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Predictions of the Relativistic Quark Model for e^+e^--Annihilation into $\omega\pi^0$

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Abstract

The relativistic quark model is applied to the transition form factor between pseudoscalar and vector mesons. We obtain a generalized vector meson dominance structure, which improves the $\rho^0(770)$ dominance result and yields the decay width $\Gamma(\omega\to\pi^0\gamma) = .76$ MeV. For the cross section $\sigma(e^+e^- \to\omega\pi^0)$ we predict $\sigma\sim 17$ nb around $Q^2 = 2.3$ GeV^2 and an asymptotic $1/Q^4$ behaviour.

1. Introduction

The measurements of the total hadronic and the one-pion inclusive cross sections at e^+-e^- storage rings have shown interesting, unexpected features. An understanding of these results may possibly come from the study of the contribution of distinguished exclusive channels.

In this paper we discuss e^+e^- annihilation into a pseudoscalar and a vector meson. One reason for the importance of this channel is that, compared to the production of two pseudoscalars, one expects to gain one power in q^2 in the cross section for purely kinematical reasons [1]. We concentrate especially on the $\omega\pi^0$ state, which contributes to the $\pi^+\pi^-\pi^0\pi^0$ final state. Actually the $\omega\pi^0\gamma$ coupling is a favorite playground of vector meson dominance, the quark model and dual resonance models. The calculations of the partial decay width for $\omega\to\pi^0\gamma$ in the VDM [2] and in the nonrelativistic quark model [3] were first, big successes of these models. In the same VDM spirit the vector-pseudoscalar-meson transition form factor was built up from the ρ,ω,ϕ poles to estimate the $e^+e^-\to V + PS$ cross section [4]. Meanwhile the VDM has been refined to the generalized vector dominance model [5-7], supported experimentally by the detection of the $\rho'(1600)$ [8]. Concerning the quarks, we have developed a relativistic, dynamical model for the mesons as boundstates of heavy quarks [9]. In this model we obtain the meson spectrum which contains a series of higher mass vector mesons which couple to the photon [7]. This leads to GVDM where the couplings of the heavy vector mesons to the photon and to the hadronic final states can be calculated [10].

The kinematics is summarized in sect. 2. The dynamical ideas of the relativistic quark model and the explicit calculation of the pseudoscalar-vectormeson-transition formfactor is presented in sect. 3. Our result contains no free parameters. In sect. 4 we give our results for the ω-π-γ coupling constant, the formfactor and the cross section $\sigma(e^+e^- \to \omega\pi)$ and compare them with $\rho^0(770)$-dominance and experiment.

2. Matrix Elements, Decay Widths, Cross Sections

a) The transition matrix element of the e.m. current between a pseudoscalar and a vector meson state is:

$$\langle P_2, 0^{-+} | j_\mu^{e.m.}(0) | P_1, 1^{--}, s_3 \rangle = (2\pi)^{-3} \cdot \varepsilon_{\mu\nu\rho\sigma}\, \epsilon_{s_3}^{\nu}\, P_1^{\rho} P_2^{\sigma} \cdot G_{VPS\gamma}(Q^2)$$

$$Q_\mu = (P_1 - P_2)_\mu . \tag{1}$$

The formfactor $G(Q^2)$ at $Q^2 = 0$ is the coupling constant $g_{VPS\gamma}$ for the decay $V \to PS+\gamma$ or $PS \to V+\gamma$ and is related to the width

$$\Gamma(V \to PS+\gamma) = \alpha \cdot g^2 \cdot \frac{P_{CM}^3}{3} \quad ; \quad P_{CM} = \frac{M_V^2 - M_{PS}^2}{2M_V}$$

$$\Gamma(PS \to V+\gamma) = \alpha \cdot g^2 \cdot P_{CM}^3 \quad ; \quad P_{CM} = \frac{M_{PS}^2 - M_V^2}{2M_{PS}} \tag{2}$$

In the spacelike region $Q^2 < 0$, the form factor $G(Q^2)$ describes the one-pion exchange contribution to the electroproduction of vector mesons.
In the timelike region we have for $4M_{Lept}^2 \leq Q^2 \leq (M_V - M_{PS})^2$ the decay with Dalitz pairs, $V \to PS + l^+ l^-$.

