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Phenomenological Fits to the Nucleon Electromagnetic Form Factors Based on Vector-Meson-Dominance

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2 HAMBURG 52 · NOTKESTIEG 1

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Abstract

Experimental data on the electromagnetic form factors of the proton and the neutron are compiled and compared with several simple model predictions based on vector meson dominance. Higher mass vector mesons are necessary to provide a quantitative description of the available data. The vector meson nucleon coupling constants obtained and the cross sections of the $e^+e^- \rightarrow p\,\bar{p}$ and $e^+e^- \rightarrow n\,\bar{n}$ processes predicted from these models are presented.

I. INTRODUCTION

In the present paper experimental data on the electromagnetic form factors of the nucleons are compiled and are compared with phenomenological relations and simple model predictions based on vector meson dominance. The present investigation differs from older ones [1-4] in two respects:

a) The analysis takes into account the coupling of the photon to vector-meson-states with masses above the one of the ϕ-meson. Evidence for this coupling has been presented by recent e^+e^--colliding-beam- [5] and photo-production- [6] experiments.

b) The analysis is based on more recent and more accurate data. Especially the new neutron data are important, since the theoretical predictions are generally formulated in terms of the isovector $(G^P - G^N)$ and the isoscalar $(G^P + G^N)$ combinations of the proton G^P and neutron G^N form factors. The experimental uncertainties of these combinations are mainly determined by the uncertainties of the neutron form factors.

Most models of the electromagnetic nucleon form factors are based on dispersion relations, and it is an open question, whether these dispersion relations should be formulated for the charge G_E and magnetic G_M, the so called Sachs form factors, or the Dirac F_1 and Pauli F_2 form factors or any other combinations. In the present paper we formulate all model expressions for the Dirac F_1 and Pauli F_2 form factors of the nucleons. The only reason for our choice is the fact that these form factors give the correct threshold behaviour for the nucleon antinucleon production by e^+e^- annihilation, which otherwise has to be imposed by additional constraints.

Considerable efforts have been made experimentally to determine the nucleon electromagnetic form factors at space-like four-momentum transfers by electron scattering experiments. In the region of time-like momentum transfers the first measurement [9] of the $e^+e^- \to p\bar{p}$ reaction has been performed recently. The latter process will be studied more extensively in the near future with e^+e^--colliding beam machines. In order to give some guidance for the design of future $e^+e^- \to p\bar{p}$ and $e^+e^- \to n\bar{n}$ experiments, we quote the cross sections, predicted by the various models for these processes.

The notations are defined, and the experimental data used are listed and discussed in Sec. II. In Sec. III the data are compared with phenomenological predictions. A discussion of the results is contained in Sec. IV, and the conclusions are presented in Sec. V.

II. THE EXPERIMENTAL DATA

II.1 Notations

The experimental data are commonly analyzed in terms of the charge G_E and magnetic G_M form factors, which at $q^2=0$ are given by the charge and the magnetic moment of the nucleons. The relation between the Sachs and the Dirac F_1 and Pauli F_2 form factors is

$$G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2} F_2(q^2)$$
$$G_M(q^2) = F_1(q^2) + F_2(q^2) \qquad (1)$$

where q^2 is the four momentum transfer given to the nucleon system and M is the nucleon mass. In our notation is $q^2 > 0$ for space-like and $q^2 < 0$ for time-like momentum transfers. The proton (P) and neutron (N) form factors are combined to isoscalar (S) and isovector form factors (V)

$$2\, G^S_{E,M} = G^P_{E,M} + G^N_{E,M}$$
$$2\, G^V_{E,M} = G^P_{E,M} - G^N_{E,M} \qquad (2)$$

and similar for the F's. The isovector form factors are real functions of q^2 for $q^2 > -(2m_\pi)^2$ and the isoscalar ones for $q^2 > -(3m_\pi)^2$. The form factors are generally complex functions beyond these thresholds.

II.2 The proton form factors

Data on the electromagnetic form factors of the proton are available in the range $0 \leq q^2 \leq 25\ (\mathrm{GeV/c})^2$ and still rather sparsely in the range $-6.8\ (\mathrm{GeV/c})^2 \leq q^2 \leq -4M^2$. In the range $0 \leq q^2 \leq 3\ (\mathrm{GeV/c})^2$ the form factors G_E and G_M have been separated. For $q^2 > 3\ (\mathrm{GeV/c})^2$ only cross section measurements have been performed which permit no clear separation of G_E and G_M.

II.2 a) G_E^P and G_M^P between $0 \leq q^2 \leq 3$ $(GeV/c)^2$

The data used in the present analysis have been compiled in ref. 7. This compilation was based on the more recent and accurate experiments. No internal beam measurements have been included. An exception is the experiment of ref. 19 which was performed at high q^2 values and large scattering angles, where the data are rather scarce.

The values are listed in table 1. Some data[21)] at low values of q^2 have been omitted in order to give not too much weight to this special q^2 region, which is probably rather sensitive to the finer details, like finite width effects etc., of the various models, which are not properly taken into account in the present investigation.

II.2 b) e-p cross section for $q^2 \geq 3$ $(GeV/c)^2$

For $q^2 > 3.0$ $(GeV/c)^2$ only separate cross section measurements, which are listed in table 2, have been used for the present analysis.
The statistical and normalisation errors, which have been quoted separately in ref. 22), were combined quadratically.

II.2 c) e-p cross sections for $q^2 < -4M^2$

Only one cross section measurement is available for time-like four momentum transfers. Two other experiments quote upper limits only. The data are listed in table 3.

II.3 The neutron form factors

Mainly three methods have been used to gain information about the neutron form factors: neutron scattering at thermal energies off atomic electrons, elastic electron deuteron scattering, and quasielastic electron deuteron scattering. In the following we list the data used in the present analysis.

II.3 a) The slope of G_E^N at $q^2 = 0$

The quantity $(d\, G_E^N/d\, q^2)_{q^2=0}$ has been determined by scattering neutrons of thermal energies off atomic electrons. Three different techniques have been used. The results of the most recent and accurate experiments are listed in table 4. We adopted the method of ref. 28, for the determination of the weighted average.

II.3 b) Neutron form factors from elastic e-d-scattering

Assuming knowledge of the deuteron wave functions, elastic e-d-scattering cross sections measured at forward angles provide data on the isoscalar charge form factor $G_E^P + G_E^N$. For a recent discussion on the theoretical analysis see ref. 29. The data used in the following are listed in table 5. They are based on recent analyses using the deuteron wave functions of Lomon and Feshbach [30] and the relativistic corrections of ref. 31. The ratios G_E^N/G_E^P of ref. 34 have been reanalysed by the authors of ref. 35 and these values are quoted. The errors however have been increased because of the theoretical uncertainties, which have been estimated from the differences resulting from the various methods used in ref. 29 and ref. 35 and from the differences between the results of ref. 29 and ref. 35 in the analysis of the same data. The data with $q^2 > 0.5\ (\mathrm{GeV/c})^2$ have been rejected, because of the uncertainties of the theoretical analysis.

