DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 72/4
February 1972

DESY-Bibliothek
23. FEB. 1972

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Vector Meson Dominance and Smoothness of Invariant Amplitudes in $\pi\Delta(1236)$ Electroproduction.

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Abstract:

The longitudinal and transverse parts of the $\pi\Delta(1236)$ electroproduction cross section are predicted from real photoproduction within the framework of vector-meson dominance, using the assumption of approximate mass independence of the invariant amplitudes for $VN \to \pi\Delta(1236)$.

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1. Introduction

Vector meson dominance of the electromagnetic current connects the observables of reactions induced by real or virtual photons with those of reactions induced by vector mesons. In order to derive the relations between these processes, the current field identity between the electromagnetic current and the fields of the vector mesons is supplemented with certain smoothness assumptions for the variation of amplitudes at high energies under extrapolation in the vector meson mass k^2 from the mass shell to $k^2 = 0$ or to spacelike values. The requirement that these smoothness assumptions should be formulated [1,2] for suitably chosen invariant amplitudes has recently led [3,4a,b] to interesting new results for the specific reactions $\gamma N \to \pi N$ and $\pi N \to VN$: Indeed, it has been shown that current conservation and the assumption of no dynamical mass dependence of the Ball invariant amplitudes [5] on the vector meson mass at high energies, allow to sucessfully predict transverse and longitudinal vector meson production from real (transverse) photoproduction. A natural extension of this procedure led to the discussion [3] of longitudinal and transverse amplitudes for single pion production by virtual spacelike photons, and it has been possible to also successfully predict [6] this reaction from real photoproduction. Clearly, the main new point in this formulation of vector meson dominance for photonreactions, is the connection between longitudinal and transverse vector meson amplitudes, and thus the prediction of the longitudinal vector meson amplitudes from the transverse real photon amplitudes. Massive vector mesons in the reaction $VN \to \pi N$ thus behave much like real photons in $\gamma N \to \pi N$: It is the same number of independent amplitudes i.e. dynamical degress of freedom which governs the behaviour of massless photons and massive vector meson in their reactions at high energies.

It is an interesting question to investigate whether the procedure of predicting longitudinal from transverse amplitudes from current conservation and mass independence of invariant amplitudes may be generalized successfully from single pion production [3,4,6] to other reactions. From the experience with single pion production it is quite obvious that current conservation and mass independence of the invariant amplitudes for arbitrary vector meson reactions $VA \to B$, (A, B any hadrons) will always lead to restrictions between the independent amplitudes. It is by no means obvious, however, that these relations do hold experimentally, and it will therefore be of great interest to investigate other reactions.

In this paper we shall consider $\pi\Delta(1236)$photo- and electroproduction, i.e. the reactions

$$\gamma_{real}\, N \to \pi\Delta(1236) \qquad (1a)$$

$$\gamma_{virt.}\, N \to \pi\Delta(1236) \qquad (1b)$$

which via vector meson dominance are related to

$$VN \to \pi\Delta \qquad (2)$$

where V stands for ρ^o, ω, ϕ.We want to emphasize, that within this procedure no crossing assumptions are necessary in addition to vector meson dominance, when relating longitudinal and transverse electroproduction to photoproduction.

This is in contrast to previous investigations where in order to directly relate the cross section for the photoproduction reaction (1a) to the cross section for a vector meson reaction measurable in the laboratory, namely

$$\pi N \to V\Delta \qquad (3)$$

simple crossing porperties, i.e. line reversal invariance, have been assumed [7]. Discrepancies found [8], when comparing cross section for (1a) with cross section from (3) multiplied with the appropriate vector meson photon coupling constants as determined in e^+e^- annihilation, may thus not uniquely be ascribed to a failure of vector meson dominance, but may rather be due to the additional assumption of line reversal invariance. Indeed, a recent calculation [9] within the electric Born term model, shows a considerable change in the cross section when passing from $\gamma p \to \pi^- \Delta^{++}$ to the line reversed reaction $\pi^+ p \to \rho^o \Delta^{++}$, thus supporting the hypothesis that it is mainly line reversal invariance and not the vector meson dominance assumption, which led to discrepancies with experiment. Good agreement with vector meson dominance predictions has been found [10] for recent measurements [11] on $\pi\Delta$ electroproduction, $\gamma_{virt}\ p \to \pi^+ \Delta^o$, where weaker crossing assumptions have been used only, thus supporting the hypothesis that vector domonance is able to reproduce at least the gross features of the data with reasonable accuracy. The reactions (1) are thus good candidates to investigate the consequences following from the assumption of no dynamical vector-meson mass dependence of suitably chosen invariant amplitudes for the vector meson reactions(2).

In section 2 a set of kinematic singularity free amplitudes is introduced for $VN \to \pi\Delta$ and the consequences from current conservation and mass independence of the invariant amplitudes are exploited. In section 3 we shall consider simple Born term models[12]. It will be shown that for sufficiently high energy the chosen invariant amplitudes in Born term models indeed fulfil the smoothness requirements under mass extrapolation, which have been assumed in section 2. In section 4 we briefly review a Born term model with absorptive corrections [13,14], which will be used as a parameterization of the real photoproduction data. In section 5 numerical predictions, as they follow from smoothness of the invariant vector-meson amplitudes and current conservation are made for $\pi\Delta$ electroproduction from $\pi\Delta$ photoproduction. Comparison with the presently available data shows qualitative agreement with the data. A discussion of our results is given in section 5.2. Some summarizing remarks are found in section 6.

2. Invariant Amplitudes, Smoothness Assumptions

The process $VN \to \pi\Delta$ (Fig. 1) is described by the amplitude

$$T \equiv T^{\nu} e_{\nu} = \bar{u}_{\mu}(P_2)\ T^{\mu\nu}\ u(P_1)\ e_{\nu}(k) \tag{4}$$

The momenta of the incoming nucleon of mass m and outgoing Δ of mass M have been denoted by p_1 and p_2, respectively, with $p_1^2 = m^2$, $p_2^2 = M^2$. The momentum of the incoming vector-meson is denoted by k_μ and the produced pion of mass μ has the momentum q_μ; e_ν with $e_\nu \cdot k^\nu = 0$ denotes the polarization vector of the incoming vector meson and $u(p_1)$ and $u_\mu(p_2)$ denote the spinors of the nucleon and Δ, respectively.

$T^{\mu\nu}$ may be expanded in terms of a set of kinematic singularity free invariant amplitudes, which are chosen as in ref. 9:

$$T^{\mu\nu} = \sum_{i=1}^{16} B_i(W^2,t,k^2)\ I_i^{\mu\nu} \tag{5}$$

We have introduced the usual variables

$$W^2 = s = (p_1+k)^2 = (p_2+q)^2,$$

$$t = (k-q)^2 = (p_1-p_2)^2. \tag{6}$$

As we shall extrapolate in the vector meson mass k^2, we have explicitly indicated that the invariant amplitudes will in general contain a dependence on k^2.

Introducing $P_\mu \equiv \frac{1}{2}(p_{1\mu}+p_{2\mu})$ the tenors $I_i^{\mu\nu}$ are given as

$$
\begin{aligned}
I_1^{\mu\nu} &= q^\mu q^\nu & I_9^{\mu\nu} &= q^\mu \gamma^\nu \not{k} \\
I_2^{\mu\nu} &= q^\mu P^\nu & I_{10}^{\mu\nu} &= k^\mu \gamma^\nu \not{k} \\
I_3^{\mu\nu} &= k^\mu q^\nu & I_{11}^{\mu\nu} &= q^\mu \gamma^\nu \\
I_4^{\mu\nu} &= k^\mu P^\nu & I_{12}^{\mu\nu} &= k^\mu \gamma^\nu \\
I_5^{\mu\nu} &= g^{\mu\nu} & I_{13}^{\mu\nu} &= k^\mu k^\nu \\
I_6^{\mu\nu} &= q^\mu P^\nu \not{k} & I_{14}^{\mu\nu} &= k^\mu k^\nu \not{k} \\
I_7^{\mu\nu} &= k^\mu P^\nu \not{k} & I_{15}^{\mu\nu} &= q^\mu k^\nu \\
I_8^{\mu\nu} &= g^{\mu\nu} \not{k} & I_{16}^{\mu\nu} &= q^\mu k^\nu \not{k}
\end{aligned}
\tag{7}
$$

We assume that the vector meson is coupled to a conserved current, i.e. we have in momentum space

$$T^{\mu\nu} k_\nu = 0 \tag{8}$$

not only on the vector mass shell $k^2 = m_V^2$, but for arbitrary values of $k^2 = (p_2+q-p_1)^2 \lesseqgtr 0$. As a consequence of eq. (8) amplitudes $B_i(s,t,k^2)$ fulfil the restrictions

$$
\begin{aligned}
&B_1 \tfrac{1}{2}(\mu^2-t) + B_2 \tfrac{1}{4}(2s-2m^2+t-\mu^2) = k^2(\tfrac{1}{4}B_2 - \tfrac{1}{2} B_1 - B_9 - B_{15}) \\
&B_3 \tfrac{1}{2}(\mu^2-t) + B_4 \tfrac{1}{4}(2s-2m^2+t-\mu^2) + B_5 = k^2(\tfrac{1}{4} B_4 - \tfrac{1}{2}B_3 - B_{10} - B_{13}) \\
&B_6 \tfrac{1}{4}(2s-2m^2+t-\mu^2) + B_{11} = k^2(\tfrac{1}{4} B_6 - B_{16}) \\
&B_7 \tfrac{1}{4}(2s-2m^2+t-\mu^2) + B_8 + B_{12} = k^2(\tfrac{1}{4} B_7 - B_{14})
\end{aligned}
\tag{9}
$$

reducing in the number of independent invariant amplitudes to 12. On the vector meson mass shell $k^2 = m_V^2$ the restrictions (9) are quite irrelevant, as the 12 helicity amplitudes can be expressed by the 12 invariant amplitudes, B_1 to B_{12}. The corresponding relations are given in Appendix 1.