The unphysical region lies between $(M_V-M_{PS})^2 \leq Q^2 \leq (M_V+M_{PS})^2$. Above the threshold $Q^2 > (M_V+M_{PS})^2$ the form factor determines the $e^+e^- \to PS+V$ annihilation. The differential cross section is:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{4Q^2}\cdot\frac{P_{cm}^3}{\sqrt{Q^2}}\cdot(1+\cos^2\theta)\cdot|G(Q^2)|^2 \; ; \qquad (3)$$

$$P_{cm}^2 = \frac{[Q^2-(M_V+M_{PS})^2]\cdot[Q^2-(M_V-M_{PS})^2]}{4Q^2}$$

The dependence on the polar angle θ(relative to the beam axis) is identical with that of $d\sigma(e^+e^-\to\mu^+\mu^-)$ in the relativistic limit:

$$\frac{d\sigma(e^+e^-\to\mu^+\mu^-)}{d\cos\theta} = \frac{\pi\alpha^2}{2Q^2}\cdot(1+\cos^2\theta) \qquad (4)$$

b) The e.m. form factors between the vector and pseudoscalar meson nonets can be decomposed into three independent SU(3) invariants [11]. Instead of this, we prefer the quark model scheme: We calculate the couplings of the e.m. current between the two mesons with help of the charge matrix of the quarks: $Q = \frac{1}{3}\,(2,-1,-1)$ and the quark wave functions for these mesons with ideal mixing for the vector mesons and an unmixed octet and singlet of pseudoscalars[+].

[+] Deviations from this simple mixing are of the order of 4° and 10° respectively, as a result of quadratic mass formulas [++]. We do not take them and the electromagnetic $\rho\omega$ and $\rho\phi$ mixing into account here, although they might give interesting phenomenological effects [13]. For example, the deviation from ideal mixing can be seen in the $\rho^0\pi^0$ form factor at the position of $\phi(1020)$.

[++] Quadratic mass formulas naturally arise in the relativistic quark model [12].

As is well known, an important part of the symmetry breaking is described correctly by using symmetric couplings, but physical values for the positions of the VDM poles (ρ,ω,ϕ-type) in the form factors. In this breaking scheme we obtain three independent form factors.

3. Form Factors from the Relativistic Quark Model

a) In the relativistic, dynamical quark model the mesons are described as bound states of heavy quarks. The spectrum and the "wave functions" are obtained from the solution of the Fermion-Antifermion Bethe-Salpeter equation with a smooth interaction [9]. A $\gamma_5 \times \gamma_5$ spin structure of this interaction yields a meson spectrum with quark spin singlet, -triplet structure. The excited states are characterized by the quantum numbers of the four-dimensional oscillator with a mass formula

$$M^2_{s,t} = M^2_{os,t} + \alpha'^{-1}(n + 2r) \; ; \; n,r = 0,\, 1,\, 2,\, \ldots$$

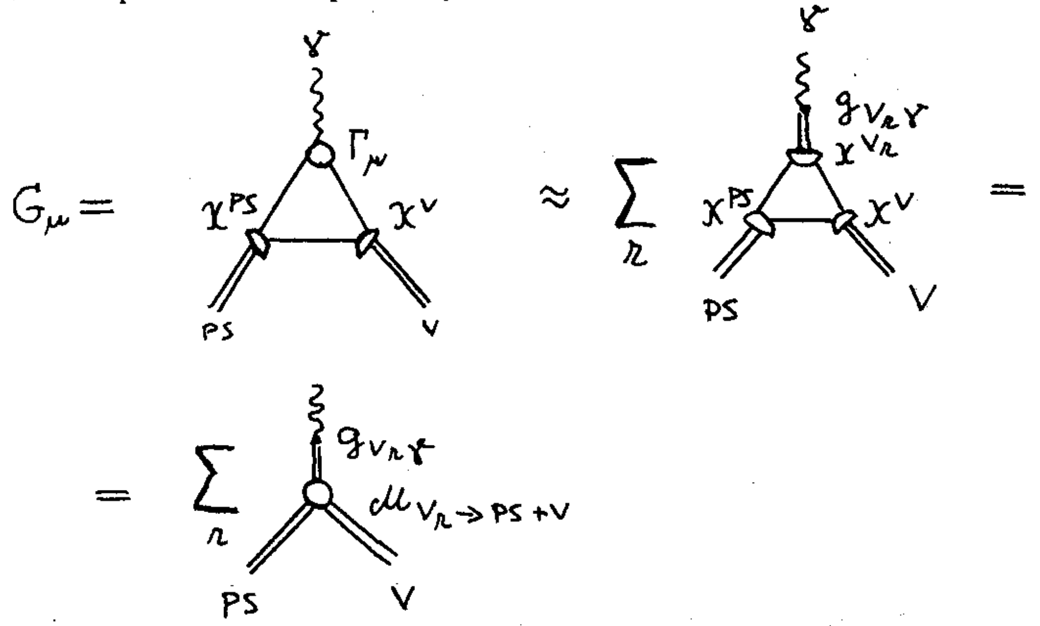
$$\alpha'^{-1} = 1.0 \text{ GeV}^2,\; M^2_{os} = 0.02 \text{ GeV}^2 \; ; \; M^2_{ot} = 0.57 \text{ GeV}^2. \tag{5}$$

