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A Simple Interpretation of Inelastic Electron-Nucleon Scattering

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A SIMPLE INTERPRETATION OF INELASTIC ELECTRON-NUCLEON SCATTERING

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Abstract: The main features of the deep inelastic scattering of electrons by neutrons and protons can be explained by a simple model of the nucleon consisting of partons. It turns out that the best results are obtained if the nucleon is assumed to be a linear combination of states with three quarks and a varying number of quark-antiquark pairs with the usual fractional charges.

The following model for inelastic electron-nucleon scattering is not original. It tries to express in simple terms ideas put forward by Feynman[1], Bjorken[2], and many others.

We assume a very fast-moving nucleon of momentum $P \simeq \gamma M$, $\gamma \gg 1$, to be a linear combination of states with different numbers N of partons or antipartons

$$\Psi_{nucleon} = \sum_N c_N \Psi_N + c_0 \Psi_0 \qquad (1)$$

where Ψ_N is a state with N freely moving partons or antipartons, and Ψ_0 is a state which cannot be so described. The total momentum in each of these states is always P so that in every state Ψ_N the sum of all parton momenta p_i is equal to P. The mass of a parton and its transverse momentum Q is assumed to be very roughly of the order M (mass of the proton). To be concrete, let us imagine that the states Ψ_N are states of three quarks plus Z quark-antiquark pairs, where $Z = 0, 1, 2 \ldots$ with $N = 3 + 2Z$. The state Ψ_0 is added in (1) in order to express the fact that the quark-model may not be a complete description of what is going on in the nucleon.

What is the probability $|c_N|^2$ of a particular state Ψ_N ? In order to estimate it, let us determine the energy difference Δ_N between the energy $\sum_i^N (p_i^2 + M_i^2 + Q_i^2)^{1/2}$ of the supposedly freely moving partons in the state Ψ_N and the energy $(P^2 + M^2)^{1/2}$ of the nucleon. Let us put approximately $p_i \simeq \frac{P}{N}$, $M_i^2 + Q_i^2 \simeq M^2$, and we get $\Delta_N \simeq (N^2-1) M/\gamma$, if $\gamma/N \gg 1$. This energy can be interpreted as the reciprocal lifetime T_N of the virtual state "N" in the nucleon. It is also the time it takes for the N partons to lag behind the center of the nucleon by a distance of the order of $(\gamma M)^{-1}$ which is the relativistically contracted dimension of the nucleon [3].

We make the simplest assumption by stating that the probability G (N) of the state N is proportional to its lifetime: $G(N) \sim C/N^2$. This assumption would be correct if the proton assumes any of the states in expression (1) in random order and stays in each state for the time T_N. The constant C can be determined by normalisation. Reverting to the special model with $N = 3 + 2Z$, we find [4] $C = 4(1-s)$ where $s = |c_0|^2$ is the relative probability that the nucleon is not an assembly of N freely-moving partons. We prefer to express G (N) as a continuous function of N and define $G(N)dN$ as the probability to find N partons when N is between N and $N + dN$. Then, considering that only odd values of N appear in our special model, we get

$$G(N) \simeq D/N^2 \qquad \text{with} \qquad D = 2(1-s) \tag{2}$$

What is the effect on the nucleon at rest, of a virtual lightquantum emitted by an inelastic electron scattering with a four-momentum transfer of $q(p,\nu)$? This is the virtual quantum emitted by an inelastic electron scattering with a four-momentum transfer of $q = (p^2 - \nu^2)^{1/2}$ and an energy loss of ν. The effect is best studied in a coordinate system in which the light quantum has a zero time-component and a space-component $-q$. In this system $\gamma = p/q$, which is large compared to unity if both q and ν are large compared to M. We assume that the partons react to light as point charges with a charge e_i ; when the proton is not an assembly of free partons, it reacts weakly with the virtual quantum, because of the well-known behavior of formfactors at large q . This is why we disregard the contribution of the substate Ψ_0 to the inelastic scattering.

In our new frame the quantum cannot transmit energy to the partons, but only momentum. Since the quantum has a momentum $-q$, the absorbing parton must have a momentum $q/2$, and will end up with a momentum $-q/2$. Hence, only partons with a momentum $q/2$ contribute to the process; that is, roughly speaking, only the substate with $N = 2P/q$. It is easily shown from our Lorentz transformation that $N = 2P/q \simeq 2M\nu/q^2$, which is the well-known Bjorken scale variable ω . Hence, the cross-section of the inelastic scattering must be proportional to the number of partons having the momentum $q/2$. This number is $N\,G(N)$.

