DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 70/56
November 1970

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Abstract

A recent dual model for diffractive two-pion photoproduction, treating the pomeron as having 0^+ structure, is generalized to two-pion electroproduction. Taking into account gauge invariance and kinematical singularity free condition in a strict way, we obtain a cross section for ρ' production which is an order of magnitude higher than the experimental value.

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1. Introduction

The Veneziano model[1] meets continued interest as it provides a simple dual prescription of resonance and Regge behaviour.[2] Although it was successful in various phenomenological applications, the experimental absence of high mass vector mesons in the diffractive photoproduction of two-pion systems[3,4] is a serious problem.

In a recent paper[5] the process

$$\gamma + p \to \pi^+ + \pi^- + p \qquad (1)$$

was treated within the framework of a dual prescription of diffraction dissociation.[6] The essential new idea was, to assume factorization for the pomeron in process (1) and to ascribe the spin-parity structure 0^+ to it making a Veneziano ansatz for the process

$$\gamma + \sigma \to \pi^+ + \pi^- , \qquad (2)$$

where σ stands for pomeron. In this way the authors of Ref.5 found good agreement with experiment. In particular they could easily explain the suppression of the ρ' and the higher vector mesons in the π-π mass spectrum relative to the ρ. However, in the treatment of gauge invariance an additional singularity was introduced which provides the essential mechanism for the suppression of the ρ'. Since gauge invariance may give servere restrictions to an amplitude and since this question is important with respect to the Veneziano model, in this note we examine again process (1) within the framework of a dual prescription, assuming the pomeron to be approximated by 0^+ particle as in Ref.5 and generalize it to the case of electroproduction. As will be shown, taking into account gauge invariance and kinematical singularity free condition, changes the situation strongly, and in particular a quite different π-π mass spectrum will result.

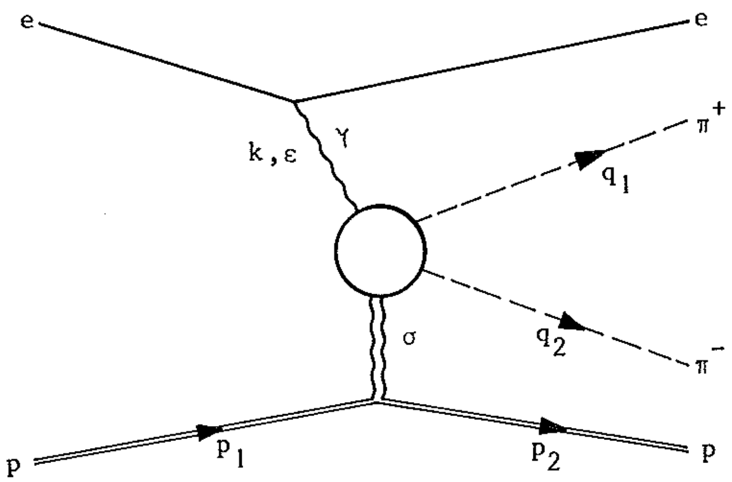
A model in which the pomeron is approximated by a 0^+ structure certainly disagrees with the observed s-channel helicity conservation in ρ photoproduction.[4,7] But the integrated quantitites such as the π-π mass spectrum may be well described by such an approximation.

2. General Form of the Amplitude

In this note we treat the process

$$e + p \to e + \pi^+ + \pi^- + p \tag{3}$$

within the framework of the dual model with the assumption[5] that the pomeron is approximated by 0^+ particle. The variables are shown in Fig. 1.



We introduce the kinematical variables* $Q_\mu = q_{1\mu} + q_{2\mu}$, $q_\mu = q_{1\mu} - q_{2\mu}$, $p_\mu = p_{1\mu} - p_{2\mu}$. $\bar{s} = (k + p_1)^2$, $\bar{t} = p^2$, $s = (k + p)^2$, $t = (k - q_1)^2$ und $u = (k - q_2)^2$. According to our assumption we write[5,6] for the amplitude of real photon process (1) as

$$M = e^{\frac{a}{2}\bar{t}}\ \bar{s}\ (\varepsilon T) \tag{4}$$

and for the amplitude of the electroproduction process (3) as

$$M = (e\ \bar{u}\ \gamma_\mu\ u)\ \frac{1}{k^2}\ T_\mu\ e^{\frac{a}{2}\bar{t}}\ \bar{s}\ , \tag{5}$$

where u is the spinor of the electron and T describes the amplitude for process (2). From Lorentz invariance we may write

$$(\varepsilon T) = A\cdot(\varepsilon q) + B\cdot(\varepsilon Q) + C\cdot(\varepsilon k)\ , \tag{6}$$

where the functions A, B, C are free from kinematical singularities.