In particular there is a set of radially excited heavy vector mesons ($n=0$, $r=0,1,2,\ldots$) which induce through their coupling to the photon a generalized VDM structure [7]. The BS amplitudes of the pseudoscalar ground state and these vector mesons are:

$$\chi^{PS}(k,P) = 12\pi\sqrt{3}\,\alpha'\left[\gamma_5 \frac{\gamma_\mu P^\mu}{M} + O(m^{-1})\right]\cdot e^{-3\alpha' k^2}\cdot |q\bar{q}\rangle_{SU_3} \tag{6}$$

$$\chi^V_r(k,P) = 12\pi\sqrt{3}\,\alpha'\left[\gamma^\mu \epsilon_\mu^{s_3} + O(m^{-1})\right]\cdot e^{-3\alpha' k^2}\cdot\frac{L^1_r(6\alpha' k^2)}{\sqrt{r+1}}\cdot |q\bar{q}\rangle_{SU_3}$$

b) The e.m. transition form factors between the meson states can be calculated in this bound state model from the triangle graph with the quark form factor and the bound state BS amplitudes as vertices. The quark form factor is determined by an inhomogeneous BS equation which can be solved with help of a closure approximation. Thereby the quark form factor Γ_μ gets expressed by the heavy vector meson poles, their residues are given by their coupling to the photon $g_{V_r\gamma}$ and their coupling to two quarks, i.e. their BS amplitudes. Graphically this reads:



(7)

This line of arguments leads naturally to the Generalized VDM structure of the e.m. form factors. As a first application we have evaluated the pion form factor, where also the derivation of the final formulas was given in detail [14].

c) The vector meson photon couplings needed in eq. (7) are given by

$$g_{V_r\gamma} = Z \cdot \frac{(-1)^r}{\pi\sqrt{3}} \cdot \alpha^{r-1} \cdot \sqrt{r+1} \cdot \langle Q_V \rangle \; ; \quad \langle Q_V \rangle_{(\rho,\omega,\phi)} = \left(\frac{1}{\sqrt{2}}, \frac{1}{3\sqrt{2}}, -\frac{1}{3}\right) \tag{8}$$

The renormalization constant Z of the e.m. current was determined from the normalization of the pion form factor to be Z=.73 [14].

The decay matrix element $\mathcal{M}_{V_r \to V+PS}$, evaluated in triangle graph approximation, is consistent with the assumption of heavy quarks only, if there is saturation of the superstrong quark forces. We have shown, that for a $\gamma_5 x \gamma_5$ interaction saturation occurs and the phenomenological applications to various decays are quite successful [10]. The result for the decay matrix element is

$$\mathcal{M}_{V_r, V, PS} = 12\sqrt{\pi}\,(\alpha')^{3/2} \cdot \frac{e^{(M_r^2 + M_{PS}^2 + M_V^2)\cdot\frac{\alpha'}{4}}}{3^{r+1}\cdot\sqrt{r+1}} \cdot \varepsilon_{\mu\nu\rho\sigma}\,\epsilon_1^{\mu}\epsilon_2^{\nu}\,P_V^{\rho}P_{PS}^{\sigma} \cdot$$

$$\cdot\left[\left(2\varepsilon_T^V + \tfrac{2}{3}\hat{\varepsilon}_T^A\right)\cdot L_r^1\!\left(-\tfrac{\alpha'}{2}(P_V - P_{PS})^2\right) + \tfrac{2}{3}\hat{\varepsilon}_T^A \cdot L_{r-1}^2\!\left(-\tfrac{\alpha'}{2}(P_V - P_{PS})^2\right)\right]. \tag{9}$$