Equations (9) become important, however, when considering the amplitude (4), or the corresponding helicity amplitudes (see Appendix 1), for $k^2 \neq m_V^2$, especially for $k^2 = 0$. Current conservation (8) then guarantees the vanishing of the longitudinal amplitudes, for $k^2 = 0$ as from (8) the longitudinal amplitudes fulfil

$$f^{long} = T^{\nu} e_{\nu}^{long} = \sqrt{k^2}\ \frac{1}{k^o} \left(T^i \frac{k^i}{|\vec{k}|} \right) . \tag{10}$$

The factor $\sqrt{k^2}$ is, of course, also obtained directly, if (9) is substituted into the expressions for the longitudinal helicity amplitudes in Appendix 1. Substituting (9) into the expressions (AI) for the transverse helicity amplitudes, the number of invariant amplitudes appearing in the transverse helicity amplitudes in the limit of $k^2 = 0$ is reduced to the correct number of 8, which number coincides with the number of independent helicity amplitudes as obtained from parity invariance for $k^2 = 0$.

Let us consider now the photoprocess $\gamma N \to \pi\Delta$, where γ may be a real or a virtual photon. Using vector meson dominance of the electromagnetic current, we have

$$e_{\nu} T^{\nu}(\gamma N\to\pi\Delta) = \sum_{V} \frac{em_V^2}{2\gamma_V} \frac{1}{(k^2-m_V^2)} e_{\nu} T^{\nu}(VN\to\pi\Delta), \tag{11}$$

where the right hand side has to be evaluated at the four momentum squared k^2 of the real or virtual photon γ using appropriate smoothness assumptions for the $VN \to \pi\Delta$ amplitude, when varying k^2.

We shall assume that for sufficiently high energies the dynamics of the vector meson induced reaction is in good approximation independent of the vector meson mass k^2, i.e. the invariant amplitudes $B_i(s,t,k^2)$ are assumed to be independent of k^2 at large s and fixed t, such that

$$s + t - m^2 - \mu^2 >> m_v^2 + |k^2| \, . \tag{12}$$

The current conservation conditions (9) consequently become a set of 8 equations, reducing the number of independent invariant amplitudes for $VN \to \pi\Delta$ to 8. The expressions for the helicity amplitudes under this constraint are simply obtained by eliminating four amplitudes (e.g. B_1, B_5, B_8, B_{11}) according to the equations now following from (9):

$$B_1 \frac{1}{2} (\mu^2-t) + B_2 \frac{1}{4} (2s-2m^2+t-\mu^2) = 0$$

$$B_3 \frac{1}{2} (\mu^2-t) + B_4 \frac{1}{4} (2s-2m^2+t-\mu^2) + B_5 = 0$$

$$B_6 \frac{1}{4} (2s-2m^2+t-\mu^2) + B_{11} = 0$$

$$B_7 \frac{1}{4} (2s-2m^2+t-\mu^2) + B_8 + B_{12} = 0. \tag{13}$$

Hence the assumption of mass independence of the dynamics contained in the invariant amplitudes B_i together with current conservation led us to a description of the vector meson induced reaction in terms of only 8 invariant amplitudes. The helicity amplitudes of reaction (2) after elimination of B_1, B_5, B_8, B_{11} by eqs. (13) then have the general structure

$$f^V_{\lambda',\lambda o} = \sqrt{+k^2} \sum_i a^i_{\lambda',\lambda o} (s,t,k^2) \, B^V_i (s,t), \tag{14a}$$

$$f^V_{\lambda',\lambda\pm 1} = \sum_i a^i_{\lambda',\lambda\pm 1} (s,t,k^2) \, B^V_i (s,t), \tag{14b}$$

where the sum runs over

$$i = 2,3,4,6,7,9,10,12 \quad (i \neq 1,5,8,11) \tag{14c}$$

and $a^i_{\lambda'\lambda}\ (s,t,k^2)$ are kinematical coefficients. λ',λ denote the helicities of the Δ and the nucleon, respectively. Explicitly, from (14b,c) the eight invariant amplitudes $B_i(s,t)$ may be expressed in terms of the transverse helicity amplitudes $f^V_{\lambda',\lambda\pm1}$, and after substitution into the longitudinal amplitude in (14a) one obtains expressions for the longitudinal amplitudes in terms of the transverse ones. In the high energy approximation equ. (12) for the coefficients $a^i_{\lambda',\lambda\mu}$, equations (14) are given explicitly in Appendix II.
With (11) we now obtain from (14) for reactions induced by real or virtual spacelike photons

$$f_{\lambda',\lambda o}\ (\gamma_{virt} N \to \pi\Delta) = \sum_V \frac{em_V^2}{2\gamma_V} \frac{1}{(k^2 - m_V^2)} \sqrt{k^2} \cdot$$

$$\cdot \sum_i a^i_{\lambda',\lambda o}\ (s,t,k^2)\ B^V_i\ (s,t), \tag{15}$$

$$f_{\lambda',\lambda\pm1}\ (\gamma_{virt} N \to \pi\Delta) = \sum_V \frac{em_V^2}{2\gamma_V} \frac{1}{(k^2 - m_V^2)} \cdot$$

$$\cdot \sum_i a^i_{\lambda',\lambda\pm1}\ (s,t,k^2)\ B^V_i\ (s,t).$$

Introducing the invariant amplitudes for the photon reaction with real photons B_i by

$$B_i = \sum_V \frac{e}{2\gamma_V} B^V_i\ (s,t\ ), \tag{16}$$

we have with $m_\rho \simeq m_\omega$ and even $m_\rho \simeq m_\phi$ for the small ϕ contribution

$$f_{\lambda',\lambda o}\ (\gamma_{virt}N\rightarrow\pi\Delta) = \frac{m_\rho^2}{(k^2-m_\rho^2)}\sqrt{-k^2}\sum_i a^i_{\lambda',\lambda o}\ (s,t,k^2)B_i(s,t),$$

$$f_{\lambda',\lambda\pm 1}\ (\gamma_{virt}N\rightarrow\pi\Delta) = \frac{m_\rho^2}{(k^2-m_\rho^2)}\sum_i a^i_{\lambda',\lambda\pm 1}\ (s,t,k^2)B_i\ (s,t). \qquad (17)$$

As only 8 invariant amplitudes appear in these equations (compare equs.(14)), the knowledge of the 8 independent transverse helicity amplitudes (directly from experiment, if available, or from a model calculation, if no complete set of experiments has been performed) for real photoproduction $k^2 = 0$.

$$f_{\lambda',\lambda\pm 1}\ (\gamma N\rightarrow\pi\Delta) = -\sum_{i\neq 1,5,8,11} a^i_{\lambda',\lambda\pm 1}\ (s,t,o)\ B_i\ (s,t) \qquad (18)$$

will allow us to predict transverse and longitudinal electroproduction from real photoproduction by substituting the 8 invariant amplitudes appearing in (18) into (17). Also, from (16) and (14), longitudinal and transverse amplitudes for the strong interaction process $\rho^o N \rightarrow \pi\Delta$ may be predicted from a knowledge of the isovector photoproduction amplitudes in (18).

So far we have retained the kinematical k^2 dependence in the coefficients $a^i_{\lambda',\lambda\mu}$, although we have been considering the high energy approximation with k^2 independence of the amplitudes $B_i(s,t)$. Explicit calculation shows that the coefficients a^i, for large s, i.e. if eq.(12) holds, are actually independent of k^2, as may be seen in the Appendix II. As we are using s-channel helicity amplitudes, we thus have the result that for sufficiently large s, the transverse s-channel helicity amplitudes are independent of k^2 and the longitudinal ones are independent of k^2 apart from the overall factor $\sqrt{\pm k^2}$. The formulation of vector meson dominance in terms of mass independence of the invariant amplitudes B_i of eq.(5), is thus consistent with the ansatz used previously [10], namely

$$f_{\lambda',\lambda o}\,(\gamma_{virt}N\to\pi\Delta) = \frac{m_\rho^2}{(k^2-m_\rho^2)}\sqrt{\frac{-k^2}{m_\rho^2}}\;\sum_V \frac{e}{2\gamma_V}\, f^V_{\lambda',\lambda o}\,(VN\to\pi\Delta)$$

$$f_{\lambda',\lambda\pm 1}\,(\gamma_{virt}N\to\pi\Delta) = \frac{m_\rho^2}{(k^2-m_\rho^2)}\;\sum_V \frac{e}{2\gamma_V}\, f^V_{\lambda',\lambda\pm 1}\,(VN\to\pi\Delta) \qquad (19)$$

3. The Smoothness Assumptions in Born Term Models

In this chapter we shall briefly demonstrate that the smoothness assumptions made for the invariant amplitudes in chapter 2 are fulfilled in simple Born term models.