Neutron form factors have been calculated using values of G_E^P which have been interpolated from table 1. For the present comparison with "the theory" however only the "measured" quantities G_E^N/G_E^P or $G_E^P + G_E^N$ have been used.

II.3 c) Neutron form factors from quasielastic e-d-scattering

The process $e+d \rightarrow e'+n+p$ has been used to gain information on the form factors of the neutron.

In the following we use the compilation of ref. 7, with the more recent data of ref. 8 added. The data obtained by the ratio method and the single arm experiments, which have been separately quoted in ref. 7, have been combined in table 6.

The errors of the neutron form factors in table 6 are mainly determined by the uncertainties of the large angle measurements. We therefore take the rather precise small angle neutron-proton cross section ratios $R = \sigma_n/\sigma_p$, which are listed in table 7, as additional data in our analysis.

III. COMPARISON WITH MODEL PREDICTIONS

III.1 The fitting procedure

The comparison between the data and the theoretical predictions is performed via the following χ^2 function which is minimized with the help of a computer program[44]:

$$\chi^2 = \sum_i^N \left(\frac{D_i - F_i(a_1 \ldots a_m)}{\Delta D_i} \right)^2$$

where D_i are the data points given in section II and ΔD_i are their quoted errors. The total number of data points is N = 121. $F_i(a_1 \ldots a_m)$ is the theoretically predicted value of the data point i, where the free parameters $a_1 \ldots a_m$ are determined by minimizing the χ^2-function. We quote the normalized function $\chi_F^2 = \chi^2/(N-m)$ as a measure for the goodness of the fit. The quoted errors ΔD_i contain, besides counting statistics the estimates of various systematic errors in many different experiments and it therefore may be unreasonable to judge the χ_F^2 on statistical grounds only.
We estimated the probably lowest value of χ_F^2 obtainable, by fitting the data with "theoretical" functions F_i of sufficient "flexibility". The resulting value $(\chi_F^2)_{min} \simeq 1.25$ was obtained with different functions F_i and it was found to be quite independent of the parameter number m, once m exceeded a minimum value. We therefore expect the ideal fit to give for χ_F^2 a value of about 1.25.

The errors Δa_i quoted in the following are obtained from the fitting program.[44] Δa_i is an estimate of the symmetrized standard deviations of a_i. The values are obtained from the computer program[44] and usually underestimate the true uncertainties.

III.2 Empirical relations

III.2.a) The "scaling law"

The relations

$$G_E^P(q^2) = \frac{G_M^P(q^2)}{\mu_P} = \frac{G_M^N(q^2)}{\mu_N} \; ; \; G_E^N(q^2) \equiv 0 \qquad (3)$$

are commonly known as the "scaling law", where μ_P and μ_N are the magnetic moments of the proton and neutron respectively.
These relations, which are predicted from the non relativistic quark-model, in general violate the threshold conditions

$$G_E^{P,N}(q^2 = -4M^2) = G_M^{P,N}(q^2 = -4M^2) \; , \qquad (4)$$

which result from eq. (1). For a comparison with the data we used

$$G_E^P(q^2) = (1.0 + a_1 q^2 + a_2 q^4 + a_3 q^6)^{-1}$$

The resulting parameter- and χ_F^2 values are listed in Table 8. Additional parameters did not improve the fit significantly. A severe discrepancy which amounts to 60% of the χ_F^2 exists between (3) and the value of $(d\,G_E^N/dq^2)_{q^2=0}$.

III.2 b) The "iso scaling law"

Eq. (3) may be modified, as emphasized in ref. 45, by scaling the isoscalar and isovector form factors separately,

$$\frac{G_M^S}{\mu_S} = 2\,G_E^S \equiv G^0$$

$$\frac{G_M^V}{\mu_V} = 2\,G_E^V \equiv G^1 \qquad (5)$$

where $2 \cdot \mu_{S(V)} = \mu_P \underset{(-)}{+} \mu_N$.

The threshold condition (4) demands a) $1/\mu_S = 2$ if not $G_E^S = G_M^S = 0$, which has to be compared with the empirical value $1/\mu_S = 2.27$, and b) $G_E^V = G_M^V = 0$ at $q^2 = -4M^2$. For comparison with the data we used the functions

$$G^0 = (1 + a_1 q^2 + a_2 q^4 + a_3 q^6)^{-1}$$

$$G^1 = (1 + b_1 q^2 + b_2 q^4 + b_3 q^6)^{-1} .$$

The parameters obtained from the fit as well as the χ_F^2 values are listed in Table 8 too.

III.3 Vector-Meson-Dominance Models

III.3. a) Simple pole models

In the following we use simple Clementi-Villi-formulae for the Dirac - and Pauli form factors F_1 and F_2 respectively:

$$F^S_{1,2}(q^2) = \sum_I \frac{a^{S,I}_{1,2}}{1 + \frac{q^2}{m^2_{S,I}}}$$

$$F^V_{1,2}(q^2) = \sum_I \frac{a^{V,I}_{1,2}}{1 + \frac{q^2}{m^2_{V,I}}} \tag{6}$$

where the summation extends over the vector mesons of isospin 0 and 1 respectively. Four of the parameters $a^{S,I}_{1,2}$, $a^{V,I}_{1,2}$ are determined by the charge and the magnetic moments of the nucleons:

$$\begin{aligned}
a_1^{S,1} &= 0.5 & &- \sum_{I\geq 2} a_1^{S,I} \\
a_2^{S,1} &= -0.060 & &- \sum_{I\geq 2} a_2^{S,I} \\
a_1^{V,1} &= 0.5 & &- \sum_{I\geq 2} a_1^{V,I} \\
a_2^{V,1} &= 1.853 & &- \sum_{I\geq 2} a_2^{V,I}
\end{aligned} \tag{7}$$

Eq. (6) usually is derived from unsubtracted dispersion relations which are assumed to be saturated by vector mesons of zero width.

Comparing eq. (6) with the data we take into account 1) only contributions from the well established vector mesons ρ,ω,ϕ, with masses $m_\rho = 0.765$, $m_\omega = 0.784$, $m_\phi = 1.019$ GeV; 2) in addition to 1) a ρ' at $m_{\rho'} = 1.5$ GeV, as indicated by recent experiments [5,6]; 3) in addition to 2) a hypothetical ω' at $m_{\omega'} = 1.5$ GeV, and finally 4) in addition to 3) a hypothetical ω' at $m_{\omega'} = 1.25$ GeV and a ρ' at $m_{\rho'} = 1.25$ GeV. The resulting parameters and χ^2_F values are listed in table 9. We abandoned adding more hypothetical mesons and with it further parameters since the fitting procedure got increasingly complicated and unstable.

III.3 b) Veneziano-Type Model

Definite predictions about the masses of higher vector mesons are made by the Veneziano model. Several authors[46,47,48] have attempted to explain the proton form factors within the framework of this model.

