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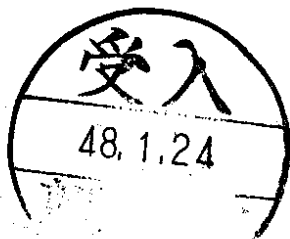


Baryon-Antibaryon Bootstrap Model of the Meson Spectrum

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G. Schierholz

II. Institut für Theoretische Physik der Universität, Hamburg, Germany



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G. Schierholz ⁺)

II. Institut für Theoretische Physik der Universität,
Hamburg, Germany

⁺) Present address: SLAC, P.O. Box 4349, Stanford, California 94305, USA.

Abstract:

In this work we present a baryon-antibaryon bootstrap model which, for the meson spectrum, we understand to be an alternative of the quark model. Starting from the baryon octets, the forces are constructed from the t-channel singularities of the nearest meson multiplets and transformed into an SU(3) symmetric potential. At this stage we assume that the baryon and meson multiplets are degenerate. Any contributions from the u-channel are neglected for it is exotic and only contains the deuteron. The dynamical equation governing the bootstrap system is the relativistic analog of the Lippmann-Schwinger equation which is an integral equation in the baryon c.m. momentum. The potential is chosen to take account of relativistic effects. Inelastic contributions such as two-meson intermediate states are neglected. Reasons why they must be small are discussed. We are looking for a self-consistent solution of the bootstrap system in which baryon-antibaryon bound state multiplets, to be interpreted as mesons, are forced to coincide with the input meson multiplets. Furthermore, the output coupling constants and F/D ratios have, to a certain extent, to agree with their input values. Practically, it is required that the bootstrap system consists of only a few multiplets, the remainder being decoupled approximately. A self-consistent solution is found comprising scalar, pseudoscalar and vector singlets and octets with masses being in good agreement with their average physical masses. The coupling constants and F/D ratios turn out to be consistent with experiment and other ideas. Possible origins of SU(3) breaking are then investigated. The spontaneous breakdown of SU(3) is ruled out by the fact that the dynamics is stable against small perturbations of the input masses. Instead, a solution of the symmetry breaking is given in terms of bootstrapped singlet-octet mixing.

1. Introduction

Various models have been proposed, mostly in the last eight years, giving dynamical understanding of the meson spectrum. The model, which has met with great success in classifying the hadrons, is the quark model [1]. The dynamics of the quark system, however, is not clearly understood although some progress has been made recently [2,3]. On the other hand, the most attractive feature of the quark model, i.e. the nonet structure of the mesons, is an ad hoc assumption without dynamical justification as long as quarks have not been found. In another class of models the meson spectrum is constructed from two-meson underlying states, a simple bootstrap principle settling the dynamics [4-6]. In these calculations self-consistency can only be achieved if higher channels are included and installed to give the main attraction [7]. This leads to the conclusion that the mesons are mainly bound states of a yet unspecified two-particle system in contradiction to the starting-point. Some earlier ideas concerning the dynamical origin of the meson spectrum [8] have been strengthened in a work by S. Wagner and the author [9]. They state that the mesons may have a baryon-antibaryon ($B\bar{B}$)-like structure which can be considered to form a possible alternative to the quark model. The main advantage of starting from $B\bar{B}$ underlying states instead of with quarks is that we already have some knowledge of the $B\bar{B}$ interaction from nucleon-nucleon scattering [10,11]. If the $B\bar{B}$ forces are likewise taken to be given by the t-channel singularities of the nearest mesons transformed into a potential (the u-channel can be neglected since it is exotic and only contains the deuteron) and the $B\bar{B}$ bound states are interpreted as mesons, then the set of input and output particles form a bootstrap system which may serve as a guide to the dynamics of the meson spectrum. It is the aim of this paper to develop a $B\bar{B}$ bootstrap model directing attention to the physical meson spectrum.

The symmetry of the system should emerge from the bootstrap condition. Several arguments have been given that the symmetry group able to fulfil the bootstrap condition is one of the groups $SU(n)$ [12]. In case of the $B\bar{B}$ bootstrap $SU(2)$ proves to be too small. There is no solution only involving $I = 0$ and $I = 1$ multiplets (I being the isospin) which is related to the fact that the crossing matrix does not have real eigenvalues. This can easily be verified by inspection of the bootstrap equation restricting to the pole terms of the wave function. In the following we assume that $SU(3)$ couples the mesons to the $B\bar{B}$ system (at least in an approximative sense) and then see if this is confirmed.

The SU(3) representations involved in the $B\bar{B}$ interaction are given by the irreducible decomposition of the two baryon octets:

$$8 \times 8 = 1 + 8_S + 8_A + 10 + \bar{10} + 27 \quad .$$

The subscripts S and A denote symmetric and antisymmetric octets respectively. Any practical exploitation of the $B\bar{B}$ bootstrap system now requires a drastic truncation of the number of multiplets (made up of the partial waves and the irreducible SU(3) representations). The unitarity equations must, furthermore, look simple. This would be the case if higher multiplets and, e.g. inelastic channels, were dynamically decoupled. An almost classical example of such a situation is the reciprocal bootstrap of the $(1/2)^+$ octet and the $(3/2)^+$ decuplet [13]. The fact that in the $B\bar{B}$ case only a few multiplets are involved can be expected from the following. At short distances there is some evidence of an almost universal $B\bar{B}$ interaction [14] which favours isosinglet-exchange dominance. Since the vector exchange is known to be dominant we have calculated the $B\bar{B}$ bound state spectrum for pure ω_0 exchange^{*)} in Ref.[9]. The results are shown in Table 1 demonstrating that for this choice of the coupling constant ω_0 exchange gives low-lying $B\bar{B}$ bound states which are already very similar to the physical meson spectrum.^{**)} Bearing in mind that the vector exchange plays a dominant role, this indicates, from the bootstrap point of view, that only the 1S_0 , $^3S_1 - ^3D_1$ and 3P_0 partial waves (lying lowest) become involved. It is now obvious that isosinglet exchange cannot be completely true since, in this case, all SU(3) multiplets are degenerate. A small attractive octet contribution, however, suffices to shift the exotics (belonging to the 10, $\bar{10}$ and 27 plet) to higher mass.

