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2 HAMBURG 52 · NOTKESTIEG 1

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Abstract:

The K_{13} vertex is considered in the technique of generalized Ward identies allowing arbitrary chiral symmetry breaking. The 1st Weinberg sum rule is derived, and constraints are found on the parameters of chiral symmetry breaking in a single particle approximation to 2-point spectral functions. In the limit $m_\pi^2 \to 0$ these constraints make all effects of general chiral symmetry breaking indistinguishable from those of a $(3,\bar{3}) + (\bar{3},3)$ scheme, if unmotivated divergences are to be avoided. We conclude that in the present framework the K_{13} vertex does not allow us to distinguish between different models of chiral symmetry breaking, except for terms of order $O(m_\pi^2)$.

I. Introduction

The phenomenological hadron dynamics seems to be approximately described by a chiral SU(3) × SU(3) symmetry. But in the real world this higher symmetry is badly broken, and one is faced with the problem of dealing with a broken symmetry which retains the validity of Gell-Mann's SU(3) × SU(3) current algebra.[1] In the past few years this has given rise to many speculations about a possible behaviour of the strong interaction Hamiltonian under chiral transformations. On the assumption that the symmetry breaking term in the Lagrangian transforms as the $(3,\bar{3}) + (\bar{3},3)$ representation of the chiral group, Glashow and Weinberg[2] have shown the natural appearance of the pseudoscalar nonent and the hypothetical kappa as would be Goldstone bosons. Gell-Mann, Oakes and Renner[3] have used the $(3,\bar{3}) + (\bar{3},3)$ scheme to parametrize the relative strength of the SU(3)-breaking and the chiral SU(3) × SU(3)-breaking and obtained a value of the relevant parameter of their model quite close to that at which the chiral SU(2) × SU(2) subsymmetry becomes exact. Several authors have also considered other possible mechanisms for chiral symmetry breaking such as the (8,8) scheme[4] and the $(6,\bar{6}) + (\bar{6},6)$ scheme[5]. Of all the models proposed so far the simple $(3,\bar{3})+(\bar{3},3)$ ansatz has proved to be most successful in spite of some of its conflicting aspects.[6]

In our present investigation we will approach the problem of CSB (chiral symmetry breaking) by asking a different type of question: Can we always consistently apply various models of CSB to situations where we assume the validity of the usual current algebra and other dynamical approximations? In other words, we will be interested to know the implication of the current algebra and dynamical assumptions alone on the possible nature of chiral symmetry breaking relevant to the problem at hand. The following considerations have chiefly motivated our approach:

(i) Simultaneous applications of specific models and apparently uncorrelated assumptions may not always be internally consistent. In some cases such a procedure may yield trivial results. For example, a consistent application of pion and kaon pole dominance to the axial current matrix element in K_{l4} transition leads to SU(3)-symmetric results.[7]

(ii) There is some evidence for overlapping of informations coming from apparently different assumptions. In a hard-meson analysis of the $A_1\pi\rho$-vertex, Azcarraga and Renner[8] have shown that a consistent application of meson pole-dominance requires the compact nature of the underlying current algebra, leaving undetermined a scale factor of the axial currents.

(iii) Barnes, Dittner and Dondi[9] have established a theorem showing a connection between the PCAC assumptions and the $(3,\bar{3}) + (\bar{3},3)$ scheme for CSB. According to this theorem, assuming the first seven divergences of the axial octet currents to be proportional to the corresponding members of a single octet of pseudoscalar fields implies the results of the $(3,\bar{3}) + (\bar{3},3)$ scheme.

Keeping the above observations and remarks in mind we will investigate the problem of general CSB in the K_{13} vertex, by using only the usual current algebra in the method of generalized Ward identities and some standard dynamical assumptions. This analysis will be carried out in two phases, differing in their levels of assumptions. In Section 2 we investigate the general character of CSB within the framework of spectral function representations of the 2-point functions in question. In Section 3 we use the the hard meson technique of Schnitzer and Weinberg[10] to see whether the constraints on CSB-parameters have some implications on the physical particle parameters.