The parameters ε were determined from the $(\omega\rho\pi)$, $(A_2\rho\pi)$, ... couplings to be [15]

$$\varepsilon_T^V = \alpha'^{-1}/3 \quad , \quad \hat{\varepsilon}_T^A = 0 .$$

Inserting the expressions (8) and (9) into eq. (7) and using the mass formula we obtain

$$G_{V\,PS\,\gamma}(Q^2) = g_{V_0\gamma}\cdot g_{V_0\to V,PS}\cdot\sum_{r=0}^{\infty}\frac{(-1/3)^r\cdot e^{r/2}\cdot L_r^1(r+c)}{M_{V_r}^2 - Q^2}\;; \tag{10}$$

$$c = -0.26 \quad \text{for } G_{\omega\pi^0\gamma}$$

In the timelike region, we have to take into account the finite width effects. We do this using Breit-Wigners with $\Gamma_{\rho(770)} = 150$ MeV, $\Gamma_{\rho'(1600)} = 500$ MeV.

4. Results, Discussion, Comparison with Experiments

We have obtained an absolute prediction for the transition form factor $G(Q^2)$. The parameters, which enter eq. (10) have already been determined from other applications of the relativistic quark model, like the meson spectrum, the strong decays and the pion form factor. The evaluation of $G(Q^2)$ at $Q^2 = 0$ gives for the $\omega\pi\gamma$ coupling constant the value

$g^{th}_{\omega\pi\gamma} = 2.40\ \mathrm{GeV}^{-1}$ (corresponding to the partial width $\Gamma^{th}_{\omega\pi\gamma} = .76$ MeV)

which is to be compared with

$$g^{exp}_{\omega\pi\gamma} = (2.55 \pm .10)\ \mathrm{GeV}^{-1}.$$

The agreement between theory and experiment is better than in the Gell-Mann Sharp Wagner calculation, where only the contribution of $\rho(770)$ is included [2], leading to a value $g_{\omega\pi\gamma} \approx 3.0\ \mathrm{GeV}^{-1}$. The possibility to lower this value by the inclusion of the $\rho'(1600)$ was discussed phenomenologically by Bramon and Greco [16].
In our model we have a well defined contribution of heavier vector mesons. In fact they give a correction to the $\rho(770)$ contribution of -20 %.

In Fig. 1 we present the form factor $G(Q^2)$ in the space- and timelike region. The prominent features are the ρ-meson peak (in the unphysical region) and the structure of the ρ'-meson. Asymptotically the form factor decreases like $-.75\ \mathrm{GeV}/Q^2$. The structure of the transition form factor

$G(Q^2)$ is rather similar to that of the pion form factor [14].

In Fig. 2 we show the cross section $\sigma(e^+e^-\to\omega\pi^o)$. Since the $\rho(770)$ pole is below the threshold of this process, the effects of the ρ' show up very drastically. In fact around $Q^2 \approx 2.5\ GeV^2$ our prediction differs from estimates [4] which include only the $\rho(770)$ by a factor 4-5. The experimental data points in Fig. 2 refer to measurements of $\sigma(e^+e^-\to \pi^+\pi^-\pi^o\pi^o)$ at Orsay [17] and Frascati [18]. The points around 1 GeV^2 show the $\omega\pi^o$ threshold. At higher values of Q^2 there is the possibility, that besides $\omega\pi$ also the $A_1\pi$ final state gives an important contribution to this channel [4]. Therefore a detailed experimental study of the 4π final state would be of great interest.

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Figure Captions

Fig. 1 The ω-π^o-γ transition formfactor in the space- and time-like region. The experimental point refers to the ω-π^o-γ decay constant.

