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Bunch Lengthening and Power Losses due to the Vacuum Chamber Walls

by

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Abstract

The longitudinal field induced by a bunched beam in a metallic or ceramic vacuum chamber is investigated. The resulting power losses and the bunch lengthening are calculated for high energies and a limit for the stability of the phase oscillations is derived.

Summary

A bunched beam produces longitudinal electric fields in the vacuum chamber walls of an accelerator. These fields react on the beam and influence the synchrotron oscillations. They occur in a metallic vacuum chamber as well as in a ceramic chamber. At high beam energies they do not depend on the energy and are considerably larger than the self-field of the beam.

Since the range of these fields is small as compared to the spacing of two bunches there is almost no coupling between the bunches and the forces depend only on the number of particles in a bunch and not on the total beam current.

In a metallic chamber power losses occur which heat the chamber walls and which must be compensated by the accelerating units. The induced fields have a gradient in the longitudinal direction which, above transition energy, enlarges the bunch length and reduces the synchrotron frequency. But only the incoherent synchrotron oscillation of a particle is influenced in this way, because the fields oscillate together with the bunch.

Power loss and bunch lengthening increase very strongly with decreasing bunch length and decrease with the square root of the conductivity of the chamber walls.

The losses in ceramic vacuum chambers are caused by the loss angle of the ceramics and by the conductive coating on the inside which prevents an accumulation of static charge. If the thickness of the coating is smaller than the skin depth and if its conductance is sufficiently large, the losses in the coating are much larger than in a metallic vacuum chamber. The gradient of the induced field is always considerably larger in a ceramic chamber.

Both power loss and gradient become still larger if the bunch length is comparable with the thickness of the ceramic wall times the square root of the relative dielectric constant. Then, the

first resonance of the induced field in the chamber walls appears which is damped by the power losses.

The bunch lengthening can be compensated by increasing the accelerating voltage maintaining stable phase oscillations. However, the defocussing force of the induced field is distributed continuously around the circumference while the focussing force of the rf-voltage is concentrated in discrete sections. With respect to stable phase oscillations, the compensation can therefore not be made exact, and it turns out that the possible range of the amplitude of the rf-voltage goes to zero with increasing bunch intensity.

1. Calculation of the electromagnetic field

The bunch may be represented by a line current which has a gaussian distribution in longitudinal direction. The density distribution of the charge is then given by

$$\rho(t-\frac{s}{v}) = \frac{N_b e}{\sqrt{2\pi}\,\sigma_s} \exp(-\frac{(s-vt)^2}{2\sigma_s^2})$$

with

N_b	=	number of particles per bunch
e	=	elementary charge
s	=	longitudinal coordinate
σ_s	=	standard deviation
v	=	velocity of the bunch

The distribution of a single bunch can be expressed by a Fourier integral

$$\rho(t-\frac{s}{v}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{\rho}(\omega)\exp(-i\omega(t-\frac{s}{v}))\, d\omega ,$$

where the Fourier transform is given by

$$\tilde{\rho}(\omega) = \frac{N_b e}{\sqrt{2\pi}\, v} \exp(-\frac{\omega^2\sigma_s^2}{2v^2}) \qquad (1)$$

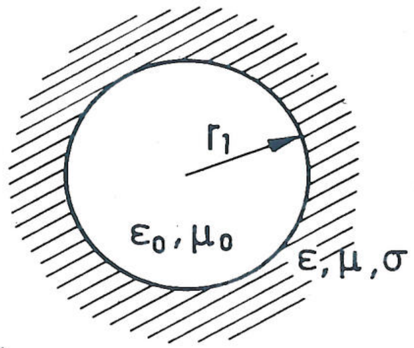
The field components can now be calculated for each frequency ω, and the total field can be represented by a Fourier integral over all frequencies:

$$E_{r,s} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{E}_{r,s}(\omega)\exp(-i\omega(t-\frac{s}{v}))d\omega$$

$$H_{\phi} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \tilde{H}_{\phi}(\omega)\exp(-i\omega(t-\frac{s}{v}))d\omega$$

The results will show that the fields drop very fast behind a bunch so that a Fourier integral is permissible instead of a Fourier series.

1.1 Metallic vacuum chamber



The rotationally symmetric field inside a circular vacuum chamber can easily be found and has the form

$r < r_1$

$$\tilde{E}_r = \frac{1}{v\varepsilon_o} \tilde{H}_{\phi} = A(K_o'(ar) + BI_o'(ar)) \qquad (2)$$

$$\tilde{E}_s = \frac{i}{\gamma}A(K_o(ar) + BI_o(ar))$$

with $a = \sqrt{\frac{\omega^2}{v^2} - \omega^2\varepsilon_o\mu_o} = \frac{\omega}{v\gamma}$

γ is the energy of a particle divided by its rest energy. K_o and I_o denote the modified Bessel functions.

The field in the chamber walls with finite conductivity σ has the form

$$r > r_1$$

$$\tilde{E}_r = \frac{1}{v\varepsilon + iv\sigma/\omega} \tilde{H}_\phi = AC\, K_o'(br)$$

$$\tilde{E}_s = \frac{ivb}{\omega} AC\, K_o(br) \qquad (3)$$

with $\quad b = \sqrt{\frac{\omega^2}{v^2} - \omega^2\varepsilon\mu - i\omega\sigma\mu}$

and $\quad \mathrm{Re}(b) > 0$

Here we have assumed that the thickness of the walls is larger than the skin depth.

The constant A must be determined such that for small r the field is that of a line current. With [1)]

$$K_o'(ar) \approx -\frac{1}{ar}$$

one obtains

$$\tilde{E}_r \approx -\frac{A}{ar} = \frac{\tilde{\rho}}{2\pi\varepsilon_o r}$$

and

$$A \approx -\frac{a}{2\pi\varepsilon_o} \tilde{\rho}(\omega) \qquad (4)$$

The constants B and C are determined such that $\tilde{E}_s$ and $\tilde{H}_\phi$ are continuous at the boundary $r = r_1$:

$$a\,(K_o(ar_1) + BI_o(ar_1)) = bCK_o(br_1)$$

$$\varepsilon_o(K_o'(ar_1) + BI_o'(ar_1)) = (\varepsilon + i\frac{\sigma}{\omega})CK_o'(br_1)$$

The influence of the walls on the beam is described completely by the constant B, which is found to be:

$$B = -\frac{K_o(ar_1)K_o'(br_1) - GK_o'(ar_1)K_o(br_1)}{I_o(ar_1)K_o'(br_1) - GI_o'(ar_1)K_o(br_1)} \qquad (5a)$$

with

$$G = \frac{b\varepsilon_o}{a(\varepsilon + i\sigma/\omega)}$$

We now assume that the particle energy is large, i.e.

$$\gamma >> \frac{r_1}{\sigma_s}$$

Then the relation

$$ar_1 = \frac{\omega r_1}{v\gamma} << 1$$

is satisfied for all frequencies of the bunch spectrum according to Eq. (1).