*) The ω_0 is the singlet component of the ω meson. The coupling constant is $g_{\omega_0}^{(V)2}/4\pi = 15.7$ ($g_{\omega_0}^{(T)} = 0$) which is adjusted to give $m_{\omega_0}^{\text{out}} = m_{\omega_0}^{\text{in}} = 784$ MeV. Note that Ref.[9] deals with the nucleon-antinucleon system. However, for isosinglet exchange the results also hold for the $B\bar{B}$ case.

***)As far as the scalar mesons are concerned the experimental situation is not clear. Even though the $\sigma(410)$ has been omitted from the tables [15] we include the σ in Table 1 since there is strong evidence for a complex pole on the second sheet with real part ~ 400 MeV [16].

	$(J^P)c_n$	mesons	output mass
1S_0	$(0^-)+$	$\pi(140)$ $K(500)$ $\eta(550)$ $\eta'(960)$	415
$^3S_1 - ^3D_1$	$(1^-)-$	$\rho(765)$ $K^*(890)$ $\omega(784)$ $\phi(1020)$	784
1P_1	$(1^+)-$	$B(1235)$	1250
3P_0	$(0^+)+$	$\delta(970)$ $\pi_A(1020)$ $\kappa(725)$ $K_N(\sim 1200)$ $\sigma(410)$ $\varepsilon(700)$ $S^*(1060)$	920
3P_1	$(1^+)+$	$A_1(1070)$ $K_A(1240-1400)$	1200
$^3P_2 - ^3F_2$	$(2^+)+$	$A_2(1300)$ $f(1260)$ $f'(1514)$	1550
1D_2	$(2^-)+$	$\pi_A(1640)$	1810

Table 1: Bound state masses (in MeV) for single isosinglet vector exchange compared to the physical meson spectrum. The $^3S_1-^3D_1$ partial wave contains a second bound state at 1530 MeV which is not quoted. The SU(3) multiplets are completely degenerate in this case.

It is now apparent that SU(3) is broken in nature. But a considerable amount of SU(3) symmetry survives the symmetry breaking mechanism so that it is a good algebra to start with. The dynamical origin of the symmetry breaking is not clearly understood although it has been discussed in various papers [17-19]. It is, however, evident that our understanding of why SU(3) works so well can only be improved if we have some insight into the symmetry breaking process. We believe that the \overline{BB} bootstrap will answer these questions since there is much room for symmetry breaking effects.

In Section 2 we give a brief review of the model and discuss our basic assumptions. Section 3 is devoted to the SU(3) symmetric bootstrap. Here a self-consistent solution is presented and compared to experiment. In Section 4 certain SU(3) symmetry breaking mechanisms are discussed and an almost quantitative description of the meson spectrum is given. Finally, in Section 5 we add some concluding remarks.

2. Outline of the Model

Our bootstrap model will be based on the dynamical equation

$$\psi_B^M(\kappa J [l_j]) = (m_B^M - E_\kappa)^{-1} \sum_{l'_j} \int d\kappa' \kappa'^2 v_{l_j, l'_j}^{J, M}(\kappa, \kappa') \psi_B^M(\kappa' J [l'_j]) \quad (2.1)$$

already employed in Refs. [9, 10]. Here M indicates the SU(3) multiplet, and B stands for the \overline{BB} channel. This type of equation is favoured by arguments arising from certain properties of the bootstrap state function under Lorentz transformations [20]. The discussion on the input forces can be restricted to the exchange of pseudoscalar, vector and scalar singlets and octets (belonging to the partial waves 1S_0 , 3S_1 , 3D_1 and 3P_0 respectively) since, as turns out later, the other multiplets are actually decoupled. Starting from the Lagrangians recorded in Appendix A the construction of the potential is standard [10]. The baryon and meson octets are assumed to be degenerate and the baryon octet mass is identified with the nucleon mass. The potential can be written in the product form

$$v_{l_j, l'_j}^{J, M}(\kappa, \kappa') = \sum_{N_\beta} C_{N_\beta}^M v_{l_j, l'_j}^{(\beta) J}(\kappa, \kappa') \quad , \quad (2.2)$$

where the factor $C_{N_\beta}^M$ carries the SU(3) crossing coefficients and $\mathcal{V}_{\ell_j, \ell_j'}^{(\beta) J}(\kappa, \kappa')$ represents the dynamical part. The t-channel multiplets are denoted by N_β

$M \backslash N_\beta$	$(1)_X$	$(8)_X$ ${}^3S_1 - {}^3D_1$ pure V or T coupling	$(8)_{3S_1 - 3D_1}$ mixed coupling
1	1	$\frac{4}{3} (14\alpha^2 - 10\alpha + 5)$	$\frac{4}{3} (14\alpha_V\alpha_T - 5(\alpha_V + \alpha_T) + 5)$
$8_S - 8_S$	1	$4\alpha^2 + 4\alpha - 2$	$4\alpha_V\alpha_T + 2(\alpha_V + \alpha_T) - 2$
$8_A - 8_A$	1	$\frac{2}{3} (14\alpha^2 - 10\alpha + 5)$	$\frac{2}{3} (14\alpha_V\alpha_T - 5(\alpha_V + \alpha_T) + 5)$
$8_S - 8_A$	0	$+ 4\sqrt{5} \alpha(1-\alpha)$	$+2\sqrt{5} (\alpha_V + \alpha_T - 2\alpha_V\alpha_T)$
10	1	$-\frac{10}{3} (1-\alpha)^2$	$-\frac{10}{3} (\alpha_V\alpha_T - \alpha_V - \alpha_T + 1)$
$\bar{10}$	1	$-\frac{10}{3} (1-\alpha)^2$	$-\frac{10}{3} (\alpha_V\alpha_T - \alpha_V - \alpha_T + 1)$
27	1	$-\frac{9}{2} (2\alpha^2 + 2\alpha - 1)$	$-\frac{9}{3} (\alpha_V\alpha_T + \alpha_V + \alpha_T - 1)$

Table 2: SU(3) crossing coefficients $C_{N_\beta}^M$. X denotes 1S_0 , ${}^3S_1 - {}^3D_1$ and 3P_0 collectively.

where β is a subscript labeling the spin-parity of the multiplets and distinguishing between different couplings. The crossing coefficients are presented in Table 2 and the dynamical part of the potential is given in Appendix A.

