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Refined Vector Meson Dominance Analysis of Single Pion Electroproduction

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Abstract

Vector meson dominance predictions using all available $\pi^- p \to \rho^0 n$ data are compared with single pion electroproduction directly relating longitudinal ρ^0 production to the longitudinal electroproduction cross section.

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Recently [1] we have made a comparison of vector meson dominance (VDM) predictions with single pion electroproduction data [2a*], the main purpose having been to show that the mass extrapolation procedure for the longitudinal amplitude together with the ratio ρ_{00}/ρ_{11} of ρ^0 density matrix elements [3] from the reaction $\pi^- p \to \rho^0 n$ is able to reproduce the gross features of the electroproduction data, which are dominated by a large longitudinal contribution. The overall normalization in the analysis of Ref. 1 has been taken from photoproduction. Predictions essentially along the same lines, but using data on $\pi^- p \to \rho^0 n$ from different experiments [4] have been given by Berends and Gastmans [5].

Whereas Ref. 1 was based on preliminary data on $\pi^- p \to \rho^0 n$ by Bulos et al. [3] as reported at the Kiev Conference, in the present work we can use the final data [6] of this group differing from the preliminary ones in the subtraction of a stronger S-wave in the $\pi^+\pi^-$-channel in $\pi^- p \to \pi^+\pi^- n$ by using experimental information from $\pi^- p \to \pi^0\pi^0 n$ [7]. The values of the ρ^0 density matrix element ρ_{11} from the preliminary data correspond [3] to the upper limit of ρ_{11} - i.e. the lower limit for the S-wave - compatible with Schwartz inequalities for density matrix elements, and the corresponding cross sections were in fairly good agreement [3] with the unpolarized real photoproduction data. The subtraction of the S-wave contribution, as obtained from data on $\pi^- p \to \pi^0\pi^0 n$ [7] for momentum transfer $|t| > 0.01\ \mathrm{GeV}^2/c^2$ reduced ρ_{11}

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[*] Data on single pion electroproduction have also been obtained in Ref. 2b, c.

to the lower limit compatible with Schwartz inequalities and led to larger deviations in the VDM predictions [8] for unpolarized real photoproduction in certain t intervals. Clearly, as transverse ρ^0 production and S-wave production are of the same order of magnitude for small t, the value of ρ_{11} obtained from experiment is strongly dependent on the S-wave subtracted - in contrast to the large quantity ρ_{00} - and changes in ρ_{11} up to 30 o/o for $0.01 \text{ GeV}^2/c^2 \leqq |t| \leqq 0.04 \text{ GeV}^2/c^2$ (for $|t| < 0.01 \text{ GeV}^2/c^2$ even larger ones) are compatible with the experimental upper and lower limits for ρ_{11} as obtained by using Schwartz inequalities for density matrix elements.

For the present predictions we shall also use all other published data [9-12] on $\pi^- p \to \rho^0 n \to \pi^+\pi^- n$ and subtract the $\pi^+\pi^-$ S-wave following the procedure of Ref. 6. This is in contrast to Ref. 5 where the S-wave has not been subtracted. Furthermore, whereas the previous predictions [1,5] for the longitudinal part σ_L of the electroproduction cross section were based on ratios of density matrix elements and photoproduction data, in this paper we shall also give absolute VDM predictions for the longitudinal part of the electroproduction cross section directly from longitudinal ρ^0 production. We shall only consider $\sigma_u + \varepsilon \sigma_L$ where σ_u is the cross section for π production by transverse virtual photons and ε characterizes the photon polarization. For the predictions for the longitudinal transverse interference contribution to electroproduction, σ_I, we refer to the previous analysis [1,5] as these predictions are not sensitive against changes in the S-wave contribution in $\pi^- p \to \pi^+\pi^- n$ and are not changed appreciably, when making absolute predictions from $\pi^- p \longrightarrow \rho^0 n$ without introducing ratios of density matrix elements and photoproduction cross sections. For the contribution σ_T to the electroproduction cross section, which is due to the linear photon polarization, and which is directly obtained from photoproduction by multiplying with a q^2 dependent factor (q_μ = virtual photon four momentum) originating from the vector meson propagator, we also refer to Ref. 1.