We further assume that the skin depth d_s is very small as compared to the chamber radius and to the shortest wavelength of the bunch spectrum. Then from

$$d_s = \sqrt{\frac{2}{\omega\mu\sigma}} << r_1, \sigma_s$$

follows the relation

$$|b| r_1 >> 1$$

With these assumptions the following approximations hold [1]

$$I_o(ar_1) \simeq 1 \qquad I_o'(ar_1) \simeq \frac{ar_1}{2}$$

$$K_o(ar_1) \simeq -\ln(ar_1) \qquad K_o'(ar_1) \simeq -\frac{1}{ar_1}$$

$$K_o(br_1) \simeq -K_o'(br_1) \simeq \sqrt{\frac{\pi}{2br_1}}\exp(-br_1)$$

and

$$B \simeq \frac{\ln ar_1 + G\frac{1}{ar_1}}{1 + Gar_1/2}$$

The quantity

$$|G|\frac{ar_1}{2} \simeq \frac{\omega^2 \varepsilon_o \mu r_1 d_s}{2\sqrt{2}}$$

is also small compared to unity, since the relation

$$d_s << \frac{\sigma_s^2}{r_1}$$

is satisfied in all practical cases. So the constant B is given by

$$B = \ln ar_1 + \frac{G}{ar_1} \qquad (5b)$$

and the Fourier component $\tilde{E}_s$ follows from Eqs. (1), (2), (4), (5) to be

$$\tilde{E}_s = - \frac{N_b e}{(2\pi)^{3/2}} \left(\frac{i\omega}{v^2 \gamma^2 \varepsilon_o} \ln \frac{r_1}{r} + \frac{b}{\sigma r_1} \right) \exp\left(- \frac{\omega^2 \sigma_s^2}{2v^2}\right) \qquad (6)$$

One finally obtains for the total longitudinal electric field the expression

$$E_s = \frac{N_b e}{2\pi} \left(\frac{u \ln(r_1/r)}{\sqrt{2\pi}\, \gamma^2 \sigma_s^2 \varepsilon_o} \exp\left(-\frac{u^2}{2}\right) + \right.$$

$$\left. + \frac{v|u|^{3/2}}{4 r_1 \sigma_s^{3/2}} \sqrt{\frac{\mu Z_o}{2\mu_o \sigma}} \exp\left(-\frac{u^2}{4}\right) \left(I_{1/4} - I_{-3/4} \mp I_{-1/4} \pm I_{3/4}\right)\right) \qquad (7)$$

with

$$u = \frac{s - vt}{\sigma_s}$$

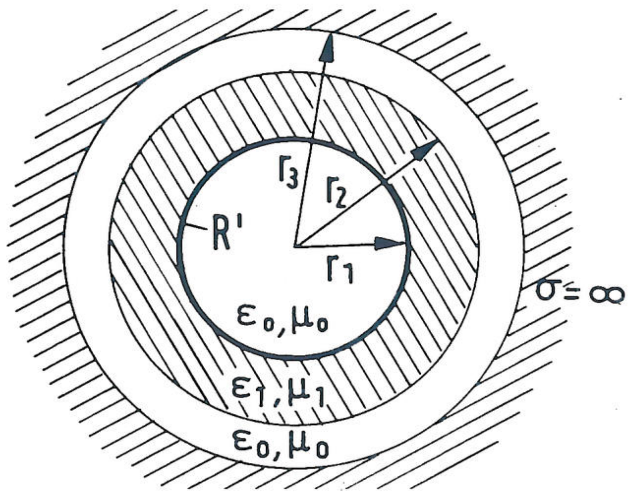
$$Z_o = \sqrt{\frac{\mu_o}{\varepsilon_o}} = 377\ \Omega$$

The upper sign is valid for positive u and the lower sign for negative u. The argument of the modified Bessel functions is $u^2/2$. The first term describes the self-field of the bunch and is independent of the wall parameters. For high energies, it is much smaller than the second term and will be neglected in the following calculations. The second term describes the influence of the walls. Fig. 1 shows the longitudinal distribution of the longitudinal electric field together with the distribution of the particle density in the bunch.

1.2 Ceramic vacuum chamber

For the calculation of the electromagnetic field in a circular ceramic vacuum chamber one may consider four different regions:

1) vacuum inside the chamber,
2) ceramic chamber walls,
3) vacuum outside the chamber,
4) metallic boundary representing the influence of the magnets.



A metallic coating at the inner side of the ceramic wall is taken into account. The thickness of the coating is assumed to be much smaller than the skin depth of all frequencies of the bunch spectrum. The resistance of a square area of the coating is denoted by R'.
Then, the electric and magnetic field is given by the following expressions:

$r < \gamma_1$

$$\tilde{E}_r = \frac{1}{v\varepsilon_o} \tilde{H}_\Phi = A(K_o'(ar) + BI_o'(ar))$$
$$\tilde{E}_s = \frac{i}{\gamma} A(K_o(ar) + BI_o(ar)) \qquad (8)$$

$r_1 < r < r_2$

$$\tilde{E}_r = \frac{1}{v\varepsilon_1} \tilde{H}\Phi = A(CN_o'(qr) + DJ_o'(qr))$$
$$\tilde{E}_s = - \frac{iqv}{\omega} A\ (CN_o(qr) + DJ_o(qr)) \qquad (9)$$

$r_2 < r < r_3$

$$\tilde{E}_r = \frac{1}{v\varepsilon_o}\tilde{H}_\phi = AW(K_o'(ar) - I_o'(ar)\,K_o(ar_3)/I_o(ar_3))$$
$$\tilde{E}_s = \frac{i}{\gamma}AW\,(K_o(ar) - I_o(ar)K_o(ar_3)/I_o(ar_3)) \qquad (10)$$

with

$$q = \frac{\omega}{v}\sqrt{v^2\varepsilon_1\mu_1 - 1}$$

J_o and N_o are the Bessel and Neumann functions, I_o and K_o the modified Bessel functions. a and A have the same meaning as in the preceding section. The constants B, C, D and W are determined by the boundary conditions for $\tilde{E}_s$ and $\tilde{H}_\phi$ at r_1 and r_2. The boundary condition at r_3 is satisfied since $\tilde{E}_s$ vanishes at r_3.

At r_1 one obtains

$$K_o(ar_1) + BI_o(ar_1) = -\frac{qv\gamma}{\omega}(CN_o(qr_1) + DJ_o(qr_1))$$

$$\varepsilon_o(K_o'(ar_1)+BI_o'(ar_1)) = \varepsilon_1(CN_o'(qr_1) + D\,J_o'(qr_1)) - \frac{i}{v\gamma R'}(K_o(ar_1) + BI_o(ar_1)).$$

The last term describes the discontinuity of the magnetic field strength which is produced by the current in the coating.

At r_2 one obtains

$$W(K_o(ar_2) - I_o(ar_2)K_o(ar_3)/I_o(ar_3)) = -\frac{qv\gamma}{\omega}(CN_o(qr_2) + DJ_o(qr_2))$$

$$\varepsilon_o W(K_o'(ar_2) - I_o'(ar_2)K_o(ar_3)/I_o(ar_3)) = \varepsilon_1(CN_o'(qr_2) + DJ_o'(qr_2)).$$

One can now solve for B and gets:

$$B = -\frac{(p_1 K_o'(ar_1)+p_2 K_o(ar_1))(M_1 Q_3+p_1 M_2 Q_1)+K_o(ar_1)(M_1 Q_4+p_1 M_2 Q_2)}{(p_1 I_o'(ar_1)+p_2 I_o(ar_1))(M_1 Q_3+p_1 M_2 Q_1)+I_o(ar_1)(M_1 Q_4+p_1 M_2 Q_2)} \qquad (11a)$$

with

$$M_1 = I_o(ar_2)K_o(ar_3) - I_o(ar_3)K_o(ar_2)$$
$$M_2 = I_o'(ar_2)K_o(ar_3) - I_o(ar_3)\,K_o'(ar_2)$$
$$Q_1 = J_o(qr_1)N_o(qr_2) - J_o(qr_2)N_o(qr_1)$$
$$Q_2 = J_o'(qr_1)N_o(qr_2) - J_o(qr_2)N_o'(qr_1)$$
$$Q_3 = J_o(qr_1)N_o'(qr_2) - J_o'(qr_2)N_o(qr_1)$$
$$Q_4 = J_o'(qr_1)N_o'(qr_2) - J_o'(qr_2)N_o'(qr_1)$$
$$p_1 = \frac{qv\gamma\varepsilon_o}{\omega\varepsilon_1} \qquad p_2 = \frac{iq}{\omega\varepsilon_1 R'}$$