Gauge invariance demands

$$A\cdot(kq) + B\cdot(kQ) + C\cdot(k^2) = 0 \tag{7}$$

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*We use the metric (+,−,−,−), and abbreviation $(A_\mu B_\mu) = (AB)$.

and one can eliminate amplitude B from Eq.(6):

$$(T\varepsilon) = \frac{A}{(kQ)}\,[(kQ)(\varepsilon q) - (kq)(\varepsilon Q)] + \frac{C}{(kQ)}\,[(kQ)(\varepsilon k) - (k^2)(\varepsilon Q)] \;. \quad (8)$$

In the case of a real photon ($k^2 = 0$, $(\varepsilon k) = 0$) the second square bracket in Eq.(8) vanishes and only amplitude A contributes. Furthermore, from Eq.(7) one can see that the amplitude $A(s, t, u)$ has to fulfill the condition

$$A(s,t,u,k^2) = 0 \quad \text{at} \quad k^2 = (kQ) = 0 \quad . \qquad (9)$$

Therefore, neglecting higher order terms in (k^2) and (kQ) we may write

$$A = A_1 \cdot (kQ) + A_2 \cdot (k^2) \qquad (10)$$

where A_1 and A_2 have to be free from kinematical singularities. In an analogous manner the constraints from Eq.(7) for the amplitudes B and C are obtained yielding

$$B = - A_1 \cdot (kq) + B_2 \cdot (k^2) \quad , \qquad (11)$$

$$C = - A_2 \cdot (kq) - B_2 \cdot (kQ) \quad . \qquad (12)$$

The Born terms read

$$A_B = N\, F_\pi(k^2) \left(\frac{1}{t - m_\pi^2} + \frac{1}{u + m_\pi^2} \right) \quad , \qquad (13)$$

$$B_B = N\, F_\pi(k^2) \left(\frac{1}{t - m_\pi^2} - \frac{1}{u - m_\pi^2} \right) \quad , \qquad (14)$$

$$C_B = N\, F_\pi(k^2) \left(- \frac{1}{t - m_\pi^2} + \frac{1}{u - m_\pi^2} \right) \quad , \qquad (15)$$

where $F_\pi(k^2)$ is the pion form factor and N is an overall constant. The Born amplitudes obey the gauge condition (7) by themselves; therefore it is convenient to single them out from A_1 and A_2 and to write

$$A = A_B \cdot D + A_3 \cdot (kQ) + A_4 \cdot (k^2) \quad , \qquad (16)$$

$$B = B_B \cdot D - A_3 \cdot (kq) + B_3 \cdot (k^2) \quad , \tag{17}$$

$$C = C_B \cdot D - A_4 \cdot (kq) - B_3 \cdot (kQ) \quad , \tag{18}$$

where the functions A_3 , A_4 , B_3 , and D must be free from kinematical singularities. Furthermore, D must be symmetric in t and u and must fulfill the condition

$$D(t = m_\pi^2, u) = D(t, u = m_\pi^2) = 1 \quad , \tag{19}$$

and the functions A_3 , A_4 and B_3 must not contain the pion poles. (For a more detailed discussion of all constraints from gauge invariance we refer to Ref.9). In the special case $k^2 = 0$, we need only the functions D and A_3.

3. Veneziano Structure of the Amplitude

We now construct the Veneziano amplitude for process (2). In the t and the u channels we have the π-trajectory

$$\alpha_\pi(x) = \alpha'(x - m_\pi^2) \quad , \quad \alpha' \approx 1 \text{ GeV}^{-2} \quad , \tag{20}$$

in the s channel the ρ-trajectory contributes:

$$\alpha_\rho(x) = 1 + \alpha'(x - m_\rho^2) + i\alpha'(x - 4m_\pi^2) \frac{m_\rho \Gamma_\rho}{(m_\rho^2 - 4m_\pi^2)} \theta(x - 4m_\pi^2) \quad , \tag{21}$$

where we have inserted an imaginary part giving the width of the ρ .

