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2 HAMBURG 52 · NOTKESTIEG 1

Information on the Dynamics of the Reaction $KN \rightarrow K^{*}N$ from $K_L N$ Interactions

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Abstract: If $K^*_{S,L}$ is a K^* resonance decaying into $K_{S,L}$ (the short-, long- lived kaon) and a neutral system S^o of pions, one can isolate the C-even and C-odd, crossed channel contributions to $KN \rightarrow K^*N$ by using the reactions $K_L N \rightarrow K^*_{S,L} N$ whether S^o is a C-eigenstate, or a mixture of C-even and C-odd states. Applications to the production of K^*_{890} and the Q-meson are discussed, and simple numerical predictions made for $Q_{S,L}$ production. Q-production data indicate approximate t-channel helicity conservation for the ω and P' exchanges at vertices involving a spin change, in similarity to the belief for the Pomeron. $Q_{S,L}$ production data can give information also on Q-decays.

1. INTRODUCTION

Dynamical information on the reaction $^{+}$ $KN \to K^{*}N$ can be obtained [1,2] in $K_L N$ interactions by considering the modes (1) and (2) of this reaction

$$K_L N \to [\, K^{\pm} (\text{pions})^{\mp} \,]\; N \qquad (1)$$

$$\to [\, k^{*\pm} (\text{pions})^{\mp} \,]\; N \qquad (2)$$

where the subsystem k^{*} decays as

$$k^{*\pm} \to K^{\pm}(\text{pions})^{o},\; K_S(\text{pions})^{\pm},\; K_L(\text{pions})^{\pm}. \qquad (2a,b,c)$$

From this, the difference of the differential cross-sections σ

$$\sigma(K^{o}N \to K^{*o}N) - \sigma(\bar{K}^{o}N \to \bar{K}^{*o}N) \qquad (3a)$$

isolates the interference between the crossed-channel C-even and C-odd contributions. The corresponding sum

$$\sigma(K^{o}N \to K^{*o}N) + \sigma(\bar{K}^{o}N \to \bar{K}^{*o}N) \qquad (3b)$$

provides an incoherent sum of the two crossed-channel contributions. Similar (complementary) information is obtained from

$$\sigma(K^{+}N \to K^{*+}N) \pm \sigma(K^{-}N \to K^{*-}N). \qquad (4)$$

This procedure still does not separate the C-even from the C-odd contribution. Our proposal * is to study the modes

$$K_L N \to K_S^{*} N \;,\; K_S^{*} \to K_S S^{o} \qquad (5a)$$

$^{+}$ We discuss mainly the non-charge exchange modes.

* We are thankful to Professor P.K. Kabir for suggesting to us that $K^{*}_{S,L}$ production is important for the dynamics of $KN \to K^{*}N$, and for encouragement.

and $K_L N \to K_L^* N$, $K_L^* \to K_L S^o$; (5b)

the channels (5a) and (5b) isolate, respectively, the C-odd (even) and C-even (odd) crossed-channel contributions when the neutral system S^o of pions is a C-even (odd) eigenstate. The separation of these two crossed-channel contributions is possible, using the modes (5), also when S^o is not a C-eigenstate. The modes (5), therefore, complement the information obtained from combinations like (3) and (4). Data on the modes (5) would be useful for a future amplitude analysis of the reaction $KN \to K^* N$.

After demonstrating the above C-odd, C-even separation, we consider applications to K^*_{890} and Q production, and make simple predictions for $Q_{S,L}$ production using $K_L p \to K_S \pi^+ \pi^- p$ data [2]. Available data are for the mode (1) for K^*_{890} production [1], and the mode (2,2b) for Q-production [2]; Q_S production requires the $K_S \pi^+ \pi^-$ system to have the characteristics of the Q, without forming the resonating subsystems $(K_S \pi^{\pm})$; $Q_S \to K_S \rho^o$ is one suitable mode. For K^*_{890}, the decay modes involved are $\pi^o K_S$ and $\pi^o K_L$.