We carry out our investigation in the following steps:

(A) We construct all possible Ward-Identities from the vertex containing two axial currents and one vector current with the K_{13} quantum numbers. In constructing the Ward identities we use a local form of the SU(3) × SU(3) current algebra, neglecting the problem of possible Schwinger terms. For the unknown equal-time commutators of current-divergences with the time components of currents we do not assume anything other than the local nature of these commutators.

(B) Next we represent all the 2-point functions appearing in the Ward identities in their non-subtracted spectral integrals. Now we compensate for neglecting the Schwinger terms in the equal-time commutators of the currents by also neglecting the non-covariant part of the spectral representation of two currents. To denote this compensating mechanism we replace the usual T-product by T^*-product. We assume, of course, that this definition of T^* does not bring in inconsistencies in the 2-point and 3-point functions of our problem.

(C) Within the framework of our assumptions we now demand consistency of all the Ward identities. This leads to some conditions to be satisfied by certain spectral function sum rules (SFSR):

I) First Weinberg sum rule for SU(3) × SU(3) spectral functions
II) A set of two independent equations containing spectral integrals related to the current-divergences and the three unknown local operators which have been introduced for the equal-time commutators between current-divergences and time-components of currents.

(D) We repeat the above steps for the hard-meson analysis assuming pole approximations, where the 2-point functions are now approximated by single-particle contributions. The consistenvy requirement yields again two types of results. The results of type (II) now give two independent relations on the physical parameters of the particles π, K and κ , involving three unknown constants.

(E) Lastly we investigate the consequences of the limit $m_\pi^2 \to 0$ in the Ward identities for the case of single particle dominance. In this special limit all the three unknown constants get fixed in terms of the decay and mass-parameters of the particles π, K and κ, in a manner which makes the case of general CSB indistinguishable from the application of a $(3,\bar{3}) + (\bar{3},3)$ model. We conclude therefore that the consequences of different models of CSB, in particular the amount of deviation from the $(3,\bar{3}) + (\bar{3},3)$ scheme, will appear only in the order $O(m_\pi^2)$ for the K_{13} vertex, within the present framework of approximations. This makes the K_{13} vertex unsuitable to put further tight constraints on the nature of CSB, once current algebra and pole dominance principles have been adopted.[11)]

II. Derivation of SFSR

In this section we will derive some spectral function sum rules from an analysis of the generalized Ward identities of our problem.
To start with let us define the three-current-vertex $W_{\mu\nu\lambda}$ with K_{13} quantum numbers by the following integral:

$$W_{\mu\nu\lambda}(p,q) \equiv \iint d^4x d^4y e^{ipx} e^{iqy} \langle 0|T^*\{A_\mu^-(x)A_\nu^3(y)V_\lambda^+(0)\}|0\rangle, \qquad (2.1)$$

with $A_\mu^-(x)$ and $A_\nu^3(y)$ carrying the outgoing 4-momenta p_μ and q_ν respectively and $V_\lambda^+(0)$ carrying an incoming 4-momentum $k_\lambda = (p+q)_\lambda$. We have abbreviated the octet indices ($4 \pm i5$) by the notations ($\pm$) on the currents. Our choice of octet indices corresponds to the decay mode $K^- \to \pi^0 l^- \bar{\nu}_l$, which is connected to other possible K_{13} decays through $|\Delta I| = \frac{1}{2}$ rule.

