I it had been possible only to check the JLS formula at the two ends of the range of photon energies.

2. Simple Formula for the Total Cross Section in the High Energy Limit (Without Screening)

In the high energy limit and in the case of no screening, the formula for pair production can be evaluated by analytical integration, and thus an exact expression for the total cross sections can be obtained. Two such formulae have been calculated by Sorenssen,[2] and by Mork[3] - who also includes the effect of retardation (i.e. pair production in the field of an electron). In the Section 3 we shall study the retardation effect. For our present purposes we need only note that as $k \to \infty$ the retardation effect vanishes. Then in this limit the Mork formula as well as the Sorrensen formula (derived with no retardation terms, but with approximations good only at high energy) agree exactly, and this result should also agree with the JLS cross section with no screening. These formulas can be further simplified in a straightforward way when k is large[4] and we present here only the final result - namely both the Mork and Sorenssen formulas reduce to:

$$\begin{aligned} \sigma_T(\text{mbarn}) &= 1.80\,\ell n\, k - 3.43 \quad k \text{ in } m_e c^2 \text{ units} \\ &= 1.80\,\ell n\, k - 2.22 \quad k \text{ in MeV units.} \end{aligned} \qquad (1)$$

Such a simple formula would not be expected to be too precise, however Table II presents the result of an evaluation of formula (1). Comparing with Table I, Column 2, the precise JLS evaluation, we find agreement to much better than 1% above 50 MeV. Formula (1) is expected to be increasingly more precise as k increases and is thus quite useful as a prediction of the total unscreened pair production cross section (the total cross section for a proton without atomic electron).

3. Calculation of an Approximate Formula for the Retardation Correction

Since a simple formula for σ_T can be quite close to the best theoretical estimates (when no screening is involved) a way is suggested to compute an approximation to the JLS formula to correct for retardation effects.

First let us briefly discuss the problem. The JLS formula is a correct prescription for pair production in the nuclear field. When the atomic electrons are considered there are two results

a) the nuclear pair production is reduced by atomic electron screening
b) the field of the atomic electrons themselves cause the photons to produce electron positron pairs.

Wheeler and Lamb and later Suh and Bethe showed that a form factor for electron field production replaces the nuclear form factor (describing nuclear shielding by electrons) in the formula for pair production.[4,5] This prescription when applied to the JLS formula is incomplete since the JLS formula does not contain any description of three types of effects that occur when a pair is produced in the field of an electron. These are exchange effects, compton (γ-e) effects and retardation effects. Mork has made an extensive study of these processes and concluded that exchange, and γ-e interactions decrease very rapidly in importance above threshold and are only 2% at ~7 MeV.[3] Retardation however has a very much larger effect. To detemine its magnitude we subtract the Mork formula for pair production with retardation, from the prediction of pair production alone (as given by Sorenssen). (Both containing no screening, a multiplicative effect that does not affect our present calculation). Our notation is such that

σ_T = pair production total cross section
$\sigma_R = \sigma_T$ + Retardation correction
ω = photon energy in $m_e c^2$ units.
σ_0 = .579 mbarn.

$$\sigma_T = \frac{2\sigma_0}{\omega^3} \{[- (\tfrac{7}{9})\omega^3 + 2\omega^2 + \tfrac{8}{9}] (1 + 2\ln \tfrac{\omega}{2}) +$$

$$[+ (\tfrac{28}{9}) \omega^3 - 4\omega^2 + \tfrac{8\omega}{3} - \tfrac{16}{9}] \ln(\omega - 1) +$$

$$[- (\tfrac{88}{27}) \omega^3 + \tfrac{64}{9} \omega^2 - \tfrac{16}{9} \omega + \tfrac{32}{27}]\}$$

σ_T becomes keeping terms of order ω^{-1} and higher,

$$= \frac{2\sigma_0}{\omega^3} \{[-(\frac{7}{9})\omega^3 + 2\omega^2][1 - 2\ell n 2 + 2\ell n\,\omega] +$$

$$[(\frac{28}{9})\omega^3 - 4\omega^2]\ell n\,\omega + [-(\frac{88}{27})\omega^3 + \frac{64}{9}\omega^2]\} .$$