This exact solution can be simplified for the interesting case of high energies, using the same approximations for the modified Bessel functions with the arguments ar_1 and ar_2 as in the preceding section. Considering in Eq. (11a) only the terms with the highest power in γ, one obtains

$$B = \frac{1}{ar_1}\,\frac{p_1\,Q_1}{(p_1 ar_1/2+p_2)Q_1+Q_2}$$
$$= \frac{\gamma^2}{qr_1}\,\frac{(\varepsilon_r-1)\,Q_1}{(qr_1/2+i\sqrt{\varepsilon_r-1}\cdot Z_o/R')Q_1+\varepsilon_r Q_2} \qquad (11b)$$

with

$$\varepsilon_r = \frac{\varepsilon_1}{\varepsilon_o}\,,\quad \mu_1 = \mu_o$$

The dependence on r_3 has disappeared, i.e. the magnets have no influcence on the investigated longitudinal fields. One should note that Q_1, Q_2 and a combination of both have an infinite number of zeros, if one neglects the loss angle of ε_r. This means that resonances appear for which the electromagnetic field has a maximum in the ceramics or inside the chamber volume. The expression Eq. (11b) is sufficiently accurate, if R' is finite so that no singularities occur for B. At such a resonance, B is zero or purely imaginary. More details can be drawn from the following calculations.

The longitudinal electric field strength per frequency interval has the form

$$\tilde{E}_s = -i \frac{N_b e Z_o}{(2\pi)^{3/2} r_1} \frac{\sqrt{\varepsilon_r - 1}\; Q_1}{(q r_1/2 + i\sqrt{\varepsilon_r - 1}\, Z_o/R')Q_1 + \varepsilon_r Q_2} \exp\left(-\frac{\omega^2 \sigma_s^2}{2v^2}\right) \qquad (12a)$$

Here the term due to the self-field of the bunch has been neglected as in the preceding section.

The total longitudinal field strength is

$$E_s = -i \frac{N_b e Z_o}{4\pi^2 r_1} \int_{-\infty}^{\infty} \frac{\sqrt{\varepsilon_r - 1}\; Q_1}{A_1 + i A_2} \exp\left(-\frac{\omega^2 \sigma_s^2}{2v^2} - i\omega(t - s/v)\right) d\omega$$

$$= \frac{N_b e Z_o}{4\pi^2 r_1} \int_{-\infty}^{\infty} \frac{A_1 \sin(\omega u \sigma_s/v) - A_2 \cos(\omega u \sigma_s/v)}{A_1^2 + A_2^2} \sqrt{\varepsilon_r - 1}\; Q_1 \exp\left(-\frac{\omega^2 \sigma_s^2}{2v^2}\right) d\omega \qquad (13a)$$

with

$$u = \frac{s - vt}{\sigma_s} \qquad \varepsilon_r = \text{real}$$

$$A_1 = \frac{q r_1}{2} Q_1 + \varepsilon_r Q_2 \qquad A_2 = \sqrt{\varepsilon_r - 1}\, \frac{Z_o}{R'} Q_1$$

The integral is solved numerically with the assumption that ε_r is constant. Fig. 2 shows the field distribution for different parameters of σ_s/r_1.

An analytical solution can be given for the case when $q(r_2 - r_1)$ is small as compared to unity for all frequencies of the bunch spectrum and when R' is not too small. If the relations

$$(r_2-r_1)\sqrt{\varepsilon_r-1} \ll \sigma_s \qquad (14)$$

$$r_1 \frac{Z_o}{R'} \ln \frac{r_2}{r_1} \ll \sigma_s \qquad (15)$$

are satisfied one obtains

$$Q_1 = \frac{2}{\pi}\left(\ln \frac{r_2}{r_1} - \frac{q^2(r_2-r_1)^3}{6r_1} + \dots\right)$$

$$Q_2 = \frac{2}{\pi q r_1}\left(-1 + \frac{q^2(r_2-r_1)^2}{2} - \dots\right)$$

The constant B simplifies to

$$B = -\gamma^2 \frac{\varepsilon_r-1}{\varepsilon_r} \ln \frac{r_2}{r_1}\left(1+i\frac{\omega r_1}{v}\frac{Z_o}{R'}\frac{\varepsilon_r-1}{\varepsilon_r}\ln\frac{r_2}{r_1}\right) \qquad (11c)$$

The Fourier transform becomes

$$\tilde{E}_s = i\frac{N_b e Z_o \omega}{(2\pi)^{3/2} v}\frac{\varepsilon_r-1}{\varepsilon_r}\ln\frac{r_2}{r_1}\left(1+i\frac{\omega r_1}{v}\frac{Z_o}{R'}\frac{\varepsilon_r-1}{\varepsilon_r}\ln\frac{r_2}{r_1}\right)\exp\left(-\frac{\omega^2\sigma_s^2}{2v^2}\right) \qquad (12b)$$

and, finally, the longitudinal field strength is given by

$$E_s = -\frac{N_b e Z_o v(\varepsilon_r-1)}{(2\pi)^{3/2}\sigma_s^2 \varepsilon_r}\ln\frac{r_2}{r_1}\left(u+\frac{r_1 Z_o(\varepsilon_r-1)}{\sigma_s R' \varepsilon_r}\ln\frac{r_2}{r_1}(1-u^2)\right)\exp\left(-\frac{u^2}{2}\right)$$

(13b)

2. Power losses

The power losses can be obtained by integrating the longitudinal electric field strength times the bunch current. This integral can also be taken over the Fourier transforms of the field strength and the bunch current according to the Fourier integral theorem. So one obtains

$$P = - v \int_{-\infty}^{\infty} E_s \rho \, ds = - v^2 \int_{-\infty}^{\infty} \tilde{E}_s \tilde{\rho} \, d\omega \tag{16}$$

For a metallic vacuum chamber this yields

$$P_m = \frac{v}{r_1 \sigma} \left(\frac{N_b e}{2\pi}\right)^2 \int_{-\infty}^{\infty} \exp\left(- \frac{\omega^2 \sigma_s^2}{v^2}\right) b \, d\omega$$

Taking into account that the real part of b is positive for all frequencies, one obtains with the same approximations as in 1.1 for the power losses per bunch

$$P_m = \frac{I_b^2 \overline{R}^2}{r_1 \sigma_s^{3/2}} \sqrt{\frac{Z_o \mu}{2\sigma\mu_o}} \, \Gamma\left(\frac{3}{4}\right) \tag{17}$$

with $I_b = \frac{N_b e v}{2\pi\overline{R}}$ = bunch current

$\overline{R}$ = mean radius of the machine

As an example, we obtain for the DESY storage ring a total power loss of 30 kW for all 480 bunches, assuming $I_b = 4A/480$, $\sigma_s = 1cm$, $\sigma = 1m/\Omega/mm^2$, $r_1 = 4cm$ and $\overline{R} = 46$ m.