A model, which in the region of very small values of the momentum transfer $|t| \lesssim \mu^2$ correctly describes the dip structure observed in the photonreaction $\gamma p \to \pi^- \Delta^{++}$, is the minimal gauge invariant extension [12] of simple one pion exchange (GIOPE model) and is based on the diagrams of Fig.2. Only the orbital current part of diagrams b and c is taken into account. Replacing the photon by a ρ^o meson with universal coupling $f_\rho = 2\gamma_\rho$ to the hadrons, one obtains a model for $\pi N \to \rho^o N$, which has been compared with some experimental data in ref. 9. The agreement with the measured ρ^o density matrix elements is good,and in the region of small $|t| \lessapprox \mu^2$, there is even agreement in the normalization of the cross section. The electric Born term model thus is not completely unrealistic at least within the narrow forward region of small $-t$.

Explicitly, the contributions from the diagrams a, b, c, d of Fig. 2 for $\rho^o p \to \pi^- \Delta^{++}$ are given by [12]

$$T_a = 2f \cdot f_\rho \frac{(\varepsilon q)}{t-\mu^2} \bar{u}_\mu (p_2) (k-q)^\mu u(p_1)$$

$$T_b = f_\rho f \bar{u}_\mu (p_2)(-q)^\mu u(p_1) \frac{2(\varepsilon p_1)}{s-m^2}$$

$$T_c = -f \cdot f_\rho \frac{2(\varepsilon p_2)}{u-M^2} \bar{u}_\mu (p_2) q^\mu u(p_1)$$

$$T_d = f \cdot f_\rho \bar{u}_\mu (p_2) \varepsilon^\mu u(p_1) \tag{20}$$

where f is the $\pi^+ p \Delta^{++}$ coupling constant ($f^2/4\pi = 18.9$ GeV^{-2}). The total transition amplitude in the Stichel-Scholz model is constructed to have only isospin I = 1 in the t-channel and is then given by:

$$T = T_a - \frac{1}{4} T_b - \frac{5}{4} T_c + T_d \tag{21}$$

An extension of this model which drops this restriction and is applied also to $\gamma p \to \Delta^o \pi^+$ is done in ref. 13.

From expressions (20) and (21) the invariant amplitude B_i^ρ are easily computed to be

$$B_1^\rho = \frac{-2f_\rho f}{t-\mu^2} + \frac{1}{4} \frac{f_\rho f}{s-m^2} - \frac{5}{4} \frac{f_\rho f}{u-M^2}$$

$$B_2^\rho = \frac{1}{2} \frac{f_\rho f}{s-m^2} + \frac{5}{2} \frac{f_\rho f}{u-M^2} \tag{22}$$

$$B_3^\rho = \frac{2f_\rho f}{t-\mu^2}$$

$$B_5^\rho = f_\rho f$$

$$B_4^\rho = B_6^\rho = B_7^\rho = B_8^\rho = B_9^\rho = B_{10}^\rho = B_{11}^\rho = B_{12}^\rho = 0 \qquad (22)$$

From current conservation (9) one obtains

$$B_{13}^\rho = \frac{-f_\rho f}{t-\mu^2} \qquad B_{14}^\rho = B_{16}^\rho = 0$$

$$B_{15}^\rho = \frac{f_\rho f}{t-\mu^2} \qquad (23)$$

From (22) and (23) we now see that the postulated smoothness assumptions for the amplitudes B_i^ρ in this particular Born term model (20) hold, as long as $(u-M^2)$ is insensitive against extrapolation in the vector meson mass from $k^2 = m_v^2$ to $k^2 < 0$, at constant s and t. This is the case for sufficiently large s such that eq.(12) is fulfilled.

One can furthermore show that smoothness in k^2 under the condition (12) even holds for a more general electric Born term model, in which the full electric coupling of the baryons and the full baryon propagators are taken into account.

4. Born Term Model with Absorption Corrections as Parameterization of Photoproduction

In section 2 we have shown that vector meson dominance with current conservation and mass independence of the invariant amplitudes allows to express longitudinal vector meson and virtual photon amplitudes in terms of transverse real photon helicity amplitudes. In order to thus obtain numerical predictions for electroproduction, as no complete set of polarization measurements has been made for photoproduction so far, a model for the transverse real photoproduction amplitudes is necessary as a parameterization of the photoproduction data.

A reasonable description of the $\gamma p \to \pi^- \Delta^{++}$ photoproduction cross section at 2.8 and 4.7 GeV photon energy for $|t| \lesssim 0.3\ \mathrm{GeV}^2/c^2$ is obtained, if the Born term model[12] described in section 3 is supplemented[13,14] with absorption corrections. In the spirit of the vector meson photon analogy, the absorptive corrections in ref. 14 have been applied to the ingoing and outgoing states in $\gamma p \to \pi^- \Delta^{++}$ by multiplying the spin J partial wave helicity amplitude with the factor (q = cms momentum of ingoing or outgoing system)

$$\{1 - C_{in} \exp(-(J-\tfrac{1}{2})^2/2A_{in}\, q_{in}^2)\}^{\frac{1}{2}} \cdot \{1-C_{out} \exp(-(J-\tfrac{1}{2})^2/2A_{out}\, q_{out}^2)\}^{\frac{1}{2}}. \qquad (24)$$

The slope parameters A_{in}, A_{out} have been assumed to be the same as measured for Compton and elastic πp scattering, respectively, i.e. $A_{in} = 6\ \mathrm{GeV}^{-2}$, $A_{out} = 8\ \mathrm{GeV}^{-2}$. For the absorption parameters $C_{in} = C_{out} = 0.8$ has been assumed as in ref. 14. It turns out to be very important to take into account the finite width of the Δ. This was done by expressing the $\pi N \Delta$ coupling f by the πN mass dependent decay width $\Gamma(s_{\pi N})$, ($s_{\pi N} \equiv$ invariant mass squared of the πN

system) then multiplying the cross section with a Breit Wigner form [15] and integrating over the π^+p mass range using the (3,3) phase shifts. The curve obtained [14] for $\frac{d\sigma}{dt}(\gamma p \to \pi^- \Delta^{++})$ is shown on Fig. 3, in order to clearly state the starting point of our VDM predictions for electroproduction.

5. Predictions for $\pi\Delta$ Electroproduction and Comparison with Experiment

5.1. The $\pi\Delta$ Electroproduction Cross Section

In the one photon exchange approximation the cross section for $\pi\Delta$ electroproduction from nucleons may be written as

$$\frac{d\sigma}{dW^2 dk^2 dt d\phi} = \frac{\alpha}{4(2\pi)^2} \frac{W^2-m^2}{E^2 m^2 k^2} \frac{1}{1-\varepsilon} \times$$

$$\times \left[\frac{1}{2}(\sigma^{xx}+\sigma^{yy}) + \varepsilon\sigma^{oo} - \varepsilon \cos 2\phi \, \frac{1}{2}(\sigma^{yy} - \sigma^{xx}) + \right.$$

$$\left. + \sqrt{2\varepsilon(1+\varepsilon)} \cos\phi \cdot \sigma^{xo}\right] \quad , \tag{25}$$

where ϕ is the angle between the electron plane defined by the three momenta of the ingoing and outgoing electrons, and the hadron plane defined by the three momenta of the outgoing pion and Δ. The convention used for the definition of the normals to these two planes and for the angle ϕ is the same as in ref. 10 . E is the energy of the incident electron. The polarization parameter ε of the virtual photon is given by

$$\varepsilon = \left[1 + 2\left(1 + \frac{(E-E')^2}{k^2} \tan^2 \frac{\delta}{2}\right)\right]^{-1}, \tag{26}$$

where E' is the energy and δ the laboratory scattering angle of the outgoing electron. The quantities $\sigma^{xx}(k^2, W^2, t)$, $\sigma^{yy}(k^2, W^2, t)$, etc. are bilinear in the amplitude for production of $\pi\Delta$ from nucleons by virtual photons, e.g.

$$\sigma^{xx} = \frac{1}{2}\, a \sum_{\lambda'\lambda} f^{x}_{\lambda'\lambda}(k^2,W^2,t)\, f^{x}_{\lambda'\lambda}(k^2,W^2,t)$$

$$a = \frac{1}{64\pi|\vec{k}|^2 s}\,|\vec{k}| = \frac{1}{2W}\left[\left((W-m)^2+k^2\right)\left((W+m)^2+k^2\right)\right]^{\frac{1}{2}} \qquad (27)$$

x, y, 0 referring to linear polarization states of the virtual photon. The indices x and y correspond to linear polarization in and perpendicular to the $\pi\Delta$ plane and 0 denotes longitudinal polarization with respect to the cms. of the outgoing π and Δ (compare also ref. 10).