Table 2 indicates that isosinglet exchange contributes the same to all multiplets so that, in this case, they would be completely degenerate. The effect of octet exchange on the singlets and octets can best be demonstrated by drawing the singlet crossing coefficient and the eigenvalues of the octet

crossing matrix for $0 \leq \alpha \leq 1$. This has been done in Fig.1 for the unmixed couplings (corresponding to $\alpha = \alpha_V = \alpha_T$ in the mixed case). The octets are degenerate for $\alpha = 1$ and are far spread in the case of $\alpha = 0$. For smaller values of α one would only then expect two low-lying bound state octets if the octet exchange proves to be small as compared to the singlet exchange. The spread between the singlet and the lowest octet has a minimum at $\alpha \approx 0.5$ and increases going to $\alpha = 0$ and $\alpha = 1$. In order to remove the exotics from the low-lying bound state spectrum one would need an overall attractive octet exchange contribution with an effective F/D ratio of $0.4 \leq \alpha_{\text{eff}} < 1$.

Some remarks have to be made concerning the form factor (see Appendix A). It was originally derived [10] from a Khuri representation of the potential being parametrized to have the "right" left-hand cut $s \leq 4m^2 - N^2\mu^2$. This corresponds to the picture that the vector mesons consist of $N = 2$ (3) pions. The cut is also present in the form factor (when it is taken on shell), and if the simple off-shell generalization (Ref.[10]) is used it also appears in the bound state wave function where it is unphysical. In order to remove this unphysical cut we set $t_0 = 0$ and

$$f(k, k') = \text{Re} \left\{ \text{arccosh} \sqrt{\frac{k^2 + m^2}{m^2}} + \text{arccosh} \sqrt{\frac{k'^2 + m^2}{m^2}} \right\} \quad (2.3)$$

The form factor is taken to be the same for all contributions to the potential, i.e. Regge-like. The trajectory is parametrized in the form

$$\alpha(t) = -0.4 + 0.9t$$

for pseudoscalar and scalar exchange and

$$\alpha(t) = 0.6 + 0.9t$$

for vector exchange. This choice is consistent with high-energy conceptions.

So far we have tacitly neglected inelastic contributions such as, e.g. two-pion intermediate states which are coupled to the $\bar{B}B$ system by means of baryon exchange. Part of these contributions are already included in the one-meson exchange terms (and in the deuteron exchange for the u-channel) since we understand the mesons to be $\bar{B}B$ bound states built up by multiple meson exchange. The diagrams, which contribute less to the extrapolated pole terms in this sense, are those with two-meson intermediate states (an extensive study of the box diagram with two-pion intermediate states is develop in Ref.[21]). They mainly

enter into the double spectral function part giving rise to very short range forces of $r \approx 0.1$ fermi (to be compared to $r \approx 1.4$ fermi for pion exchange). On the other hand, the diagrams with larger numbers of intermediate mesons contribute much to the one-meson-exchange amplitudes, and that part belonging to the double spectral function is frozen at the expected bound state energies because of the high thresholds. In order to estimate the effect of the inelastic channels on the mass of the bound state and on the wave function we study the case of two coupled channels $\bar{B}\bar{B} \leftrightarrow \bar{M}\bar{M}$ where M stands for meson. The Hamiltonian is taken to be

$$\begin{pmatrix} h_{OB} + v_B & v_{BM}^{(1)} \\ v_{BM}^{(1)} & h_{OM} \end{pmatrix}, \quad (2.4)$$

i.e. the forces in the $\bar{M}\bar{M}$ channel are ignored. We assume that low-order perturbation theory can be employed for $v_{BM}^{(1)}$ because of the short-range nature of these forces. Suppose there is a low-lying bound state in the $\bar{B}\bar{B}$ channel, then first order perturbation theory gives a contribution to the bound state wave function in the meson channel (for simplicity the SU(3) specification is dropped)

$$\psi_M^{(1)} = (m_B - h_{OM} + i\epsilon)^{-1} v_{BM}^{(1)} \psi_B, \quad (2.5)$$

where we have set $(\psi_B, \psi_B) = 1$. A mass shift at first occurs in second order yielding

$$m^{(2)} = (\psi_B, v_{BM}^{(1)} \psi_M^{(1)}). \quad (2.6)$$

It is complex for m_B lying above the two-meson production threshold. We will discuss this case first. The imaginary part of (2.6) represents the width

$$\Gamma = 2\pi (\psi_B, v_{BM}^{(1)} \delta(m_B - h_{OM}) v_{BM}^{(1)} \psi_B), \quad (2.7)$$

and the real part

$$\text{Re } m^{(2)} = (\psi_B, v_{BM}^{(1)} P(m_B - h_{OM}) v_{BM}^{(1)} \psi_B) \quad (2.8)$$

contributes a correction to the mass of the bound state. The $v_{BM}^{(1)}$ has to be constructed to give the correct width and to exclude double counting. Since the widths are small (the ρ width would be smaller than its experimental value in a SU(3)-symmetric world which we discuss here) $\text{Re } m^{(2)}$ should be small and even smaller than the width if the major contribution to the principle value integral (2.8) comes from $E \approx m_B$ as is expected. The partial wave function (2.5), which describes the two-meson decay of the bound state may, however, contribute a large amount to the norm of the wave function because the propagator occurs to the second power there. On the other hand, the part of the higher mass $M\bar{M}$ channels closed at $E = m_B$ belonging to the double spectral function is suppressed relative to $m_B^2/(\text{inelastic threshold})$. Although this is no proof we may assume that inelastic channels are approximately decoupled from the eigenvalue equations but can operate considerably on the wave function. Note, however, that this situation may change in the case of loosely bound states (in the $B\bar{B}$ channel). Aldrovani and Caser [22] have recently shown that inelastic channels may lower the bound state mass, but they make a number of assumptions which are not well founded and do not hold true for deeply bound $B\bar{B}$ states. Inelastic effects should first be studied in the case of the ρ where the dynamics is constrained to give the physical width. The various attempts to understand the mesons as two meson resonances (mentioned in the introduction), however, already show that the two-meson channel is almost negligible as far as the output meson masses are concerned.

3. SU(3) Symmetric Bootstrap Solution

The self-consistency requirement inherent in this model extends to the mass, the coupling constant and the F/D ratio (if any) of the multiplets involved. Multiplets not included in the input may occur as bound states at higher energy such as to contribute only a little to the input.