For the ceramic vacuum chamber we will neglect the loss angle of the ceramics, because these losses are much smaller than the losses in the metallic coating. So one obtains from Eqs. (16), (1), (12a)

$$P_{ce} = \frac{i}{r_1 \varepsilon_o}\left(\frac{N_b e}{2\pi}\right)^2 \int_{-\infty}^{\infty} \exp\left(-\frac{\omega^2\sigma_s{}^2}{v^2}\right) \frac{Q_1\sqrt{\varepsilon_1 - 1}\, d\omega}{Q_1(qr_1/2 + iZ_o\sqrt{\varepsilon_r - 1}/R') + \varepsilon_r Q_2}$$

$$= \frac{(\varepsilon_r - 1)Z_o}{r_1 \varepsilon_o R'}\left(\frac{N_b e}{2\pi}\right)^2 \int_{-\infty}^{\infty} \exp\left(-\frac{\omega^2\sigma_s{}^2}{v^2}\right) \frac{Q_1{}^2\, d\omega}{(Q_1 q r_1/2 + \varepsilon_r Q_2)^2 + Q_1^2 Z_o^2(\varepsilon_r - 1)/R'^2}$$

This integral can be solved analythically for the limiting case when the bunch length is large, i.e. when the relations Eq.(14) and Eq.(15) are satisfied. In that case one can use Eq.(12) instead of Eq.(12a) for the Fourier transform of the electric field. Then the power losses per bunch are given by

$$P_{ce} = \frac{\sqrt{\pi}}{2}\, \frac{r_1 Z_o^2}{\sigma_s^3 R'} \left(\frac{N_b e v(\varepsilon_r - 1)}{2\pi\varepsilon_r} \ln \frac{r_2}{r_1}\right)^2$$

Now one can write the power losses for an arbitrary bunch length in the form

$$P_{ce} = \frac{\sqrt{\pi}}{2}\, \frac{I_b^2 Z_o^2 \bar{R}^2 r_1}{R'\sigma_s^3}\left(\frac{\varepsilon_r - 1}{\varepsilon_r} \ln\frac{r_2}{r_1}\right)^2 f_L\left(\frac{r_1}{\sigma_s}, \frac{Z_o}{R'}, \frac{r_2}{r_1}, \varepsilon_r\right), \qquad (18)$$

where the number of particles per bunch is expressed by the bunch current I_b and f_L is given by the integral

$$f_L\left(\frac{r_1}{\sigma_s}, \frac{Z_o}{R'}, \frac{r_2}{r_1}, \varepsilon_r\right) = \frac{2\sqrt{\varepsilon_r - 1}\,\sigma_s^3}{\sqrt{\pi}\, r_1^3}\left(\frac{\varepsilon_r}{(\varepsilon_r - 1)\ln(r_2/r_1)}\right)^2 \cdot$$

$$\cdot \int_{-\infty}^{\infty} \frac{Q_1^2}{(Q_1 x/2 + \varepsilon_r Q_2)^2 + Q_1^2 Z_o^2(\varepsilon_r - 1)/R'^2} \exp\left(-\frac{\sigma_s^2 x^2}{r_1^2(\varepsilon_r - 1)}\right) dx$$

with

$$qr_1 = x$$

f_L is solved numerically and plotted in Fig. 3. It can be seen that the losses can increase due to resonances. On the other hand the losses decrease when the conductance of the coating becomes very large. In that case the voltage induced in the ceramics is reduced by the currents in the coating.

To compare the power losses in metallic chambers and in ceramic chambers with a metallic coating one can consider the ratio of the expressions Eq.(17) and Eq.(18):

$$\frac{P_{ce}}{P_m} = \frac{\sqrt{\pi}\, r_1^2\, Z_o}{\Gamma(3/4)\ \sigma_s^{3/2} R'} \sqrt{\frac{\sigma Z_o}{2}}\ \left(\frac{\varepsilon_r - 1}{\varepsilon_r} \ln\frac{r_2}{r_1}\right)^2 \cdot f_L$$

with usual parameters ($\sigma = 1\ m/\Omega/mm^2$, $\varepsilon_r = 7$, $r_2 - r_1 = 1$ cm, $r_1 >> r_2 - r_1$) one obtains

$$\frac{P_{ce}}{P_m} = 1.5 \cdot 10^3 \cdot \frac{Z_o}{R'} \left(\frac{cm}{\sigma_s}\right)^{3/2} \cdot f_L$$

Thus the losses in the coating of a ceramic chamber can be larger by orders of magnitude than the losses in a pure metallic chamber. The reason is that the longitudinal voltage induced in the ceramics is considerably larger than the longitudinal voltage induced by the image currents in a metallic chamber.

3. Bunch lengthening

There are two mechanism which lead to an increase of the bunch length. To compensate the power losses in the chamber walls the particles need a higher accelerating voltage and receive a larger phase angle. Thus they see a smaller gradient of the voltage and the bunch length becomes larger.

Secondly the induced field itself has a gradient as can be seen from Fig. 1 and Fig. 2. Since the fields are shown for positive charged particles this gradient has, for not too small bunch lengths , the opposite sign as the gradient of the accelerating voltage and therefore enlarges the bunch length. Although the gradient varies along the bunch we will, in a first approximation, assume that the gradient is constant and take its value in the center of the bunch.

If one is far away from the stability limit of the synchrotron oscillation as derived in the following section, the dependence of the bunch length on the gradient is given by

$$\frac{1}{\sigma_s^{2}} \sim \frac{\partial U}{\partial \psi}$$

where ψ is the phase angle of the synchrotron oscillation. The gradient without beam is

$$\left.\frac{\partial U}{\partial \psi}\right|_O = U_{ac} \cos\psi_{so}$$

where U_{ac} is the maximum accelerating voltage per circumference, and ψ_{so} is the equilibrium phase angle without beam. The gradient with beam is

$$\frac{\partial U}{\partial \psi} = U_{ac} \cos\psi_s + \frac{\partial U_{in}}{\partial \psi}$$

The equilibrium phase follows from the relation

$$U_{ac} \sin\psi_s = \frac{P_\gamma + P_w}{e f_o}$$

$$= U_{ac} \sin\psi_{so} + \frac{P_w}{e f_o}$$

with P_γ = radiation losses
P_w = power losses in the walls
f_o = revolution frequency

With these relations one obtains for the bunch length the expression

$$\frac{\sigma_{so}^{2}}{\sigma_s^{2}} = \sqrt{1 - \tan^2\psi_{so} \cdot \frac{P_w}{P_\gamma}\left(2 + \frac{P_w}{P_\gamma}\right)} + \frac{1}{U_{ac}\cos\psi_{so}} \frac{\partial U_{in}}{\partial \psi} \qquad (19)$$

In this derivation we have neglected the change of the amplitude of the acceleration voltage due to the change of the power consumption of the beam or due to the change of the beam loading.

The phase angle can be eliminated by the relation

$$\frac{\partial U_{in}}{\partial \psi} = \lambda \bar{R} \frac{\partial E_s}{\partial s} \qquad (20)$$

with λ = wavelength of the accelerating field.

For the metallic vacuum chamber one obtains with Eqs. (7) and (20)

$$\frac{\partial U_{in\ m}}{\partial \psi} = - \frac{I_b \lambda \bar{R}^2}{\pi r_1 \sigma_s^{5/2}} \sqrt{\frac{Z_o}{\sqrt{2}\sigma}}\ \Gamma(\tfrac{5}{4}) \qquad (21)$$

As an example, we obtain for the DESY storage ring $|\partial U/\partial \psi| = 125$ kV, assuming the same parameters as in the preceding section.