We obtain[9] for the function D the following dual form

$$D(s,t,u) = \frac{\Gamma\big(1 - \alpha_\pi(t)\big)\Gamma\big(1 - \alpha_\pi(u)\big)}{\Gamma\big(1 - \alpha_\pi(t) - \alpha_\pi(u)\big)} \quad . \tag{22}$$

If we want to have the residues of all recurrences with correct polynomical behaviour, we cannot include a term containing the ρ-trajectory without introducing a kinematical singularity in D , or without disturbing the $t - u$ symmetry or spoiling condition (19). In the case $k^2 = 0$ we need only the functions

D and A_3 . So we include the ρ-trajectory in A_3 which on the other hand must not contain a pion pole:

$$A_3(s,t,u) = N\left[a_3 \frac{\Gamma\big(1-\alpha_\pi(t)\big)\Gamma\big(1-\alpha_\rho(s)\big)}{\Gamma\big(2-\alpha_\pi(t)-\alpha_\rho(s)\big)} + \frac{\Gamma\big(1-\alpha_\pi(u)\big)\Gamma\big(1-\alpha_\rho(s)\big)}{\Gamma\big(2-\alpha_\pi(u)-\alpha_\rho(s)\big)} + \right.$$

$$\left. + a_3' \frac{\Gamma\big(1-\alpha_\pi(t)\big)\Gamma\big(1-\alpha_\pi(u)\big)}{\Gamma\big(2-\alpha_\pi(t)-\alpha_\pi(u)\big)} \right] . \qquad (23)$$

Amplitude $A_4(s,t,u)$ has exactly the same Veneziano structure as $A_3(s,t,u)$ in Eq.(23) with different parameters. On the other hand, amplitude $B_3(s,t,u)$ does not contain the ρ pole. So we simply put $B_3(s,t,u) = 0$. In our expressions for the amplitudes D, A_3 , A_4 , and B_3 we have neglected higher Veneziano terms.

One can see that in this case we cannot have any Veneziano term which contains both the π-pole and the ρ-pole simultaneously. In the most interesting kinematical region, namely $s \approx m_\rho^2$ and $t \leq 0$, the main contribution to A_3 is the ρ-pole at $s \approx m_\rho^2$. So we see that our amplitude A given by Eqs.(14), (20) and (21) has more resemblance with the interference model of Söding[8] than with a model where π and ρ are dual.

We now calculate the π-π mass distribution at $k^2 = 0$. We may safely put $a_3' = 0$; for obtaining an estimate of the parameter a_3 we may consult Ref.10. There approximately $\sigma_\rho/\sigma_{Drell} \approx 4$ has been obtained for the ratio between σ_ρ and σ_{Drell}. In our notation this corresponds to $\sigma_\rho/\sigma_{Drell} \simeq (a_3 m_\rho^2)^2$ and this gives us as an estimate $a_3 \simeq 4\ \mathrm{GeV}^{-2}$. Calculating now the π-π mass distribution with our amplitudes (4), (8), (16) and (23) we get in serious troubles in the ρ' region. We obtain a total cross section for diffractive ρ' production according to process (1) of about 8 μb. This is at least one order of magnitude higher than the experimental upper limit of 0.5 μb given in Ref.10. In principle one could overcome this discrepancy by adding satellites to the amplitude A_3 in Eq.(21). But one would need quite a large number of satellites to obtain a π-π mass spectrum which is in reasonable agreement with experiment. So this is not really a way out of the difficulty.

Another possible choice for the function D(s,t,u) instead of (22) might be

$$D(s,t,u) = \frac{1}{\alpha_\pi(t)-\alpha_\pi(u)} \left[\alpha_\pi(t) \frac{\Gamma\left(1-\alpha_\pi(u)\right)\Gamma\left(-\alpha_\rho(s)\right)}{\Gamma\left(-\alpha_\pi(u) - \alpha_\rho(s)\right)} - \right.$$

$$\left. - \alpha_\pi(u) \frac{\Gamma\left(1-\alpha_\pi(t)\right)\Gamma\left(-\alpha_\rho(s)\right)}{\Gamma\left(-\alpha_\pi(t) - \alpha_\rho(s)\right)} \right] \quad . \tag{24}$$

