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Decay Properties of the New Vector Mesons in Broken SU(4)

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Abstract: We calculate mass spectra for vector, pseudoscalar and tensor mesons on the basis of singlet and fifteenplet mixing in broken SU(4) and study the dependence of wave functions on input masses. With these wave functions we compute various two-body decays of $\Psi(3.1)$ with SU(4) invariant couplings.

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1. Introduction.

A narrow resonance Ψ at a mass of 3.095 GeV and a total width Γ_{Ψ} = 69 KeV has been found recently by the BNL [1], SLAC [2], Frascati [3] and two Desy [4] groups. A second narrow resonance with a mass of 3.684 GeV has also been observed by the SLAC [5] and the Desy [6] groups.

An interesting problem in connection with these new particles is their relatively small total width. The value of the total width of the Ψ(3095) and presumably also of the Ψ(3684) is several orders of magnitude smaller than what one expects of a heavy strongly interacting meson. One of the possible ways to understand the new resonances in the e^+e^- system is their interpretation as vector mesons composed predominantly of charmed quarks and antiquarks. Then the basic symmetry of strong interactions becomes SU(4) instead of SU(3), with the four quarks u, d, s and c now belonging to the fundamental four dimensional representation. The new fourth quark c has charge 2/3, isospin 0 and one unit of a new quantum number C called charm which is conserved in strong and electromagnetic interactions. In such a scheme the narrow width of the Ψ's could be naturally explained if the Ψ's are rather pure $c\bar{c}$ states with masses below the threshold for decay into charmed mesons or baryons and which couple predominantly only to charmed particles.

In a previous paper [7], we studied the general framework for SU(4) symmetry breaking. It was found that, if one places the usual vector mesons (ρ, K^*, ω, Φ) together with the

$\Psi(3.095)$ into an $1 \oplus 15$ representation of SU(4), the Ψ comes out as a rather pure $c\bar{c}$ state and the masses of charmed vector mesons D^* and F^*, which belong to a representation 3 in SU(3), are above 2 GeV [8].

In this paper we continue this analysis including pseudoscalar and tensor meson multiplets. Then we consider the two-body decays of $\Psi(3.1)$ based on SU(4) invariant couplings. Here, similar to SU(3), we assume, that the couplings deviate less from their SU(4) symmetric values than the masses. This means that in a first approximation, the breaking of SU(4) appears only through breaking of masses in phase space factors.
In section 2 we present the masses and wave functions for the mixed vector, pseudoscalar and tensor multiplets for several choices concerning input masses. With these wave functions we calculate two- body decays of the $\Psi(3.1)$ and some other vector mesons in section 3 (strong decays) and section 4 (radiative decays).
In section 5 we draw our conclusions.

2. Masses and Wave Functions.

In this section we shall give the numerical results of our mixing analysis given in a previous paper [7]. We start with the vector mesons ρ, K^*, ω, ϕ, Ψ, D^* and F^* belonging to the representation $\underline{15} \oplus \underline{1}$ of SU(4) and neglect electromagnetic mass splitting effects. The mass matrix is diagonal except for the subspace spanned by $|\omega_8\rangle$, $|\omega_{15}\rangle$ and $|\omega_0\rangle$, where $|\omega_8\rangle$ and $|\omega_{15}\rangle$ denote the isoscalar wave-functions belonging to $\underline{15}$, with SU(3)-octet and SU(3)-singlet quantum numbers respectively, and $|\omega_0\rangle$ denotes the wavefunction belonging to $\underline{1}$. The mass matrix in this subspace is given in eq. (3.1) of I, with the constraint I (3.2). The eigenvalues are the physical masses of the ω, ϕ and ψ meson. The corresponding wave functions can be expressed either in the basis $\{|\omega_0\rangle, |\omega_8\rangle, |\omega_{15}\rangle\}$:

$$|V\rangle = \alpha_0^{(V)}|\omega_0\rangle + \alpha_8^{(V)}|\omega_8\rangle + \alpha_{15}^{(V)}|\omega_{15}\rangle , \qquad (V = \omega, \phi, \psi) \tag{2.1}$$

or in the basis $\{|\omega_\sigma\rangle, |\omega_s\rangle, |\omega_c\rangle\}$ corresponding to ideal mixing and defined in I(3.11):

$$|V\rangle = \alpha_\sigma^{(V)}|\omega_\sigma\rangle + \alpha_s^{(V)}|\omega_s\rangle + \alpha_c^{(V)}|\omega_c\rangle , \qquad (V = \omega, \phi, \psi) . \tag{2.2}$$