Mork gives, for the total cross section with retardation effects included to the same approximation, i.e., terms in ω^{-1} included,

$$\sigma_R = \sigma_0\{3.111\ \ell n\ 2\omega - 8.074 - [1.333\ (\ell n\ 2\omega)^3$$

$$- 3(\ell n\ 2\omega)^2 + 6.84\ \ell n\ 2\omega - 21.51]\ \omega^{-1}\} .$$

We then form the normalized difference

$$\frac{\sigma_T - \sigma_R}{\sigma_T} = \{(\frac{82}{9} - 4\ \ell n\ 2) + 1.333\ G^3 - 3G^2$$

$$+ 6.84\ G - 21.51\} \times \frac{1}{\omega(3.111\ G - 8.014)} = \Delta \qquad (2)$$

with $G = \ell n\ 2\omega$ and ω is in units of $m_e C^2 = 1$.

Using the above formula we have constructed Table III. The retardation effect above 1 GeV is less than 2%. This is interpreted of course, as 2%/(Z + 1) error in the total cross section (or 1/Z in relation to the coherent case). By the use of this formula, we can make this correction quite accurately.

By noting that lim $k \to \infty$, $\Delta \to 0$ we see that these results confirm the arguments of Suh and Bethe[5] that at sufficiently high energies the expression for triplet and for pair production should be identical (the screening terms are of course still different). Alternately, the conclusion of Joseph and Rohrlich[6] that about a -8% correction existed even at the highest energies is shown to be invalid. In Ref.4, the error in the Joseph and Rohrlich argument is shown. When their arguments are corrected, one arrives at the identical formulation presented here.

In order to check Equation (2) which contains approximations good only at large energies the explicit evaluation of the full Mork formula (taken from Ref.3) was subtracted from the results of the JLS formula (both with no screening) presented here. Table III shows this exact calculation of the difference, as well as the results of Equation (2). The results indicate that Equation (2) is capable of giving results precise to 2% in σ_T (unscreened) above 50 MeV (note 2% difference in σ_T(unscreened) yields 1% difference in σ_T(Hydrogen)).Table IV presents an evaluation of Eq.(2) in MeV units.

4. Radiative and Other Corrections

The radiative correction to the total pair production cross section has been calculated by Mork and Olsen.[7] They give the formula

$$\sigma_{Rad} = (.93 \pm .05)\% \ \sigma_T$$

$$\sigma = \sigma_T + \sigma_{RAD}$$

This is good in the limit of complete screening i.e. high energies. In the case of no screening, i.e. low energies the correction is a little larger being 1.12% at 15 MeV photon energy. For the purpose of providing accurate cross sections above 50 MeV one may add simply 0.93% to the total pair cross section as a very close approximation.

In addition to the corrections mentioned there is an uncertainty in the case of Hydrogen as to the effect in the screening calculation of the molecular form. The value of this correction term depends sensitively upon the particular wave function of Hydrogen molecule used. In a test of several molecular forms the author found that the correction was consistent with 0 ± .5% for a selection of several models. Further details of this correction will be published later.

5. Conclusion

In conclusion, it is possible now to present a table of total cross section values for Hydrogen and Helium that will be precise to about ± 0.5% for Hydrogen, and ±0.3% for Helium at all energies. These cross sections are computed by the following prescription.

$$Z^2\sigma_T(\text{JLS, with coherent screening}) +$$
$$Z\{\sigma_T(\text{JLS, with incoherent screening}) - \text{Retardation correction}\} +$$
$$\text{Radiative Correction} = \sigma$$