For completeness we give the relation between σ^{xx} etc. and the conventional quantities used in ref. 11

$$\sigma_u \equiv \frac{1}{2}(\sigma^{xx}+\sigma^{yy})$$

$$\sigma_L \equiv \sigma^{oo}$$

$$\sigma_T \equiv \frac{1}{2}(\sigma^{xx}-\sigma^{yy})$$

and

$$\sigma_I \equiv -\sigma^{xo}, \qquad (28)$$

as our choice [10] of the angle ϕ differs from the one in ref. 11 by 180^o. The normalization is such that σ_U at $q^2 = 0$ corresponds to the unpolarized photoproduction cross section $d\sigma/dt$ integrated over the azimuthal production angle.

5.2. Predictions for $\pi\Delta$ Electroproduction and Comparison with Experiment

We now make use of the parameterization of the $\pi\Delta$ real photoproduction helicity amplitudes in the Born term model with absorption corrections, which was reviewed in section 4, in order to predict electroproduction from photoproduction. Inserting this parameterization of the helicity amplitudes into eq.(18), we numerically solve for the invariant amplitudes $B_i(s,t)$ $(i = 2,3,4,6,7,9,10,12)$. Substituting the resulting amplitudes into eqs.(17) yields the predictions for the longitudinal and transverse virtual photoproduction amplitudes resulting from vector dominance and smoothness of the invariant amplitudes. In computing σ_U, σ_L, σ_T and σ_I from (28) the finite width of the Δ was taken into account in the same way as described for the case of photoproduction in chapter 4. This finite width correction lowers the (uncorrected) theoretical value of the cross section by roughly 30 %.

A comparison with the experimental data[(11)] on $\gamma_{virt}\, p \rightarrow \pi^+\Delta^o$ is shown on figures 4 and 5. From these figures, it is seen that qualitative agreement with the data is achieved only. In particular, the prediction for the longitudinal part σ_L seems to be too low.

The discrepancies on figs. 4 and 5 are somewhat surprising in view of the rather good agreement of VDM predictions with $\pi^+\Delta^o$ electroproduction, which has recently been found by two of the present authors [10]. In ref. 10 the model has been formulated in terms of s channel helicity amplitudes. The predictions for σ_U from photoproduction thus coincide* with the predictions of the present work.

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* The variation of the phase space factor a in eq.(27) with k^2 has been taken into account here in contrast to previous work [10] and results in a small(10-20 %) decrease of σ_U in comparison with ref. 10.

The longitudinal cross section, however, has been predicted from the ratio of the ρ^o density matrix elements $\frac{\rho_{oo}}{\rho_{11}}$ produced in $\pi^+ p \rightarrow \rho^o \Delta^{++}$ - assuming line reversal invariance for this ratio and no I = 2 exchange contribution - by using the $\frac{k^2}{m_\rho^2}$ extrapolation procedure and taking the normalization from photoproduction. Let us discriminate between different possible explanations for the differences of the results between this and the previous work [10]:

(1) There is the possibility that vector meson dominance as formulated previously for the s channel helicity amplitudes is correct, whereas the longitudinal transverse connection as implied by smooth invariant amplitudes does not hold. In view of the close connection between smoothness of the transverse helicity amplitudes and of the invariant amplitudes, it seems that this possibility is a somewhat remote one, however.

(2) Also, that previous agreement [10] is fortuitous and a consequence of a fortuitous interplay of a violation of crossing and smoothness seems rather remote.

(3) Assuming that previous results [10] are not fortuitous and excluding possibility (1), one is inclined to believe that shortcomings in the parameterization of photoproduction in terms of the absorptive GIOPE model used as input to the present work, are the reason for larger discrepancies as seen on figs. 4 and 5. Indeed, although no better model for Δ photoproduction is available at present, and the unpolarized cross section is reasonable well reproduced by the GIOPE with absorption, there are discrepancies in some of the density matrix elements [14] of the Δ and in the recently measured asymmetry parameter[17]. As all the helicity amplitudes are important for the prediction of the longitudinal amplitudes in terms of the transverse ones, discrepancies may well result from discrepancies between theory and experiment for the photoproduction

input.

(4) No attempt has been made in this work to study the dependence of the results obtained from smoothness on the choice of the invariant amplitudes. Smoothness of the s channel helicity amplitudes and of the invariant amplitudes in simple Born term models most probably do not uniquely determine the set of invariant amplitudes to be used.

In order to clarify the situation from the theoretical side, while keeping in mind this last mentioned possibility, work along the lines of point 3, namely a better parameterization of photoproduction, seems to us most likely to yield improved predictions. Also, let us keep in mind that we are using vector meson dominance at quite low energies, and from the experimental side, further $\pi\Delta$ electroproduction data at higher energies and higher k^2, a measurement of different charge states, and finally longitudinal transverse separation would be most valuable.

6. Summarizing and Concluding Remarks

1. Generalizing results obtained in simple Born term models (section 3), we assumed that the chosen kinematic singularity free invariant amplitudes for the reaction $VN \to \pi\Delta$ at sufficiently high energies are independent of the vector-meson mass k^2. From this assumption we obtained the result that the transverse s channel helicity amplitudes for $VN \to \pi\Delta$ are also independent of the vector meson mass at sufficiently high energies. The longitudinal amplitudes are independent of the vector meson mass k^2 apart from a factor $\sqrt{k^2}$ originating from current conservation. These results are consistent with the formulation of vector meson dominance in terms of s channel heli-

city amplitudes used previously [7,10].

2. From current conservation and vector-meson mass independence at high energies of the invariant amplitudes we obtained restrictions, reducing the number of independent invariant amplitudes from 12 to 8. The longitudinal vector meson reaction amplitudes e.g. for $\rho^o N \to \pi\Delta$ may thus be obtained from the transverse ones, which are related to the isovector photoproduction amplitudes by multiplication with the usual coupling constant. Quite similarly, longitudinal electroproduction is obtained from transverse real photoproduction.

3. A comparison with experiment has been carried through for $\gamma_{virt}\, p \to \pi^+\Delta^o$, where some experimental work [11] has come out recently. A parameterization of the real (transverse) photoproduction data by a Born term model with absorption corrections [12,13,14] has been used as input, leading to a prediction for transverse and longitudinal electroproduction. No parameter has been introduced in going from photo- to electroproduction. The model reproduces the data only qualitatively. The possible reasons for this discrepancy are summarized at the end of chapter 5. Further experimental material for the reactions $ep \to e\Delta^o\pi^+$, $e\Delta^{++}\pi^-$, particularly at higher energies, would be of great value in order to test the vector meson dominance model based on smoothness in the photon mass of the invariant amplitudes.

4. A further test of the model is of course provided by comparing predictions obtained from $\gamma N \to \pi\Delta$ with data on $\pi N \to V\Delta$, using a model for photoproduction which allows to explicitly perform the crossing transformation from the s channel to the u channel. This test has not yet been carried through.

5. In summary: The general results for $VN \rightarrow \pi\Delta$ and $\gamma\binom{real}{virt} N \rightarrow \pi\Delta$ are in complete analogy to $VN \rightarrow \pi N$ and $\gamma\binom{real}{virt} N \rightarrow \pi N$: Smoothness with respect to the s channel helicity frame, if smoothness is required for a specific set of kinematic singularity free invariant amplitudes, which fulfil smoothness in simple Born term models. The status as regards the specific prediction of longitudinal and transverse $\pi^+\Delta^o$ electroproduction is not as satisfactory as in single pion production, but this may be a reflection of the less reliable input photoproduction model for the case of $\pi\Delta$ production.

Acknowledgements:

We would like to thank Dr. G. Wolf for providing us with his computer program for $\pi\Delta(1236)$ photoproduction.
Useful discussions with Drs. K. Heinloth, G. Hofmann and D. Schmidt on the DESY $\pi\Delta$ electroproduction data and also with Drs. D. Lüke, P. Söding and G. Wolf on $\pi\Delta$ photoproduction are gratefully acknowledged.

W. M. expresses his gratitude to Profs. H. Joos, E. Lohrmann and W. Paul for the hospitality extended to him at DESY. A. B. thanks Prof. W. Sandhas for discussions and DESY for financial support of several stays in Hamburg while this work has been performed.