Next by employing the usual trick of partial integration and taking the 4-derivative inside the time-ordered product, we construct all possible Ward identities from different contractions of $W_{\mu\nu\lambda}$ with the momenta p^μ, q^ν and k^λ. The 12 such identities involve 7 further vertices containing one or more current divergences, and a number of 2-point functions. We define these vertices in a way analogous to (2.1) and denote them by $W_{\cdot\nu\lambda}$, $W_{\mu\cdot\lambda}$, $W_{\mu\nu\cdot}$, $W_{\mu\cdot\cdot}$, $W_{\cdot\nu\cdot}$, $W_{\cdot\cdot\lambda}$, $W_{\cdot\cdot\cdot}$, where a dot in place of a Lorentz index stands for the corresponding current-divergence in the time-ordered product; for example,

$$W_{\cdot\nu\lambda} \equiv \iint d^4x\, d^4y\, e^{ipx}\, e^{iqy} \langle 0|T^*\{\partial A^-(x)A^3_\nu(y)V^+_\lambda(0)\}|0\rangle \tag{2.2}$$

The 2-point functions in the 12 Ward identities of our problem will encounter the following equal-time commutators (ETC):

$$\begin{aligned}&\text{(a)}\quad [A^-_o,A^3_\lambda],\ [A^-_o,V^+_\lambda],\ [A^3_o,A^-_\lambda],[A^3_o,V^+_\lambda],\ [V^+_o,A^-_\lambda],\ [V^+_o,\ A^3_\lambda]\\ &\text{(b)}\quad [A^-_o,\partial A^3],\ [A^-_o,\partial V^+],\ [A^3_o,\partial A^-],\ [A^3_o,\partial V^+],\ [V^+_o,\partial A^-],\ [V^+_o,\partial A^3]\end{aligned} \tag{2.3}$$

The ETC's of the type (a) are evaluated from the usual local form of the Gell-Mann current algebra, while the ETC's of the type (b) are all unknown in our general treatment. For 3 of these unknown ETC's we introduce the following local operators:

$$\left.\begin{aligned}[][A^-_o(x)\partial A^3(y)\ ]\delta(x_o-y_o) &\equiv \tfrac{1}{2}\mathscr{S}^-(x)\ \delta^4(x-y)\\ [A^3_o(x),\partial V^+(y)]\delta(x_o-y_o) &\equiv \tfrac{1}{2}\ P^+(x)\ \delta^4(x-y)\\ [V^+_o(x),\partial A^-(y)]\delta(x_o-y_o) &\equiv\ P^3(x)\ \delta^4(x-y).\end{aligned}\right\} \tag{2.4}$$

The remaining 3 ETC's in 2.3(b) can be expressed in terms of these new operators and current-divergences by utilizing the relation

$$[Q^i,\partial J^j] + [\partial J^i,Q^j] = if^{ijk}\ \partial J^k \tag{2.5}$$

for the current-divergences ∂A^-, ∂A^3, ∂V^+ as local operators. We want to stress here that we do not assume single octet behaviour for the operators P^+ and P^3 in spite of the apparent octet symbols which we have used to denote the appropri-

ate isospin and C-conjugation properties only.
For illustration we write down 3 typical Ward identities here:

$$p^\mu W_{\mu\nu\lambda} = iW_{\cdot\nu\lambda} + \frac{i}{2}\int d^4y\, e^{iky} \langle 0|T^*\{V^-_\nu(y)\, V^+_\lambda(0)\}|0\rangle$$
$$-i\int d^4y\, e^{iqy} \langle 0|T^*\{A^3_\nu(y)A^3_\lambda(0)\}|0\rangle \quad (2.6a)$$

$$p^\mu W_{\mu\nu\cdot} = iW_{\cdot\nu\cdot} + \frac{i}{2}\int d^4y\, e^{iky} \langle 0|T^*\{V^-_\nu(y)\partial V^+(0)\}|0\rangle$$
$$+i\int d^4y\, e^{iqy} \langle 0|T^*\{A^3_\nu(y)P^3(0)-A^3_\nu(y)\partial A^3(0)\}|0\rangle \quad (2.6b)$$

$$p^\mu W_{\mu\cdot\cdot} = i\, W_{\cdot\cdot\cdot} + \frac{i}{2}\int d^4y\, e^{iky} \langle 0|T^*\{\mathcal{S}^-(y)\partial V^+(0)\}|0\rangle$$
$$+i\int d^4y\, e^{iqy} \langle 0|T^*\{\partial A^3(y)P^3(0)-\partial A^3(y)\partial A^3(0)\}|0\rangle \quad (2.6c)$$