The present comparison is based on an extended version of the Frampton formula[48]:

$$F^{S,V}_{1,2}(q^2) = F^{S,V}_{1,2}(0)\,\frac{\Gamma(c^{S,V}_{1,2}-\alpha)\;\Gamma(1-\alpha+\beta q^2)}{\Gamma(1-\alpha)\;\;\Gamma(c^{S,V}_{1,2}-\alpha+\beta q^2)} \tag{8}$$

where the $c^{S,V}_{1,2}$ are free parameters, which determine the asymptotic behaviour of the form factors

$$F(q^2) \sim (q^2)^{1-c}$$

and $\alpha-\beta\cdot q^2$, with $\alpha = 0.5$ and $\beta=1$, representing the ρ and ω trajectory functions, which we assume to be degenerate. The contributions related to the ϕ-meson are neglected. In the space-like region eq. (8) is a smooth function, being the collective tail of an infinite number of poles of the factor $\Gamma(1-\alpha+\beta q^2)$ at negative integer values of $-\alpha + \beta q^2$. The coefficient a^n of the n^{th} pole $a^n/(1+q^2/_{n-0.5})$ is given by

$$a^n = (-1)^n\,\frac{F(o)\;\Gamma(c-0.5)}{n!\;\Gamma(c-n-1)\;\Gamma(0.5)\;(n-0.5)} \tag{9}$$

The parameter values obtained from a fit to the data as well as the coefficients a of the lower mass poles are listed in table 10.

III.3 c) Models with vector meson nucleon form factors

Massam and Zichichi[2] modified eq. (6) by the introduction of another "form factor" describing the vector meson nucleon coupling. The authors proposed the following expression

$$F^S_{1,2} = \sum_I \frac{a^{S,I}_{1,2}}{(1 + \frac{q^2}{m^2_I})} \cdot \frac{1}{(1 + \frac{q^2}{\Lambda^2_I})}$$

$$F^V_{1,2} = \sum_I \frac{a^{V,I}_{1,2}}{(1 + \frac{q^2}{m^2_I})} \cdot \frac{1}{(1 + \frac{q^2}{\Lambda^2_I})} \qquad (10)$$

where in addition to the a^I which are handled as in III.3 a), the Λ_I's are treated as free parameters.

It has been claimed in the past[2,3], that eq. (10) yields a satisfactory fit to the form factor data, using ρ,ω and ϕ meson terms only. This is not the case with the present more precise set of data as can be seen from table 11, where the parameters and the χ^2_F value obtained are listed (model 1).

The fit improves significantly if we add a $\rho'(1500)$-term in eq. (10), even if we use again only 5 parameters by demanding $\Lambda_\rho = \Lambda_\omega = \Lambda_\phi = \Lambda$. The results of this fit are listed in table 11, they are indicated as model 2.

IV. DISCUSSION

A detailed description of the nucleon form factors, including a "normal" threshold behaviour at $q^2 = -4M^2$, is rather complicated, despite of the fact, that the Sachs form factors show quite "simple properties" at space-like momentum transfers namely a) the "scaling law" eq. (3), which, except for the slope of G^N_E at $q^2 = 0$, roughly describes the data at $q^2 > 0$ and b) the "iso scaling law" eq.(5), which is an almost satisfactory representation of the $q^2 > 0$ data.

It is well known that formula (6), which is based on vector meson dominated dispersion relations, fails badly to describe the data if only contributions from the well known ρ, ω, ϕ vector mesons are taken into account. The fit is considerably improved if the contributions of the $\rho'(1500)$[5,6] and a hypothetical $\omega'(1500)$ are added, and it is further improved ($\chi_F^2 = 3.9$) if contributions of two hypothetical vector mesons $\omega'(1250)$ and $\rho'(1250)$ are taken into account. A better fit ($\chi_F^2 = 3.1$) is obtained if one includes an infinite series of vector mesons with coupling constants determined by the 4 parameters of eq.(8) and (9), and a fit of similar quality ($\chi_F^2 = 2.8$) is obtained if only the ρ, ω, ϕ and ρ' contributions are included together with an additional vector meson nucleon form factor as formulated in eq.(10). The best fit curves of the three different approaches III.3 a) - 3 c) are compared with the data in Figs. 1 - 3.

The coefficients $a_{1,2}^V$ resulting from the fit can be regarded as the following product of coupling constants

$$2\, a_{1,2}^V = \frac{g_{1,2}^{VN\bar{N}}}{f_V} \tag{11}$$

where $g_{1,2}^{VNN}$ are the vector meson nucleon coupling constants, which cannot be determined experimentally for $m_V^2 < 4M^2$, and em_V^2/f_V is the photon vector meson coupling constant, which is experimentally known for the ρ,ω,ϕ-mesons and has been estimated for the ρ'-meson. The values of $f_V^2/4\pi$ are listed in table 12 together with estimates of $g^2_{VN\bar{N}}/4\pi$ obtained from theoretical analysis of pion nucleon and nucleon scattering data. Sakurais[55)] universality normalisation is used.

Comparing the $a_{1,2}^V$ listed in table 12 with our best fit results listed in tables 9-11 we note the following:

1) The coefficients a_2 obtained from the Veneziano model differ significantly from those of table 12.

2) The ρ-meson coefficients a_1 and a_2 of the simple pole models Sec.III.a) are roughly within the limits of table 12. The same is true for the $\rho'(1500)$. The coefficients a_V of the modified pole models eq.(10) should be multiplied for this comparison by the factor $(1-m_V^2/\Lambda_V^2)^{-1}$, since the

intermediate vector meson has to be on the mass shell. The ρ and ρ' coefficients of model 2 in table 11 are then also roughly within the limits of table 22.

3) The coefficients of the isoscalar mesons are rather model dependent. Only for model 2 in Sec.III.3c are the values roughly within the limits of table 12.

Making this comparison one should however keep in mind that both the coefficients a_V obtained from the fit and the vector meson nucleon coupling constants $g_{1,2}^{V,N\bar{N}}$ listed are rather model dependent and that the models used to fit the present form factor data are rather crude. Finite width effects as well as non resonant background contributions, which are certainly present, may change the parameters appreciably. However a more thorough analysis is difficult and has so far been done for the 2π intermediate state only[53].

The $e^+e^- \rightarrow p\,\bar{p}$ and $n\,\bar{n}$ cross sections predicted by the three models, the results of which for space-like four momentum transfers are shown in fig. 1 - 3, are plotted in fig. 4. The quantity shown is $d\sigma/d\Omega$ at 90^o in the CM-System. A special treatment was necessary for the Veneziano model eq. (8), since it predicts poles in the physical region at $q^2 = -n + 0.5$ $(n = 1,2,\ldots)$. In fig. 4 we plotted the quantity

$$\int_{-n+0.5+\varepsilon}^{-n+1.5-\varepsilon} dq^2 \frac{d\sigma}{d\Omega}(90^o) \Bigg/ (1.0 - 2\varepsilon)$$

at $q^2 = -n + 1$ and connected these points by straight lines. A value $\varepsilon = 2.0$ MeV was chosen to fit the Frascati results roughly. The relative q^2 dependence of the cross section however turned out to be quite independent of ε for $q^2 > -7.0\ (\mathrm{GeV}/c)^2$. It is evident from fig. 4, that these models, which give similar cross sections over a large range of space-like four momentum transfers, give predictions for $q^2 \leq -4M^2$ which differ by orders of magnitude.