For the ceramic chamber one obtains with Eqs. (13a) and (2o)

$$\frac{\partial U_{in\ ce}}{\partial \psi} = \frac{N_b e Z_o \lambda \bar{R} \sqrt{\varepsilon_r - 1}}{4\pi^2\ v\ r_1} \int_{\infty}^{\infty} \exp\left(- \frac{\omega^2 \sigma_s^2}{2v^2}\right) \frac{(Q_1 q r_1/2 + Q_2 \varepsilon_r) Q_1 \omega d\omega}{(Q_1 q r_1/2 + Q_2 \varepsilon_r)^2 + Q_1^2 Z_o^2 (\varepsilon_r - 1)/R'^2}$$

This integral can again be solved for a large bunch length satisfying Eq. (14) and Eq. (15) and yields

$$\frac{\partial U_{in\ ce}}{\partial \psi} = - \frac{N_b e Z_o v \lambda \bar{R} (\varepsilon_r - 1)}{(2\pi)^{3/2} \sigma_s^3 \varepsilon_r} \ln \frac{r_2}{r_1}$$

Thus, for an arbitrary bunch length one can write

$$\frac{\partial U_{in\ ce}}{\partial \psi} = - \frac{I_b \lambda \bar{R}^2 Z_o (\varepsilon_r - 1)}{\sqrt{2\pi}\ \sigma_s^3\ \varepsilon_r} \ln \frac{r_2}{r_1} \cdot f_g\left(\frac{r_1}{\sigma_s}, \frac{Z_o}{R'}, \frac{r_2}{r_1}, \varepsilon_r\right)$$

with

$$f_g\left(\frac{r_1}{\sigma_s}, \frac{Z_o}{R'}, \frac{r_2}{r_1}, \varepsilon_r\right) = - \sqrt{\frac{2}{\pi}} \frac{\sigma_s^3 \varepsilon_r}{r_1^3 (\varepsilon_r - 1)^{3/2} \ln(r_2/r_1)} .$$

$$\cdot \int_{-\infty}^{\infty} \exp\left(- \frac{x^2 \sigma_s^2}{2 r_1^2\ (\varepsilon_r - 1)}\right) \frac{(Q_1 x/2 + Q_2 \varepsilon_r) Q_1 x dx}{(Q_1 x/2 + Q_2 \varepsilon_r)^2 + Q_1^2 Z_o^2 (\varepsilon_r - 1)/R'^2}$$

and $\qquad q r_1 = x$

Fig. 4 shows the function f_g for different parameters.

It can be seen how the gradient is influenced by resonances at short bunch lengths and how a larger conductance of the coating reduces the gradient.

The ratio of the gradients in ceramic and metallic vacuum chambers is

$$\frac{\partial U_{in\ ce}}{\partial \psi} / \frac{\partial U_{in\ m}}{\partial \psi} = \frac{\sqrt{\pi}}{\Gamma(\frac{5}{4})} \sqrt{\frac{\sigma Z_o}{\sqrt{2}}} \frac{\varepsilon_r - 1}{\varepsilon_r} \frac{r_1}{\sqrt{\sigma_s}} \ln \frac{r_2}{r_1} f_g$$

With the same parameters as in the preceding section ($\sigma = 1\text{m}/\Omega/\text{mm}^2, \varepsilon_r = 7$, $r_2 - r_1 = 1$ cm, $r_1 >> r_2 - r_1$) this ratio becomes

$$\frac{\partial U_{in\ ce}}{\partial \psi} / \frac{\partial U_{in\ m}}{\partial \psi} = 2.7 \cdot 10^3 \frac{\sqrt{cm}}{\sigma_s} f_g$$

4. Stability limit for synchrotron oscillations

The focusing force of the accelerating units and the defocusing force of the induced field do not act simultaneously on the particles. For large beam currents and also for high synchrotron frequencies it is necessary to take into account the exact distribution of the different fields [2]. Therefore we will write down separately the equations for the phase oscillation in the two different sections. We assume that the ring consists of N equal accelerating units and N equal sections, in which the curvature and the gradient of the induced field are uniformly distributed.

In an accelerating unit without curvature the linearized equations for the phase oscillation are

$$\frac{d\Delta\psi}{ds} = 0 \qquad \frac{d\eta}{ds} = \frac{eU_{ac}}{Nl_{ac}} \cos\psi_s \, \Delta\psi$$

with $\Delta\psi$ = deviation from the equilibrium phase

$\eta = \frac{\Delta E}{E}$ = relative deviation from the equilibrium energy

l_{ac} = length of an accelerating unit.

The matrix notation for the motion through an accelerating unit is

$$\begin{pmatrix} \eta \\ \Delta\psi \end{pmatrix}_1 = \begin{pmatrix} 1 & \frac{eU_{ac}\cos\psi_s}{NE} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \eta \\ \Delta\psi \end{pmatrix}_0 \qquad (23)$$

In a section between two accelerating units the equations for the synchrotron oscillation are

$$\frac{d\Delta\psi}{ds} = -\frac{k\alpha}{\overline{R}}\eta \qquad \frac{d\eta}{ds} = -\frac{e}{2\pi\overline{R}E}\left|\frac{\partial U_{in}}{\partial\psi}\right|\Delta\psi$$

with k = harmonic number, E = equilibrium energy

α = momentum compaction factor

Combining the two equations one obtains a differential equation for η

$$\frac{d^2\eta}{ds^2} = \frac{k\alpha e}{2\pi\overline{R}^2E}\left|\frac{\partial U_{in}}{\partial\psi}\right|\eta$$

with the solution

$$\eta = B_1\cosh\left(\frac{yNs}{\pi\overline{R}}\right) + B_2\sinh\left(\frac{yNs}{\pi\overline{R}}\right)$$

The constant y is given by

$$y = \frac{1}{N}\sqrt{\frac{\pi k\alpha e}{2E}\left|\frac{\partial U_{in}}{\partial\psi}\right|} \qquad (24)$$

A similar solution follows for $\Delta\psi$

$$\Delta\psi = -\frac{\pi k\alpha}{yN}\left(B_1\sinh\left(\frac{yNs}{\pi\overline{R}}\right) + B_2\cosh\left(\frac{yNs}{\pi\overline{R}}\right)\right)$$

Replacing the integration constant B_1 and B_2 by the initial values at $s = 0$ yields

$$B_1 = \eta_o \qquad B_2 = - \frac{yN}{\pi k \alpha} \Delta\psi_o$$

The transfer matrix for one curved section can be written in the form

$$\begin{pmatrix} \eta \\ \Delta\psi \end{pmatrix}_1 = \begin{pmatrix} \cosh(2y) & -\frac{yN}{\pi k\alpha} \sinh(2y) \\ -\frac{\pi k \alpha}{yN} \sinh(2y) & \cosh(2y) \end{pmatrix} \begin{pmatrix} \eta \\ \Delta\psi \end{pmatrix}_0$$

Here we have replaced s by $2\pi\bar{R}/N$ and have neglected the length of the accelerating units.

The transfer matrix for an accelerating unit and the following curved section has the form

$$M_{tr} = \begin{pmatrix} \cosh(2y) & \frac{eU_{ac}\cos\psi_s}{N^2 E} \cosh(2y) - \frac{yN}{\pi k\alpha} \sinh(2y) \\ -\frac{\pi k\alpha}{yN} \sin(2y) & \cosh(2y) - \frac{\pi k \alpha e U_{ac} \cos\psi_s}{yN^2 E} \sinh(2y) \end{pmatrix}$$

The eigenvalues of the transfer matrix are

$$\lambda_{1,2} = \frac{1}{2} Sp(M_{tr}) \pm \sqrt{\frac{1}{4} Sp^2(M_{tr}) - 1}$$

with
$$Sp(M_{tr}) = 2 \cosh(2y) - \frac{\pi k \alpha e U_{ac} \cos\psi_s}{yN^2 E} \sinh(2y)$$
$$= 2 \cosh(2y) - \frac{2\pi^2 \Omega_o^{\ 2}}{yN^2 \omega_o^{\ 2}} \sinh(2y)$$

where $\Omega_o/2\pi$ denotes the synchrotron frequency, calculated without the gradient of the induced field, and $\omega_o/2\pi$ the revolution frequency. The stability of the synchrotron oscillation requires that the eigenvalues do not have an absolute value greater than or equal to one. This yields the two inequalities

$$-1 < \cosh(2y) - \frac{\pi^2 \Omega_o^2}{y N^2 \omega_o^2} \sinh(2y) < 1$$

which can also be written as

$$y\tanh y < \frac{\pi^2 \Omega_o^2}{N^2 \omega_o^2} < y\coth y \qquad (25)$$

Fig. 5 shows the stable region for the synchrotron oscillation. For small bunch currents the first inequality becomes

$$\left| \frac{\partial U_{in}}{\partial \psi} \right| < \frac{2\pi E \Omega_o^2}{k \alpha e \, \omega_o^2} = U_{ac} \cos \psi_s$$

which means that the gradient of the accelerating voltage must be larger than the induced gradient. However, this condition is valid only for small currents. As can be seen from Eq. (25) for large currents the gradient of the accelerating voltage must be smaller than the induced gradient.