[A complete list of these identities can be found in Ref. 12] .
Our next step consists in expressing all the 2-point functions occurring in the Ward identities in their non-subtracted spectral representations. For this purpose 9 different spectral functions have to be introduced - 6 for representing 3 current-propagators and 3 for the propagators containing the unknown operators P^+, P^3, $\mathcal{S}^-$. Because of the assumed property of the T*-product, we are concerned only with covariant terms in the spectral representations. We write for example

$$\int d^4x\, e^{ipx} \langle 0|T^*\{A^-_\mu(x)A^+_\nu(0)\}|0\rangle = i\int_0^\infty dm^2\left\{ \frac{\frac{p_\mu p_\nu}{m^2} - g_{\mu\nu}}{p^2 - m^2}\, \rho_K^{(1)}(m^2) + \frac{p_\mu p_\nu}{p^2-m^2}\, \rho_K^{(0)}(m^2)\right\}, \quad (2.7)$$

where the spectral functions $\rho_K^{(1)}$ and $\rho_K^{(0)}$, receiving contributions respectively from spin (1) and spin (0) intermediate states with the quantum numbers of charged kaon, are defined through[12)]

$$\sum_n \langle 0|A^-_\mu|n,k\rangle \langle n,k|A^+_\nu|0\rangle \equiv \left(\frac{k_\mu k_\nu}{k^2} - g_{\mu\nu}\right)\rho_K^{(1)}(k^2) + k_\mu k_\nu \rho_K^{(0)}(k^2). \quad (2.8)$$

The spectral functions $\rho_\pi^{(1)}$, $\rho_\pi^{(0)}$, $\rho_\kappa^{(1)}$ and $\rho_\kappa^{(0)}$ are defined similarly, the notations being obvious. The coupling of the operators P^+, P^3, $\mathcal{S}^-$ to the current-divergences is represented by three functions ω_K, ω_π, ω_κ, defined through

$$\left.\begin{aligned} &\sum_n \langle 0|\partial A^-|n,k\rangle \langle n,k|P^+|0\rangle \equiv \omega_K(k^2)\\ &\sum_n \langle 0|\partial A^3|n,k\rangle \langle n,k|P^3|0\rangle \equiv \omega_\pi(k^2)\\ &\sum_n \langle 0|\mathcal{S}^-|n,k\rangle \langle n,k|\partial V^+|0\rangle \equiv \omega_\kappa(k^2)\end{aligned}\right\} \quad (2.9)$$

On having expressed all the 2-point functions of our problem in spectral integrals, we demand consitency of all the Ward identities within our framwork. This means that all possible ways of taking Ward identities by multiplying $W_{\mu\nu\lambda}$ with p^μ, q^ν and k^λ should be equivalent and lead to the same Lorentz-scalar function of the variables p^μ, q^ν and k^λ. In other words, we must have in particular

$$p^\mu q^\nu k^\lambda W_{\mu\nu\lambda} = k^\lambda p^\mu q^\nu W_{\mu\nu\lambda} = q^\nu k^\lambda p^\mu W_{\mu\nu\lambda} \qquad (2.10)$$

As conditions for (2.10) to hold, we find two sets of relations among some spectral integrals, when we explicitly utilize the Ward identities and the spectral representations as discussed above. The first set is the well-known spectral function sum rule [SFSR] of Weinberg:[13)]

$$\begin{aligned}\int_0^\infty dm^2 \frac{\rho_K^{(1)}}{m^2} + \int_0^\infty dm^2\, \rho_K^{(0)} &= \int_0^\infty dm^2 \frac{\rho_\kappa^{(1)}}{m^2} + \int_0^\infty dm^2 \rho_\kappa^{(0)} \\ &= 2\int_0^\infty dm^2 \frac{\rho_\pi^{(1)}}{m^2} + 2\int_0^\infty dm^2\, \rho_\pi^{(0)}\end{aligned} \qquad (2.11)$$