V. CONCLUSIONS

The present comparison shows that the available form factor data can be quantitatively described by these simple models based on vector meson dominance if the contributions of higher mass vector mesons are taken into account. The resulting coupling constants and the assumed masses are not in gross disagreement with known experimental facts, with the possible exception of the Veneziano type model Sec. III.3 b) which yields ρ- and ω-meson tensor coupling constants a_2 in disagreement with those obtained from pion-nucleon and nucleon-nucleon scattering analysis. At the present time however these models cannot be regarded as a satisfactory description of the observed behaviour since little is known about the higher mass vector meson states.

The possible theoretical choices of the nucleon form factors will be much more constrained by accurate data at time-like four momentum transfers. The single data point measured at $q^2 = -4.41\ (\mathrm{GeV}/c)^2$ is of negligible weight in the present analysis, due to it's large error of about 25%. But if this value is confirmed by further experiments the simple models discussed in Sec. II.3 a) and c) have to be modified by the addition of higher mass vector meson terms or possibly by a modification of the simple pole terms along the lines of ref. 54.

Finally one should mention that the "iso scaling law" eq. (5) provides an almost perfect representation of the available data in the space-like region. This may indicate that the vector-meson dominated dispersion relations of the Dirac and Pauli form factors are intricate means to explain the nucleon form factors and that one should look for simpler ways.

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$q^2\,(\frac{GeV}{c})^2$	$\mu_p\left(\frac{G_E^P}{G_M^P}\right)^2$		$\frac{G_M^P}{\mu_p(1+\frac{q^2}{0.71})^2}$		Ref.
0.018	1.03 ±	0.07	0.97 ±	0.03	10, 11)
0.023	1.07	0.06	0.957	0.019	10, 12)
0.039	0.97	0.04	1.00	0.015	11, 11, 12)
0.062	0.97	0.11	1.02	0.038	12)
0.078	0.99	0.04	0.980	0.013	10, 13)
0.117	1.01	0.05	0.978	0.010	10, 13)
0.155	1.10	0.06	0.95	0.014	10, 14)
0.195	0.944	0.053	0.994	0.011	10, 13, 14)
0.273	1.02	0.067	0.958	0.015	14, 15, 16)
0.311	0.962	0.070	0.979	0.014	10, 13, 14)
0.390	1.04	0.05	0.972	0.010	10, 13, 14, 15, 16, 17)
0.584	0.968	0.048	1.00	0.010	13, 14, 15, 16)
0.78	0.851	0.064	1.02	0.011	13, 14, 15, 16)
1.00	0.98	0.10	1.02	0.014	13, 14, 18, 7)
1.17	0.94	0.11	1.02	0.014	13, 14, 15, 16, 17, 7)
1.56	0.92	0.21	1.038	0.023	13, 17, 18, 7)
1.75	0.570	0.17	1.07	0.017	13, 15, 16, 17, 7)
2.00	0.79	0.19	1.047	0.017	13, 17, 18, 7, 19)
2.33	0.51	0.29	1.06	0.021	7)
2.50	1.32	0.52	1.01	0.039	18)
3.00	0.424	0.29	1.075	0.017	15, 7, 19, 20)
3.75	2.08	0.86	0.983	0.045	18)

TABLE 1

The charge- and magnetic form factors of the proton. Listed are the ratio $\mu_p(G_E^P/G_M^P)^2$, which is independent of normalisation errors, and $G_M^P/(\mu_p \cdot G_D)$. Where μ_p is the magnetic moment of the proton and $G_D = (1 + q^2/0.71)^{-2}$ is the dipole fit.

q^2 $(GeV/c)^2$	E_1 (GeV)	Θ (degr.)	dσ/dΩ cm^2/ster.	Ref.
3.504	6.00	21.66	$3.64 \pm 0.16 \cdot 10^{-34}$	15)
3.759	9.998	12.45	$9.48 \quad 0.48 \cdot 10^{-34}$	22)
3.893	5.50	26.30	$1.35 \quad 0.076 \cdot 10^{-34}$	15)
4.087	5.886	25.0	$1.113 \quad 0.056 \cdot 10^{-34}$	17)
4.477	6.00	26.30	$7.25 \quad 0.41 \cdot 10^{-35}$	15)
5.061	6.00	29.25	$2.98 \quad 0.21 \cdot 10^{-35}$	15)
5.075	10.70	13.99	$1.86 \quad 0.10 \cdot 10^{-34}$	22)
5.839	6.00	33.70	$1.04 \quad 0.10 \cdot 10^{-35}$	15)
6.270	11.35	15.10	$5.46 \quad 0.30 \cdot 10^{-35}$	22)
7.498	12.00	16.07	$1.95 \quad 0.11 \cdot 10^{-35}$	22)
8.752	12.69	16.85	$7.13 \quad 0.42 \cdot 10^{-36}$	22)
9.556	6.13	75.78	$6.04 \quad 0.85 \cdot 10^{-38}$	19)
9.982	13.33	17.59	$3.62 \quad 0.20 \cdot 10^{-36}$	22)
12.50	14.66	18.80	$8.49 \quad 0.65 \cdot 10^{-37}$	22)
15.10	16.06	19.72	$2.73 \quad 0.28 \cdot 10^{-37}$	22)
20.00	17.31	24.04	$3.13 \quad 0.61 \cdot 10^{-38}$	22)
25.03	17.31	35.09	$4.80 \quad 2.10 \cdot 10^{-39}$	22)

TABLE 2

Elastic electron-proton cross sections for $q^2 > 3.5 \ (GeV/c)^2$

q^2 $(GeV/c)^2$	$d\sigma/d\Omega$ $(\Theta = 90^o)$ (cm^2/sr)	Ref.
4.41	$7.2 \pm 1.8 \cdot 10^{-35}$	9)
5.1	$< 4.0 \cdot 10^{-35}$	23)
6.8	$< 2.0 \cdot 10^{-35}$	23, 24)

TABLE 3

Differential cross sections of the $e^+e^- \rightarrow p\,\bar{p}$ reaction at $\Theta = 90^o$ in the CM-system. The data have been obtained from the $e^+e^- \rightarrow p\,\bar{p}$ reaction [9] and from the $p\,\bar{p} \rightarrow e^+e^-$ reaction [23,24] via the relation $d\sigma/d\Omega\ (e^+e^- \rightarrow p\,\bar{p}) = \beta^2\ d\sigma/d\Omega\ (p\,\bar{p} \rightarrow e^+e^-)$, where β is the velocity of the proton in units of c.