The output coupling constants and F/D ratios result from the residues of the bound state wave functions (which have a pole at $E_k = m_B$) making use of Eq.(A.1-A.6). The normalization of the wave function is, however, not determined by the eigenvalue equation (the same problem is met with in the Bethe-Salpeter equation; for a review see Ref.[23]) so that the residue only permits a calculation of the F/D ratio and $g^{(T)}/g^{(V)}$ (in the case of the 3S_1 - 3D_1 partial wave). One way of normalizing the wave function (being intuitive for our aim) is by calculating the electromagnetic form factor at zero momentum transfer which

gives the total charge. This is unambiguous although this is not the case for finite q^2 [24]. It is obvious that this procedure is only relevant if the form factor can be explained by the diagram shown in Fig.2a. From the preceding discussion on the inelastic channels we know, however, that the diagrams like that in Fig.2b may contribute a large amount to the form factor (and the norm of the wave function) even if they do not effect the position of the bound state. For this reason we can only assume that the normalization condition applies to the bound states which are not allowed to decay into two mesons. These are the pseudoscalar octet mesons (if the physical masses are taken). The normalization method is completely equivalent to setting $(\psi_B, \psi_B) = 1$ and can, therefore, be taken over by the neutral bound states.

Table 1 leads us to suppose that the partial waves playing an active part in the bootstrap are essentially 1S_0 , $^3S_1 - ^3D_1$ and 3P_0 . Their effect on the output is shown in Table 3 (for later references) where + and - stand for attractive and repulsive respectively (this refers to the dynamical part only).

$\begin{matrix} t \\ s \end{matrix}$	1S_0	$\begin{matrix} ^3S_1 - ^3D_1 \\ V \quad T \end{matrix}$	3P_0
1S_0	-	+ +	+
$^3S_1 - ^3D_1$	+	+ -	+
3P_0	+	+ +	+

Table 3: Signs of the pseudoscalar, vector and scalar meson exchange contributions to the lowest partial waves. Here + (-) means attractive (repulsive) and V(T) denotes vector (tensor) coupling.

On the input side $V(T)$ means pure vector (tensor) coupling. In our bootstrap calculation we only take these partial waves into account and shall comment on the others later. We then start from the corresponding low-lying singlet and octet mesons taking their average physical masses [15] and the coupling constants derived in Ref.[10] as initial values. For admissible F/D ratios this choice does not lead to deeply bound states in the exotic channels which may justify removing the exotics from the input at this early stage (we shall come back to the exotics later). The π and η'_0 coupling constants^{*)} are subject to the bootstrap condition (in reality the π coupling constant need not be varied because the output one already agrees fairly well with the physical value) since we believe that the pseudoscalar bound states can be normalized by only considering the diagram in Fig.2a (i.e. $(\psi_B, \psi_B) = 1$). This is supported by the fact that the $\pi^0 \rightarrow \gamma\gamma$ decay can be quantitatively described by a single nucleon loop diagram [25]. The scalar and vector meson coupling constants are left open ($g^{(T)}/g^{(V)}$ and the F/D ratios are, however, fixed), but the input values may not exceed the output ones in any scale. Experiment, however, sets a limit to the coupling constants and any model which claims to have to do with physics should stay within this limit. A solution to this program exists. The result is given in Table 4 and will be discussed now (the bootstrap program was stopped here although some smaller refinements in the input parameters had to be made). The experimental values recorded in Table 4 refer to Refs.[26,27].

The (partial) bootstrap solution exhibits the nonet structure of the low-lying pseudoscalar, vector and scalar mesons. The masses are in good agreement with the average physical meson masses except for the η'_0 and the ω_0 which seem to be too low. In the case of the 3P_0 partial wave we assign the singlet to the σ meson and the octet to δ , $\{\kappa, K_N\}$ and $\{\epsilon, S^{**}\}$. As far as the coupling constants and F/D ratios are concerned we are faced with the following situation:

^{*)} the octet coupling constants are labeled by their $I = 1, Y = 0$ members.

	1_{S_0}		$3_{S_1} - 3_{D_1}$		3_{P_0}	
	1	8	1	8	1	8
m_{in}	580	400	590	905	460	860
m_{out}	580	400	590	905	460	860
\bar{m}_{exp}	-900	~400	~880	~850	400-1100	400-1100
$(g^2/4\pi)_{in}$	10.0	14.4	V 7.5 T 5.0	0.7 5.0	7.0	0.5
$(g^2/4\pi)_{out}$	12.4	15.7	?	?	?	?
$(g^2/4\pi)_{exp}$	0. - 15.	14.4	V 5.-15. T ?	0.5-1.0 4. -15.	0.-15.	0.-15.
$(g^{(T)}/g^{(V)})_{in}$	-	-	0.82	2.7	-	-
$(g^{(T)}/g^{(V)})_{out}$	-	-	0.7	2.6	-	-
$(g^{(T)}/g^{(V)})_{exp}$	-	-	?	2.4-5.0	-	-
α_{in}	-	0.15	-	V 1. T 0.5	-	0.6
α_{out}	-	0.15	-	V 0.97 T 0.43	-	0.6
α_{exp}	-	0.2-0.4	-	V ~1 T 0.3-0.4	-	?

Table 4: SU(3) symmetric bootstrap solution compared with experiment. V(T) stands for vector (tensor) coupling.

(α) 1S_0 : The π coupling constant is fairly well reproduced. The F/D ratio turns out somewhat lower than that derived from the reciprocal bootstrap [28], but it is consistent with experiment. The η'_0 coupling constant is not well known and the situation is complicated because of $\eta_0 - \eta'_0$ mixing.

(β) $^3S_1 - ^3D_1$: The F/D ratio of the vector coupling is in agreement with isovector current conservation [29]. The F/D ratio of the tensor coupling seems reasonable compared to experiment. In addition, the ratios $g^{(T)}/g^{(V)}$ are fixed. For the ρ it is close to the nucleon form factor prediction [30]. The overall strength of the singlet and octet couplings turns out to be nearly the same as was found in low-energy nucleon-nucleon scattering [10] if the ω_0 be assigned to the ω . Moreover, as far as $g_\rho^{(V)}$ is concerned, we are in agreement with the universal vector coupling hypothesis [31] predicting $g_\rho^{(V)2}/4\pi \approx 0.5 - 0.7$. If ordinary ω - ϕ mixing [32] is assumed (we shall come back to this problem later), we obtain the physical coupling constants $g_{\phi NN}^{(V)2}/4\pi \approx 0$, $g_{\phi NN}^{(T)2}/4\pi \approx 0$ and $g_{\omega NN}^{(V)2}/4\pi = 13.7$, $g_{\omega NN}^{(T)2}/4\pi = 9.7$. This reproduces the known fact that the ϕ is decoupled from the nucleons.