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Appendix I:

The s-channel helicity amplitudes in terms of the invariant amplitudes for the process $\rho^0 N \to \pi\Delta$ have the following form:

Transversal:

$$N.\,f^{V}_{\frac{3}{2},\frac{1}{2}1} = \sin^2\frac{\Theta}{2}\cos\frac{\Theta}{2}\Big\{\eta_-\big[(2B_3-B_4)kq\cos^2\frac{\Theta}{2}-B_5\big]-(k_0\eta_++k\xi_+)(B_7kq\cos^2\frac{\Theta}{2}+B_8)+$$

$$+2k\big[B_{10}(k_0\xi_-+k\eta_-)-B_{12}\xi_+\big]\Big\}$$

$$N.\,f^{V}_{\frac{3}{2},\frac{1}{2}-1} = \cos^3\frac{\Theta}{2}\Big\{\eta_-\big[(-2B_3+B_4)kq\sin^2\frac{\Theta}{2}-B_5\big]+(k_0\eta_++k\xi_+)\big(B_7kq\sin^2\frac{\Theta}{2}-B_8\big)\Big\}$$

$$N.\,f^{V}_{-\frac{3}{2},\frac{1}{2}-1} = \sin^3\frac{\Theta}{2}\Big\{\eta_+\big[(2B_3-B_4)kq\cos^2\frac{\Theta}{2}-B_5\big]-(k_0\eta_-+k\xi_-)(B_7kq\cos^2\frac{\Theta}{2}+B_8)\Big\}$$

$$N.\,f^{V}_{-\frac{3}{2},\frac{1}{2}1} = \sin\frac{\Theta}{2}.\cos^2\frac{\Theta}{2}\Big\{\eta_+\big[(-2B_3+B_4)kq\sin^2\frac{\Theta}{2}-B_5\big]+(k_0\eta_-+k\xi_-)\big(B_7kq\sin^2\frac{\Theta}{2}-B_8\big)-$$

$$-2k\big[-B_{10}(k_0\xi_++k\eta_+)+B_{12}\xi_-\big]\Big\}$$

$$N.\,f^{V}_{\frac{1}{2},\frac{1}{2}1} = \frac{1}{\sqrt{3}}\sin\frac{\Theta}{2}\Big\{\sin^2\frac{\Theta}{2}\big\{\eta_+\big[(2B_3-B_4)kq\cos^2\frac{\Theta}{2}-B_5\big]-(k_0\eta_-+k\xi_-)(B_7kq\cos^2\frac{\Theta}{2}+B_8)\big\}+$$

$$+2\cos^2\frac{\Theta}{2}.\,k\big[-B_{10}(k_0\xi_++k\eta_+)+B_{12}\xi_-\big]-\frac{1}{M}\cos^2\frac{\Theta}{2}\big\{-\eta_-\big[q^2\sqrt{5}(-2B_1+B_2)+$$

$$+(-2B_3+B_4)q(k_0q+kE_2\cos\Theta)+2B_5E_2\big]-(k_0\eta_++k\xi_+)\big[B_6q^2\sqrt{5}+$$

$$+q(k_0q+kE_2\cos\Theta)B_7+2B_8E_2\big]\big\}-\frac{2}{M}\big\{q\sqrt{5}\big[B_9(k_0\xi_-+k\eta_-)-B_{11}\xi_+\big)+$$

$$+(k_0q+kE_2\cos\Theta)\big[B_{10}(k_0\xi_-+k\eta_-)-B_{12}\xi_+\big]\big\}\Big\}$$

$$N.\,f^{V}_{-\frac{1}{2},\frac{1}{2}-1} = \frac{1}{\sqrt{3}}\sin^2\frac{\Theta}{2}\cos\frac{\Theta}{2}\Big\{-\frac{1}{M}\big\{\eta_+\big[q^2\sqrt{5}(-2B_1+B_2)+(-2B_3+B_4)q(k_0q+kE_2\cos\Theta)+$$

$$+2B_5E_2\big]+(k_0\eta_-+k\xi_-)\big[B_6q^2\sqrt{5}+B_7q(k_0q+kE_2\cos\Theta)+2B_8E_2\big]\big\}+$$

$$+\eta_-\big[(2B_3-B_4)kq\cos^2\frac{\Theta}{2}-B_5\big]-(k_0\eta_++k\xi_+)(B_7kq\cos^2\frac{\Theta}{2}+B_8)\Big\}$$

$$N\cdot f^V_{\frac12,\frac12,-1} = \frac{1}{\sqrt3}\sin\frac{\Theta}{2}\cos^2\frac{\Theta}{2}\Big\{\eta_+\left[(-2B_3+B_4)kq\sin^2\frac{\Theta}{2}-B_5\right]+(k_0\eta_-+k\xi_-)(B_7kq\sin^2\frac{\Theta}{2}-B_8)-$$

$$-\frac{1}{M}\Big\{\eta_-\left[q^2\sqrt{s}(-2B_1+B_2)+(-2B_3+B_4)q(k_0q+kE_2\cos\Theta)+2B_5E_2\right]+$$

$$+(k_0\eta_++k\xi_+)\left[B_6q^2\sqrt{s}+B_7q(k_0q+kE_2\cos\Theta)+2B_8E_2\right]\Big\}\Big\}$$

$$N\cdot f^V_{-\frac12,\frac12,1} = \frac{1}{\sqrt3}\cos\Theta/2\Big\{\frac{1}{M}\sin^2\frac{\Theta}{2}\Big\{\eta_+\left[q^2\sqrt{s}(-2B_1+B_2)+(-2B_3+B_4)q(k_0q+kE_2\cos\Theta)+\right.$$

$$\left.+2B_5E_2\right]+(k_0\eta_-+k\xi_-)\left[B_6q^2\sqrt{s}+B_7q(k_0q+kE_2\cos\Theta)+2B_8E_2\right]\Big\}-$$

$$-\frac{2}{M}\Big\{q\sqrt{s}\left[-B_9(k_0\xi_++k\eta_+)+B_{11}\xi_-\right]+(k_0q+kE_2\cos\Theta)\left[-B_{10}(k_0\xi_++k\eta_+)+\right.$$

$$\left.+B_{12}\xi_-\right]\Big\}-\cos^2\frac{\Theta}{2}\Big\{\eta_-\left[(2B_3-B_4)kq\sin^2\frac{\Theta}{2}+B_5\right]-(k_0\eta_++k\xi_+)\cdot$$

$$\cdot(B_7kq\sin^2\frac{\Theta}{2}-B_8)\Big\}-2k\sin^2\frac{\Theta}{2}\left[B_{10}(k_0\xi_-+k\eta_-)-B_{12}\xi_+\right]\Big\}$$

Longitudinal:

$$N\cdot f^V_{\frac32,\frac12 0} = \sqrt2\cos^2\frac{\Theta}{2}\sin\frac{\Theta}{2}\cdot\frac{1}{m_\rho}\Big\{-\eta_-\left[B_3k(qk_0\cos\Theta-kq_0)-B_4\frac12k\left[k(\sqrt{s}+E_2)+qk_0\cos\Theta\right]-\right.$$

$$\left.-B_5k_0\right]+(k_0\eta_++k\xi_+)\left[B_8k_0+B_7\frac12k\left[k(\sqrt{s}+E_2)+qk_0\cos\Theta\right]\right]+$$

$$+k\left[-B_{10}\xi_-m_\rho^2+B_{12}(k\eta_++k_0\xi_+)\right]\Big\}$$

$$N\cdot f^V_{-\frac32,\frac12 0} = \sqrt2\cos\frac{\Theta}{2}\cdot\sin^2\frac{\Theta}{2}\frac{1}{m_\rho}\Big\{\eta_+\left[B_3k(qk_0\cos\Theta-kq_0)-B_4\frac12k\left[k(\sqrt{s}+E_2)+qk_0\cos\Theta\right]-\right.$$

$$\left.-B_5k_0\right]-(k_0\eta_-+k\xi_-)\left[B_7\frac12k\left[k(\sqrt{s}+E_2)+qk_0\cos\Theta\right]+B_8k_0\right]-$$

$$-k\left[-B_{10}\xi_+m_\rho^2+B_{12}(k\eta_-+k_0\xi_-)\right]\Big\}$$

$$f^V_{\frac12,\frac12 0} = -\frac{1}{\sqrt3}f^V_{-\frac32,\frac12 0}-\sqrt{\frac23}\frac{\cos\frac{\Theta}{2}}{N}\frac{1}{Mm_\rho}\Big\{\eta_-\Big[q\sqrt{s}\left[B_1(kq_0-qk_0\cos\Theta)+\frac12B_2\left(k(\sqrt{s}+E_2)+qk_0\cos\Theta\right)\right]+$$

$$+(qk_0+E_2k\cos\Theta)\left[B_3(kq_0-qk_0\cos\Theta)+\frac12B_4\left(k(\sqrt{s}+E_2)+qk_0\cos\Theta\right)\right]+$$

$$+B_5(qk+E_2k_0\cos\Theta)\Big]+(k_0\eta_++k\xi_+)\Big[\frac12\left(k(\sqrt{s}+E_2)+qk_0\cos\Theta\right)\left[B_6q\sqrt{s}+\right.$$

$$\left.+B_7(qk_0+E_2k\cos\Theta)\right]+B_8(qk+E_2k_0\cos\Theta)\Big]-m_\rho^2\xi_-\left[B_9q\sqrt{s}+\right.$$

$$\left.+B_{10}(qk_0+E_2k\cos\Theta)\right]+(k\eta_++k_0\xi_+)\left[B_{11}q\sqrt{s}+B_{12}(qk_0+E_2k\cos\Theta)\right]\Big\}$$