SLOPE $(GeV/c)^2$	Ref.	Method
0.579 ± 0.018	25)	transmission method
0.512 ± 0.049	26)	mirror reflection method
0.495 ± 0.010	27)	asymmetry measurement
0.514 ± 0.024	average	

TABLE 4

Values of the slope $(dG_E^N(q^2)/dq^2)_{q^2=0}$ determined by three different methods. The average value and its error have been calculated by the method of ref. 23. The scaling factor is S = 2.8.

q^2 $(GeV/c)^2$	$G_E^P + G_E^N$	G_E^N / G_E^P	G_E^N	$\frac{G_M^N}{\mu_N \cdot G_D}$	Ref.
0.00389		−0.0020 ± 0.0055	−0.0020 ± 0.0056		32,29)
0.00778		+0.0052 ± 0.0032	−0.0051 ± 0.0033		32,29)
0.0117		+0.0050 ± 0.0038	0.0012 ± 0.0036		32,33,29)
0.0156		+0.0105 ± 0.0038	0.0101 ± 0.0040		32,29)
0.0195		+0.0069 ± 0.0079	0.0067 ± 0.0080		32,29)
0.0233		+0.0048 ± 0.0065	0.0047 ± 0.0070		32,33,29)
0.0311		+0.0197 ± 0.0065	0.0184 ± 0.0070		32,29)
0.0389		+0.0166 ± 0.0070	0.0144 ± 0.0061		32,29)
0.0622		+0.0179 ± 0.0090	0.0150 ± 0.0075		32,29)
0.0856		+0.0126 ± 0.0110	0.0100 ± 0.0087		32,29)
0.117		+0.110 ± 0.055	0.08 ± 0.035	1.00±0.11	34,35)
0.156		+0.074 ± 0.040	0.05 ± 0.030	0.94±0.08	34,35)
0.195		+0.081 ± 0.040	0.05 ± 0.030	0.99±0.10	34,35)
0.239	0.594±0.016		0.055 ± 0.03		35)
0.292	0.556±0.020		0.070 ± 0.03		35)
0.379	0.478±0.018		0.05 ± 0.03		35)
0.429	0.430±0.014		0.05 ± 0.03		35)
0.505	0.408±0.018		0.07 ± 0.03		35)

TABLE 5

Data on the neutron form factors obtained from elastic e-d scattering using the deuteron wave functions of Lomon and Feshbach. The low q^2 experiments determined essentially the ratio G_E^N/G_E^P and the isovector combination $G_E^P + G_E^N$ has been determined in the experiment of ref. 35). Neutron form factors have been calculated, for orientation purposes only, using proton form factors G_E^P interpolated from table 1.

q^2 $(GeV/c)^2$	$(G_E^N)^2$	$G_M^N/\mu_N G_D$	Ref.
0.039		1.39 ± 0.30	36,37)
0.058		1.05 ± 0.056	36,37)
0.097		1.10 ± 0.032	36,37)
0.113		1.12 ± 0.08	38,37)
0.160		1.04 ± 0.07	38,37)
0.178		1.03 ± 0.035	36,37)
0.214	0.002 ± 0.0084		39)
0.292		0.94 ± 0.026	36,37)
0.389	-0.005 ± 0.007	1.00 ± 0.03	36,39,40,41,42,7,8)
0.565	-0.004 ± 0.006	1.01 ± 0.08	39, 7)
0.600	-0.012 ± 0.006	1.04 ± 0.036	36,40,41,42, 8)
0.780	0.0120± 0.0077	0.93 ± 0.054	36,40,42, 7, 8)
1.00	0.0045± 0.0045	1.00 ± 0.06	36,42,43, 7)
1.17	0.0052± 0.007	0.96 ± 0.094	36,40,41, 8)
1.53	0.0021± 0.0025	1.055± 0.055	7)
1.80	+0.0000± 0.0041	1.06 ± 0.10	40,41,43, 8)
2.70	0.0029± 0.0034	0.84 ± 0.26	40,41,43)

TABLE 6

Neutron form factors from quasielastic (e-d)-scattering. The quantities $(G_E^N)^2$ and $G_M^N/(\mu_N \cdot G_D)$, where μ_N is the magnetic moment of the neutron and $G_D = (1 \cdot 0 + q^2/0.71)^{-2}$, are listed. For $q^2 < 0.3\ (GeV/c)^2$ the data as re-analysed by Kramer and coworkers[37)] have been taken.

q^2 $(GeV/c)^2$	Θ (degrees)	$R = \frac{\sigma_n}{\sigma_p}$	Ref.
0.339	10	0.221 ± 0.014	7)
0.565	10	0.258 0.013	7)
0.780	10	0.338 0.019	7)
1.00	12	0.344 0.018	7)
1.50	12	0.435 0.024	7)
1.75	20	0.395 0.061	40)
2.72	20.16	0.458 0.11	40)
3.33	47.9	0.474 0.067	43)
3.92	47.8	0.450 0.087	43)

TABLE 7

Neutron proton cross section ratios $R = \sigma_n/\sigma_p$ deduced from quasielastic (e-d)-scattering.

Model	χ_F^2		i = 1	i = 2	i = 3
"scaling law"	5.88	a_i	3.04 ± 0.04	1.54 ± 0.05	0.068 ± 0.007
"iso scaling law"	1.53	a_i	2.29 ± 0.05	2.76 ± 0.18	-0.097 ± 0.008
		b_i	3.32 ± 0.03	1.13 ± 0.02	0.132 ± 0.004

TABLE 8

Best fit parameters and χ_F^2 values for the "scaling law" and "iso scaling law" relations.

Model	χ_F^2		ρ	ω	ϕ	$\rho'(1500)$	$\omega'(1500)$	$\rho'(1250)$	$\omega'(1250)$
1)	3630	a_1	0.5	1.36	$-0.862\pm.001$				
		a_2	1.853	2.45	$-2.51\pm.001$				
2)	73	a_1	0.74	1.30	$-0.80\pm.01$	$-0.237\pm.003$			
		a_2	2.63	-0.22	$0.16\pm.01$	$-0.772\pm.003$			
3)	9.5	a_1	0.66	0.837	$-0.169\pm.004$	$-0.155\pm.003$	$-0.168\pm.003$		
		a_2	2.97	2.23	$-3.72\pm.004$	$-1.12\pm.002$	$1.43\pm.003$		
4)	3.9	a_1	0.698	0.766	-0.085 ± 0.001	0.128 ± 0.001	-0.425 ± 0.001	-0.326 ± 0.001	0.244 ± 0.001
		a_2	3.76	0.804	-1.48 ± 0.04	0.69 ± 0.002	0.569 ± 0.003	-2.60 ± 0.001	0.047 ± 0.01

Table 9

Best fit parameters and χ_F^2 values for the vector meson pole models (eq. 6) described in the text. The ρ and ω coefficients are obtained from the normalization conditions at $q^2 = 0$.