The result of the VDM prediction for $\sigma_u + \varepsilon \sigma_L$ $(\varepsilon \cong 0.75)$ according to [1]

$$\sigma_u = \frac{m_\rho^4}{(q^2 - m_\rho^2)^2} \cdot \frac{d\sigma}{dt}(\gamma p \rightarrow \pi^+ n)$$

$$\sigma_L = \frac{-q^2}{m_\rho^2} \, \frac{m_\rho^4}{(q^2 - m_\rho^2)^2} \, \frac{\rho_{00}^{\rho^0}}{\rho_{11}^{\rho^0}} \, \frac{1}{2} \left(\frac{d\sigma}{dt}(\gamma p \rightarrow \pi^+ n) + \frac{d\sigma}{dt}(\gamma n \rightarrow \pi^- p) \right) \qquad (1)$$

(m_ρ = ρ^0 mass, $\frac{d\sigma}{dt}$ = real photoproduction cross section) is compared with the data [2a] on Fig. 1, together with the previous analysis [1]. The $\rho^0 \omega$ interference term has been taken into account by assuming maximal interference, but it contributes less than 10 o/o in the range of t under consideration. The new predictions based on data from Ref. 6 approximately coincide with the previous ones based on data from Ref. 3 for $|t| > 0.05$ GeV^2/c^2, as ρ_{11} in this region of t has not changed significantly. For $|t| < 0.05$ GeV^2/c^2 the inclusion of the experimental S-wave gave a drastic reduction of ρ_{11}, compared to the upper limit of ρ_{11} used previously, resulting in a change up to almost a factor of 2 in the ratio ρ_{00}/ρ_{11} at certain values of t. Although the data for ρ_{00}/ρ_{11} at lower energies lie lower than the 15 GeV points, no systematic energy dependence of ρ_{00}/ρ_{11} is observed. A comparison with Ref. 5 shows that our predictions for $\sigma_u + \varepsilon\sigma_L$ lie somewhat higher than those of Ref. 5, although our mass extrapolation factor $-q^2/m_\rho^2$ for the longitudinal amplitude is smaller than the expression used in Ref. 5, which is given in formula (3) below. Our larger values are due to the inclusion of the S-wave, which has been neglected in Ref. 5. The large error bars * in Fig. 1 show the limits of the procedure used.

As the final data of Bulos et al. [6] yield discrepancies between VDM predictions and real photoproduction going up to almost 50 o/o at some values of t [8], we think it is more reliable to base our predictions for σ_L not on the ratio ρ_{00}/ρ_{11} as in (1), but on the longitudinal ρ^0 production cross section directly. In Figs. 2 and 3 we show $\sigma_u + \varepsilon\sigma_L$ with σ_L obtained from longitudinal ρ^0 production ($\gamma_\rho^2/4\pi = 0.5$) **

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* As the statistical error $\Delta\rho_{11}$ is comparable to ρ_{11} in magnitude we have calculated the asymmetric error $(\rho_{00} \pm \Delta\rho_{00})/(\rho_{11} \mp \Delta\rho_{11})$ for the ratio, which however should be taken as upper limit of the true error.

** i.e. we use the "old" Orsay storage ring value[13] $\gamma_\rho^2/4\pi = 0.50 \pm 0.03$. After completion of this work, at the Cornell Conference, a new value of $\gamma_\rho^2/4\pi = 0.64 \pm 0.06$ has been reported [14], obtained in a second Orsay experiment, designed to carefully measure and analyse the $\rho^0\omega$ interference effect.