(γ) 3P_0 : The coupling constants are consistent with what is known from low-energy nucleon-nucleon interaction [11], not only in the order of magnitude but also in the fact that g_δ is small in comparison to g_σ . The F/D ratio is not known from other investigations.

If ψ_B is normalized to one for $^3S_1 - ^3D_1$ and 3P_0 the corresponding output coupling constants are larger by a factor of 2-5 than the input ones.

Our bootstrap solution gives rise to a second octet in all three partial waves. Their masses are quoted in Table 5. In the case of the pseudoscalar and vector octet this is supported by experiment although the experimental situation is by no means settled.

	1S_0	$^3S_1 - ^3D_1$	3P_0
mass	900	1200	1500
experimental indications	H(990) ? E(1422)	ρ (1540) K^* (1270)	

Table 5: Bound state masses (in MeV) of the second octet not included in the input.

The second pseudoscalar octet turns out to be rather low and should actually have been included in the bootstrap. However, the output coupling constant is $g^2/4\pi = 5(\alpha \approx 1)$ which already proves that this contribution is smaller by a factor of at least 15 than the π exchange.

In the exotic channels only a 27-plet appears in the ${}^3S_1 - {}^3D_1$ partial wave at 1400 MeV. This is fairly high and not excluded by experiment. There are some indications [15] that a $I = 3/2, Y = 1$ meson exists at 1200 MeV. The other partial waves not included in the bootstrap contain bound states above 1300 MeV as shown in Table 6. The tensor singlet and octet (${}^3P_2 - {}^3F_2$) is in good agreement with experiment. The axial vector bound states (3P_1) are compatible with the D and K_A mesons but are far too high compared to the A_1 . This would support the Deck-effect interpretation of the A_1 [33]. The first recurrence of the π , the B meson, is also fairly well reproduced (if the width is taken into account).

	1P_1	3P_1	${}^3P_2 - {}^3F_2$
singlet mass	1480	1570	1300
octet mass	1500	1530	1320
experiment	B(1235)	A_1 (1070) D (1285) K_A (1240 - 1400)	f (1260) f' (1514) A_2 (1300)

Table 6: Bound state masses (in MeV) for some higher partial waves not involved in the bootstrap.

The coupling constants indicate that the isosinglet exchange is by far the dominant one. In addition, this is improved by the fact that the pseudo-scalar and vector octet exchange contribution to the 1S_0 and ${}^3S_1 - {}^3D_1$ singlets and octets cancel for the main part as a consequence of the special parametrization of the vector exchange (see Table 3). For this reason we do not believe that our choice of the form factor parameters is crucial. Small

changes of the form factor parameters merely effect the isosinglet exchange and can be cancelled by simply readjusting the vector and scalar singlet coupling constants which are not subject to the self-consistency condition.

Since the scale of the vector and scalar coupling constants is left open by our bootstrap model one could argue that further solutions exist. The dependence of the masses on these coupling constants, however, gives a closed curve (in the mass-coupling constant plane) of small extension so that a much different solution is not expected.

4. SU(3) Breaking

So far we have assumed SU(3) invariance in an ideal sense. The mass splittings indicate, however, that SU(3) is broken in nature and that the symmetry breaking emerges in a definite way. In our model the binding energy of the bound state multiplets (1. - 1.4 GeV) sets the scale of the SU(3) violations. According to that they are of the order of 25 - 30% which cannot be neglected. On the other hand, there are many reasons for believing that the special pattern of SU(3) symmetry breaking is a result of the properties of the SU(3) symmetric interaction itself [34]. This means that our understanding of SU(3) and the dynamics it governs could be advanced if the deviations be carefully examined. We are now going to discuss possible origins of SU(3) symmetry breaking.

In a bootstrap model the symmetry breaking chooses that direction leading to unstable dynamics. Some general features of the bootstrap theory of SU(3) breaking have been pointed out in the literature [35]. But in order to favour one mechanism more than the other one has to make detailed calculations. We shall first discuss the conjecture of spontaneously broken SU(3) [17]. This arises if the bootstrap system has a self-supporting mass splitting solution but exhibits the full symmetry. The mass splittings are then expected to go mainly like $\delta m_B D_8 (\langle D_8 \rangle = \frac{1}{\sqrt{3}} \{I(I+1) - \frac{1}{4} Y^2 - 1\})$ since only octets are involved and first order perturbation theory should apply. We have attempted to construct a solution of this type. The first order perturbation on the output is proportional to $\delta m_B D_8 2m_B / (t - m_B^2)^2$ being of the order $(\delta m_B D_8 / m_B) \cdot (v/\text{binding energy})$. In the case of the pseudoscalar octet it is even less because the pseudoscalar exchange vanishes at $t = 0$. For the physical mass splittings ($\delta m_{PS} = 450$ MeV, $\delta m_V = 160$ MeV, the scalar octet has not been taken into account because of the

very small contribution) first order perturbation theory (taking SU(3) symmetric couplings) gives only a minute correction to the 1S_0 and $^3S_1 - ^3D_1$ bound state masses. The reason for this is threefold. First, the pseudoscalar exchange term is strongly suppressed by the fact that it vanishes at $t = 0$. Second, the vector octet is not far split and third, the pseudoscalar and vector octet contributions have opposite signs in the 1S_0 and $^3S_1 - ^3D_1$ partial waves (for our parametrization) so that they are partly cancelled (see Table 3). This rules out spontaneous breakdown of SU(3). There would also have been no room for a (massless) Goldstone boson in strong interactions. The form of the symmetry breaking, however, points into the right direction.