χ_F^2	C_1^S	C_2^S	C_1^V	C_2^V
3.1	2.82±0.03	2.14±0.05	2.68±0.02	3.68±0.02
M_i^2 (GeV^2)	a_{i1}^S	a_{i2}^S	a_{i1}^V	a_{i2}^V
0.5	$-7.2 \cdot 10^{-1}$	-2.0	$-6.8 \cdot 10^{-1}$	$1.1 \cdot 10^{-1}$
1.5	$2.0 \cdot 10^{-1}$	$9.3 \cdot 10^{-2}$	$1.5 \cdot 10^{-1}$	$-6.0 \cdot 10^{-2}$
2.5	$1.0 \cdot 10^{-2}$	$2.4 \cdot 10^{-2}$	$1.5 \cdot 10^{-2}$	$1.2 \cdot 10^{-2}$
3.5	$2.9 \cdot 10^{-3}$	$1.1 \cdot 10^{-2}$	$4.6 \cdot 10^{-3}$	$9.4 \cdot 10^{-4}$
4.5	$1.2 \cdot 10^{-3}$	$6.0 \cdot 10^{-3}$	$2.1 \cdot 10^{-3}$	$1.1 \cdot 10^{-4}$
5.5	$6.2 \cdot 10^{-4}$	$3.8 \cdot 10^{-3}$	$1.1 \cdot 10^{-3}$	$9.4 \cdot 10^{-5}$
6.5	$3.8 \cdot 10^{-4}$	$2.6 \cdot 10^{-3}$	$7.0 \cdot 10^{-4}$	$4.3 \cdot 10^{-5}$

TABLE 10

Best fit coefficients C from eq. (8) and the coefficients a^n from eq. (9) of the lower mass vector mesons are listed. The maximum uncertainty of the a's listed is about 20%.

Model	χ^2		ρ	ω	ϕ	Λ_ρ	Λ_ω	Λ_ϕ
1	8.2	a_1	0.5	2.61	-2.11 ± 0.06	0.87 ± 0.01	1.44 ± 0.03	1.02 ± 0.02
		a_2	1.853	-3.11	3.05 ± 0.12			
			ρ	ω	ϕ	$\rho'(1500)$	Λ	
2	2.8	a_1	0.28	0.29	0.21 ± 0.03	0.22 ± 0.02	1.03 ± 0.02	
		a_2	2.29	0.19	-0.25 ± 0.03	-0.44 ± 0.02		

TABLE 11

Best fit parameters of the models 1 and 2 discussed in section II.3 c). The ρ and ω coefficients have been determined from the normalization conditions at $q^2 = 0$.

V	$f_V^2/4\pi$	$(g_1^{VN\bar{N}})^2/4\pi$	$g_2^{VN\bar{N}}/g_1^{VN\bar{N}}$	$g_1^{VN\bar{N}}+g_2^{VN\bar{N}}$	$\lvert a_1^V \rvert$	$\lvert a_2^V \rvert$	Ref.
ρ	2.56±0.27	1.3 - 7.5	1.9 - 4.8	36 - 64	0.35-0.85	0.7 - 4.0	51,49)
ω	18.4 ±1.8	4.0 - 14	∿0	-	0.22 - 0.45	∿0	51,49)
ϕ	11.5 ±0.9	0 - 5	-	-	0 - 0.4		51,50)
ρ'	∿15	-	-	∿8	0 / 0.3	0.3 / 0	5),52)

TABLE 12

Coupling constants $f_V^2/4\pi$, determined from storage ring experiments and estimates of $(g_{1,2}^{VN\bar{N}})^2/4\pi$ obtained from theoretical analysis of pion-nucleon and nucleon nucleon scattering data. The coefficients a calculated from these data are listed too; they have to be compared with the best fit coefficients a listed in tables 9 - 11.

FIGURE CAPTIONS

Fig. 1 The ratios $(\mu_p G_E^P/G_M^P)^2$ and $G_M^P/\mu_p G_D$, where $G_D = (1.0+q^2/0.71)^{-2}$, versus q^2 in a logarithmic scale. The high q^2 points indicated by triangles were plotted for orientation purposes only. They have been calculated from the cross section measurements of ref. 22 assuming $G_E^P = G_M^P/\mu_p$. The curves labeled A are the best fit results based on the simple Clementi - Villi formulae eq. (6) assuming contributions from the ρ, ω, ϕ, $\rho'(1500)$ and the hypothetical mesons $\omega'(1500)$, $\rho'(1250)$, $\omega'(1250)$. Curves B represent the best fit result on the Veneziano expression eq.(8), and curves C were obtained from the best fit with the Massam - Zichichi formula eq.(10) including contributions from the ρ, ω, ϕ and ρ' mesons.

Fig. 2 $(G_E^N)^2$ obtained from quasielastic e-d-scattering and $G_M^N/\mu_N G_D$ versus q^2. The curves A, B and C correspond to the curves of Fig. 1.

Fig. 3 Values of G_E^N obtained from elastic e-d-scattering. The model results A, B and C are shown together with the extrapolated slope $(dG_E^N/dq^2)_{q^2=0}$. The curves A, B and C correspond to the curves of Fig. 1.

Fig. 4 The model results A, B and C for $d\sigma/d\Omega$ $(\Theta_{CM} = 90^\circ)$ of the reactions $e^+ + e^- \rightarrow n + \bar{n}$ and $e^+ + e^- \rightarrow p + \bar{p}$. Curve B has been adjusted to the data point of ref. 9 as described in the text. The curves A, B and C correspond to the curves of Fig. 1.