$$\sigma_L = \frac{-q^2}{m_\rho^2}\,\frac{m_\rho^4}{(q^2-m_\rho^2)^2}\,\frac{\alpha\pi}{\gamma_\rho^2}\left(\rho_{00}\frac{d\sigma}{dt}\right)_{\pi^- p\to\rho^0 n} + \tag{2}$$

$$+\ (\rho^0\omega)\ \text{interference term}$$

extrapolating in energy by using an $(s-M^2)^{-2}$ behaviour for $\pi^- p \longrightarrow \rho^0_{\text{long.}} n$. This prediction is less sensitive against the subtracted S-wave, as the S-wave is of the order of magnitude of a few percent of the longitudinal production cross section only. The procedure is more sensitive, however, against systematic errors in the normalization of the hadronic cross section. At $q^2 = -0.75\ \text{GeV}^2/c^2$ (Fig. 3) the VDM predictions lie only slightly below the data. For $q^2 = -0.26\ \text{GeV}^2/c^2$ (Fig. 2) we have a discrepancy of roughly 30 o/o for small $-t$. It should be kept in mind, however, that we are applying vector meson dominance at rather low energies (πN c.m.s. energy $W \cong 2.2$ GeV), and that systematic uncertainties in the hadronic data can go up to 25 o/o [6], mainly due to uncertainties in the ρ^0 line shape [6]. Furthermore, whereas at infinite energies different extrapolation procedure for the longitudinal amplitude coincide with each other, there are appreciable differences at the present energy between our procedure using $-q^2/m_\rho^2$ [15] and other possibilities suggested in the literature [5,16]

$$c_1 = -\frac{q^2}{m_\rho^2}\left(\frac{q^0(m_\rho^2)}{q^0(q^2)}\right)^2_{\text{c.m.s.}} = -\frac{q^2}{m_\rho^2}\left(\frac{W^2-M^2+m_\rho^2}{W^2-M^2+q^2}\right)^2 \qquad \text{(Ref. 5)}$$

$$c_2 = -\frac{q^2}{m_\rho^2}\left(\frac{q^0(m_\rho^2)}{q^0(q^2)}\right)^2_{\text{lab.}} = -\frac{q^2}{m_\rho^2}\left(\frac{W^2-M^2-m_\rho^2}{W^2-M^2-q^2}\right)^2 \qquad \text{(Ref. 16)} \tag{3}$$

As C_1 develops a pole in the spacelike region at $q^2 = M^2 - W^2$, we refrain from attempting to improve agreement with experiment by using a different extrapolation procedure.

Let us make a comment on the discrepancies between the VDM predictions shown on Fig. 1 from ρ_{00}/ρ_{11} according to (1) and on Fig. 2 from ρ_{00} according to (2). Comparison shows that both predictions based on the 11.2 GeV data [12] agree rather well and also reasonably well for the 4.0 GeV data [10], but the predictions on Fig. 1 are systematically higher than the ones on Fig. 2 especially for the 15 GeV data [6]. The reason is clear: agreement with photoproduction is assumed when using (1) instead of (2) to predict σ_L , this agreement being better fulfilled for the 11.2 GeV data than for the data by Bulos et al. [6]. The preliminary "upper limit ρ_{11}" data of Ref. 3 lie in between. The widely scattered distribution of points on Fig. 1 reflects the uncertainties inherent in the determination of ρ_{11} from experiment for small values of $|t|$.

Finally we would like to make a comparison with calculations based on dynamical models [17,18,19] to predict the electroproduction data, and also to extract the pion formfactor. The authors of Refs. 17,18,19 use fixed t dispersion relations with the pole terms and Δ (1236) as main contributions. Effects of higher resonances are investigated in Ref. 17 and the isoscalar part is taken into account in Ref. 18 by using ρ^0 exchange. If ρ^0 dominance is assumed for the pion formfactor in these calculations, the results agree with our VDM predictions.

Concluding Remarks

1) The dominant longitudinal contribution to single pion electroproduction is in good qualitative agreement with the prediction from the dominant longitudinal ρ^0 production in $\pi^- p \longrightarrow \rho^0 n$. In the quantitative comparison between VDM predictions and electroproduction data some discrepancies are seen especially at $q^2 = -0.26\ \mathrm{GeV}^2/c^2$ and at small $|t|$, similar to the discrepancies found in dispersion theoretic calculations with ρ^0 dominance for the pion formfactor. Let us remark, however, that we are applying vector meson dominance at rather low energies, where even kinematic effects due to the ρ^0 mass are in general not negligible * and that systematic errors, which are certainly present, are not included in the Figures.

2) The predictions from $\rho_{00} \frac{d\sigma}{dt} (\pi^- p \rightarrow \rho^0 n)$ in this paper should be taken to be superior to the previous predictions [1,5] based on ρ_{00} / ρ_{11} , as discrepancies, which exist between VDM predictions from some of the ρ^0 data and real transverse photoproduction to not enter.