2. THE C-SEPARATION

Assume, temporarily, CP-invariance for K^o decays; then, the initial state (omitting the proton for simplicity) $|K_L\rangle = |K^o\rangle - |\bar{K}^o\rangle$ results in

$$f\,|K^{*o}\rangle - \bar{f}\,|\bar{K}^{*o}\rangle = \frac{f+\bar{f}}{2}|K^{*o}-\bar{K}^{*o}\rangle + \frac{f-\bar{f}}{2}|K^{*o}+\bar{K}^{*o}\rangle \qquad (6)$$

where f and $\bar{f}$ are, respectively, the $K^o p \to K^{*o} p$ and $\bar{K}^o p \to \bar{K}^{*o} p$ amplitudes, suppressing spin indices. For $K^* \to K\pi$ decay, the $K^+\pi^-$ decays (from K^{*o}) can be physically distinguished from $K^-\pi^+$ decays (from $\bar{K}^{*o}$) and determine $|f|$ and $|\bar{f}|$ respectively; however, interference effects arise in the $K^o\pi^o$ and $\bar{K}^o\pi^o$ modes. If $C|K^{*o}\rangle = |\bar{K}^{*o}\rangle$, the $\pi^o K_S$ and $\pi^o K_L$ modes are produced by the amplitudes $\frac{1}{2}(f-\bar{f})$ and $\frac{1}{2}(f+\bar{f})$ respectively. The argument of this paragraph is essentially from ref. [3].

While $|f|$ and $|\bar{f}|$ can be determined by the channels (1) and (2) or (assuming charge-symmetry) by $K^{\pm}n \rightarrow K^{*\pm}n$, the relative phases (see Eq. (16) for example) of the corresponding f and $\bar{f}$ amplitudes (implied by a knowledge of $f \pm \bar{f}$) cannot be so determined; hence the usefulness of the channels (5). The combinations $(f \pm \bar{f})$ have definite charge-conjugation. Polarisation and K^* decay density-matrix data help to separate the various helicity amplitudes.

As for ordinary regeneration $K_Lp \rightarrow K_Sp$ [4], the above C-separation does not [5] require CP-invariance for neutral kaon decays; this is because with only CPT-invariance, the C-eigenstates $K\bar{K}+\bar{K}K$ and $K\bar{K}-\bar{K}K$ correspond to $K_LK_L-K_SK_S$ and $K_LK_S-K_SK_L$ respectively, apart from overall factors. With only CPT-invariance, (6) gets replaced by (retaining K^* decays involving neutral kaons only)

$$f^+\sum_i \left[\alpha_i |K_L P_i^o\rangle + \beta_i |K_S M_i^o\rangle\right] + f^-\sum_i\left[\alpha_i |K_S P_i^o\rangle + \beta_i |K_L M_i^o\rangle\right] \qquad (7)$$

where $f^{\pm} = \frac{1}{2}(f \pm \eta \bar{f})$ are the C-even and C-odd amplitudes; $C|K^{*o}\rangle = \eta|\bar{K}^{*o}\rangle$; the amplitude f (and similarly, $\bar{f}$) now refers to K^{*o} production where $K^{*o} \rightarrow K^oS^o$, $S^o = \sum_i \left[\alpha_i P_i^o + \beta_i M_i^o\right]$, the system S^o is a mixture (with coefficients α_i and β_i) of the C-even (P_i^o) and C-odd (M_i^o) pionic components.

The C-separation follows from (7) which holds for both $\eta=+1$ and -1. When S^o is a C-eigenstate, the statement immediately following (5) results from (7). For K^*_{890}, S^o is just π^o. If P^o and M^o are physically distinguishable (e.g., $Q^o \rightarrow K^o\omega$, $K^o\pi^o\pi^o$), the production rates * for K_LP^o and K_SM^o components in (7) give $|f^+|^2$, while those for K_LM^o and K_SP^o give $|f^-|^2$, assuming α and β to be known (from $K^{*\pm}$ decays, for example). The third case is when P^o and M^o are not different; for $Q \rightarrow K^o\pi^+\pi^-$, the $\pi^+\pi^-$ may be isovector (M^o) or isoscalar (P^o). To illustrate the C-separation in such cases, we consider $K_Lp \rightarrow Qp$, $Q \rightarrow K(\pi\pi)$; taking α and β to be known.+

* One can easily get $|\alpha/\beta|^2$ also therefrom.