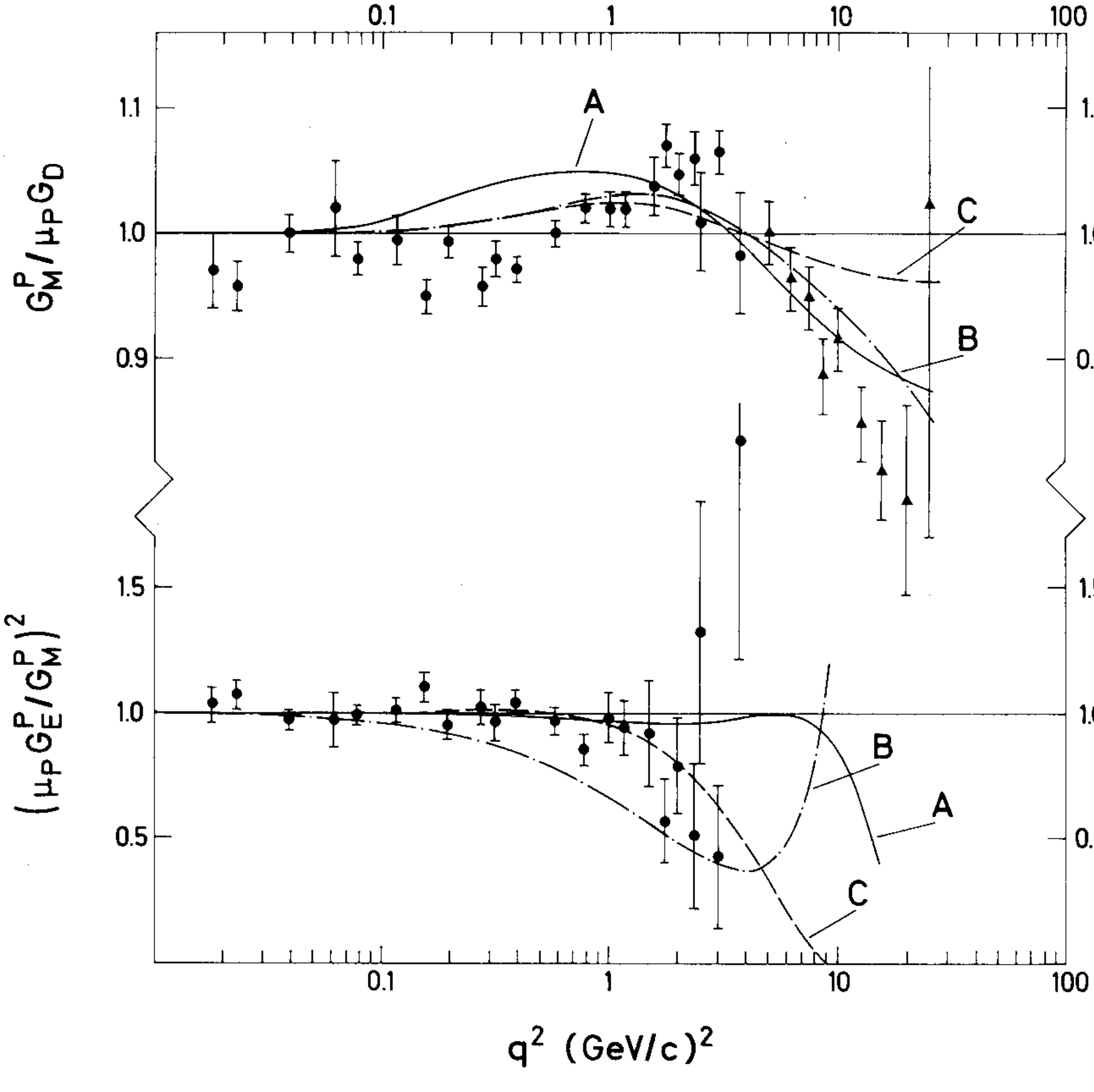

Fig.1

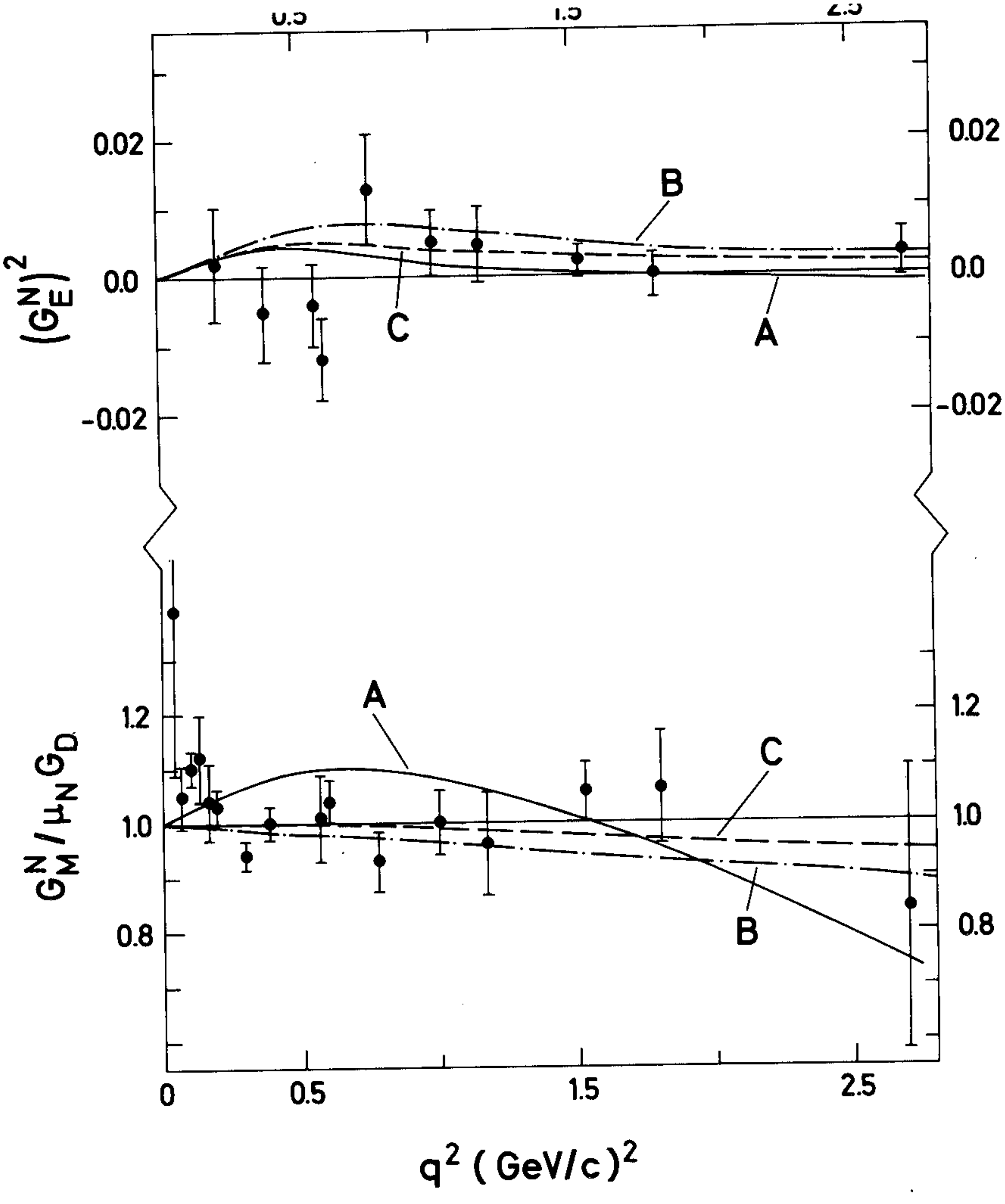

Fig.2

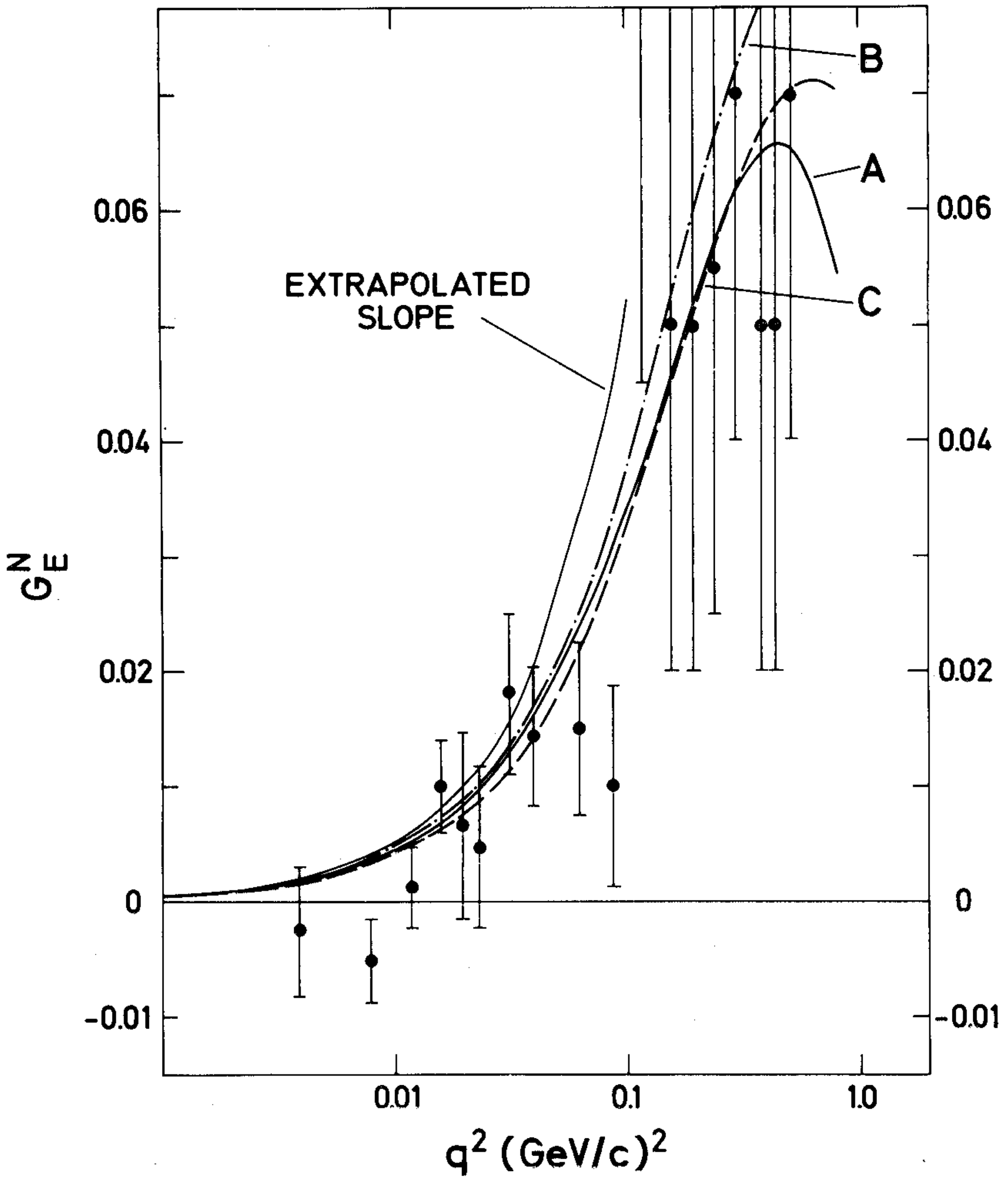