In any but the spontaneous symmetry breaking case the SU(3) violation is started by an extraneous SU(3) noninvariant interaction. Such an interaction becomes apparent, e.g. in the ω - ϕ mixing mechanism [36]. Henceforth, we shall pursue the idea that the "driving term" is given by singlet-octet mixing effects alone. In view of the fact that spontaneous SU(3) breaking is dropped out the "driving term" must play the dominant role. For the present we only consider vector nonet mixing. To begin with we introduce ω - ϕ mixing by hand through the symmetry breaking interaction

$$v' = m_{18} \{ |\omega_0\rangle \langle \phi_0| + |\phi_0\rangle \langle \omega_0| \} . \quad (3.1)$$

If this is included in the potential (A.7) there arises an explicit SU(3) violating contribution recorded in Appendix B. It corresponds to the diagrams shown in Fig.3. We now evaluate the ω - ϕ mixing mechanism in second order perturbation theory. It causes an output mass correction

(a) $(I, Y) \neq (0, 0)$:

$$\delta m_{3\sigma}^8 = (\psi_{3\sigma}^8, \tilde{v}_{\omega\phi} D_8 \psi_{3\sigma}^8) , \quad \psi_{3\sigma}^8 = \psi_3^8 |\sigma\rangle , \quad \psi_3^8 = \begin{pmatrix} \psi_3^{8S} \\ \psi_3^{8A} \end{pmatrix} \quad (3.2)$$

(b) $(I, Y) = (0, 0)$:

$$\delta m_3^1 = (m_3^1 - m_3^8)^{-1} / (\psi_3^1, v_{\omega\phi} \psi_3^8)^2 , \quad (3.3)$$

$$\delta m_{36}^8 = (\psi_{36}^8, \tilde{v}_{\omega\phi} D_8 \psi_{36}^8) - (m_3^1 - m_3^8)^{-1} / (\psi_3^1, v_{\omega\phi} \psi_3^8)^2 , \quad (3.4)$$

where σ denotes (I, I_3, Y) collectively and $\sigma = 6$ means $I = I_3 = Y = 0$. The wave functions are normalized to $(\psi_B^{M'}, \psi_B^M) = \delta_{M'M}$. Note that the result, however, is independent of the norm. Apart from the singlet-octet mixing the mass shift transforms like the eighth component of an octet just as described by the Gell-Mann-Okubo mass formula. The strength of the symmetry violating

	Experimental Mesons	Output Mass
1S_0	π (140)	145
	K (500)	525
	η (550)	505
	η' (960)	600
$^3S_1 - ^3D_1$	ρ (765)	760
	K^* (890)	975
	ω (784)	520
	ϕ (1020)	1120
3P_0	$\left\{ \begin{array}{l} \delta (970) \\ \pi_A (1020) \end{array} \right\}$	600
	$\left\{ \begin{array}{l} \kappa (725) \\ K_N (\sim 1200) \end{array} \right\}$	995
	σ (410)	300
	$\left\{ \begin{array}{l} \varepsilon (\sim 700) \\ S^* (1060) \end{array} \right\}$	1300

Table 7: Bound state mass spectrum (in MeV) arising from bootstrapped ω - ϕ mixing compared to the physical meson spectrum.

part is given by m_{18} . Since the ω - ϕ mixing is self-generating, m_{18} is subject to the bootstrap condition. In second order we have

$$m_{18}^{\text{out}} = (\psi_3^1, \tilde{\nu}_{\omega\phi} \psi_3^8) - (m_3^1 - m_3^8)^{-1} (\psi_3^1, \tilde{\nu}_{\omega\phi} \psi_3^8) (\psi_{36}^8, \tilde{\nu}_{\omega\phi} \psi_{36}^8) \quad (3.5)$$

which corresponds to the diagrams in Fig.4. On the input side the pseudoscalar nonet mixing proves to be small for physical mixing parameters. Scalar mixing is negligible because of the very small octet coupling constant. It is, therefore, expected that the self-consistency condition already applies to ω - ϕ mixing. Evaluating Eq.(3.5) we obtain $m_{18} = 240$ MeV. This allows to calculate the SU(3) breaking unambiguously. It leads to the mass spectrum shown in Table 7 for the 1S_0 , $^3S_1 - ^3D_1$ and 3P_0 partial waves. The magnitude of the mass shifts agrees almost quantitatively with experiment apart from the scalar octet, in which case the experimental situation is not settled. The deviations from the physical meson spectrum have its origin in the SU(3) symmetric interaction rather than in the symmetry breaking mechanism. They are caused by the somewhat too low singlet mass. The mixing angles turn out to be $\theta_V = 26.2^\circ$ for the vector nonet and $\theta_{PS} = -23^\circ$ for the pseudoscalar nonet. This has to be compared with the experimental values $\theta_V = 35^\circ$ and $\theta_{PS} = -23^\circ$ derived from the linear mass formula (quadratic mass formula: $\theta_V = 39^\circ$, $\theta_{PS} = -11^\circ$). The second octet does, of course, also mix with the singlet giving rise to a further mass shift which, however, we will not discuss here.

Our calculation shows that the dynamics is unstable against ω - ϕ mixing. Since m_{18} is determined by self-consistency the ω - ϕ mixing mechanism is phenomenological rather than fundamental. That means ω - ϕ mixing emerges from the bootstrap system instead of being isolated [18]. We now have a fully symmetric solution of the bootstrap system and an asymmetric one. The bootstrap principle, however, only supports that solution which is stable against small perturbations. In this way the symmetry of the system reduces to SU(2), but the symmetry breaking is too small as to yield decoupling of hypercharge-one channels. The fact that the characteristic pattern of SU(3) breaking follows from ω - ϕ mixing is closely connected to the dominant role played by the vector exchange. The strength of the SU(3) breaking forces depends on the symmetric solution and the dynamics it arises from. One of the main reasons why the deviations from SU(3) are that small lies in the isosinglet-exchange dominance corresponding to relatively small octet couplings.

5. Conclusions

Our model is a first attempt to treat the meson spectrum starting from $B\bar{B}$ underlying states. The results indicate that the physical mesons can be explained as $B\bar{B}$ composite particles. In view of this one should consider the possibility of attributing the fundamental entities "quarks" to the baryons. The good features of our model voting for a $B\bar{B}$ underlying structure can be summarized as follows:

1. The dynamical concept and the solution of our $B\bar{B}$ bootstrap model is consistent with what is known from low-energy nucleon-nucleon interaction and other models being supported by experiment. This gives an impression of the seriousness of the model.
2. The physical meson spectrum is well reproduced. The nonet structure of the low-lying mesons (being one of the pillars of the quark model) here is a consequence of isosinglet-exchange dominance. We have always a second octet despite of small changes of the parameters. In the quark model a further nonet will only arise for very high quark mass [2,38].
3. The SU(3) breaking mechanism is guided by the dynamics of the SU(3) symmetric solution. The form of the symmetry breaking therefore is a critical test of the dynamical concept. Our model has an SU(3) breaking bootstrap solution which is consistent with experiment.