Let us estimate the value of this cross-section. In order to avoid angular complications, we consider only small angles. We assume that scattering cross-section of the electron with the partons is given by the Mott-cross-section, which depends only on q .

The Mott-cross-section is evaluated for a particle of charge e . The partons or quarks may have a different charge e_i . This is why we introduce a function $f(N) = (\overline{e_i^2/e^2})_N$ which is the average parton charge square in the substate N . We then get

$$\frac{d\sigma(q,\nu)}{d\nu} = \sigma_{Mott}\, f(N)\, N\, G(N)\, \frac{dN}{d\nu}$$

where N is the number of the substate which can absorb the virtual quantum in question: $N = 2M\nu/q^2$. This becomes with the help of (2):

$$\frac{d\sigma}{d\nu} = \sigma_{Mott} W_2 \;, \qquad \nu W_2 = D f(N) \tag{3}$$

The values of $f(N)$ are easily calculated on the basis of the current quark-model. When $N = 3$, if $f(N) = 1/3$ for protons and 2/9 for neutrons; for higher N, we determine $f(N)$ by assuming that, on the average, e_i^2/e^2 for a quark-antiquark pairs is 2/9. We then get the expressions

$$f(N) = \frac{2}{9} \qquad \text{for Neutrons}$$

$$f(N) = \frac{2}{9} + \frac{1}{3N} \qquad \text{for Protons} .$$

$f(N)$ is different for protons and neutrons, but they become close for large N. Fig. 1 shows νW_2 as a function of $\omega \simeq N$, as measured for protons at SLAC [5]. The large dots are the values calculated with our expression (2) by making the plausible assumption $s = 0.4$. The upper dots are for the proton, the lower ones for the neutron. The difference $\nu(W_2^P - W_2^N)$ is plotted in Fig. 2, together with the experimental values from SLAC [5]. Our expression does not give any values for $N < 3$. However, we must keep in mind that values of $\omega < 3$ correspond to partons with momenta $p_i > P/3$. We excluded those values by our average assumption: $p_i = P/N$ and $N \geq 3$. Actually, we expect that the values of our function νW_2, as given by (3), must be averaged over an interval of $\Delta N \simeq 1$ or 2. In particular, the curve must go smoothly to zero at $N = 1$.

It is surprising that this simple and crude model reproduces pretty well the observed shape of νW_2 and the difference between protons and neutrons. In particular, the fact that this difference has a maximum at $\omega = 3$ and that the ratio of νW_2 for Protons and Neutrons indeed was found to be about 3/2, is a surprising support of the quark model. We also get a decrease for larger ω in the case of the proton. Even the absolute value is of the right order of magnitude, but one must keep in mind that it depends on our very arbitrary, but natural assumption on the value of s. It is worthwhile to point out that a higher value of e_i^2/e^2 , -- non-fractional charges for quarks -- would require a much larger value for s, which would be equivalent to stating that the quark model is not important in the make-up of the nucleon.

Furthermore, the observations relative to the polarization also fit this picture: The extreme relativistic situation requires the same helicity before and after absorption of the virtual photon if the partons have a spin 1/2. Since the direction of the partons is reversed in the absorption process, there would be a total helicity change of one unit, which in turn would require a virtual photon with helicity of unity. This would imply a zero-ratio between the longitudinal and transverse cross-section, as the experiments seem to indicate.

The question arises, what secondary particles would be observed after the inelastic collision process. It is highly improbable that the ejected quark will be observed as an independent particle. It is more probable that the quark will pick-up an anti-quark and emerge in form of a meson of high energy. Obviously, this final state interaction may considerably change the cross-section from the value estimated above. It could be, however, that the semi-quantitative features of our crude model may be preserved.

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2. J.D. Bjorken, Proceedings of 1967 Int. School of Physics at Varenna (Academic Press, New York and London 1968), J.D. Bjorken and E.A. Paschos, Phys. Rev. 185, 1975 (1969).

3. See also H. Cheng and T.T. Wu, Phys. Rev. Lett. 23, 670 (1969).

4. The sum of reciprocal squares of odd numbers beginning with 3 is almost exactly 0.25.

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Figure captions:

1. The function νW_2 versus ω as measured for protons (small dots with error-bars, Ref. 5) and the predictions of our model (large dots, upper dots for protons, lower dots for neutrons).