B
A
C
EXTRAPOLATED
SLOPE
0.06
0.04
0.02
0
-0.01
G_E^N
0.01
0.1
1.0
q^2 (GeV/c)^2

Fig.3

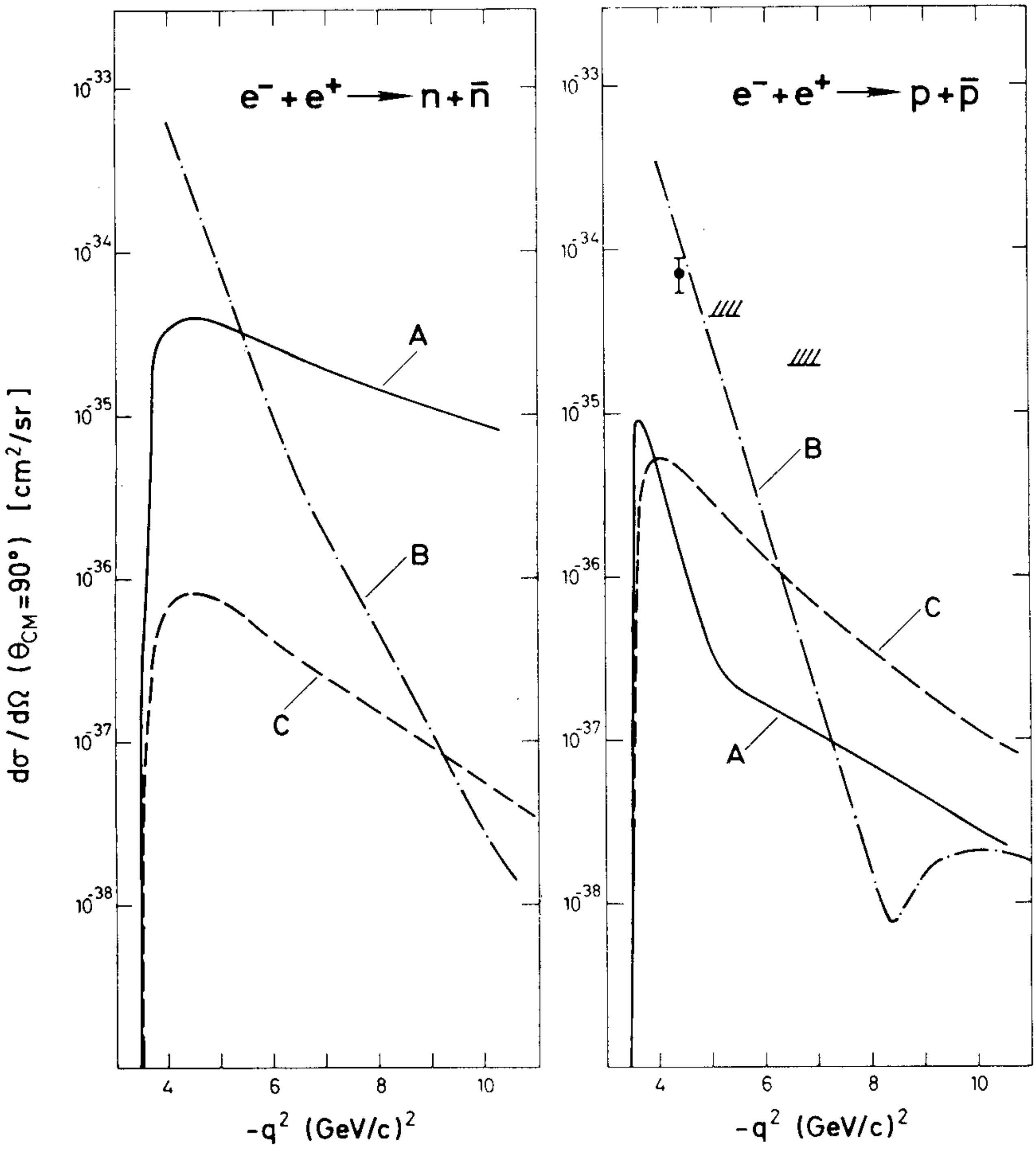

Fig.4