The values σ_T(JLS) were presented in Part I of this work, and the retardation and radiative corrections are discussed in Part II. Using this prescription Table V has been constructed for selected photon energies. More significant figures than are justified are carried to the final answer in order to show how each correction is computed. Numbers from this table then represent the correct cross section for pair production in Hydrogen (± .5% accuarcy) and in Helium (± .3%). These numbers are useful in obtaining photon flux values in a number of high energy reactions. It should be noted that recent experimental work at DESY has obtained values for the total cross section for pair production in Hydrogen and Deuterium[8] (which should have the identical cross section to Hydrogen except for possible differences in the molecular wave function of Deuterium). Table VI and Figure 1 shows a comparison of these numbers, with the theoretical calculations presented here. In general the Hydrogen and Deuterium data are the same within statistical errors, showing to the level of about 1.0% in the total cross section that the molecular correction differences between the two different states are not observed. Comparison with the theoretical calculations presented here shows that the data tends to give a slightly larger cross section, by about .2 mb, or 1% on average, than theory would predict. Since this is only about one standard deviation, one must regard the comparison as showing agreement between experiment and theory. Refinement of the DESY measurements could in principle allow a .1% absolute measurement[4].

Summary

The theory of Quantum Electrodynamics can be used to make a very precise estimate of the total cross section for photon interactions which produce electron positron pairs. At high energies this is the predominant reaction contributing to the attenuation of photons. The most complete calculation of the pair production cross section had never been integrated nor numerically evaluated to compare with experiment. In Part I of this paper this formula was integrated including screening corrections necessary when applying the result to measurement in Hydrogen and in Helium. In this form the values so obtained were found to agree with two approximate cross section expressions, at the limits of very low and very high energy for the photon. In Part II the JLS values for no screening were compared to recent calculations of Maximon and found to agree exactly. In addition to screening two other small corrections are considered. For pairs produced in the field of an atomic electron an additional correction to the JLS formula, (which assumed a heavy nucleus for recoil) is necessary (retardation correction). By using an expression for the unscreened total cross section with this correction (due to Mork) and subtracting a formula σ_T without this correction (both good at high energies) we were able to obtain a formula for the retardation correction, that agreed well with the difference of values given by the unscreened atomic electron formula for pair production (also due to Mork) and the JLS formula (both unscreened but good at all energies) The radiative correction has been already calculated and was applied here. The cross section for Hydrogen and Helium are given in a corrected form. Recent experimental data on Hydrogen confirms the cross sections presented here but only to ±1.0%. For other elements another experiment found agreement to 0.3%.[4] The JLS total pair production cross section has been verified by both theoretical and experimental cross-checks, and represents the most precise way to compute the total pair production cross section presently available.

Table VII shows the corrected values of σ_T for Hydrogen and Helium at various energies. The errors an ± 0.5%, and ±0.3% respectively, arising mainly from uncertainties in the screening corrections. These numbers are the best estimates currently available for the total pair production cross sections in Hydrogen and Helium.

TABLE I

Total Cross Section for Pair Production (no screening)
For a Z = 1 nucleus, with no atomic electrons. ϕ = 0.57938 mbarn

MeV Photon Energy	JLS mb	JLS ϕ units	Maximon ϕ units
2	0.197	0.340	0.3030
3	0.506	0.873	0.8716
4	0.815	1.407	1.416
5	1.094	1.889	1.900
6	1.343	2.318	2.328
8	1.764	3.044	3.052
10	2.110	3.641	3.647
20	3.255	5.617	5.618
50	4.855	8.380	8.377
100	6.092	10.514	10.511
200	7.336	12.662	12.659
500	8.986	15.510	15.507
1000	10.247	17.685	17.662

TABLE II

Evaluation of $\sigma_T(\mathrm{mb}) = 1.80\,\ell n\, k - 2.22$ (k in MeV)

k(MeV)	σ_T(mb)
10	1.93
50	4.83
100	6.084
200	7.333
500	8.985
1000	10.253
5000	13.14
10000	14.39

TABLE III

$k(m_eC^2$ units)	Mork σ_T mb	JLS σ mb	δ mb	(δ/JLS)%	Equation (2) Δ %
50	2.77	3.68	0.91	24.7	26.3
60	3.12	3.99	.87	21.8	23.0
70	3.43	4.26	.83	19.5	20.6
80	3.69	4.50	.81	18.0	18.7
90	3.93	4.71	.78	16.6	17.2
100	4.14	4.89	.75	15.3	16.0
200	5.56	6.13	.57	9.3	9.6
300	6.38	6.86	.48	7.0	7.1
500	7.40	7.77	.37	4.8	4.9
1000	8.76	9.03	.27	3.0	2.9
5000	11.84	12.03	.19	1.6	0.8