+ Possible determination of α and β is discussed later.

The amplitudes for the modes $K_S\pi^{\pm}\pi^{\mp}$ and $K_L\pi^{\pm}\pi^{\mp}$ are, from (7),

$$A(K_S\pi^{\pm}\pi^{\mp}) = \alpha f^- \pm \beta f^+ \quad , \tag{8a}$$

$$A(K_L\pi^{\pm}\pi^{\mp}) = \alpha f^+ \pm \beta f^- \tag{8b}$$

which gives, symbolically, the production cross-sections σ as

$$\sigma(Q_S) \equiv \sigma(K_S\pi^+\pi^-) + \sigma(K_S\pi^-\pi^+) = 2\left[|\alpha f^-|^2 + |\beta f^+|^2\right] , \tag{9a}$$

$$\sigma(Q_L) \equiv \sigma(K_L\pi^+\pi^-) + \sigma(K_L\pi^-\pi^+) = 2\left[|\alpha f^+|^2 + |\beta f^-|^2\right] , \tag{9b}$$

$$R = \frac{\sigma(Q_S) + \sigma(Q_L)}{\sigma(Q_S) - \sigma(Q_L)} = \frac{(|\alpha|^2 + |\beta|^2)(|f^-|^2 + |f^+|^2)}{(|\alpha|^2 - |\beta|^2)(|f^-|^2 - |f^+|^2)} \quad . \tag{9c}$$

Given $|\alpha|$ and $|\beta|$, R gives $|f^-/f^+|^2$; using $\sigma(Q_S)$ or $\sigma(Q_L)$ gives $|f^-|^2$ and $|f^+|^2$ separately, Q.E.D. One may determine $|f^+|^2 + |f^-|^2$ needed in R also from the separate data on Q^0 and $\bar{Q}^0$ channels, and combine it with $\sigma(Q_S)$ or $\sigma(Q_L)$ to get the needed C-separation.

3. $K^*(890)$ PRODUCTION

Since pion is the dominant unnatural parity Regge pole in $KN \rightarrow K^*_{890}N$, the present analyses (see, for example,[6]) cannot uniquely determine the other unnatural parity exchanges (isovectors of odd-C, and isoscalars) which are usually neglected. The C-separation from $K^*_{S,L}$ production would be obviously useful here. For natural parity exchanges also, the C-separation would be useful. The question of the fast energy dependence [7] of the $K^{\pm}p \rightarrow K^{*\pm}p$ cross-sections would be usefully illumined by knowing the energy-dependence of separately the C-odd (even) contributions from $K^*_S(K^*_L)$ production.

Unlike $K_Lp \rightarrow K_Sp$ [4], model-independent predictions for the phases of $K^*_{S,L}$ production are difficult because now, the optical theorem is not applicable and, in general, more helicity amplitudes contribute. We illustrate the qualitative expectation for forward K^*_{890} production. For K^*_S production, ρ-exchange ≫ ω-exchange [+], in contrast to $K_L p \to K_S p$ where the ω dominates [4]. The reason now is that the relevant s-channel amplitude $(0\,\frac{1}{2}\,,-1\,-\frac{1}{2})$ involves nucleon helicity flip, and is stronger [8] for the ρ; at the meson vertex, the ω/ρ coupling ratio is 1 for an ideal vector nonet. Similarly, A_2 exchange should dominate over P'. In fact, the P',ω and Pomeron contributions may be rather small because they seem [8] to conserve s-channel helicity at the nucleon vertex.