This function is symmetric in t and u , fulfills condition (19), has no poles at $\alpha_\pi(t) - \alpha_\pi(u) = 0$ and $\alpha_\rho(s) = 0$ and contains the π and the ρ in a dual manner. But it has the defect that, because of the factor $\left(\alpha_\pi(t) - \alpha_\pi(u)\right)^{-1}$, the residues of the recurrences of the π trajectory, $\alpha_\pi(t) = 1,2,3,...$ do not have the correct polynomial behaviour in s . The form of the residues at $\alpha_\rho(s) = 1,2,3,...$ is correct, therefore this function may be used for the description of $\pi\pi$ photoproduction. However, it would predict a cross section for ρ' production even larger than that of ρ production. It is, however, possible to drasticall reduce the residues of the ρ' and of the other high mass states by addition of only one satellite term. The ansatz

$$D(s,t,u) = \left\{ \frac{1}{\alpha_\pi(t)-\alpha_\pi(u)} \left[\alpha_\pi(t) \frac{\Gamma(1-\alpha_\pi(u))\Gamma(-\alpha_\rho(s))}{\Gamma\left(-\alpha_\pi(u)-\alpha_\rho(s)\right)} - \alpha_\pi(u) \frac{\Gamma\left(1-\alpha_\pi(t)\right)\Gamma\left(-\alpha_\rho(s)\right)}{\Gamma\left(-\alpha_\pi(t)-\alpha_\rho(s)\right)} \right] \right.$$

$$+ \frac{\beta_1}{\alpha_\pi(t)-\alpha_\pi(u)} \left[\alpha_\pi(t) \frac{\Gamma\left(1-\alpha_\pi(u)\right)\Gamma\left(1-\alpha_\rho(s)\right)}{\Gamma\left(1-\alpha_\pi(u)-\alpha_\rho(s)\right)} - \alpha_\pi(u) \frac{\Gamma\left(1-\alpha_\pi(t)\right)\Gamma\left(1-\alpha_\rho(s)\right)}{\Gamma\left(1-\alpha_\pi(t)-\alpha_\rho(s)\right)} \right] \tag{25}$$

$$\left. + \beta_2 \frac{\Gamma\left(1-\alpha_\pi(t)\right)\Gamma\left(1-\alpha_\pi(u)\right)}{\Gamma\left(1-\alpha_\pi(t)-\alpha_\pi(u)\right)} \right\} \cdot \frac{1}{1 + \beta_1 + \beta_2}$$

with $\beta_1 = -0.7$ and $\beta_2 = 0.15$ and with $A_3(s,t,u) = 0$ not only suppresses the ρ' and the higher vector mesons but also reproduces the asymmetry of the

ρ line shape reasonably well. This is shown in Fig.2. The reason for this suppression of the higher vector mesons lies simply in the fact that the term in the second square bracket in Eq.(25) does not contain the ρ pole at all but contains the higher states with residues approximately equal to the term in the first square bracket. So by a proper choice of the parameter β_1 one can subtract away all the higher vector mesons. Of course this does not explain the absence of the high mass vector mesons.

In Ref.5 a Veneziano model has been constructed in which the π and the ρ are dual partners. This was possible by introducing an extra kinematical factor $1/K\cdot Q = 2/(s-\bar{t})$. The situation there corresponds to $A_3 = 0$ and

$$D(s,t,u) = \frac{1}{\alpha_\pi(u)+\alpha_\pi(t)} \left[\alpha_\pi(u) \frac{\Gamma\left(1-\alpha_\rho(s)\right)\Gamma\left(1-\alpha_\pi(t)\right)}{\Gamma\left(1-\alpha_\rho(s)-\alpha_\pi(t)\right)} + \alpha_\pi(t) \frac{\Gamma\left(1-\alpha_\rho(s)\right)\Gamma\left(1-\alpha_\pi(u)\right)}{\Gamma\left(1-\alpha_\rho(s)-\alpha_\pi(u)\right)} \right];$$

showing a singularity at $\alpha_\pi(t) + \alpha_\pi(u) = -2\alpha'.(K.Q) = 0$. Correspondingly this amplitude does not fulfill condition (9). It is just this singularity, however, which reduces the amplitude for production of high mass vector mesons.

4. Conclusion

We have constructed an amplitude describing diffractive two-pion photoproduction taking into account all requirements of gauge invariance. The main assumptions have been

(a) 0^+ spin structure of the pomeron

(b) Veneziano ansatz for the production amplitude.

We could not explain the experimental π-π mass spectrum in the region of the higher vector mesons. In principle this difficulty could be overcome by adding higher Veneziano terms or by including a more complicated spin structure of the pomeron. In our opinion the problem of the higher vector mesons within the context of the Veneziano model is still unsolved.

