DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 72/7
February 1972

Self-trapping Field of Quarks in Hadrons

by

H. Suura

2 HAMBURG 52 · NOTKESTIEG 1

Self-trapping Field of Quarks in Hadrons *

H. Suura +

Deutsches Elektronen-Synchrotron DESY, Hamburg

Abstract

A formulation is given for a physical situation in which a large self mass of quarks acts as a trapping barrier for bare quarks. A vector cohesive field in a hadron is considered in the classical limit and a proper field is assumed to be three dimensional. From $1/\kappa$ expansion of the self-field of the quark an asymptotic hadron mass spectrum follows almost uniquely. In the limit of an infinitely heavy quark, the mass spectrum reduces to that of a single four-point Veneziano amplitude.

* Work assisted in part by the U.S. Atomic Energy Commission.

+ On leave from School of Physics and Astronomy, University of Minnesota, Minneapolis, Minn. 55455

Negative results of numerous searches [1] for a quark up to the present time indicate that the quark must have a very large mass if it ever exists. A direct method to treat a tightly bound system of high mass quarks relativistically has been developed by Böhm, Joos and Krammer [2], who used a perturbation in the ratio of level spacing to quark mass, starting from a $O(4)$ symmetric unperturbed state for a vanishing hadron mass. On the other hand, the known size of a proton and the baryon level spacing require an effective quark mass of around 300 MeV. [3] Recently, B-L. Young and the author [4] were also lead to a physical picture where quarks almost permanently trapped in a hadron propagate locally with a small mass. Thus one may hope to justify the quark dynamics in terms of a small effective mass by incorporating the large real mass into the theory and also by explaining the nature of trapping mechanism necessary to prevent the low mass quarks from propagating asymptotically. In the present paper we would like to sketch one such possibility, in which a large quark self-energy itself acts as a trapping barrier against bare-massed quarks.

Our formulation is based on three basic assumptions.

(I) The proper configuration of a hadron is three dimensional. Namely, the wave function of a hadron and the field inside of it in the rest frame of the hadron are functions of relative spacial distances only, independent of relative times.

(II) Main cohesive field of the quarks is a neutral vector.[5]

(III) The gluon field is so strong that we can represent it by a c-number distribution. In other word we are considering some kind of coherent state for the gluons.

Equation of motion for the quark field $q(x)$ interacting with the vector gluon field $\varphi_\mu(x)$ is

$$(i\gamma\cdot\partial - M_0)\, q(x) = g\,\gamma\cdot\varphi(x)\, q(x)\,, \tag{1}$$

where M_0 is the bare mass of the quark. We introduce a 4×4 wave function $\chi(\xi)$ for a $q\bar{q}$ system of total momentum P_μ by

$$e^{-iP\cdot X}\chi_{\alpha\beta}(\xi) = (\Phi_0, T[\bar{q}_\beta(x_2), q_\alpha(x_1)]\,\Phi)\,, \tag{2}$$

with

$$X = \frac{1}{2}(x_1 + x_2)\,, \qquad \xi = x_1 - x_2\,.$$

Using (1) and (2), we obtain

$$\begin{aligned}&(\tfrac{1}{2}\gamma\cdot P + i\gamma\cdot\partial - M_0)\,\chi(\xi)\\ &= e^{iP\cdot X}(\Phi_0, T[\bar{q}(x_2), g\gamma\cdot\varphi(x_1)\, q(x_1)]\,\Phi)\\ &= g\,\gamma\cdot f(\xi)\,\chi(\xi)\end{aligned} \tag{3}$$

where the last step to replace $\varphi_\mu(x_1)$ by a c-number function $f_\mu(\xi)$ was made by the assumption (III)[6] and an observation that f_μ at x_1 can be a function of only ξ from translational invariance. We have also neglected an annihilation term in the spirit of a quark model. From the assumption (I), $g f_\mu(\xi)$ must have a form [7] $P_\mu V(r) + \xi_\mu W(r)$ with a proper distance r defined by

$$r^2 = -\xi^2 + (P\cdot\xi)^2/P^2 , \qquad (4)$$

which reduces to $\vec{\xi}^{\,2}$ for $\vec{P}=0$. Since $(\xi_\mu - P_\mu P\cdot\xi/P^2)W(r)$, a perfect derivative, can be transformed away by a Gauge transformation on $\chi(\xi)$, $\xi_\mu W(r)$ is equivalent to $P_\mu P\cdot\xi\, W(r)$ which we drop under the assumption (I). We have then