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[*] In fact, after completion of this work we received a preprint by Kellett[20] with predictions for electroproduction by using the invariant amplitudes approach to vector meson dominance treating the kinematical coefficients exactly. The predictions for $\sigma_u + \varepsilon \sigma_L$ are somewhat higher and the transverse-longitudinal interference term is correctly predicted.

Figure Captions

Fig. 1: Comparison of VDM-predictions from $\left(\frac{\rho_{00}}{\rho_{11}}\right)_{\pi^- p \to \rho^0 n}$ as obtained from data in Refs.3,6 and 9 to 12 with electroproduction data of Ref.2a at $W^2 = 4.84$ GeV^2 and $q^2 = -0.26$ GeV^2/c^2 ($\varepsilon = 0.75$).

Fig. 2: Comparison of VDM-predictions from $\left(\rho_{00} \frac{d\sigma}{dt}\right)_{\pi^- p \to \rho^0 n}$ as obtained from ρ^0 data in Refs. 6 and 9 to 12 with electroproduction data of Ref.2a at $W^2 = 4.84$ GeV^2 and $q^2 = -0.26$ GeV^2/c^2 ($\varepsilon = 0.75$).

Fig. 3: The same as Fig. 2, but for $q^2 = -0.75$ GeV^2/c^2.

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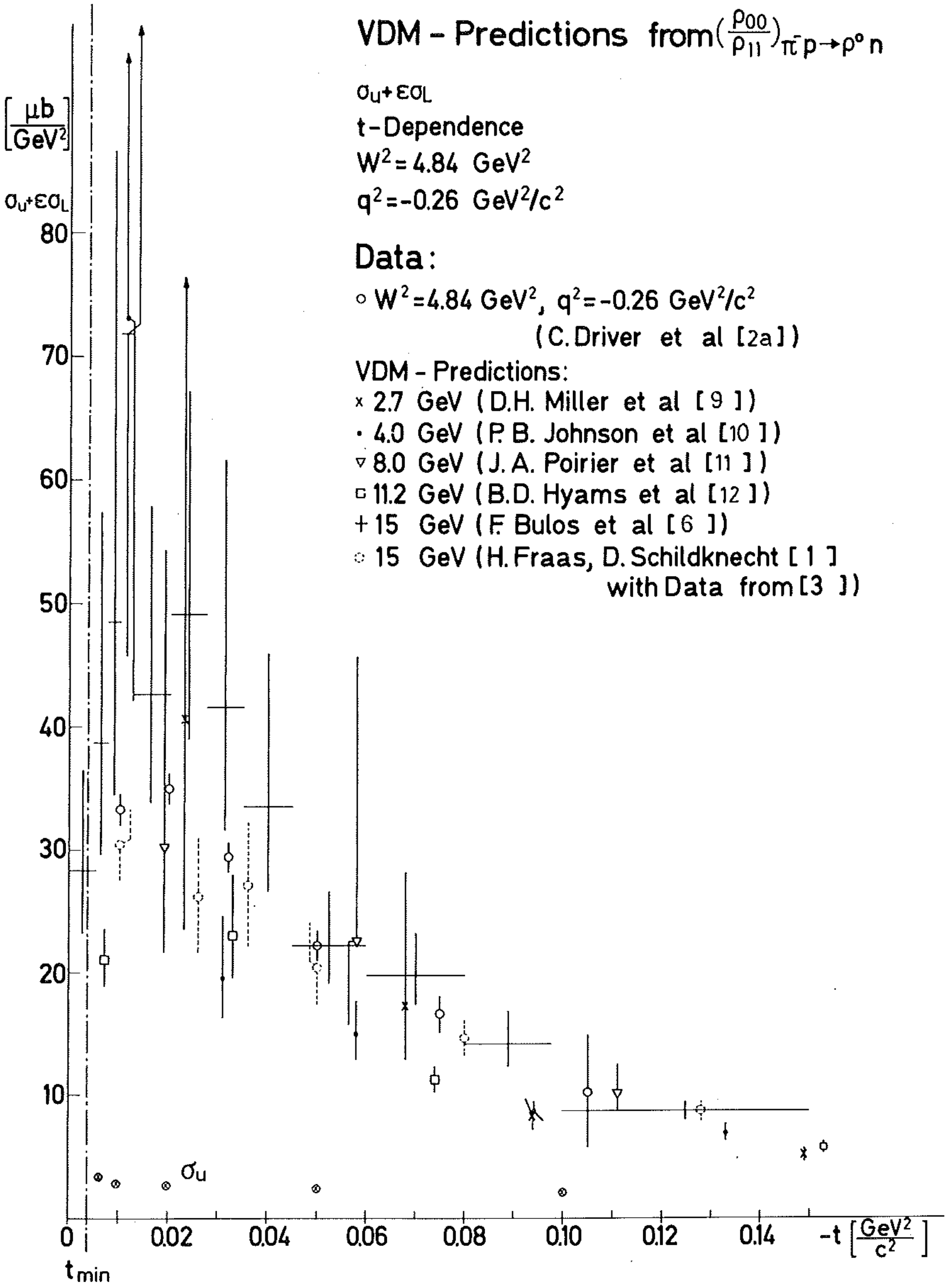
VDM - Predictions from$(\frac{\rho_{00}}{\rho_{11}})_{\pi^- p \to \rho^\circ n}$
$\sigma_u + \varepsilon \sigma_L$
t-Dependence
$W^2 = 4.84$ GeV^2
$q^2 = -0.26$ GeV^2/c^2
Data:
○ $W^2 = 4.84$ GeV^2, $q^2 = -0.26$ GeV^2/c^2
(C. Driver et al [2a])
VDM - Predictions:
× 2.7 GeV (D.H. Miller et al [9])
• 4.0 GeV (P. B. Johnson et al [10])
▽ 8.0 GeV (J. A. Poirier et al [11])
□ 11.2 GeV (B.D. Hyams et al [12])
+ 15 GeV (F. Bulos et al [6])
◌ 15 GeV (H. Fraas, D. Schildknecht [1]
with Data from [3])
$[\frac{\mu b}{GeV^2}]$
$\sigma_u + \varepsilon \sigma_L$
80
70
60
50
40
30
20
10
σ_u
0
0.02
0.04
0.06
0.08
0.10
0.12
0.14
$-t\ [\frac{GeV^2}{c^2}]$
t_{min}