The second inequality becomes for small currents

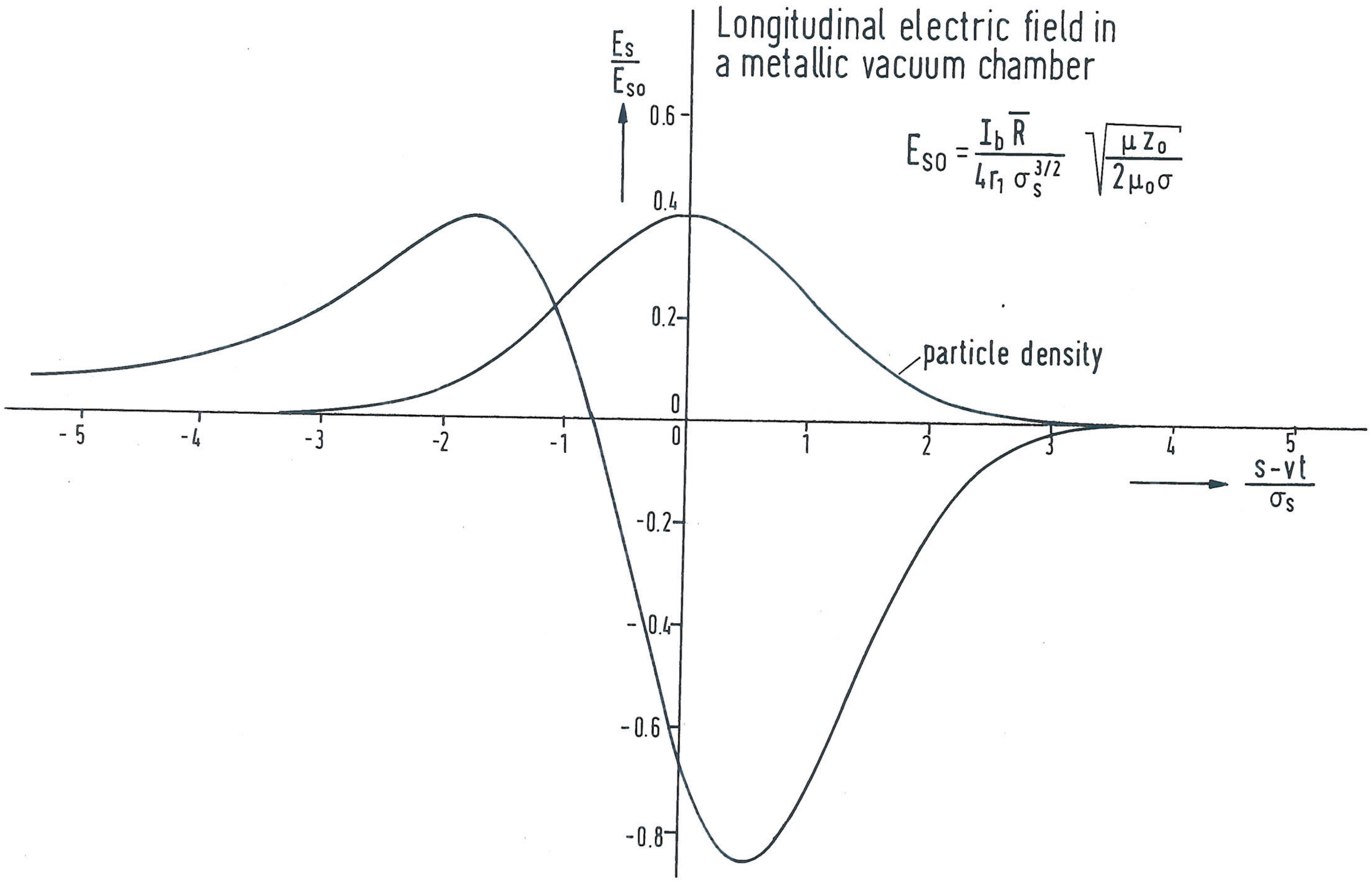
$$\frac{\pi \Omega_o}{N \omega_o} < 1$$

which corresponds to the known limit for the synchrotron frequency given in [2].

References

1) G.N. Watson; Theory of Bessel functions, Cambridge 1958

2) A. Piwinski; Nucl. Instr. Meth. 72, 79 (1969)



Longitudinal electric field in
a metallic vacuum chamber
$E_{so} = \frac{I_b \overline{R}}{4 r_1 \sigma_s^{3/2}} \sqrt{\frac{\mu Z_o}{2 \mu_o \sigma}}$
$\frac{E_s}{E_{so}}$
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
particle density
-5
-4
-3
-2
-1
0
1
2
3
4
5
$\frac{s - vt}{\sigma_s}$