The coefficients appearing in (2.1) and (2.2) are related by

$$\begin{aligned} \alpha_\sigma &= \frac{1}{\sqrt{2}}\alpha_0 + \frac{1}{\sqrt{3}}\alpha_8 + \frac{1}{\sqrt{6}}\alpha_{15} , \\ \alpha_s &= \frac{1}{2}\alpha_0 - \sqrt{\frac{2}{3}}\alpha_8 + \frac{1}{\sqrt{12}}\alpha_{15} , \\ \alpha_c &= \frac{1}{2}\alpha_0 \qquad\qquad - \frac{\sqrt{3}}{2}\alpha_{15} , \end{aligned} \tag{2.3}$$

and , inversely, by

$$\begin{aligned} \alpha_0 &= \frac{1}{\sqrt{2}}\alpha_\sigma + \frac{1}{2}\alpha_s + \frac{1}{2}\alpha_c , \\ \alpha_8 &= \frac{1}{\sqrt{3}}\alpha_\sigma - \sqrt{\frac{2}{3}}\alpha_s , \\ \alpha_{15} &= \frac{1}{\sqrt{6}}\alpha_\sigma + \frac{1}{\sqrt{12}}\alpha_s - \frac{\sqrt{3}}{2}\alpha_c . \end{aligned} \tag{2.4}$$

Since the quark content of the states $|\omega_\sigma\rangle$, $|\omega_s\rangle$ and $|\omega_c\rangle$ is given by

$$|\omega_\sigma\rangle = \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle , \qquad |\omega_s\rangle = |s\bar{s}\rangle , \qquad |\omega_c\rangle = |c\bar{c}\rangle \tag{2.5}$$

the content of usual (u,d), strange (s) and charmed (c) quarks of $V = \omega, \phi, \psi$ can be read off directly from the expansion (2.2). The corresponding coefficients $\alpha_\sigma^{(V)}$, $\alpha_s^{(V)}$ and $\alpha_c^{(V)}$ essentially determine also the decay properties of V if we assume SU(4)-invariant three-body vertices.

In our numerical analysis we use the masses of ρ, K^*, ω, ϕ and ψ as input and consider only quadratic mass formulas. The results, especially for the "off-diagonal" coefficients $\alpha_s^{(\omega)}$, $\alpha_c^{(\omega)}$, $\alpha_\sigma^{(\phi)}$, $\alpha_c^{(\phi)}$, $\alpha_\sigma^{(\psi)}$ and $\alpha_s^{(\psi)}$, which measure the deviation from ideal mixing, turn out to depend very sensitively on the input masses, particularly on the ρ-mass. Changing m_ρ within one standard deviation (from 0.76 to 0.78 GeV) may result in changes of $\alpha_\sigma^{(\psi)}$, $\alpha_s^{(\psi)}$, $\alpha_c^{(\omega)}$ and $\alpha_c^{(\phi)}$ by several orders of magnitude. For the decay widths the changes are then even more substantial. In order to reduce drastically the partial decay widths of the ψ-meson, the mixing in (2.2) has to be nearly ideal, i.e. $|\omega\rangle \approx |\omega_\sigma\rangle$, $|\phi\rangle \approx |\omega_s\rangle$, $|\psi\rangle \approx |\omega_c\rangle$. This can be achieved by lowering the ρ-mass. Thus, if one believes in SU(4) symmetry of strong interactions, one might get the idea to use the sensitive dependence of the SU(4) - predictions on the ρ-mass as a way to fix m_ρ around 760 MeV. But because of electromagnetic mass shifts, which have been neglected here and because the ρ has a rather larger width, which makes the mass determination uncertain, we shall not pursue this any further. At the moment we shall study only the dependence of wave functions and decays on this particular parameter.

In table 1-3 we have summarized our numerical results. Since they do not depend sensitively on the other input masses m_{K^*}, m_ω, m_ϕ and m_ψ we held them fixed and allowed only m_ρ to take on different values. The implications for the strong and radiative decay modes are consiedered in the following sections. Before we come to this we discuss the mass splitting of the other meson sixteenplets : pseudo-scalar mesons $(\pi, K, \eta, \eta', \psi_P, D, F)$ and tensor mesons $(A_2, K^{**}, f, f', \psi_T, D^{**}, F^{**})$.