[The numerical factor appearing in the last line of (2.11) is due to the normalization used here.]
The second set of relations involves the functions ω_K, ω_π, ω_κ, and $\rho_K^{(0)}$, $\rho_\pi^{(0)}$ and $\rho_\kappa^{(0)}$;

$$\begin{aligned}\int_0^\infty dm^2 \frac{\omega_K}{m^2} + 2\int_0^\infty dm^2 \frac{\omega_\pi}{m^2} &= 2\int_0^\infty dm^2\, m^2\, \rho_\pi^{(0)} - \int_0^\infty dm^2\, m^2\, \rho_\kappa^{(0)} \\ \int_0^\infty dm^2 \frac{\omega_K}{m^2} + \int_0^\infty dm^2 \frac{\omega_\kappa}{m^2} &= 2\int_0^\infty dm^2\, m^2\, \rho_\pi^{(0)} - \int_0^\infty dm^2\, m^2\, \rho_K^{(0)}\end{aligned} \qquad (2.12)$$

The SFSR's (2.12) put some conditions to be satisfied by certain integrals containing the spectral functions corresponding to the operators P^+, P^3, $\mathcal{S}^-$, ∂A^-, ∂A^3 and ∂V^+; but the constraints on the CSB are not uniquely determined because of the lack of one more independent equation in the set of relations (2.12). This is all what we can learn about the possible nature of CSB in our general approach.

III. Hard-meson Analysis with Pole Approximations

In this section we apply the standard hard-meson technique for our investigation of the CSB in K_{13} vertex. Following Schnitzer and Weinberg,[10] the 3-point functions $W_{\mu\nu\lambda}$, $W_{\cdot\nu\lambda}$, etc. are first reduced to their respective meson pole terms and the "proper vertices" which are some "smooth" polynomials expressing the residual momentum dependence. Then by saturating the spectral functions with single meson contributions, we obtain from the Ward identities (2.6) a set of Ward-like identities for these proper vertices.

We define our 8 proper vertices $\Gamma^{\mu\nu\lambda}$ etc. through $W_{\mu\nu\lambda}(p,q) =$

$$= \frac{f_{K_A}(p_\mu p_{\mu'} - g_{\mu\mu'} m_{K_A}^2)}{m_{K_A}^2 (p^2 - m_{K_A}^2)} \frac{f_{A_1}(q_\nu q_{\nu'} - g_{\nu\nu'} m_{A_1}^2)}{m_{A_1}^2 (q^2 - m_{A_1}^2)} \frac{f_{K^*}(k_\lambda k_{\lambda'} - g_{\lambda\lambda'} m_{K^*}^2)}{m_{K^*}^2 (k^2 - m_{K^*}^2)} \Gamma^{\mu'\nu'\lambda'}$$

$$+ \frac{f_{K_A}(p_\mu p_{\mu'} - g_{\mu\mu'} m_{K_A}^2)}{m_{K_A}^2 (p^2 - m_{K_A}^2)} \frac{F_\kappa k_\lambda}{(k^2 - m_\kappa^2)} \left\{ \frac{f_{A_1}(q_\nu q_{\nu'} - g_{\nu\nu'} m_{A_1}^2)}{m_{A_1}^2 (q^2 - m_{A_1}^2)} \Gamma^{\mu'\nu'\cdot} + \frac{iF_\pi q_\nu}{(q^2 - m_\pi^2)} \Gamma^{\mu'\cdot\cdot} \right\}$$