Fig. 2 The cross section for $e^+e^- \to \omega\pi^o$ from the relativistic quark model (solid line). Data points refer to $\sigma(e^+e^- \to \pi^+\pi^-\pi^o\pi^o)$ [17,18]. For comparison: ρ^o(770)-dominance prediction with $g_{\omega\rho\gamma} = 2.40\ \mathrm{GeV}^{-1}$ (— . —) and $\sigma(e^+e^- \to \mu^+\mu^-)$ (- - -).

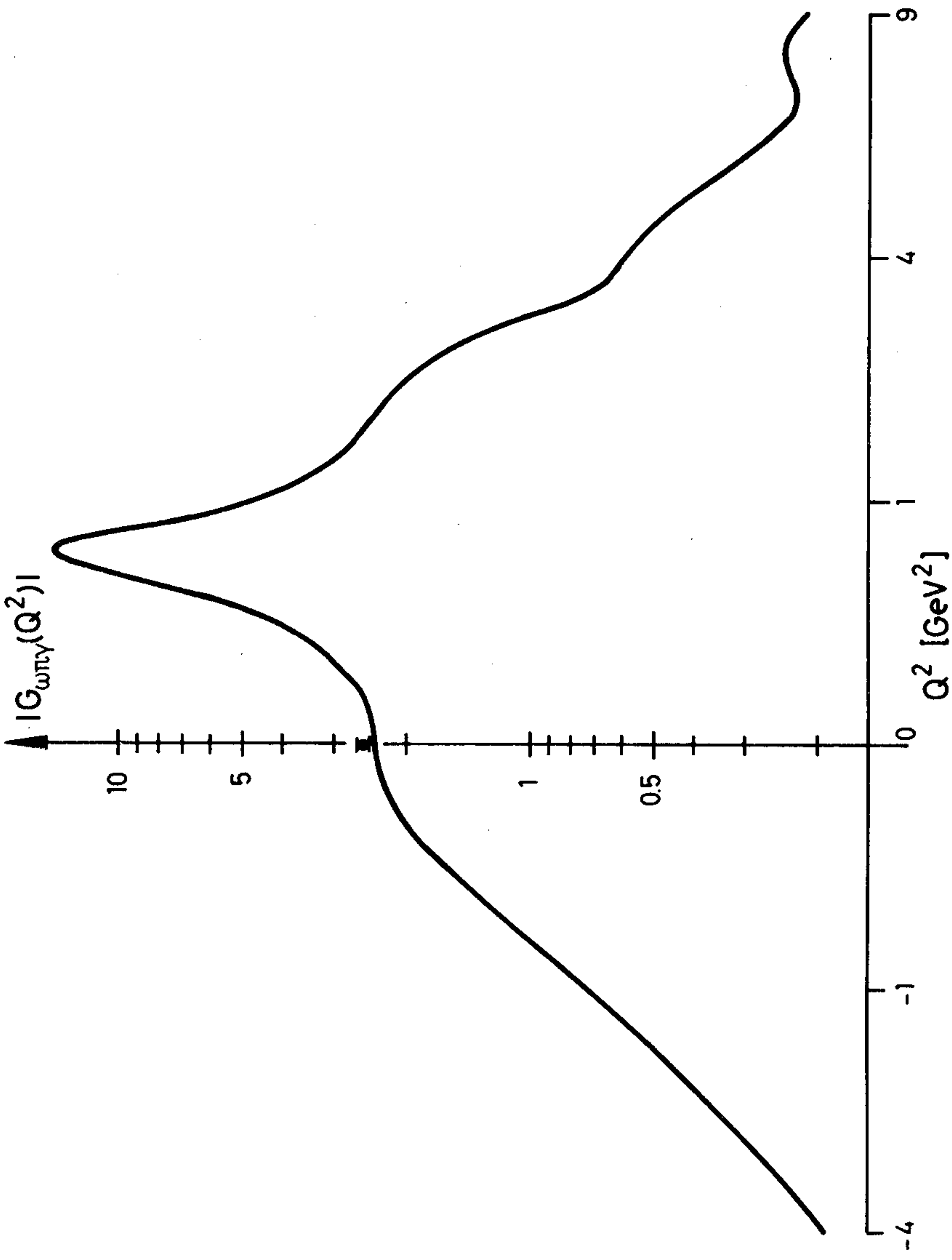

Fig. 1

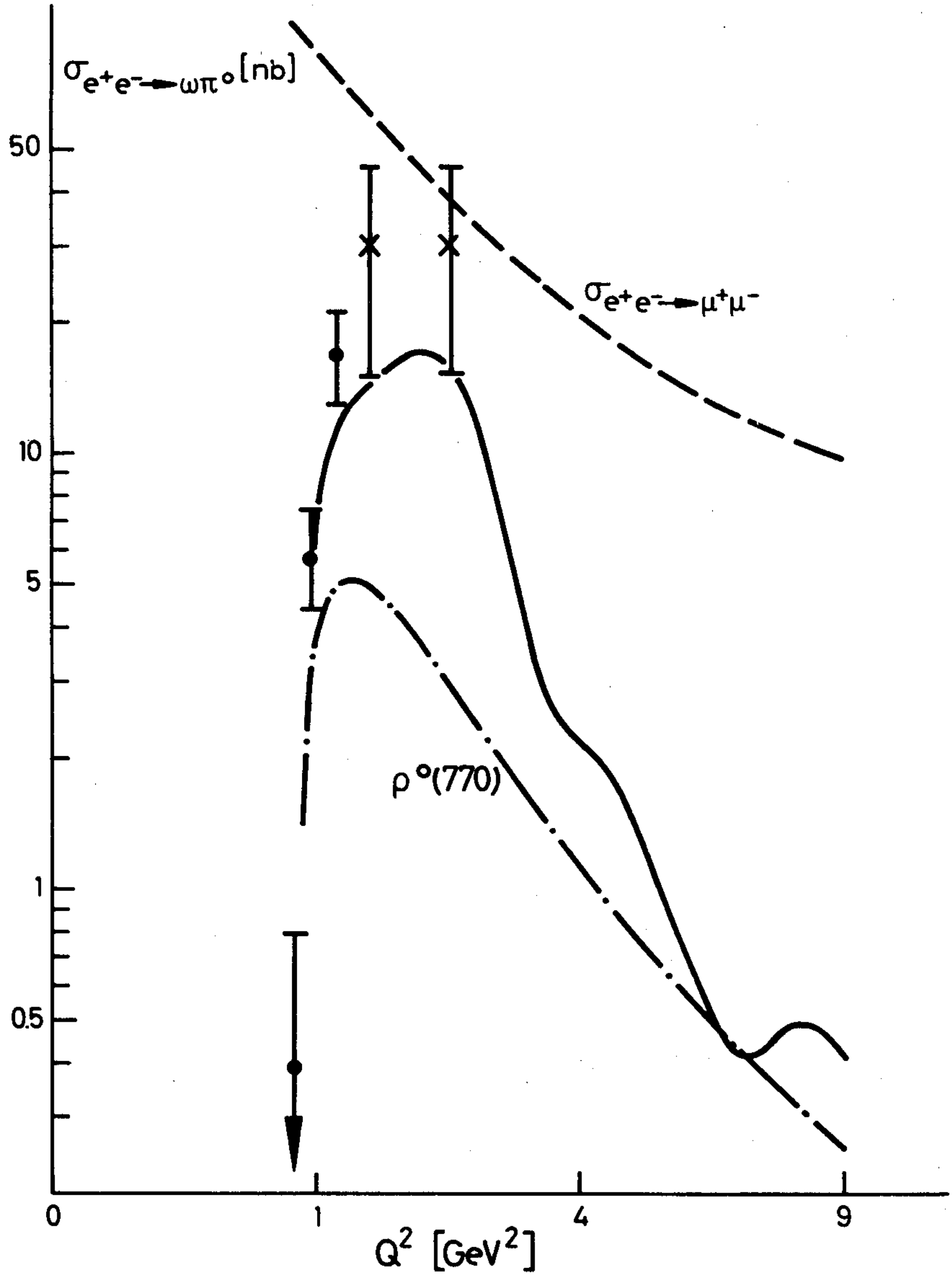
$\sigma_{e^+e^- \to \omega\pi^\circ}$ [nb]
50
10
5
1
0.5
$\sigma_{e^+e^- \to \mu^+\mu^-}$
ρ°(770)
0
1
4
9
Q^2 [GeV^2]

Fig. 2