TABLE IV

Evaluation of Formula (2)

k (MeV)	Δ %
20	31
30	23
50	16
70	13
100	9.8
120	8.6
140	7.6
160	6.9
200	5.9
300	4.3
400	3.5
500	2.9
750	2.1
1000	1.7
2000	.98
4000	.56
10000	.27

TABLE V

Cross Sections in mb/atom

k MeV	σ_T(JLS,H) (coherent)	σ_T(JLS,H) (incoherent)	Retardation correction	Radiation Correction	σ mb/atom
50.0	4.850	4.854	− 0.777	+ .089	9.0
100.0	6.052	6.088	− 0.596	+ .115	11.7
200.0	7.142	7.305	− 0.431	+ .140	14.2
400.0	8.001	8.414	− 0.295	+ .161	16.3
1000.0	8.741	9.543	− 0.162	+ .182	18.3
α	9.529	11.031	0	+ .206	20.7

[Correlated wave function used for He]

	$Z^2\sigma_T$(JLS, He) (coherent)	$Z\sigma_T$(JLS, He) (incoherent)			
50.0	19.312	9.706	− 1.562	+ .275	27.70
100.0	23.788	12.141	− 1.218	+ .347	35.10
200.0	27.468	14.426	− 0.852	+ .410	41.40
400.0	30.114	16.344	− 0.572	+ .459	46.30
1000.0	32.247	18.137	− 0.308	+ .508	50.50
α	34.386	20.275	0	+ .547	55.20

TABLE VI

Photon Energy GeV	Theory σ_T mb	Hydrogen σ_T mb	Deuterium σ_T mb	Average of H_2 and D_2
0.55	17.1	15.54 ± 0.5		15.54 ± 0.5
0.87	18.0	17.52 ± 0.8		17.52 ± 0.8
1.18	18.6	18.63 ± 0.9		18.63 ± 0.9
1.46	18.9	18.91 ± 0.20	18.89 ± 0.18	18.90 ± 0.14
1.98	19.2	19.06 ± 0.33	19.70 ± 0.23	19.38 ± 0.20
2.55	19.5	19.61 ± 0.28	19.62 ± 0.26	19.62 ± 0.20
2.99	19.7	19.57 ± 0.30	20.60 ± 0.19	20.08 ± 0.20
3.46	19.8	19.70 ± 0.24	19.97 ± 0.23	19.84 ± 0.17
3.98	19.9	20.02 ± 0.30	20.49 ± 0.21	20.26 ± 0.20
4.55	19.9	20.19 ± 0.33	20.34 ± 0.25	20.27 ± 0.20
4.99	20.0	19.58 ± 0.18	20.28 ± 0.15	19.93 ± 0.10
5.46	20.1	19.90 ± 0.25	20.25 ± 0.15	20.07 ± 0.15
5.98	20.1	20.17 ± 0.21	20.34 ± 0.20	20.25 ± 0.15
6.55	20.2	20.50 ± 0.24	20.76 ± 0.18	20.63 ± 0.15

Table VII

Corrected Total Cross Sections

Cross sections in mb/atom

Photon Energy k (MeV)	σ_T Hydrogen	σ_T Helium (Correlated w.f.)
100.0	11.66	35.1
150.0	13.15	39.0
175.0	13.69	40.3
200.0	14.15	41.4
300.0	15.45	44.5
400.0	16.28	46.3
500.0	16.85	47.6
600.0	17.28	48.5
700.0	17.62	49.2
800.0	17.88	49.7
900.0	18.10	50.2
1000.0	18.29	50.5
1250.0	18.65	51.3
1500.0	18.91	51.8
1750.0	19.11	52.1
2000.0	19.26	52.4
3000.0	19.65	53.2
4000.0	19.87	53.6
5000.0	20.02	53.9
8000.0	20.25	54.3
10000.0	20.33	54.4