$$+ \frac{f_{A_1}(q_\nu q_{\nu'} - g_{\nu\nu'} m_{A_1}^2)}{m_{A_1}^2 (q^2 - m_{A_1}^2)} \frac{iF_K p_\mu}{(p^2 - m_K^2)} \left\{ \frac{f_{K^*}(k_\lambda k_{\lambda'} - g_{\lambda\lambda'} m_{K^*}^2)}{m_{K^*}^2 (k^2 - m_{K^*}^2)} \Gamma^{\cdot\nu'\lambda'} + \frac{F_\kappa k_\lambda}{(k^2 - m_\kappa^2)} \Gamma^{\cdot\nu'\cdot} \right\}$$

$$+ \frac{f_{K^*}(k_\lambda k_{\lambda'} - g_{\lambda\lambda'} m_{K^*}^2)}{m_{K^*}^2 (k^2 - m_{K^*}^2)} \frac{iF_\pi q_\nu}{(q^2 - m_\pi^2)} \left\{ \frac{f_{K_A}(p_\mu p_{\mu'} - g_{\mu\mu'} m_{K_A}^2)}{m_{K_A}^2 (p^2 - m_{K_A}^2)} \Gamma^{\mu'\cdot\lambda'} + \frac{iF_K p_\mu}{(p^2 - m_K^2)} \Gamma^{\cdot\cdot\lambda'} \right\}$$

$$+ \frac{iF_K p_\mu \; iF_\pi q_\nu \; F_\kappa k_\lambda}{(p^2 - m_K^2)(q^2 - m_\pi^2)(k^2 - m_\kappa^2)} \Gamma^{\cdot\cdot\cdot}, \tag{3.1}$$

where we have introduced the decay constants of the mesons K_A, A_1, K^*, K, π and κ in the following amplitudes of the currents:

$$<0|A_\mu^-|K_A^+> = f_{K_A}\, \varepsilon_\mu^{(K_A)}, \qquad <0|A_\mu^-|K^+> = iF_K\, p_\mu^{(K)},$$

$$<0|A_\mu^3|A_1> = f_{A_1}\, \varepsilon_\mu^{(A_1)}, \qquad <0|A_\mu^3|A_1> = iF_\pi\, p_\mu^{(\pi)}, \tag{3.2}$$

$$<K^*|^+\, V_\mu^+|0> = f_{K^*}\varepsilon_\mu^{(K^*)}, \qquad <\kappa^+|V_\mu^+|0> = F_\kappa\, p_\mu^{(\kappa)};$$

ε_μ denotes the polarization vector of the spin(1) particle concerned. One can write down similar expressions as (3.1) for all other 3-point functions ($W._{\nu\lambda}$ etc.), showing their pole structures.[12)]

The 2-point functions are now approximated by keeping the single-particle contributions. Certain 2-point functions will involve the unknown operators P^+, P^3 and $\mathcal{S}^+$, whose single-particle contributions can come only through the amplitudes of the form

$$<0|P^+|K^-> \equiv Z_K, \quad <0|P^3|\pi> \equiv Z_\pi, \quad <\kappa|\mathcal{S}^-|0> \equiv i\, Z_\kappa, \tag{3.3}$$

where the real constants Z_K, Z_π, Z_κ represent three unknown parameters of general CSB.

As a result of the above approximations the Ward identities (2.6) reduce to a set of identities among the 8 proper vertices. Here we write down 4 such identities which we will need later in our discussions. [For a complete list, see Ref. (12)].