$$f^{V}_{-\frac{1}{2},\frac{1}{2}0} = -\sqrt{\frac{2}{3}}\,\frac{\sin\frac{\theta}{2}}{N}\,\frac{1}{Mm_\rho}\Big\{\eta_+\big[q\sqrt{s}\,\big[B_1(kq_0 - qk_0\cos\Theta) + \tfrac{1}{2}B_2\big(k(\sqrt{s}+E_2)+qk_0\cos\Theta\big)\big] +$$

$$+ (qk_0 + E_2 k\cos\Theta)\big[B_3(kq_0 - qk_0\cos\Theta) + \tfrac{1}{2}B_4\,[k(\sqrt{s}+E_2)+qk_0\cos\Theta]\big] +$$

$$+ B_5(qk + E_2 k_0\cos\Theta)\big] + (k_0\eta_- + k\xi_-)\Big[\tfrac{1}{2}\big(k(\sqrt{s}+E_2)+qk_0\cos\Theta\big)\big[B_6\, q\sqrt{s} +$$

$$+ B_7(qk_0 + E_2 k\cos\Theta)\big] + B_8(qk + E_2 k_0\cos\Theta)\Big] - m_\rho^2\,\xi_+\,\big[B_9\, q\sqrt{s} + B_{10}\,\cdot$$

$$\cdot\,(qk_0 + E_2 k\cos\Theta)\big] + (k\eta_- + k_0\xi_-)\,\big[B_{11}\, q\sqrt{s} + B_{12}(qk_0 + E_2 k\cos\Theta)\big]\Big\} - \frac{1}{\sqrt{3}}\,f^{V}_{\frac{3}{2},\frac{1}{2}0}$$

In $f_{\lambda',\lambda\mu}$ λ,μ,λ' denotes the helicity of the nucleon, the pion and the Δ, respectively. The other quantities are: Θ the c.m.s. scattering angle in the s-channel, k_1, q the incoming and outcoming c.m.s. momentum, respectively; k_0 is the energy of the photon, E_2 that one of the Δ in the c.m.s.. Finally the following abbreviations are used:

$$\eta_\pm = (E_2+M)(E_1+m) \pm kq \qquad\qquad N = [(E_1+m)(E_2+M)]^{\frac{1}{2}}$$

$$\xi_\pm = k(E_2+M) \pm q(E_1+m)$$

where E_1 is the energy of the nucleon in the c.m.s.

Appendix II :

For completeness we shall give below also the helicity amplitudes for $s \to \infty$ where we eliminated in addition four invariant amplitudes (here B_1, B_5, B_8, B_{11}) by the smoothness equations (13):

$$N.f^V_{\frac{3}{2},\frac{1}{2}1} = \sin^2\frac{\Theta}{2}\cos\frac{\Theta}{2}\,\frac{\sqrt{s}}{2}\left\{\frac{m_+}{2}sB_3 + \frac{m_+}{4}sB_4 + \frac{1}{4}s^2B_7 + sMB_{10}\right\}$$

$$N.f^V_{\frac{3}{2},\frac{1}{2}-1} = \cos^3\frac{\Theta}{2}\,\frac{\sqrt{s}}{2}\left\{\frac{1}{2}m_+\mu^2B_3 + \frac{1}{2}m_+sB_4 + \frac{1}{2}s^2B_7 + sB_{12}\right\}$$

$$N.f^V_{-\frac{3}{2},\frac{1}{2}1} = \sin\frac{\Theta}{2}\cos^2\frac{\Theta}{2}\cdot\frac{s}{2}\left\{\frac{\mu^2}{2}B_3 + \frac{1}{2}sB_4 + \frac{1}{2}MsB_7 + sB_{10} + mB_{12}\right\}$$

$$N.f^V_{-\frac{3}{2},\frac{1}{2}-1} = \sin^3\frac{\Theta}{2}\cdot\frac{s}{2}\left\{\frac{s}{2}B_3 + \frac{1}{4}sB_4 + \frac{1}{4}MsB_7 + MB_{12}\right\}$$

$$N.f^V_{\frac{1}{2},\frac{1}{2}1} = \frac{1}{\sqrt{3}}\sin\frac{\Theta}{2}\cdot\frac{s}{2}\left\{-\frac{m_+}{2M}\frac{s^2}{(t-\mu^2)}B_2 - \frac{m_+}{M}\cdot\frac{s}{2}B_3 - \frac{m_+}{M}\cdot\frac{s}{4}B_4 - \frac{s^2}{4M}B_6 - \frac{s^2}{4M}B_7 - \right.$$
$$\left. - sB_9 - 2sB_{10} - 2mB_{12}\right\}$$

$$N.f^V_{\frac{1}{2},\frac{1}{2}-1} = \frac{1}{\sqrt{3}}\sin\frac{\Theta}{2}\cos^2\frac{\Theta}{2}\cdot\frac{s}{2}\left\{\frac{m_+}{2M}\frac{s^2}{(t-\mu^2)}B_2 + \frac{m_+}{2M}sB_3 + \frac{s}{2}\left(1+\frac{m_+}{2M}\right)B_4 - \frac{s^2}{4M}B_6 + \right.$$
$$\left. + \frac{s^2}{4M}B_7 + \frac{s}{M}B_{12}\right\}$$

$$N.f^V_{-\frac{1}{2},\frac{1}{2}1} = \frac{1}{\sqrt{3}}\cos\frac{\Theta}{2}\,\frac{\sqrt{s}}{2}\left\{\frac{t}{2M}\frac{s^2}{(t-\mu^2)}B_2 + \frac{st}{2M}B_3 + \frac{s}{2}\left(\frac{t}{2M}+m_+\right)B_4 + \frac{m_-}{M}\cdot\frac{s^2}{2}B_6 + \right.$$
$$\left. + \frac{s^2}{2}B_7 + \frac{s^2}{M}B_9 + \frac{s^2}{M}B_{10} + s\left(1-\frac{m_-}{M}\right)B_{12}\right\}$$

$$N.f^V_{-\frac{1}{2},\frac{1}{2}-1} = \frac{1}{\sqrt{3}}\sin^2\frac{\Theta}{2}\cos\frac{\Theta}{2}\,s\sqrt{s}\left\{\frac{s^2}{4M}\frac{1}{(t-\mu^2)}B_2 + \frac{s}{4M}B_3 + \frac{s}{8M}B_4 - \frac{s}{8}B_6 + \frac{s}{4}B_7 + B_{12}\right\}$$

$$N.f^V_{\frac{3}{2},\frac{1}{2}0} = \sqrt{2}\cos^2\frac{\Theta}{2}\cdot\sin\frac{\Theta}{2}\cdot\frac{s}{8}m_\rho\left\{B_3m_+ + \frac{5}{2}m_+B_4 + \frac{5s}{2}B_7 + 2m_-B_{10} + 4B_{12}\right\}$$

$$N.f^V_{-\frac{3}{2},\frac{1}{2}0} = \sqrt{2}\cos\frac{\Theta}{2}\cdot\sin^2\frac{\Theta}{2}\,\frac{\sqrt{s}}{8}(-m_\rho)\left\{B_3s + \frac{5}{2}sB_4 + \frac{5}{2}sMB_7 + 2sB_{10} + 4m_+\cdot B_{12}\right\}$$

$$N.f^V_{\frac{1}{2},\frac{1}{2}0} = -\frac{1}{\sqrt{3}}f^V_{-\frac{3}{2},\frac{1}{2}0} - \frac{1}{N}\sqrt{\frac{2}{3}}\cos\frac{\Theta}{2}\frac{m_\rho}{M}\left\{B_2\frac{s^2\sqrt{s}}{8}\frac{m_+}{(t-\mu^2)} + \frac{s\sqrt{s}}{8}m_+\cdot\left(B_3+\frac{1}{2}B_4\right) + \right.$$
$$\left. + \frac{s^2\sqrt{s}}{16}(B_6+B_7) + \frac{s\sqrt{s}}{4}m_-\cdot(B_9+B_{10}) + \frac{\sqrt{s}}{2}(Mm_+ + t)B_{12}\right\}$$

$$N.f^V_{-\frac{1}{2},\frac{1}{2}0} = -\frac{1}{N}\sqrt{\frac{2}{3}}\sin\frac{\Theta}{2}\,\frac{s}{8M}m_\rho\left\{B_2\frac{s^2}{(t-\mu^2)} + s\left(B_3+\frac{1}{2}B_4\right) - \frac{s}{2}(4m-M)B_6 + \right.$$
$$\left. + \frac{s}{2}MB_7 + 2s(B_9+B_{10}) + 4mB_{12}\right\} - \frac{1}{\sqrt{3}}f^V_{\frac{3}{2},\frac{1}{2}0}$$

$$m_\pm = M \pm m$$

Figure Captions

Fig. 1 The process $VN \to \pi\Delta$, where V denotes a vector meson.

Fig. 2 Lowest order Feynman diagrams for the process $VN \to \pi\Delta$

Fig. 3 Comparison of theory and experiment for $\pi^-\Delta^{++}$ photoproduction . The experimental points are taken from ref. 14. The curve is the prediction of the gauge-invariant OPE model with absorption corrections.