Acknowledgement:

We thank Profs. W. Jentschke, H. Joos and E. Lohrmann for their kind hospitality at DESY. We also thank Drs. H. Satz and K. Schilling for a discussion in which they explained to us their point of view.

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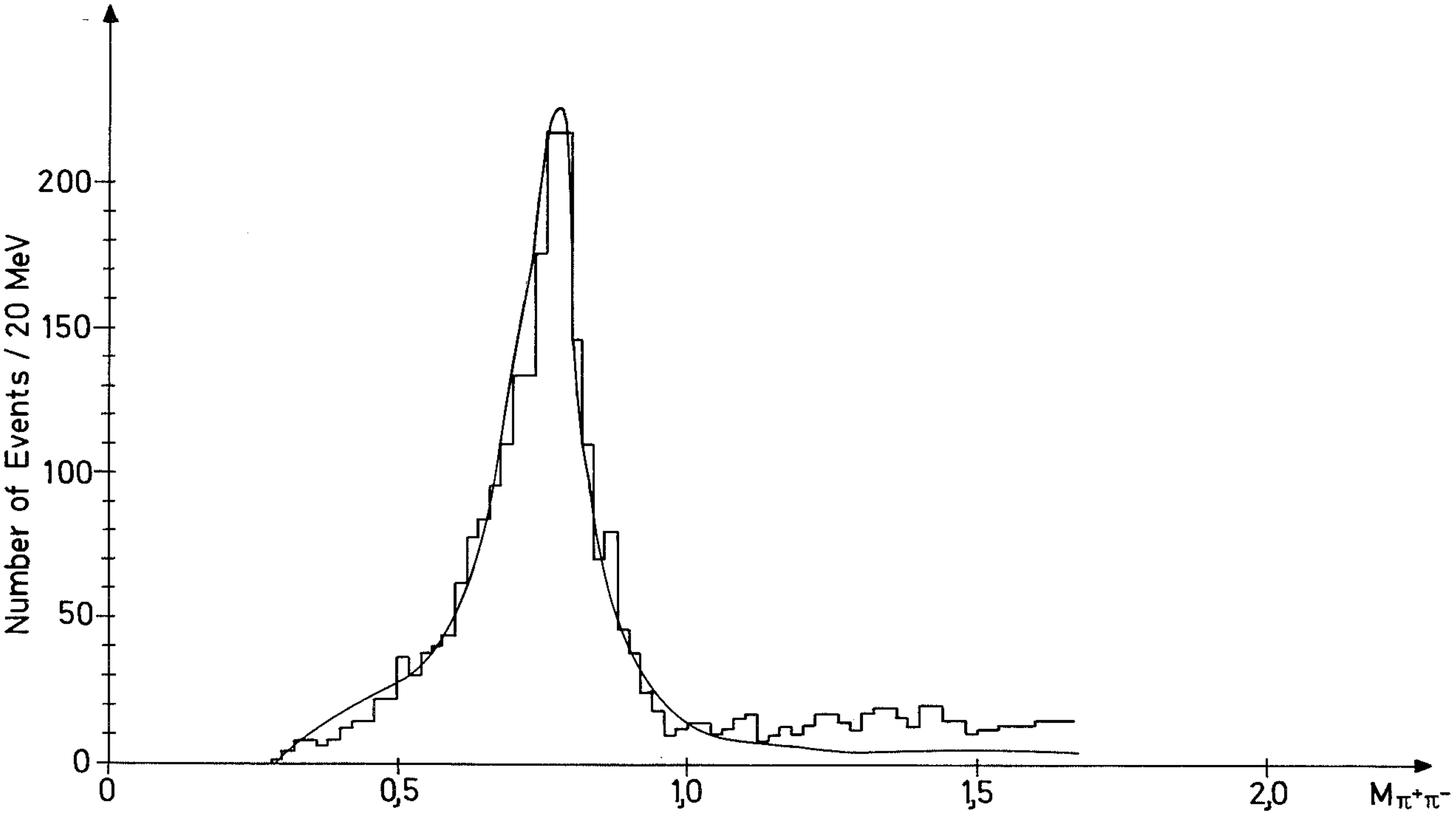
Number of Events / 20 MeV
200
150
100
50
0
0
0,5
1,0
1,5
2,0
$M_{\pi^+\pi^-}$