4. Q-PRODUCTION

The main decay mode of interest here is $Q^0 \to K^0(\pi\pi)$. It is believed [9] that there is no evidence for Q decay modes other than πK^*_{890} and Kρ. The $K^*\pi$ mode implies both the K^0P^0 and the K^0M^0 types; the $K^0\rho^0$ is purely K^0M^0 type. Even if there be no other decay modes (like $K^0\epsilon^0$), the parameters α and β of (7) are unknown because the strength of the $K^*\pi$ and Kρ modes is not established [9]. The amplitude β can be determined, through isospin, from the $K^{\pm}(\pi^{\mp}\pi^0)$ modes because both M^0 and $(\pi^{\pm}\pi^0)$ here are isovectors. The amplitudes α and β can be determined, by charge symmetry, from $Q^{\pm} \to K^{\pm}(\pi^+\pi^-)$ decays. In general, one needs detailed decay distributions in the various angles to deduce α and β. In fact, one may determine (α/β) from $Q_{S,L}$ production data; see point (D) below.

Our simple predictions for $Q_{S,L}$ production are an average over the laboratory momentum interval 4 → 12 GeV/c because they are based on similarly averaged $K_L p \to K_S \pi^+\pi^- p$ data [2].

(A) Density Matrix - Because t-channel states of nonzero Q-helicity are weakly produced [2], we retain only the dominant t-channel amplitudes $g_{00,\frac{1}{2}\pm\frac{1}{2}}$ $(K^0p \to Q^0p)$ and $\bar{g}_{00,\frac{1}{2}\pm\frac{1}{2}}$ $(\bar{K}^0p \to \bar{Q}^0p)$. Amplitudes of a definite charge-conjugation other than $g^{\pm}_{00,\frac{1}{2}\pm\frac{1}{2}}$ are then small. This is interesting

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+ An s-channel helicity conserving absorption would not alter this statement made for pure Regge-poles.

information, for C-even and C-odd exchanges separately, on the helicity structure at the meson vertex in this diffractive process. Since, in general, the amplitudes for $Q_{S,L}$ production are combinations like $(\alpha g^{\mp} + \beta g^{\pm})$, see (7), the results [2)]

$$\rho^t_{00} \simeq 1 \ , \quad \mathrm{Re}\,\rho^t_{10} \simeq 0 \ , \quad \rho^t_{1-1} \simeq 0 \tag{10}$$

for Q^0 and $\bar{Q}^0$ production should hold also for $Q_{S,L}$ production over the same (s,t) range.

Only natural parity effective t-channel contributions are henceforth needed because unnatural parity exchanges populate states of only [+] nonzero Q-helicity. In the forward direction [*], the only contributions $g^{\pm}_{00,\frac{1}{2}\frac{1}{2}}$ are dominated [8)] by the isoscalars (P'+Pomeron) and ω; the above evidence for t-channel helicity conservation is then mainly for isoscalars. Because of the absence [2)] of a turnover in the near-forward differential cross-sections for Q^0 and $\bar{Q}^0$ production, the amplitudes $g^{\pm}_{00,\frac{1}{2}-\frac{1}{2}}$ may [+*] be rather small. To that extent, since the isovectors (ρ and A_2) are not negligible [8)] as compared to the isoscalars in these (t-channel nucleon-flip) amplitudes, the above indication for approximate t-channel helicity conservation for isoscalar exchanges holds for the whole t-range over which the amplitudes for nonzero Q-helicity are found [2)] small.

The statement for ω and P' exchanges is interesting. At vertices involving no spin change, they are believed to conserve s-channel helicity,[8)] like the Pomeron. The indication now is that also at vertices involving a spin change, they behave like the Pomeron [10)] - i.e., conserve t-channel helicity. More accurate and complete data are, however, needed to establish this firmly; the argument is especially weak for the ω because of its comparatively small contribution, Eqs. (14,15b).

[+] In general, it holds for pure Regge-poles; but in the forward direction where we shall mainly use it, it holds also in the presence of s-channel helicity conserving absorption.

[*] There, t- and s-channel helicity conservation are equivalent.

[+*] This can be checked by baryon polarisation data.