Fig. 1

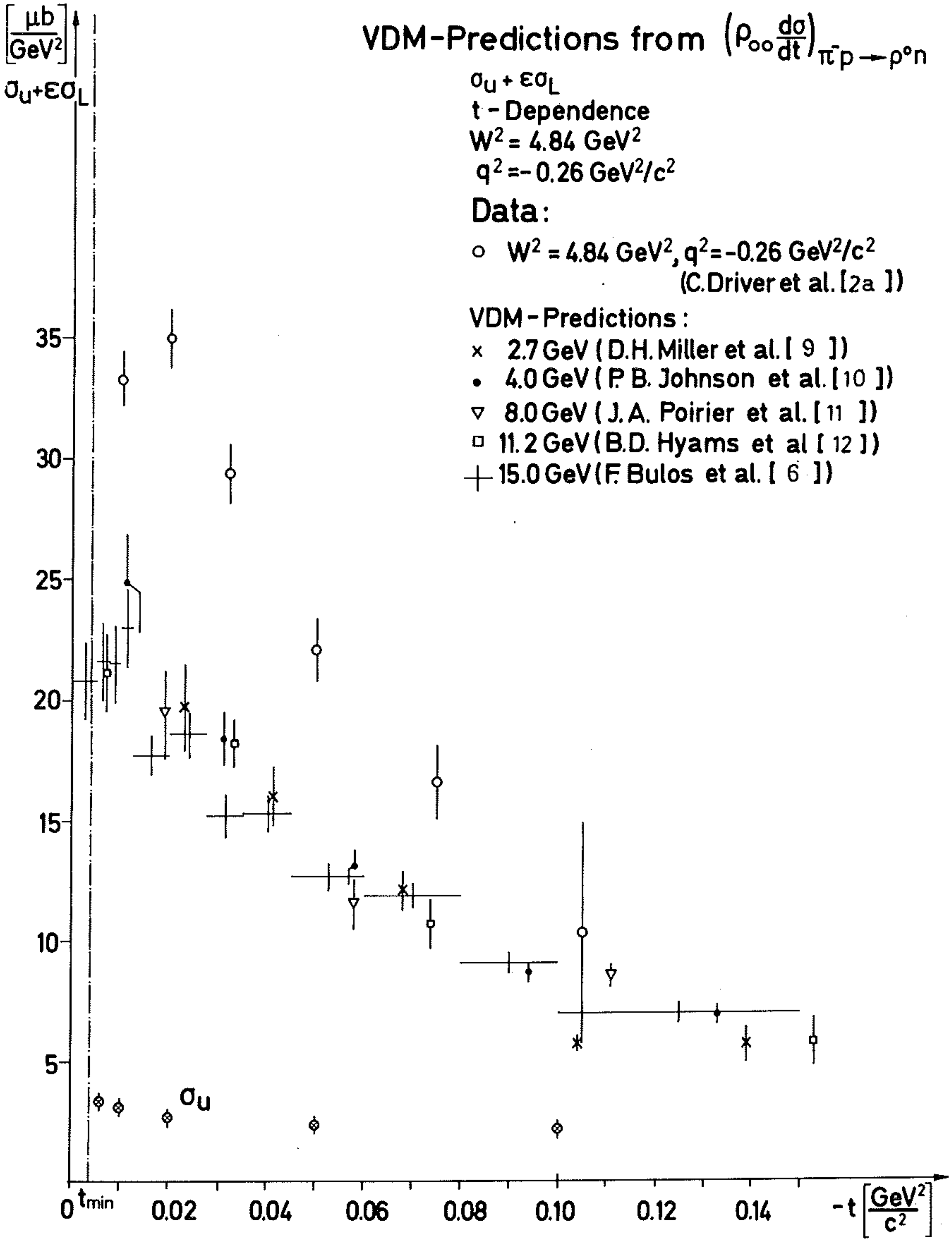
VDM-Predictions from $\left(\rho_{00}\frac{d\sigma}{dt}\right)_{\pi^- p \to \rho^0 n}$
$\sigma_u + \varepsilon\sigma_L$
t - Dependence
$W^2 = 4.84\ GeV^2$
$q^2 = -0.26\ GeV^2/c^2$
Data:
○ $W^2 = 4.84\ GeV^2$, $q^2 = -0.26\ GeV^2/c^2$ (C. Driver et al. [2a])
VDM-Predictions:
× 2.7 GeV (D.H. Miller et al. [9])
• 4.0 GeV (P. B. Johnson et al. [10])
▽ 8.0 GeV (J. A. Poirier et al. [11])
□ 11.2 GeV (B.D. Hyams et al [12])
+ 15.0 GeV (F. Bulos et al. [6])
$\left[\frac{\mu b}{GeV^2}\right]$
$\sigma_u + \varepsilon\sigma_L$
35
30
25
20
15
10
5
σ_u
0
t_{min}
0.02
0.04
0.06
0.08
0.10
0.12
0.14
$-t\left[\frac{GeV^2}{c^2}\right]$