Fig. 4 Comparison of theory and experiment ($\pi^+\Delta^o$ electroproduction) for $\sigma_u + \varepsilon\sigma_L$, σ_I, σ_T vs. $t-t_{min}$ for $s = 5.5\ GeV^2$, $k^2 = -0.5\ GeV^2/c^2$, $\varepsilon = 0.75$

The data are from ref. 11.

Fig. 5 Comparison of theory and experiment ($\pi^+\Delta^o$ electroproduction) for $\sigma_u + \varepsilon\sigma_L$, σ_I, σ_T vs. k^2 for $s = 5.5\ GeV^2$, $\varepsilon = 0.75$ $t-t_{min} = -0.05\ GeV^2/c^2$

The data are from ref. 11.

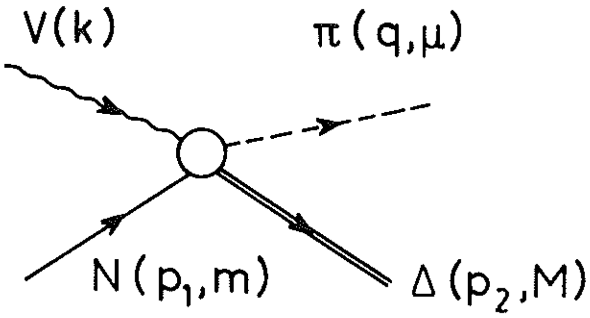


Fig. 1

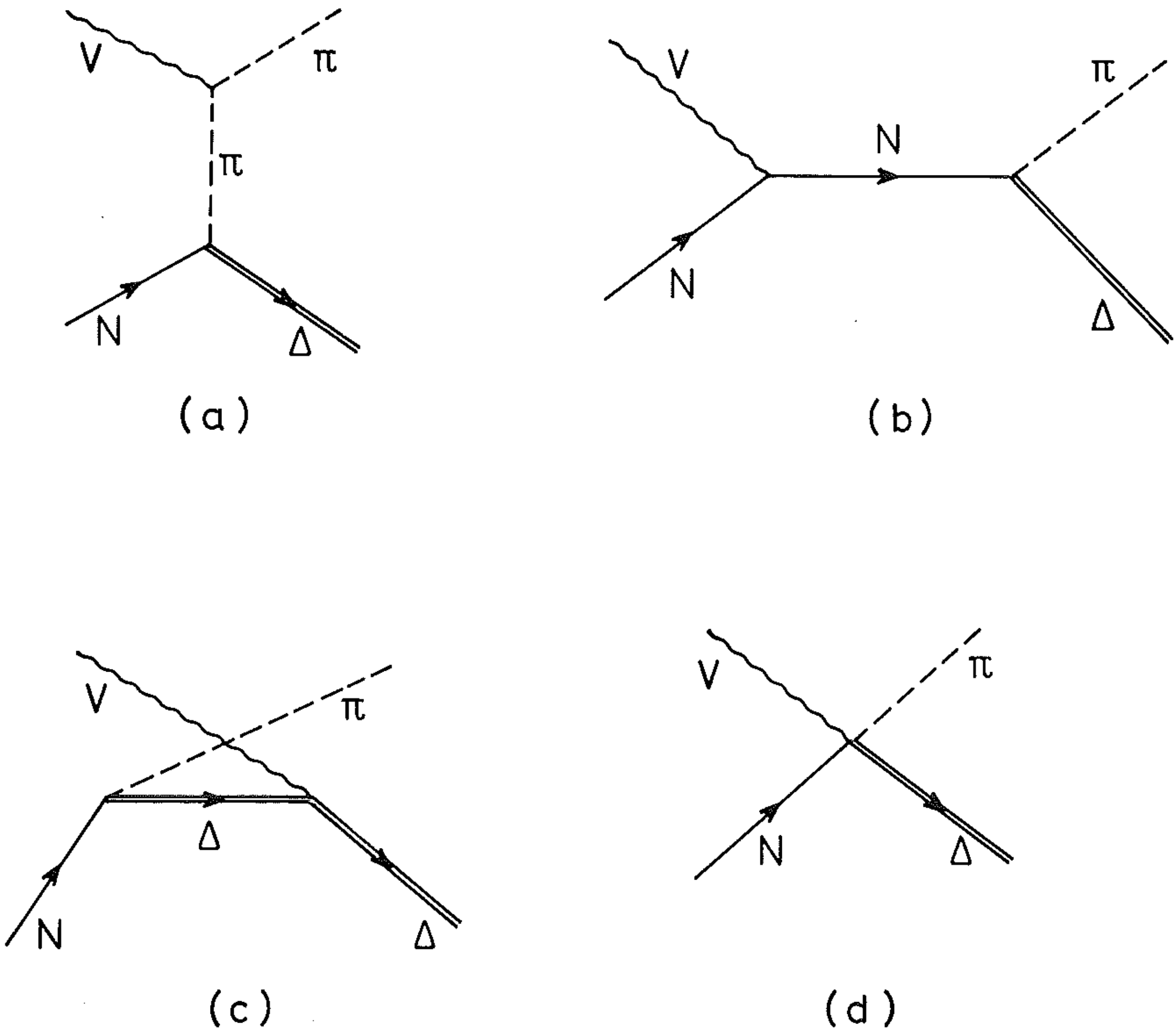


Fig. 2

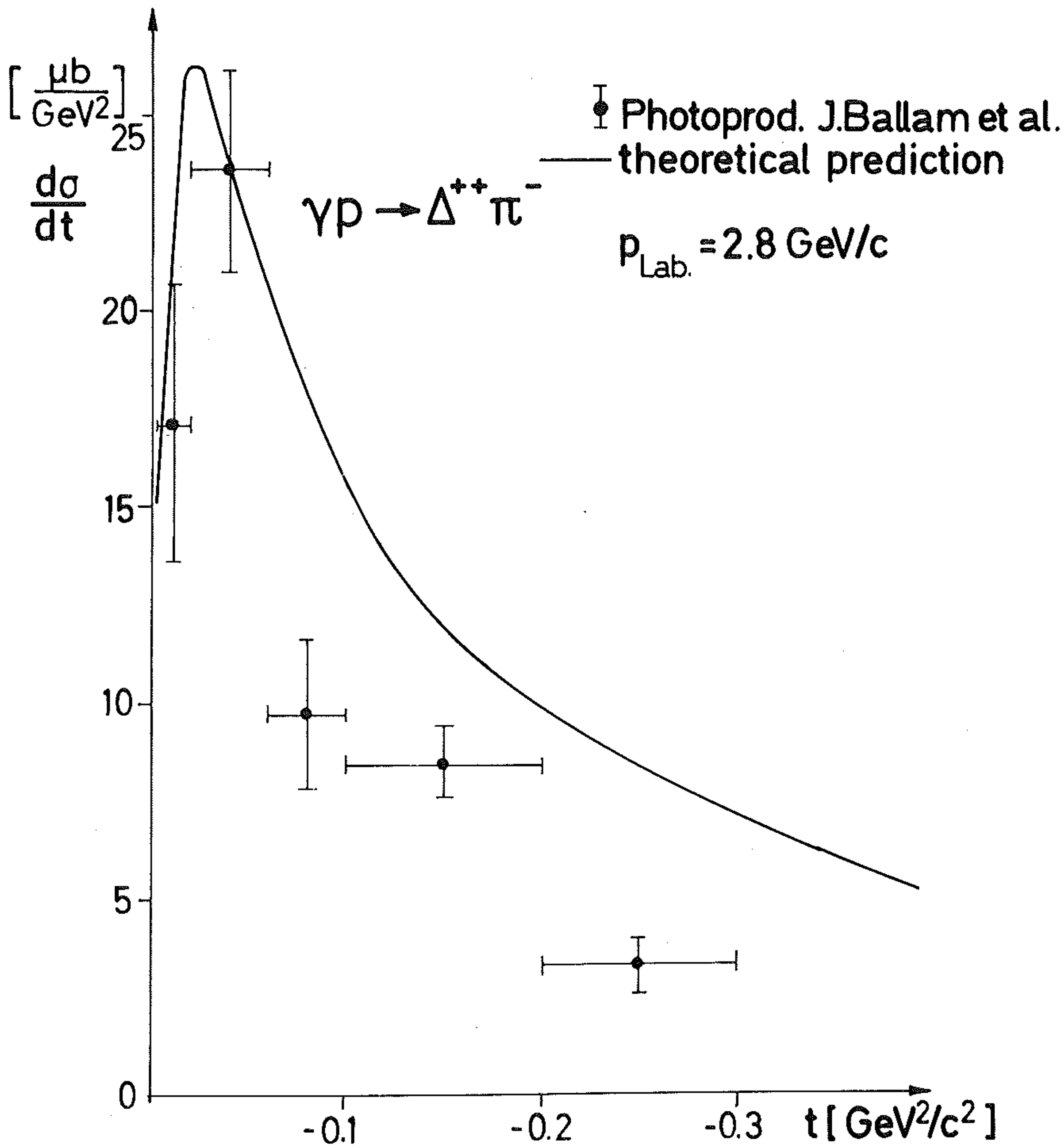
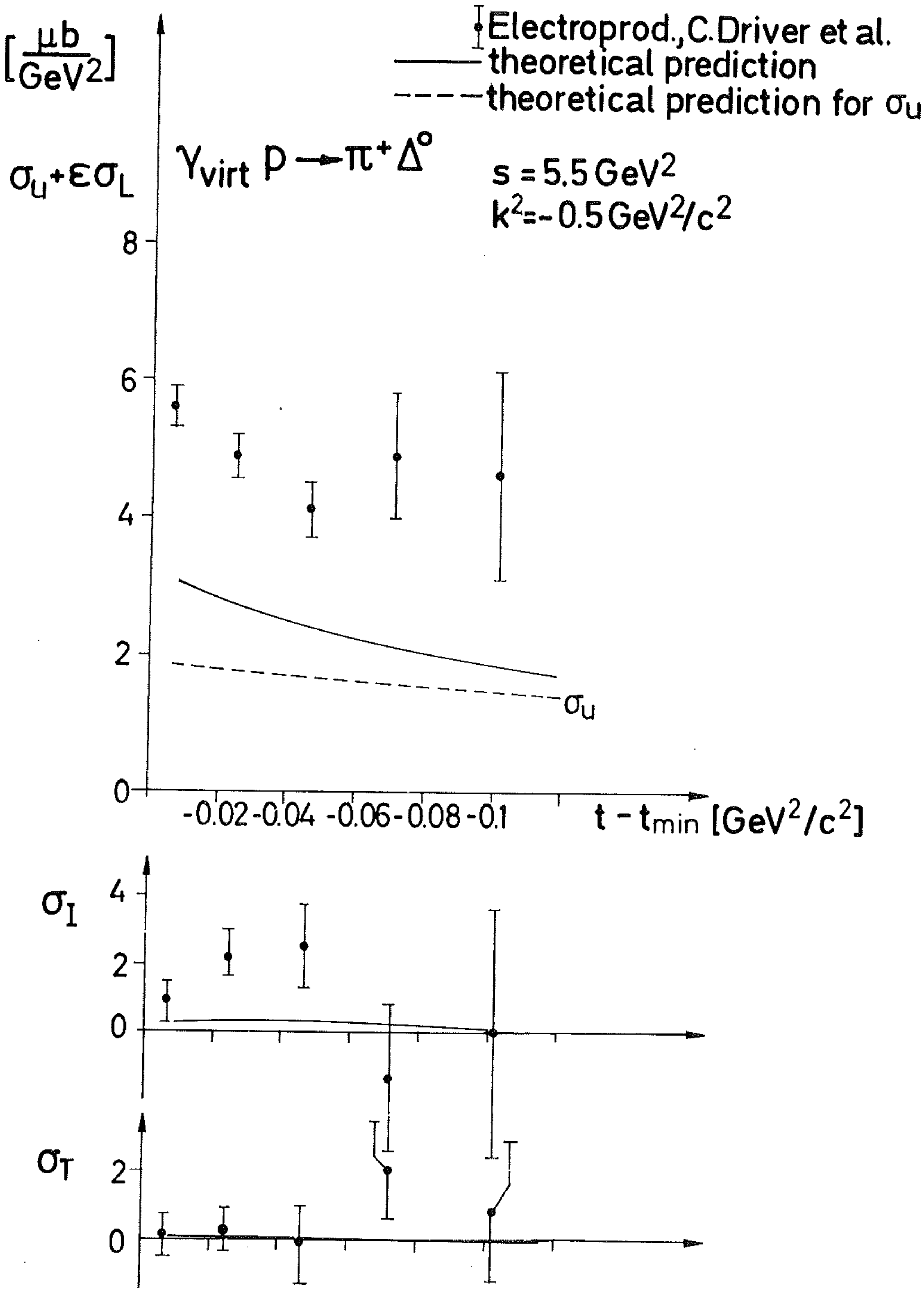
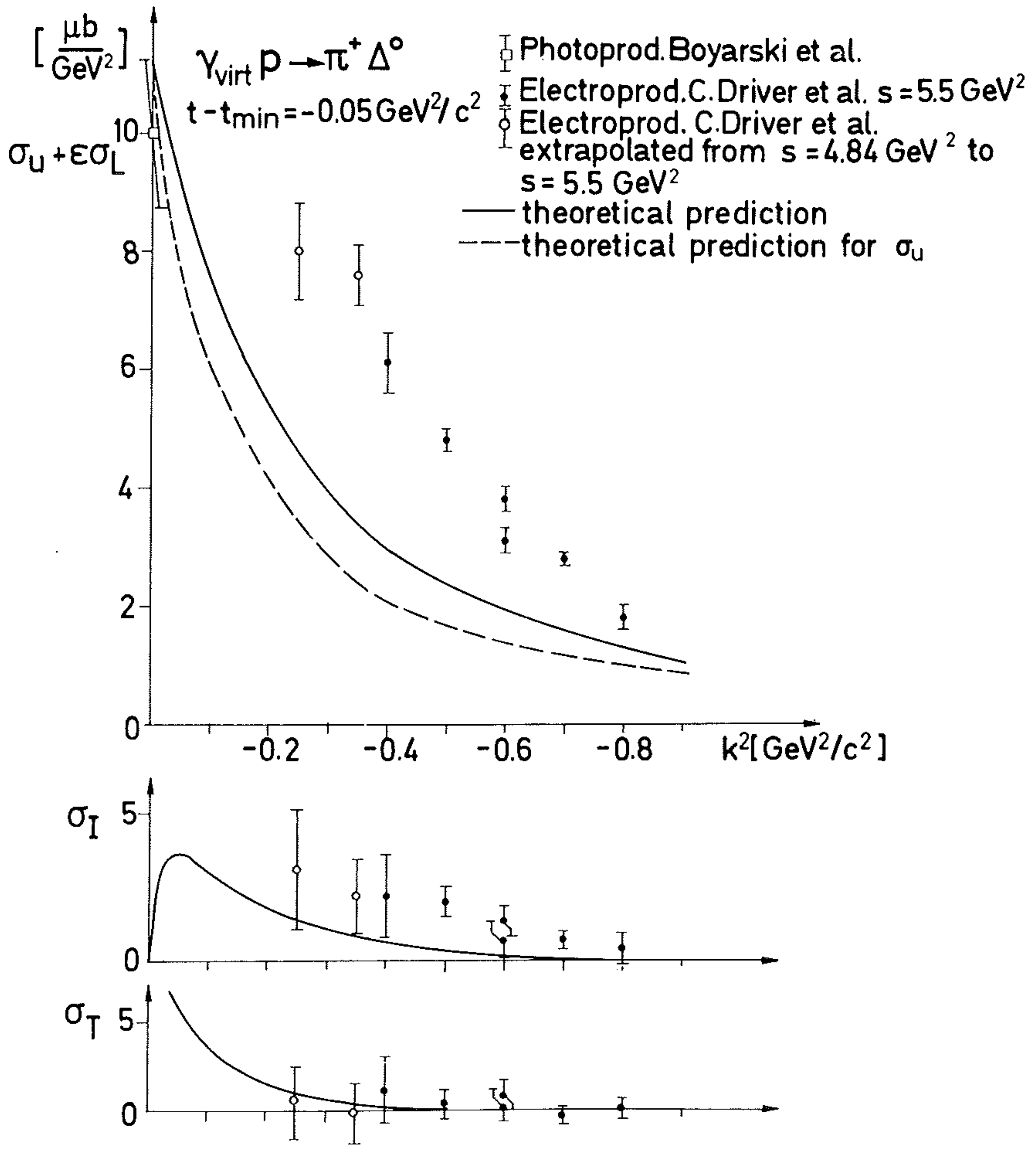

[μb/GeV²]
dσ/dt
25
20
15
10
5
0
-0.1
-0.2
-0.3