(B) The C-Separation and Relative Phase of the Forward ($t'=o$) Amplitudes for ($K^o p \to Q^o p$) and ($\bar{K}^o p \to \bar{Q}^o p$) - The only independent amplitudes for forward production are $F \equiv \mathcal{G}^{t}_{oo,\frac{1}{2}\frac{1}{2}}$ and $\bar{F} \equiv \bar{\mathcal{G}}^{t}_{co,\frac{1}{2}\frac{1}{2}}$. Using particle symbols for isospin and charge-conjugation, and keeping all decay modes,

$$F(K^o p \to Q^o p) = f^o + \omega + \rho + A_2 \quad , \tag{11a}$$

$$\bar{F}(\bar{K}^o p \to \bar{Q}^o p) = \eta_Q (f^o - \omega - \rho + A_2) \quad , \tag{11b}$$

$$F^- = \frac{1}{2}(F - \eta_Q \bar{F}) = \omega + \rho \quad , \tag{11c}$$

$$F^+ = \frac{1}{2}(F + \eta_Q \bar{F}) = f^o + A_2 \quad , \tag{11d}$$

$C|Q^o\rangle = \eta_Q |\bar{Q}^o\rangle$, $\eta_Q = +1$ or -1. From ref. [2], normalising $|F|^2 = \sigma(Q)$, one gets

$$|\mathrm{Im}(\omega+\rho)| \simeq \frac{|\bar{F}|^2 - |F|^2}{4|F|} = \left.\frac{\sigma(\bar{Q})-\sigma(Q)}{4\sqrt{\sigma}(Q)}\right|_{t'=o} \tag{12a}$$

$$= 0.17\sqrt{(3.9 \pm 0.8 \text{ mb.GeV}^{-2})} \tag{12b}$$

where $\sigma(Q) \equiv \frac{d\sigma}{dt'}(K^o p \to Q^o p)$ and $\sigma(\bar{Q}) \equiv \frac{d\sigma}{dt'}(\bar{K}^o p \to \bar{Q}^o p)$. This gives

$$|\mathrm{Im}(\omega+\rho)|^2 = (0.11 \pm 0.02)\text{ mb./ GeV}^2 \ . \tag{12c}$$

With an intercept $\frac{1}{2}$ for the ρ and ω trajectories *,

$$\mathrm{Re}\,(\omega+\rho) = \mathrm{Im}\,(\omega+\rho) \ . \tag{13}$$

This gives

$$|F^-|^2 = |\omega + \rho|^2 = (0.23 \pm 0.05)\text{ mb. GeV}^{-2} \ , \tag{14}$$

$$|F^+|^2 = |f^o + A_2|^2 = \frac{1}{2}(|F|^2 + |\bar{F}|^2) - |F^-|^2 \tag{15a}$$

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* Since the ω dominates [8] the ρ for the relevant (nucleon nonflip) amplitude, the ρ/ω coupling ratio being 1 at the meson vertex for an ideal vector nonet, Eq. (13) uses mainly $\alpha_\omega(o) = 1/2$. This, and (13), are very analogous [4] to $K_L p \to K_S p$.

$$\cong (5.0 \pm 1.1)\, mb.\, GeV^{-2} \qquad (15b)$$

where $\sigma(Q)$ and $\sigma(\bar{Q})$ from [2] have been used.

So, the C-separation is available in Eqs. (14) and (15b). This, in fact, implies that the dominant amplitudes F and $\bar{F}$ are consistent with being relatively real; their relative phase ϕ is given by

$$\mathrm{Cos}\,\phi = \frac{1}{4}\eta_Q\,(|F + \eta_Q\bar{F}|^2 - |F - \eta_Q\bar{F}|^2)/|F\bar{F}| \qquad (16a)$$

$$\cong \eta_Q\,(0.94 \pm 0.29) \qquad (16b)$$

using $\sigma(Q)$ and $\sigma(\bar{Q})$ from [2], and Eqs. (14,15b).