Acknowledgement

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Appendix A

In order to establish both the dynamics and define the coupling constants we write down the Lagrangians for the scalar, pseudoscalar and vector singlets and octets interacting with the baryon octets:

Singlet

(α) scalar

$$\mathcal{L} = \sqrt{4\pi} g \bar{B} B \phi \quad (\text{A.1})$$

(β) pseudoscalar

$$\mathcal{L} = \sqrt{4\pi} g \bar{B} \gamma_5 B \phi \quad (\text{A.2})$$

(γ) vector

$$\mathcal{L} = \sqrt{4\pi} (g^{(V)} \bar{B} \gamma_\mu B \phi^\mu + \frac{g^{(T)}}{2m_V} \bar{B} \sigma_{\mu\nu} B (\partial^\mu \phi^\nu - \partial^\nu \phi^\mu)) \quad (\text{A.3})$$

Octet

(α) scalar

$$\mathcal{L} = 2\sqrt{4\pi} g \bar{B} (\alpha \mathcal{D}_\ell + (1-\alpha) \mathcal{F}_\ell) B \phi_\ell \quad (\text{A.4})$$

(β) pseudoscalar

$$\mathcal{L} = 2\sqrt{4\pi} g \bar{B} (\alpha \mathcal{D}_\ell + (1-\alpha) \mathcal{F}_\ell) \gamma_5 B \phi_\ell \quad (\text{A.5})$$

(γ) vector

$$\begin{aligned} \mathcal{L} = & 2\sqrt{4\pi} (g^{(V)} \bar{B} (\alpha_V \mathcal{D}_\ell + (1-\alpha_V) \mathcal{F}_\ell) \gamma_\mu B \phi_\ell^\mu \\ & - \frac{g^{(T)}}{2m_V} \bar{B} (\alpha_T \mathcal{D}_\ell + (1-\alpha_T) \mathcal{F}_\ell) \sigma_{\mu\nu} B (\partial^\mu \phi_\ell^\nu - \partial^\nu \phi_\ell^\mu)) \end{aligned} \quad (\text{A.6})$$

Note that the coupling constant of the vector octet (i.e. the $\rho N \bar{N}$ coupling constant) differs by a factor of 2 from that of the compilation [26].

Following Ref.[10] the Lagrangians (A.1 - A.6) give rise to a $B\bar{B}$ potential, symmetric under SU(3) transformations. The dynamical part of the potential is apart from the form factor given by

$$\begin{aligned}
V_{j_3 j_3', j_3 j_3'}^{(\beta)}(\vec{k}, \vec{k}') &= \frac{1}{(2\pi)^3} \frac{4m^2}{E_k E_{k'}} \sum_{\substack{s_1^3, s_2^3, s_3^3, s_3'^3 \\ (\mu, \nu)}} \langle \frac{1}{2} s_1^3, \frac{1}{2} s_2^3 | j_3 j_3 \rangle \langle \frac{1}{2} \bar{s}_1^3, \frac{1}{2} \bar{s}_2^3 | j_3' j_3' \rangle \\
&\times \left\{ \tilde{w}^{s_1^3}(\vec{k}) \sigma_{(\mu,+)}^\beta w^{s_2^3}(\vec{k}') \right\} \frac{I_\beta^{(\mu\nu)}}{t - m_\rho^2} \left\{ \tilde{w}^{s_3^3}(-\vec{k}) \sigma_{(\nu,-)}^\beta w^{s_3'^3}(-\vec{k}') \right\} \quad (\text{A.7})
\end{aligned}$$

and the partial wave expansion employed in Ref.[37]. The σ^β and I_β are

$$(\alpha) \text{ scalar} \quad : \quad 0 = g, I = 1$$

$$(\beta) \text{ pseudoscalar} \quad : \quad 0 = g \delta_5^4, I = 1$$

$$(\gamma) \text{ vector} \quad : \quad 0_{\mu,\pm} = g^{(\nu)} \delta_\mu^\nu - \frac{g^{(\tau)}}{4m} [\delta_\mu^\nu, \delta_\sigma^\tau] \Delta_\pm^\sigma, \quad I^{\mu\nu} = g^{\mu\nu} - \frac{1}{m_\rho^2} \Delta_+^\mu \Delta_-^\nu$$

where $\Delta_\pm = (\frac{1}{2}(E_k - E_{k'}), \pm(\vec{k} - \vec{k}'))$. Compared to the baryon-baryon interaction the vector exchange contribution only changes signs. The form factor is taken to be of the form [10]

$$F(\vec{k}, \vec{k}') = \exp \{ (\alpha_\beta(t) - J_\beta) \mathcal{F}(k, k') \}, \quad (\text{A.8})$$

where J_β denotes the spin of the exchanged particle.

Appendix B

The SU(3) violating part of the potential corresponding to Fig.3 arises if v' is taken into account in the vector-meson propagators. This changes the pole-term $\frac{1}{t - m_v^2}$ for the noninvariant part into

$$\frac{m_{18} (m_{\omega_0} + m_{\phi_0})}{(t - (m_{\omega_0}^2 + m_{18}^2)) (t - (m_{\phi_0}^2 + m_{18}^2)) - (m_{\omega_0} + m_{\phi_0})^2 m_{18}^2} \quad (\text{B.1})$$

At the same time, the vertices become mixed, one belonging to the singlet and the other to the octet. The noninvariant contribution to the potential which we call $v_{\omega\phi}$ can be written in the form (2.2). The crossing coefficients are

(α) singlet-octet transition

$$C = \left(\frac{10}{\sqrt{30}} (1-\alpha), \sqrt{6} \alpha \right) \quad (\text{B.2})$$

(β) octet-octet transition

$$C = \begin{pmatrix} -\frac{4}{\sqrt{2}} (1-\alpha) & \frac{27}{2\sqrt{10}} \alpha \\ \frac{27}{2\sqrt{10}} \alpha & \frac{9}{\sqrt{8}} (1-\alpha) \end{pmatrix} \cdot \sqrt{2} D_8 \quad (\text{B.3})$$

where α refers to the vector and tensor coupling respectively. There is no contribution to the singlet-singlet transition. The dynamical part is given by the vector exchange contribution with $O_{\mu,+}$ and $O_{\nu,-}$ (see Appendix A) referring to the singlet and octet couplings respectively and the propagator substituted by (B.1). In case of the octet-octet transition we write

$$v_{\omega\phi} = \tilde{v}_{\omega\phi} \cdot D_8 .$$

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Figure Captions

- Fig.1 Octet-singlet crossing coefficient (1) and eigenvalues of the octet-octet crossing matrix (8_1 and 8_2).
- Fig.2 Diagrams for the electromagnetic form factor and the normalization condition:
(a) nucleon loop and (b) meson loop contribution.
- Fig.3 Diagrams arising from ω - ϕ mixing to be included in the potential.
- Fig.4 Self-generating ω - ϕ mixing diagrams in second order.