For all these multiplets neither the analogous states to the ψ (ψ_P, ψ_T), nor the mesons with charm (D, F or D^{**}, F^{**}) have been discovered yet. Therefore predictions of the corresponding spectra are not possible with our five parameter mass matrix I(3.1) together with I (3.2). We need an additional assumption concerning the parameters m, m_1, m_2, m_0 and A. As in section 4 of I we fix the ratio of the contributions due to F_{15} and F_8 in the mass formula. There we kept α the same for the coupling of the 15-plet to the singlet as inside the fifteenplet. It seems natural to take α equal to the value as determined from the vector meson fifteenplet, also for all other meson fifteenplets. Actually for the vector, pseudoscalar and tensor mesons the parameter $m_1 = \frac{1}{6}(K^* - \rho) \approx \frac{1}{6}(K - \pi) \approx \frac{1}{6}(K^{**} - A_2)$ has roughly the same value. Therefore our fixing of α amounts to fix the parameter m_2 (see I (3.6)). With m_1 and m_2 equal for these meson fifteenplets we just keep the splitting of the SU(4) fifteenplets constant and vary only the contribution of the SU(4) invariant term in I (2.12) given by m. As was already emphasized in paper I the input masses of the vector multiplet are such that the eigenfunctions deviate only very little from ideal mixing.

As was already mentioned these deviations in the wave functions depend very sensitively on the input masses. For the vector mesons we show in table 1 two versions: version 1 corresponding to an input ρ-mass $m_\rho = \frac{1}{3}(2m_{\rho+} + m_{\rho o}) = 0.7674\,\text{GeV}$ as given in the data card listings of ref. 9 , and version 2 with an input mass $m_\rho = 0.76009$ GeV. This value was chosen to obtain for the Ψ a small enough admixture of the usual quarks u,d,s. Together with the coefficients α_σ, α_s and α_c of the eigenfunctions we have exhibited in table 1 also the other input mass values and the results for m_{D*} and m_{F*}. As to be expected the output values of m_{D*} and m_{F*} change only very little for the two versions 1 and 2. The important quantities for the decay of the Ψ are $\alpha_\sigma^{(\Psi)}$ and $\alpha_s^{(\Psi)}$. We see that $\alpha_\sigma^{(\Psi)}$ and $\alpha_s^{(\Psi)}$ are already small in version 1, of the order of 2%. They change by two orders of magnitude if we go from version 1 to version 2. Likewise this also reduces the α_c for ω and ϕ by the same amount. On the other hand the coefficient $\alpha_\sigma^{(\phi)}$ is not changed appreciably, so that the decay of the ϕ in nonstrange mesons is not effected by reducing the $c\bar{c}$ content of the Ψ, although $\alpha_\sigma^{(\phi)}$ and $\alpha_s^{(\omega)}$ are reduced somewhat so that ω and ϕ are more ideally mixed in version 2 than in version 1. We notice that $\alpha_\sigma^{(\Psi)}$ and $\alpha_s^{(\Psi)}$ (likewise $\alpha_c^{(\omega)}$ and $\alpha_s^{(\phi)}$) go through zero if m_ρ changes from 0.767 to 0.760. To study the dependence of these two coefficients in the vicinity of this zero we have plotted $\alpha_\sigma^{(\Psi)}$ and $\alpha_s^{(\Psi)}$ as a function of m_ρ in fig. 1. The input masses are the same as in table 1. We remark that both coefficients $\alpha_\sigma^{(\Psi)}$ and $\alpha_s^{(\Psi)}$ vanish near $m_\rho = 0.760$, so that the coupling of Ψ to nonstrange and to strange particles is reduced by the same amount. $\alpha_\sigma^{(\phi)}$ is rather unsensitive to the variation of m_ρ in this limited range.

The mass spectra and the wave functions of the pseudoscalar mesons are given in table 2a and 2b, also for two values of α , one corresponding to $m_\rho = 0.7674$ (version 1) and one to $m_\rho = 0.76009$ (version 2). Unfortunately we have two candidates for the SU(3) - partner of the η particle: the $\eta'(0.9576)$ and the E(1.416). Up to now[9] it is not excluded that η' has spin-parity 2^- , so that both assignments have to be discussed . In table 2a the spectrum and the wave functions with η' are shown. We see that with this input η, η' and ψ_P deviate appreciably from ideal mixing. In particular η' has a large $c\bar{c}$ component (roughly 25%). The 0^- partner of the ψ, called ψ_P has a mass around 2.7 GeV and also has a rather large admixture of noncharmed quarks, so that ψ_P should be a broad resonance compared to $\psi(3.1)$. If we transform the coefficients in table 2a according to (2.4) into α_0, α_8 and α_{15}, it turns out that η', η and ψ_P are predominantly $|\omega_0\rangle$, $|\omega_8\rangle$ and $|\omega_{15}\rangle$, respectively. The large $c\bar{c}$ content of the $\eta'(0.958)$ will have important consequences for the radiative decay of the ψ as will be shown in section 4.

The result for the other assignment E (1.416) as partner of the η is given in table 2b. With this choice the mass of the ψ_P is much higher (around 3.0 GeV) and the admixture of noncharmed quarks in ψ_P is small, around 4%. Similarly the $c\bar{c}$ content of E and η is now also much smaller. We notice that the D and F masses are the same in table 2a and table 2b. They depend only on the parameters m_1 and α, which remain unchanged if η' is replaced by E.