(C) Forward $K_L p \to Q_{S,L} p$: Phases of the Amplitudes; the differential cross-section $d\sigma/dt'$; Energy-Dependence - For definiteness, we consider $Q^0 \to K^0(\pi^+\pi^-)$ with amplitudes α and β for the $\pi^+\pi^-$ system to be isoscalar [+] (P^0) and isovector (M^0) respectively. Denoting $F^{\pm}$ by $f^{\pm}$ and detecting (as assumed hereafter throughout) charges symmetrically, Eqs. (9a,b) give $\frac{d\sigma}{dt'}$ $(K_L p \to Q_{S,L} p)$. In general, the unknown parameters α and β may be comparable; it is the case if the supposedly [9] dominant mode πK^*_{890} is the only one. Since $|f^+|^2 \simeq 25\,|f^-|^2$ from Eqs. (14,15b), both Q_S and Q_L production should be determined mainly by $|f^+|^2$ for comparable values of α and β. Because of the known [2] energy-dependence of the integrated cross-sections for Q^0 and $\bar{Q}^0$ production, the Pomeron seems dominant over P' and A_2 exchanges in $|f^+|^2$; the phase of f^+ is predominantly imaginary. The energy dependence of f^- is $\sim S^{-\frac{1}{2}}$ and its phase given by Eq. (13). Thus, for $|\alpha| \approx |\beta|$, the energy-dependence of both Q_S and Q_L production would be rather flat; and the phase predominantly imaginary. For illustration, take $|\alpha| = |\beta|$; Eqs. (9a,b) give

$$\frac{d\sigma}{dt'}\,(K_L p \to Q_{S,L} p) = 2|\alpha|^2\,(|F^+|^2 + |F^-|^2) \qquad (17a)$$

$$\simeq (10.5 \pm 2.3)\,|\alpha|^2\, mb.\, GeV^{-2} \qquad (17b)$$

[+] This implies, of course, the $K^0(\pi^0\pi^0)$ mode also.

using Eqs. (14,15b); for $|\alpha|^2 = \frac{1}{4}$, (17b) is comparable to the measured $\sigma(Q) = (3.9 \pm 0.8)$ mb. GeV^{-2}.

On the other hand, if the $\pi^+\pi^-$ system in $Q^o \rightarrow K^o(\pi^+\pi^-)$ is overwhelmingly C-even ($|\alpha| \gg 5|\beta|$) or C-odd ($5|\alpha| \ll |\beta|$), one can get the $|F^-|^2$ contribution from Q_S or Q_L production respectively: With $\frac{d\sigma}{dt'} = A\ e^{Bt'}$, the value (14) gives an integrated cross-section of $|\alpha|^2$ or $|\beta|^2$ times $(46 \pm 10)\mu b$ for $B = 10\ GeV^{-2}$, using the B value [4] for the similar reaction $K_L p \rightarrow K_S p$. This value for the "charge-conjugation exchange" cross-section is similar to that ($\sim 30\mu b$) expected [2] for charge or hypercharge exchange Q-production reactions in the momentum range 5-10 GeV/c.

<u>(D) Relative Strength α/β of the C-even and C-odd $\pi\pi$ Components in $Q^o \rightarrow K^o(\pi\pi)$ and Possible Determination of the Branching Ratio $Q \rightarrow (\pi K^*_{890}/K\rho)$ -</u>

Together with $K^o p \rightarrow Q^o p$ and $\bar{K}^o p \rightarrow \bar{Q}^o p$ data, Eqs. (8) can be used to get information on α and β in various ways. We only mention the determination of $|\alpha/\beta|$ from Eq. (9c) in the easier experiment of detecting $\pi\pi$ charges symmetrically: Since for forward $Q_{S,L}$ production, the amplitudes $|f^{\pm}|^2$ of Eq. (9c) are given by Eqs. (14,15b), knowing R gives $|\alpha/\beta|$.

Because nonzero α arises from only the πK^* decay mode and because a given α from this mode implies a definite contribution to β from this mode, the above determination of α/β would help to determine (two possible values for) the branching ratio $(Q \rightarrow \pi K^*/Q \rightarrow K\rho)$, assuming πK^* and $K\rho$ to be the only [9] decay modes, for α/β real.

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