t [GeV²/c²]
γp → Δ⁺⁺π⁻
Photoprod. J.Ballam et al.
theoretical prediction
p_Lab. = 2.8 GeV/c

Fig. 3



Electroprod., C. Driver et al.
theoretical prediction
theoretical prediction for σu
[μb/GeV2]
σu+εσL
γvirt p → π+ Δ°
s = 5.5 GeV2
k2 = -0.5 GeV2/c2
8
6
4
2
0
σu
-0.02 -0.04 -0.06 -0.08 -0.1
t - tmin [GeV2/c2]
σI
4
2
0
σT
2
0

Fig. 4



$[\frac{\mu b}{GeV^2}]$
$\gamma_{virt}\, p \rightarrow \pi^+ \Delta^0$
$t - t_{min} = -0.05\, GeV^2/c^2$
Photoprod. Boyarski et al.
Electroprod. C. Driver et al. s = 5.5 GeV^2
Electroprod. C. Driver et al. extrapolated from s = 4.84 GeV^2 to s = 5.5 GeV^2
theoretical prediction
theoretical prediction for σ_u
$\sigma_u + \varepsilon\sigma_L$
10
8
6
4
2
0
-0.2
-0.4
-0.6
-0.8
$k^2\,[GeV^2/c^2]$
σ_I
5
0
σ_T
5
0

Fig. 5