Fig.2

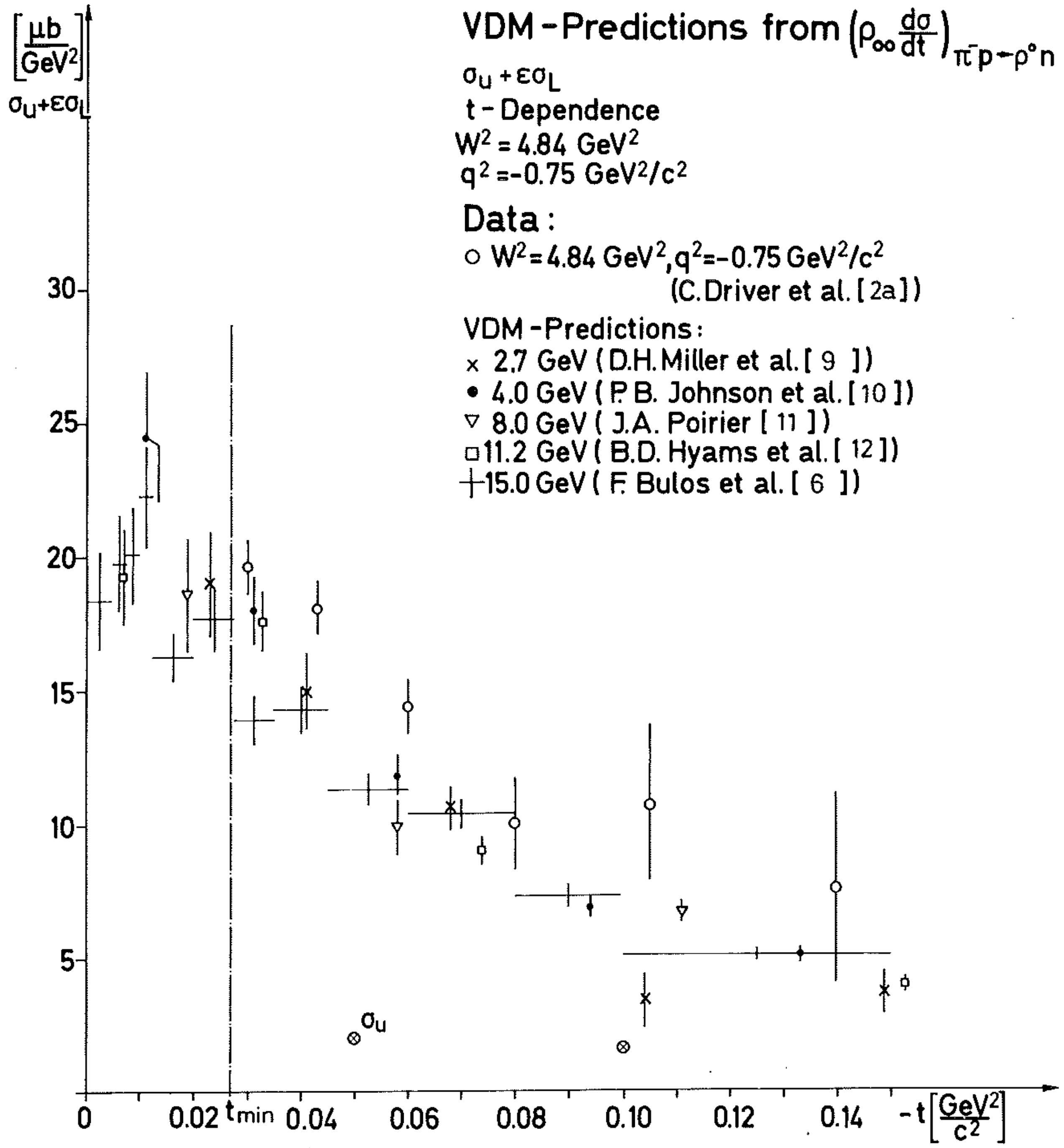
VDM - Predictions from $\left(\rho_{00}\frac{d\sigma}{dt}\right)_{\pi^- p \to \rho^0 n}$
$\sigma_u + \varepsilon\sigma_L$
t - Dependence
$W^2 = 4.84$ GeV^2
$q^2 = -0.75$ GeV^2/c^2
Data :
○ W^2=4.84 GeV^2, q^2=-0.75 GeV^2/c^2 (C.Driver et al. [2a])
VDM - Predictions :
× 2.7 GeV (D.H. Miller et al. [9])
• 4.0 GeV (P. B. Johnson et al. [10])
▽ 8.0 GeV (J.A. Poirier [11])
□ 11.2 GeV (B.D. Hyams et al. [12])
+ 15.0 GeV (F. Bulos et al. [6])
$\left[\frac{\mu b}{GeV^2}\right]$
$\sigma_u + \varepsilon\sigma_L$
30
25
20
15
10
5
σ_u
0
0.02
t_{min}
0.04
0.06
0.08
0.10
0.12
0.14
$-t\left[\frac{GeV^2}{c^2}\right]$

Fig.3