fig.1

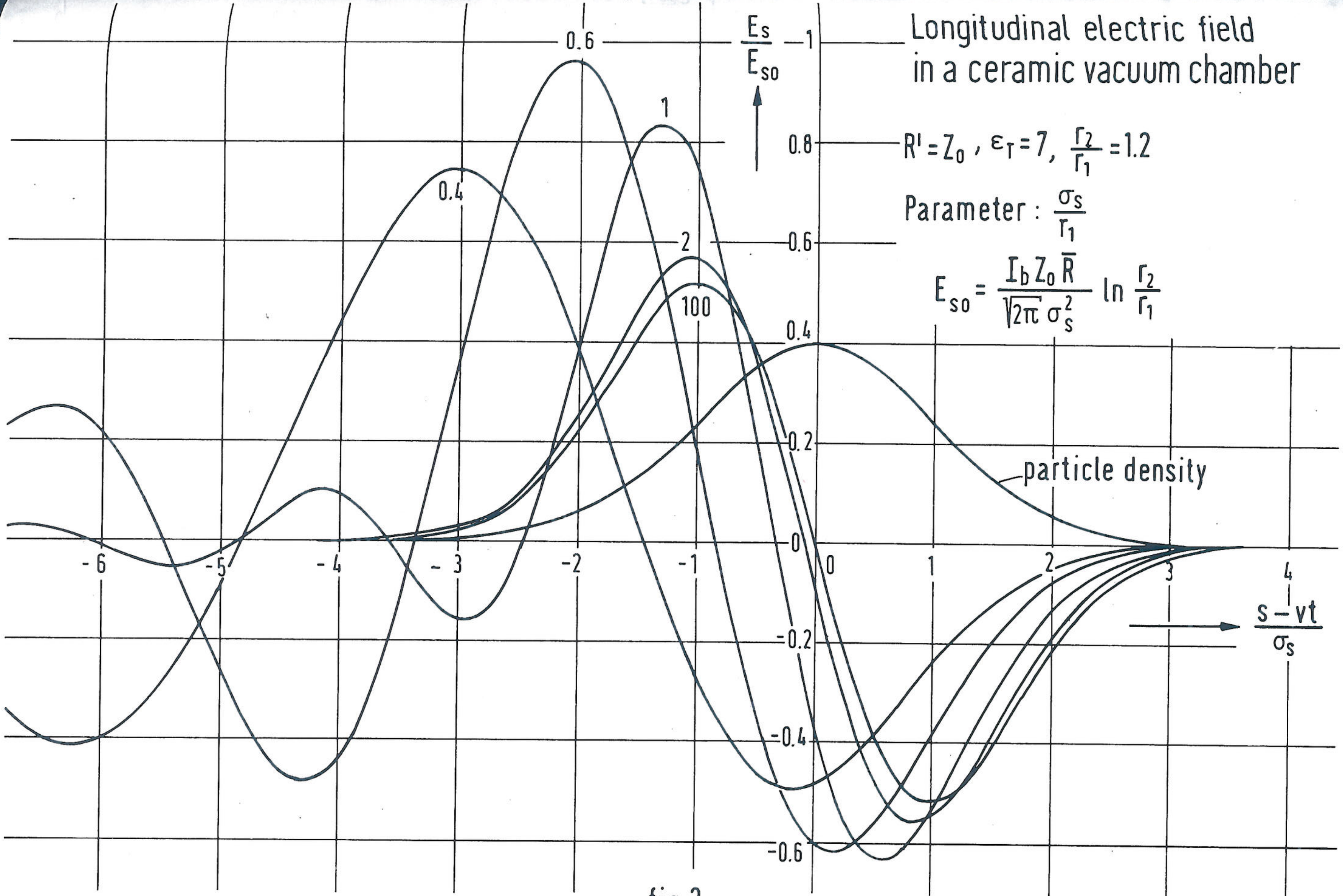
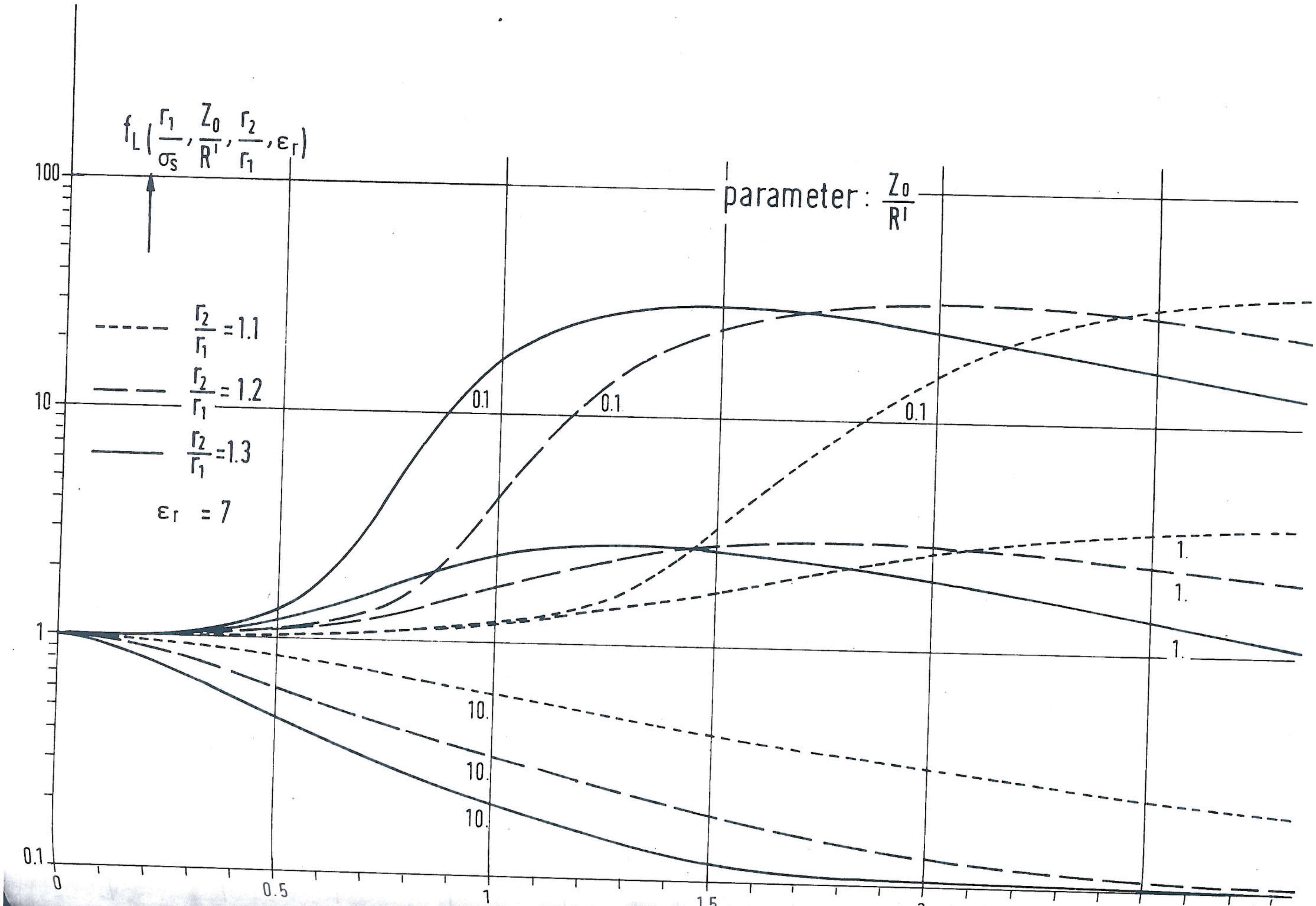
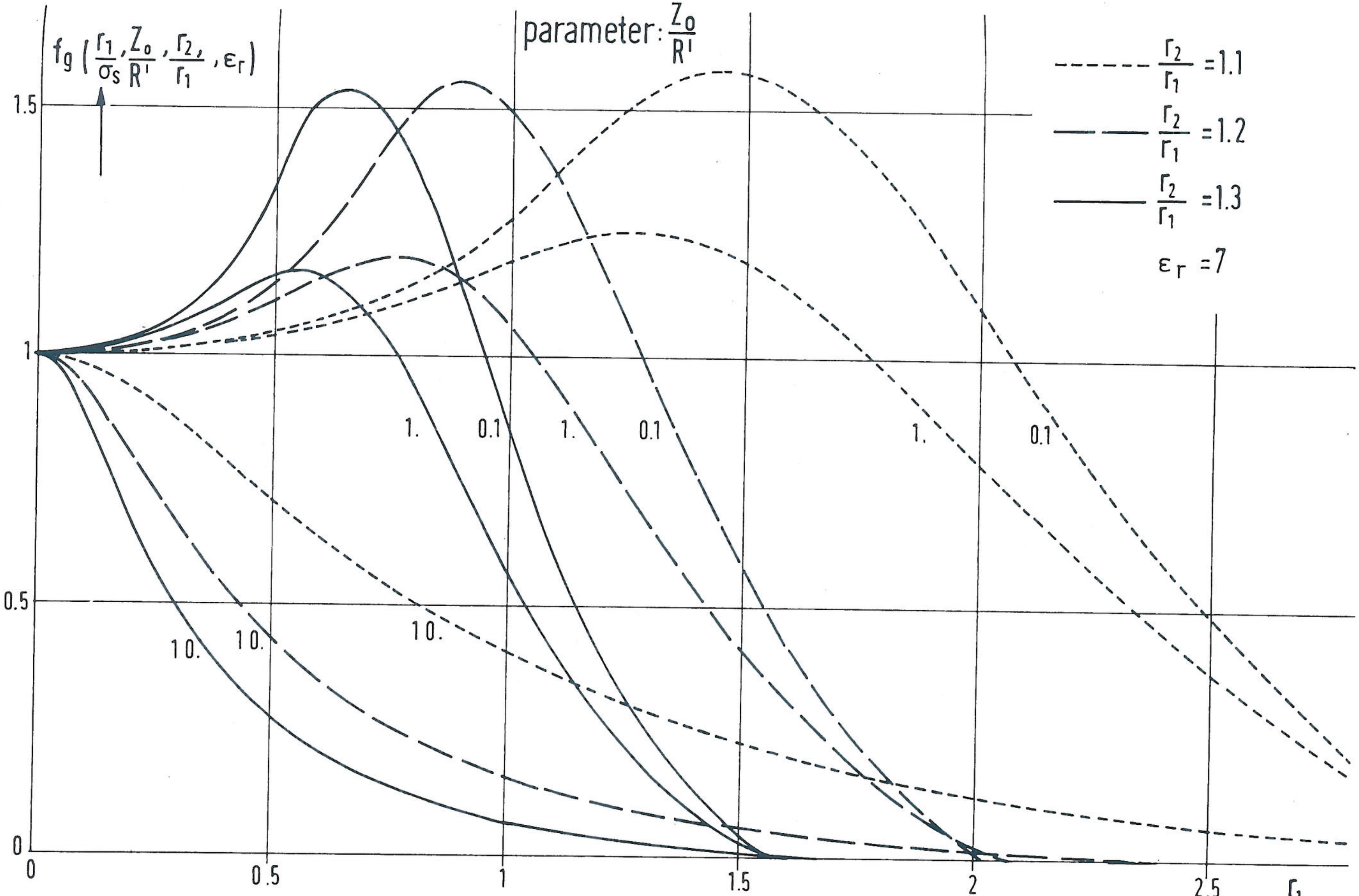