2. The difference of νW_2 for protons and neutrons versus ω . Small dots with error-bars are experimental results (Ref. 5), large dots are the predictions of our model.

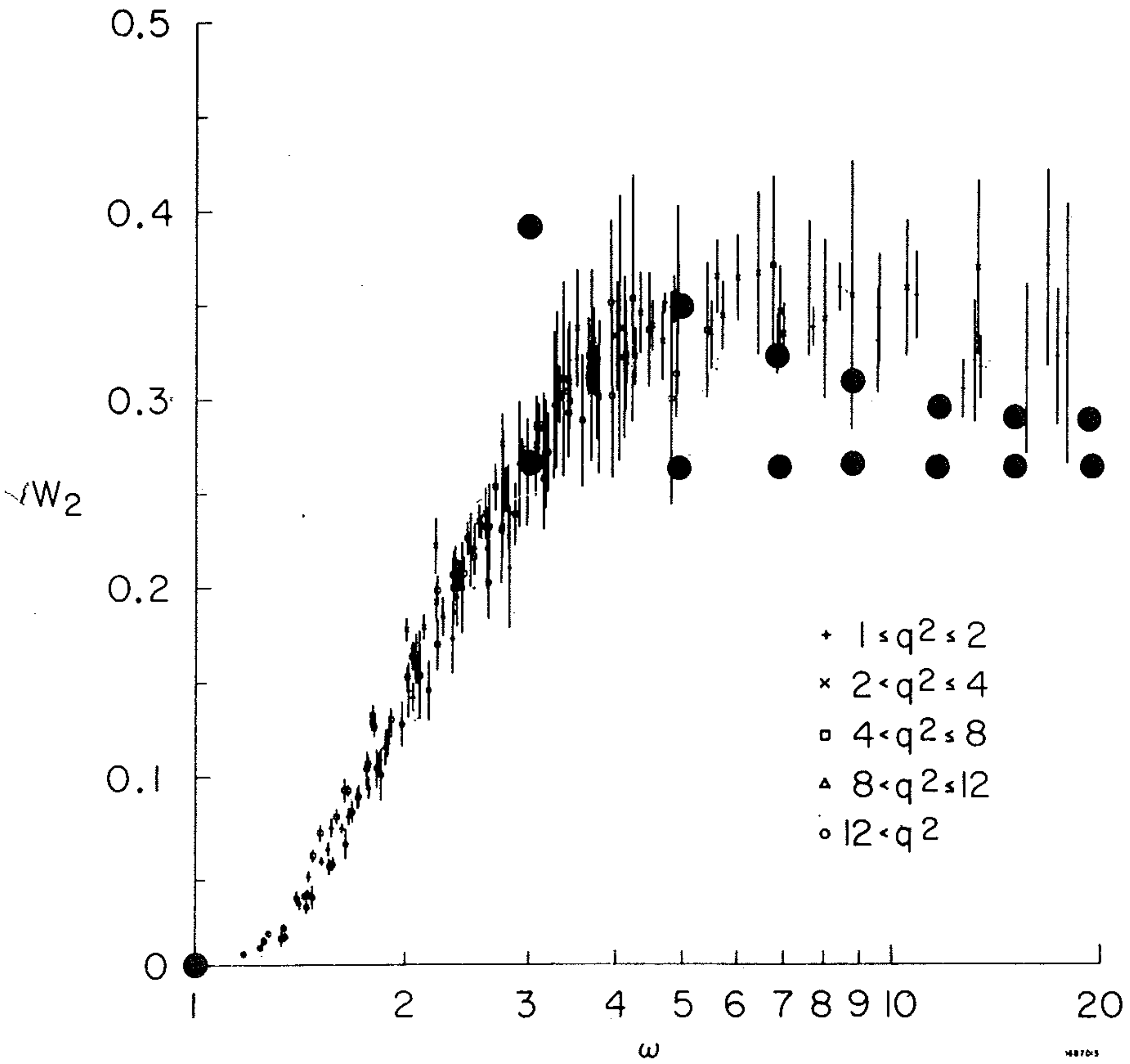
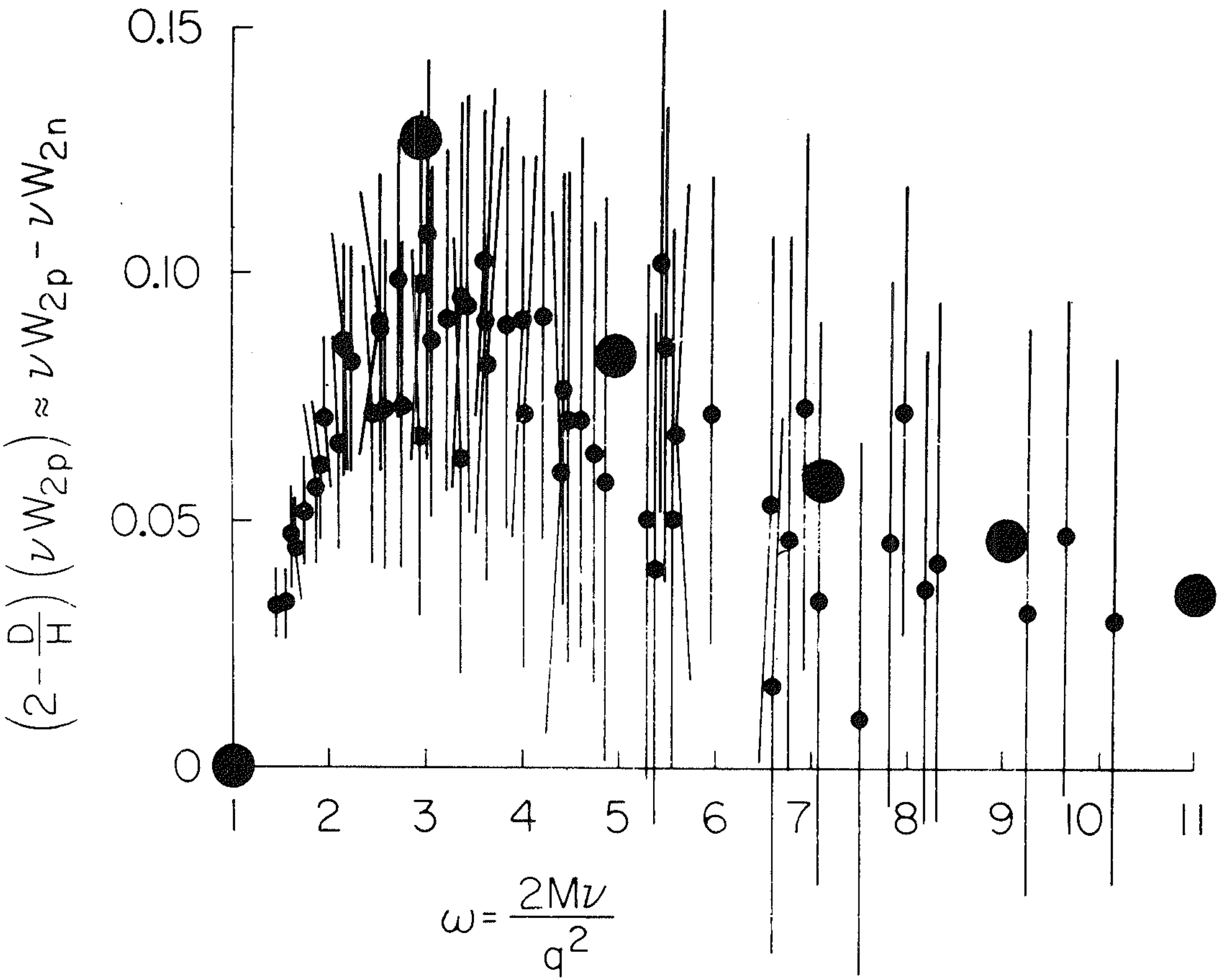

0.5
0.4
0.3
0.2
0.1
0
νW_2
1
2
3
4
5
6
7
8
9
10
20
ω
+ $1 \leq q^2 \leq 2$
× $2 < q^2 \leq 4$
□ $4 < q^2 \leq 8$
△ $8 < q^2 \leq 12$
○ $12 < q^2$

Fig.1



0.15
0.10
0.05
0
$\left(2-\frac{D}{H}\right)\left(\nu W_{2p}\right) \approx \nu W_{2p} - \nu W_{2n}$
1
2
3
4
5
6
7
8
9
10
11
$\omega = \frac{2M\nu}{q^2}$

Fig. 2