$$g f_\mu(\xi) = P_\mu V(r) \qquad (5)$$

The field f_μ must include the self-field in addition to those generated by the other (anti-) quark and possibly by a continuous source due to gluon-gluon interactions. In an asymptotic region where short range effects have died down, $V(r)$ is assumed to have a form

$$V(r) = V(\infty) - \frac{b}{r} + O\left(\frac{1}{r^2}\right) \qquad (6)$$

The constant term $V(\infty)$ is related to the self-mass as we will see below. The existence of the long range $1/r$ term may be explained on different grounds. First, the vector field may be massless, as would be the case for magnetic-monopole fields in the model of Schwinger [8] and Nambu [9]. Or the vector field may have a very small mass μ , and the mean q-$\bar{q}$ distance is smaller than $1/\mu$, so that the Yukawa field can be approximated by $1/r$. This set the upper limit for μ at about 300 MeV. Alles and Pati [10] considered such a vector gluon in connection with the $K_L \to \mu^+\mu^-$ puzzle. Finally we may consider a self-consistent scheme in which the wave equation (3) is coupled to an equation for f_μ , expressing it as the self-field of the quark in the same state χ moving in the same field f_μ . Such a self-consistent scheme may have a singular solution like (6), a self-trapping solution, other than a regular solution $V(r) = V(\infty)$, which corresponds to a free quark. If such a solution exists, we may not be forced to introduce real zero mass particles. In any case, we will simply assume (6) in the present paper. Eq. (3) can be rationalized by multiplying $\frac{1}{2}\gamma\cdot P + i\gamma\cdot\partial + M_0$ from the left. Derivatives of f_μ can be neglected as they are of higher order in $1/r$. We can drop $P\cdot\partial\chi$ also since χ is essentially a function of r only [11]. Thus, keeping terms of order $1/r$ for large r , we obtain

$$[P^2(\varepsilon^2 + \frac{a}{r}) - \partial^2 - M_0^2]\,\chi(r) = 0\,, \tag{7}$$

where

$$\varepsilon = \frac{1}{2} - V(\infty)\,,$$
$$a = 2\varepsilon b \tag{8}$$

This is an universal equation in the sense that it holds for main amplitudes at a large r regardless of the spin of the $q\bar{q}$ system.

The equation can be solved most conveniently in the rest frame, where we have

$$(M_0'^2 - \nabla^2 - m^2 a/r)\,\chi(r) = 0, \tag{10}$$

with

$$P^2 = m^2 \quad , \quad M_0'^2 = M_0^2 - \varepsilon^2 m^2 \tag{11}$$

Eq. (10) is exactly the Schrödinger equation for 'a hydrogen atom', with charge $e = \sqrt{m^2 a}$, energy eigenvalue $E = -M_0'^2$ and mass 1/2. The Bohr spectrum is then $M_0'^2 = m^4 a^2/4n^2$ which, combined with (11), yields a hadronic mass spectrum

$$P^2 = m_n^2 = \frac{2M_0^2 n}{\varepsilon^2 n + \sqrt{\varepsilon^4 n^2 + M_0^2 a^2}} \quad (n = 1, 2, \cdots). \tag{12}$$

The degeneracy of the levels is exactly the same as in the hydrogen atom. Since we did not consider any short range correlations in deriving eq. (7), the spectrum may be shifted substancially for low orbital states. The levels accumulate at

$$m_\infty^2 = M_0^2/\varepsilon^2 \tag{13}$$

which we may naturally identify as a threshold for production of a real quark and anti-quark pair. Hence the real quark mass M will be given by

$$M = m_\infty/2 = M_0/2\varepsilon . \tag{14}$$

It should be stressed that the bare mass cannot be eliminated from our picture although superficially it can be vanished from the mass spectrum (12) in terms of the real mass M and the original coupling parameter b . This is because the wave functions are scaled by $M_0' \sim M_0$ rather than by M. Our size of a hadron is of the order of 10^{-13}cm even though M may be as large as 10 GeV. The other parameter a is related to the level spacing. For a large real mass and a small bare mass we have $\varepsilon \ll 1$ from (14), and the level spacings between low lying states are almost constant and equal to $2M_0/a$ from (12). We may temporarily set it equal to the actual spacing of the ρ-f trajectory, so that

$$m_\rho^2 = M_0/a = M/b \, . \tag{15}$$

Again some reservation must be made here because of the asymptotic nature of our spectrum. As a byline, we may apply our spectrum to the Schwinger dyon model [8]. Expanding (12) in $1/n^2$, we identify the $1/n^2$ term as the Bohr spectrum of two-dyon system, $-g^4 M/4n^2$, for a magnetic charge g. This determines the coupling b as $b = g^2/4M$. From (15) we then determine the dyon mass as

$$(2M)^2 = g^2 m_\rho^2$$

which is equal to Schwinger's intuitive derivation to within a factor 4. For the elementary magnetic charge, $g^2 = g_0^2 = 4 \times 137$, we have M ~ 9 GeV. Of the discrepancy of the factor 4, one is due a difference of a factor 2 in eq. (15), namely $m_\rho^2 = 2M_0/a$ which is equivalent to $m_1^2 = m_\rho^2$. The genuine difference is then a factor 2.