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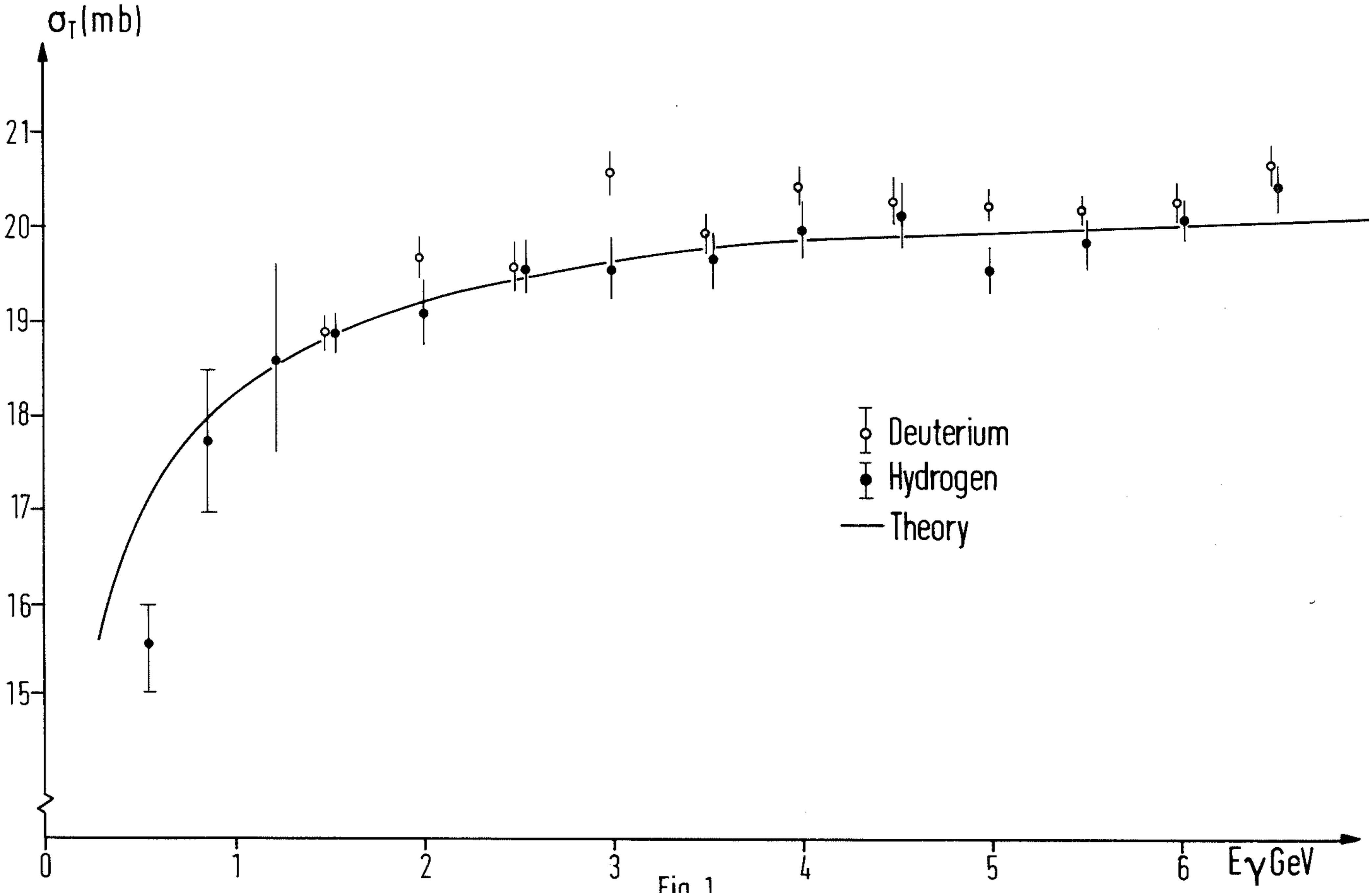
σ_T (mb)
21
20
19
18
17
16
15
Deuterium
Hydrogen
Theory
0
1
2
3
4
5
6
EγGeV
Fig. 1