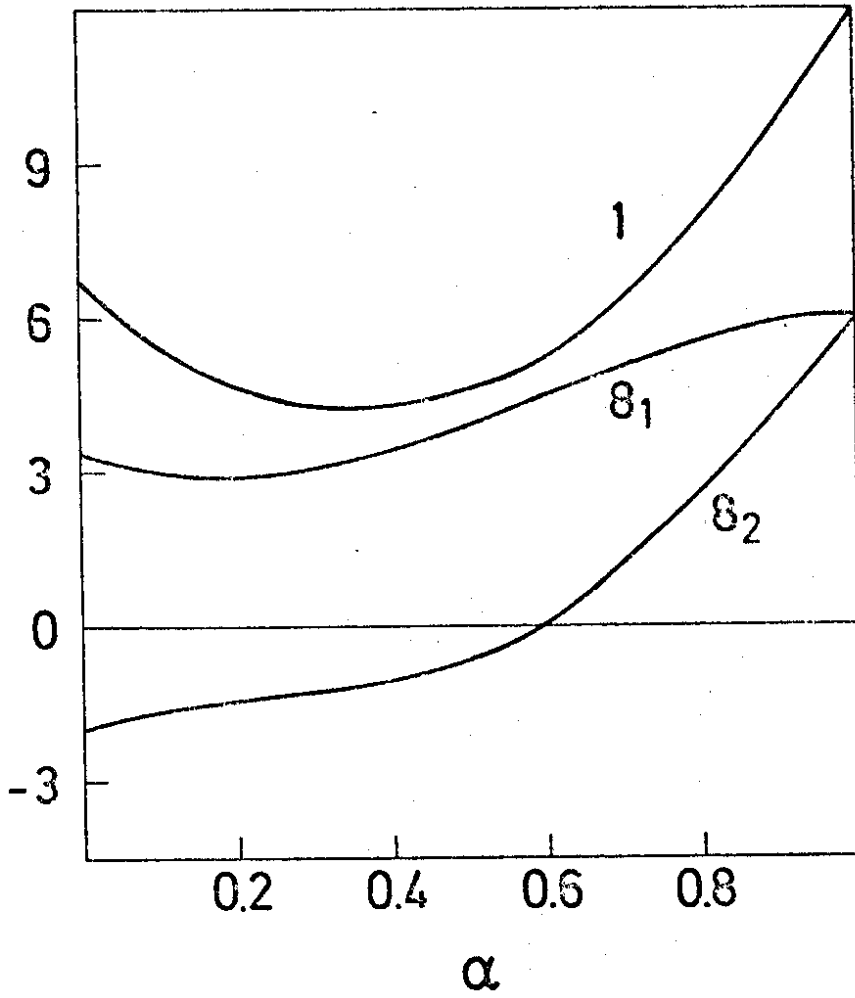
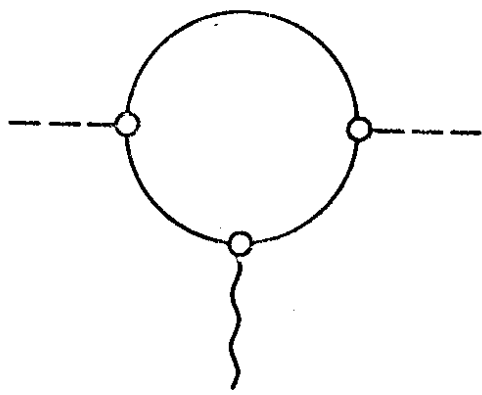
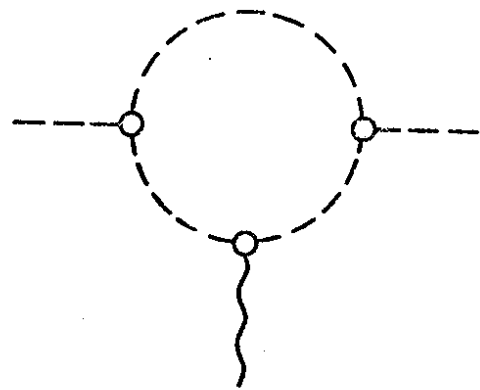


Fig. 1



(a)



(b)

Fig. 2

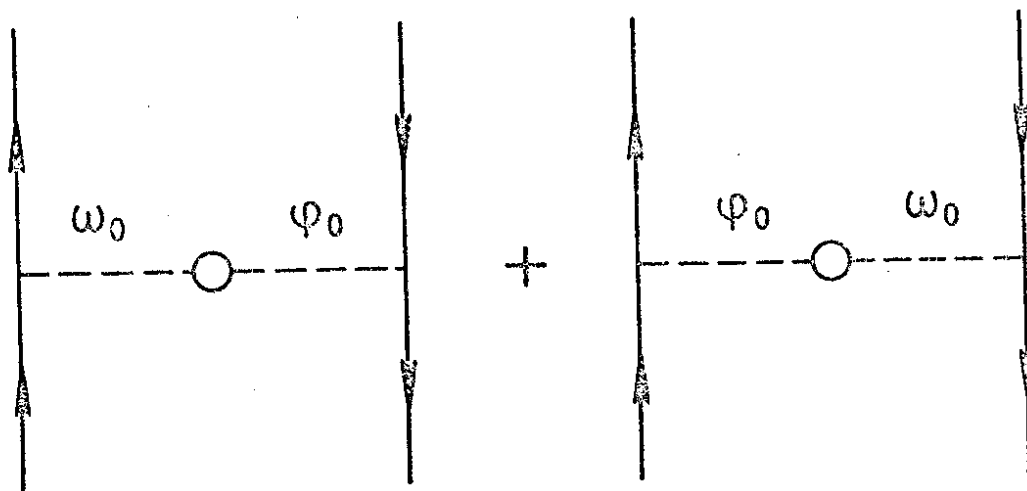


Fig. 3



Fig. 4