An interesting limit is $\varepsilon \to 0$ (or $V(\infty) = 1/2$) keeping a finite. According to (14) and (12) this limit corresponds to an infinite quark mass and a linearly rising spectrum

$$m_n^2 = 2(M_0/a)\,n \quad (n = 1, 2, \cdots). \tag{16}$$

According to (8) and (14), the limit also corresponds to $b \propto M \to \infty$, meaning the coupling constant responsible for the asymptotic field to increase with M. The spectrum and the degeneracy is the same as those of a single four-point Veneziano amplitude [12]. Hence if we couple each state (n, ℓ) with two external particles with an appropriate strength, then we should be able to produce the dual four-point amplitude by taking our spectrum as the s-channel intermediate states.

A remark is due concerning the charge conjugation property of χ. From (2) we have

$$C^{-1}\chi(\xi)\,C = \pm\,\chi^T(\xi) \tag{17}$$

where $\pm$ refers to C parity of the $q\bar{q}$ system. Although our original equation (3) has a rigorous solution for the field (5) in the form consistent with the assumption (I), this solution has a small component (at large r) which does not satisfy (17). It is only when we neglect terms of order $1/r^2$ relative to the main amplitude that we obtain a solution consistent with (17).

The author would like to thank Professors H. Joos, E. Lohrmann and W. Paul for their hospitality extended to him at DESY. He is indebted to Professors H. Joos and B. Renner and Dr. F. Gutbrod for very critical and useful discussions. The author's study of the hadronic spectrum by the vector gluon field was initiated, from a somewhat different angle, during his stay at the International Centre for Theoretical Physics in Trieste in the Fall of 1969. He would like to thank Professors A. Salam and P.Budini for their hospitality given him at that time and for very stimulating discussions.

1. A. Böhm et al., Phys. Rev. Letters 28, 326 (1972), and references therein.

2. M. Böhm, H. Joos, M. Krammer, Nuovo Cim. 7A, 21 (1972) B-S equation for spin 1/2 quarks has also been solved in the same method by these authors, private communication.

3. See for instance, G. Morpurgo, 'The quark model', Proceeding of the 14th International Conference on High-Energy Physics, Vienna 1968.

4. H. Suura and B-L. Young, to be published.

5. This was first proposed by Fujii, Y. Fijii, Prog. Theor. Phys. 21, 232 (1959)

6. A more general statement is $(\Phi_0, T[\bar{q}(2), q(1), \varphi_\mu(x)]\Phi) = F_\mu(x; x_1, x_2)\, e^{-iP\cdot X}\, \chi(\xi)$. Charge conjugation (cf. Eq. (17)) requires that $F_\mu(x;x_1,x_2) = -F_\mu(x;x_2,x_1)$. We see that the field at x_2 , $F_\mu(x_2, x_1, x_2)$, is equal to $-f_\mu(-\xi)$

7. We may also consider a term $\gamma_\mu S$ in $g f_\mu$, which inserted into (3), changes M_0 into $M_0 + 4S$. Since we want to keep the effective mass small, we shall introduce an auxiliary scalar field such as to cancel the term $4S$. With this understanding we will not consider the $\gamma_\mu S$ term in this paper.

8. J. Schwinger, Science 165, 757 (1969).

9. Y. Nambu, Univ. of Chicago Preprint (1970)

10. W. Alles and J.C. Pati, CERN Preprint TH. 1429

11. A boosted wave function with an orbital angular momentum ℓ can be expressed by

$$\chi \sim (n_1\cdot\xi)^{\ell_1}(n_2\cdot\xi)^{\ell_2}(n_3\cdot\xi)^{\ell_3}\, R(r) \qquad (\ell_1+\ell_2+\ell_3=\ell)$$

with $n_i\cdot P = 0$. Hence $P\cdot\partial\chi = 0$. The wave function may also involve ξ in covariants like $\gamma\cdot\xi$, $\gamma\cdot P\,\gamma\cdot\xi$ etc. It turns out that these are small amplitudes, namely higher order in $1/r$ relative to the main amplitude, for which (7) holds.

12. G. Veneziano, Nuovo Cim. A 57, 190 (1968)