$$\frac{f_{K_A}}{m_{K_A}^2}\, p_\mu\, \Gamma^{\mu\nu\cdot} = -\, iF_K\, \Gamma^{\cdot\nu\cdot} + \frac{Z_\kappa}{2m_\pi^2}\, q + \frac{F_\kappa}{f_{A_1}}\{q^\mu q^\nu - g^{\mu\nu}(q^2 - m_{A_1}^2)\}k_\mu \tag{3.4a}$$

$$\frac{f_K}{m_K^2}\, k_\lambda\, \Gamma^{\cdot\nu\lambda} = -F_\kappa\, \Gamma^{\cdot\nu\cdot} - \frac{i(F_K Z_K - F_K^2 m_K^2)}{2m_\pi^2}\, \frac{m_{A_1}^2}{f_{A_1}}\, q^\nu - \frac{iF_K}{2f_{A_1}}\, \{q^\nu q^\lambda - g^{\nu\lambda}(q^2 - m_{A_1}^2)\}\, p_\lambda \tag{3.4b}$$

$$\frac{f_{K_A}}{m^2_{K_A}} p_\mu \Gamma^{\mu\cdot\cdot} \doteq -iF_K \Gamma^{\cdot\cdot\cdot} + \frac{iZ_K(q^2-m^2_\pi)}{2F_\pi m^2_\pi} + \frac{i(F_\pi Z_\pi - F^2_\pi m^2_\pi)(k^2-m^2_K)}{F_K m^2_K} \tag{3.4c}$$

$$\frac{f_{K*}}{m^2_{K*}} k_\lambda \Gamma^{\cdot\cdot\lambda} = - F_K \Gamma^{\cdot\cdot\cdot} + \frac{(F_K Z_K - F^2_K m^2_K)(q^2-m^2_\pi)}{2F_\pi m^2_\pi} - \frac{Z_\pi(p^2-m^2_K)}{F_K m^2_K} \tag{3.4d}$$

As in the Section 2 we now demand consistency among the Ward-like identities for the proper vertices. The equality

$$p_\mu q_\nu k_\lambda \Gamma^{\mu\nu\lambda} = k_\lambda p_\mu q_\nu \Gamma^{\mu\nu\lambda} = q_\nu k_\lambda p_\mu \Gamma^{\mu\nu\lambda}$$

now yields the following conditions:

A) 1st Weinberg sum rules:

$$F^2_K + \frac{f^2_{K_A}}{m^2_{K_A}} = F^2_K + \frac{f^2_K}{m^2_K} = 2 F^2_\pi + 2 \frac{f^2_{A_1}}{m^2_{A_1}} \tag{3.5}$$

B) Sum rules on CSB:

$$\left.\begin{aligned} F_K Z_K - 2 F_\pi Z_\pi &= F^2_K m^2_K - 2 F^2_\pi m^2_\pi \\ F_K Z_K + 2 F_\pi Z_\pi &= F^2_K m^2_K - F^2_K m^2_K \end{aligned}\right\} \tag{3.6}$$

One can easily see that the constraints (3.6) are not strong enough to uniquely determine the unknown Z-parameters in terms of the particle parameters, so that these relations must be satisfied in all CSB models which preserve the usual current algebra. In other words, for consitency of the current algebra and the approximations used, we require of a possible model for CSB the compatibility of the relations (3.6). It can be easily shown that the $(3,\bar{3}) \oplus (\bar{3},3)$ model is compatible with them.

Now we consider one interesting case. If we take the limit $m_\pi^2 \to 0$ in (3.6) and in (3.4) (no other identities of this type contain the m_π^2-term), one must demand

$$
\begin{aligned}
Z_\kappa &= 0 \\
Z_K &= F_K\, m_K^2 \qquad\qquad (3.7) \\
Z_\pi &= (F_K^2 m_K^2 - F_\kappa^2 m_\kappa^2)/2F_\pi
\end{aligned}
$$

to avoid unwanted divergences of harmless expressions like the Γ's appearing in (3.4). So in this limiting case the parameters Z_K, Z_π, Z_κ get fixed, independent of special models of CSB. Let us now consider this same limit in the $(3,\bar{3}) + (\bar{3},3)$ model. The limit $m_\pi^2 \to 0$ would correspond to the Hamiltonian $H_B = u_o - \sqrt{2}\, U_8$ and to the vanishing of the operator ∂A^π in this model, so that

$$
\mathcal{S}^- = 2[A_o^-, \partial A^3] = 0 \text{ and } P^+ - \partial A^+ = 2[V_o^+, \partial A^3] = 0 \qquad (3.8)
$$

$$
\text{ie.} \qquad Z_\kappa = 0 \text{ and } Z_K - F_K m_K^2 = 0 \text{ in the lim } m_\pi^2 \to 0
$$