Longitudinal electric field
in a ceramic vacuum chamber
R' = Z0 , εT = 7, r2/r1 = 1.2
Parameter : σs/r1
Eso = Ib Z0 R̄ / (√2π σs²) ln r2/r1
Es/Eso
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
0.6
0.4
1
2
100
particle density
- 6
-5
- 4
- 3
-2
-1
0
1
2
3
4
s − vt / σs

fig.2



$f_L\left(\frac{r_1}{\sigma_s}, \frac{Z_0}{R'}, \frac{r_2}{r_1}, \varepsilon_r\right)$
100
10
1
0.1
parameter : $\frac{Z_0}{R'}$
$\frac{r_2}{r_1} = 1.1$
$\frac{r_2}{r_1} = 1.2$
$\frac{r_2}{r_1} = 1.3$
$\varepsilon_r = 7$
0.1
0.1
0.1
1.
1.
1.
10.
10.
10.
0
0.5
1
1.5
2
2.5
$\frac{r_1}{\sigma_s}$

fig 3



$f_g\left(\frac{r_1}{\sigma_s}, \frac{Z_0}{R'}, \frac{r_2}{r_1}, \varepsilon_r\right)$
parameter: $\frac{Z_0}{R'}$
$\frac{r_2}{r_1} = 1.1$
$\frac{r_2}{r_1} = 1.2$
$\frac{r_2}{r_1} = 1.3$
$\varepsilon_r = 7$
1.5
1
0.5
0
0.5
1
1.5
2
2.5
$\frac{r_1}{\sigma_s}$
1.
0.1
1.
0.1
1.
0.1
10.
10.
10.

fig.4

Stability limits for the synchrotron oscillation

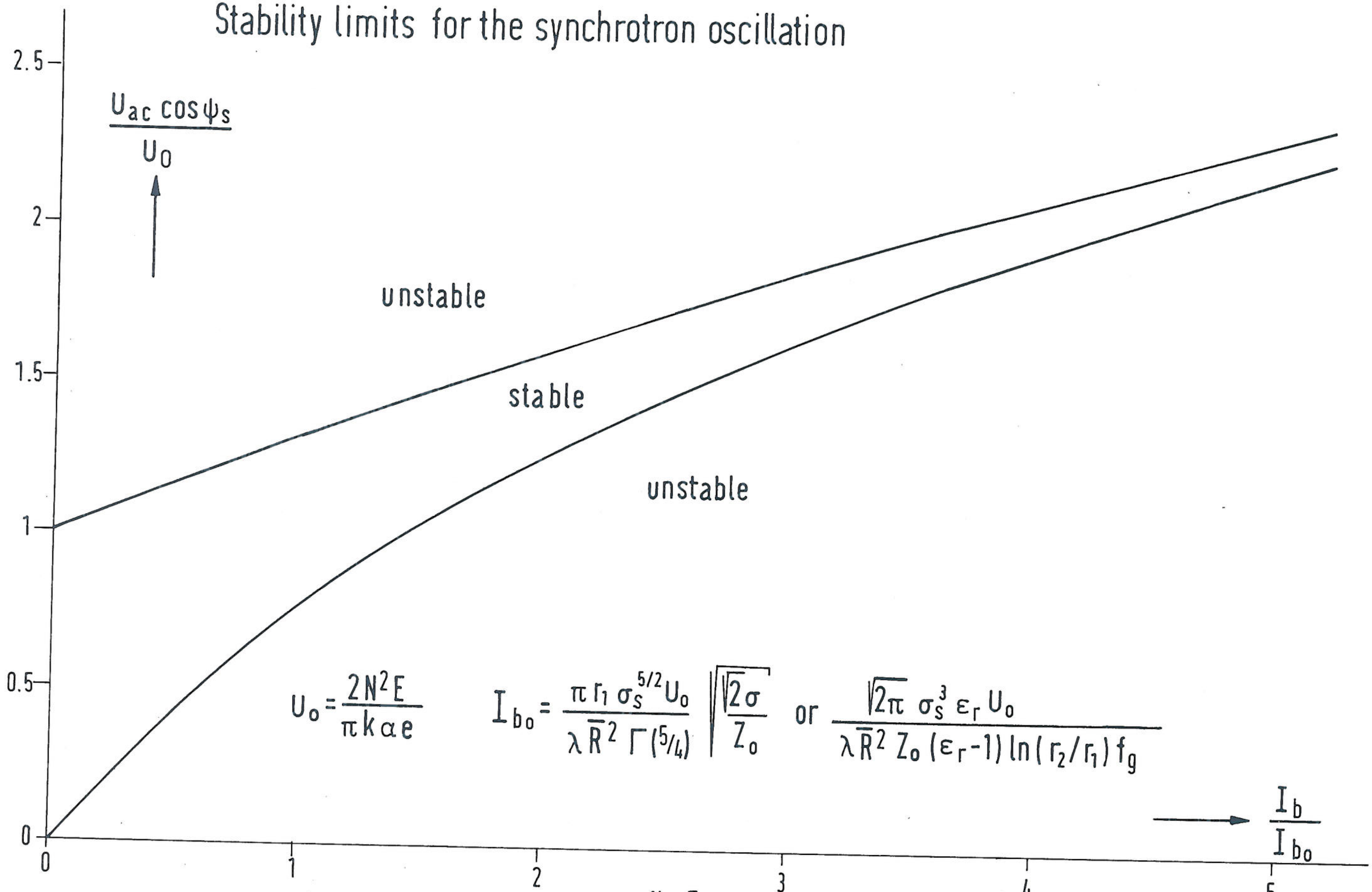


fig.5