In other models the vanishing of the parameters Z_κ and $(Z_K - F_K m_K^2)$ in the limit $m_\pi^2 \to 0$ is not obvious without further dynamical consistency conditions. (See, for example, the Ref. 4 for a discussion on (8,8)-model in this context)

IV. Conclusions

In this note we have made an attempt to investigate the problem of chiral symmetry breaking from general considerations. Starting from the generalized Ward identities utilizing only the usual current algebra, we have analysed the general CSB in the framework of some standard dynamical assumptions. We have found that the complete nature of the CSB cannot be constrained by our consideration of the K_{13} vertex. But we have also found that the limit $m_\pi^2 \to 0$, as applied in the single particle approximation of our problem (this approximation in the case of axial divergences is the natural consequence of regarding π and K as Goldstone bosons), leads to results which make the $(3,\bar{3}) + (\bar{3},3)$ model more plausible than the other models. For $m_\pi^2 \neq 0$ the predictions of different CSB models can be in general different for the CSB parameters, satisfying the constraints (3.6). For example, in the $(3,\bar{3}) + (\bar{3},3)$ model we have for $c \neq -\sqrt{2}$

$$Z_\pi = \frac{\sqrt{2}-c/2}{\sqrt{2}+c} F_\pi m_\pi^2; \; Z_K = \frac{-3c}{2\sqrt{2}-c} F_K m_K^2 \; ; \; Z_\kappa = \frac{-(\sqrt{2}+c)}{3c} F_\kappa m_\kappa^2. \tag{4.1}$$

The values of these parameters in other models can be different from (4.1) only in the order of $O(m_\pi^2)$, because for $m_\pi^2 = 0$ the Z-parameters are fixed by (3.7). But at the same time one would expect corrections of order $O(m_\pi^2)$ to the pion-pole approximation, which cannot be calculated in general. So we conclude that for all practical purposes there is no way to distinguish between the different CSB models by considering only the K_{13} vertex in the framework of current algebra, Ward identities and other general dynamical principles.

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References and Footnotes

1. M.Gell-Mann, Phys.Rev. 125, 1067 (1962).
2. S.L.Glashow, S.Weinberg, Phys.Rev.Lett. 4, 380 (1968).
3. M.Gell-Mann, R.J.Oakes, B.Renner, Phys.Rev. 175, 2195 (1968).
4. H.Genz, J.Katz, Phys.Rev. D8, 908 (1972).
5. P.Dittner, P.H.Dondi, S.Eliezer, Imperial College Preprint, London (1972).
6. T.Cheng, R.Dashen, Phys.Rev.Lett. 26, 594 (1971).
7. See, for example, the discussion on K_{14} decay in the article "Current Algebra and Weak Interactions" by B.Renner, Springer Tracts 52, 60 (1970).
8. J.A. DeAzcárraga, B.Renner, Nucl.Phys. B23, 236 (1970).
9. K.J.Barnes, P.Dittner, P.H.Dondi, Nucl.Phys. B31, 195 (1971).
10. H.J.Schnitzer, S.Weinberg, Phys.Rev. 164, 1828 (1967).
11. A similar remark has been made in the context of considering the $(6,\bar{6}) + (\bar{6},6)$ model by the authors of Ref. 5.
12. A.H.Kazi, Diplomarbeit (unpublished), II. Institut für Theoretische Physik, Universität Hamburg (1972).
13. S.Weinberg, Phys.Rev.Lett. 18, 507 (1967).
    T.Das, V.S.Mathur, S.Okubo, Ibid. 18, 761 (1967).