Finally the spectrum and wave functions of the tensor mesons are given in table 3. The mass of ψ_T is near 3.8 GeV. The wave functions of f, f' and ψ_T are near to ideal mixing. The $c\bar{c}$ content of f and f' and the admixture of noncharmed quarks in ψ_T are around 2%. In both cases, for the pseudoscalar and tensor mesons a change of α corresponding to version 1 and 2 in choosing the ρ-mass has little influence on the spectra and wavefunctions. α , which a priori can be different for the different multiplets (vector, pseudoscalar, tensor) must be varied over a wider range in order to see an effect on the spectrum. This is shown in fig. 2, where the masses of the new particles are plotted as a function of α . From this figure further interesting conclusions can be drawn. For example, to have charmed pseudoscalar particles with masses below 2.0 GeV, the α parameter for pseudoscalars must be below 17. In this case the threshold for production of $D\bar{D}$ and $F\bar{F}$ would be low enough to account for the rise in σ_{tot} $(e^+e^- \to \text{hadrons})$ around 4.1 GeV. [10] Another interesting aspect is the mass of the ψ_T . For $\alpha > 20$ the $\psi(3.7)$ cannot decay radiatively into ψ_T.

3. Strong Decays.

In our scheme the observed $\psi(3095)$ resonance is a member of a $15 \oplus 1$ representation of SU(4). In such a symmetry scheme the couplings of the ψ to other particles in SU(4) multiplets are determined. It is of interest to see whether these symmetry relations between coupling constants are compatible with the known partial decay widths of the other members of the vector meson multiplet and the small decay width of the $\Psi(3095)$. To begin with we consider only the decays $V \longrightarrow PP$ and $V \longrightarrow VP$, where V and P are vector and pseudoscalar mesons, respectively.

(a) $V \longrightarrow PP$

The SU(4) invariant coupling H_{VPP} has the following form

$$H_{VPP} = g_{VPP}\, f_{abc}\, V^{a}_{\mu} \left(P^{b} \overleftrightarrow{\partial}^{\mu} P^{c} \right) , \tag{3.1}$$

where a,b,c = 1,2,...,15 and the f_{abc} are the usual structure constants of SU(4), which can be found for example in ref. 11. The connection of the fields V^{a} and P^{b} with the physical particles is exhibited in appendix A. Substituting these relations in (3.1) gives us the following SU(4) invariant VPP couplings:

$$\begin{aligned}
H_{VPP} = g_{VPP} \Big\{ & \left[i\vec{\pi} \times \overleftrightarrow{\partial}^{\mu} \vec{\pi} + i\mathcal{K}^{\dagger} \vec{\tau} \overleftrightarrow{\partial}^{\mu} \mathcal{K} + i\mathcal{D}^{\dagger} \vec{\tau} \overleftrightarrow{\partial}^{\mu} \mathcal{D} \right] \cdot \vec{\rho}_{\mu} \\
& + \Big[i\vec{\pi} \cdot \overleftrightarrow{\partial}^{\mu} \mathcal{K}^{\dagger} \vec{\tau} \mathcal{K}^{*}_{\mu} - i\sqrt{2}\, F^{-} \overleftrightarrow{\partial}^{\mu} \mathcal{D}^{\dagger} \mathcal{K}^{*}_{\mu} \\
& \qquad + i(\eta_{\sigma} - \sqrt{2}\,\eta_{s}) \overleftrightarrow{\partial}^{\mu} \mathcal{K}^{\dagger} \mathcal{K}^{*}_{\mu} + h.c. \Big] \\
& + i\sqrt{2} \left[-\mathcal{K}^{\dagger} \overleftrightarrow{\partial}^{\mu} \mathcal{K} + F^{+} \overleftrightarrow{\partial}^{\mu} F^{-} \right] \omega_{s\mu} \\
& + \Big[i\vec{\pi} \cdot \overleftrightarrow{\partial}^{\mu} \mathcal{D}^{\dagger} \vec{\tau} \mathcal{D}^{*}_{\mu} - i\sqrt{2}\, F^{+} \overleftrightarrow{\partial}^{\mu} \mathcal{K}^{\dagger} \mathcal{D}^{*}_{\mu} \\
& \qquad + i(\eta_{\sigma} - \sqrt{2}\,\eta_{c}) \overleftrightarrow{\partial}^{\mu} \mathcal{D}^{\dagger} \mathcal{D}^{*}_{\mu} + h.c. \Big] \\
& + \left[i\sqrt{2} \left(F^{+} \overleftrightarrow{\partial}^{\mu} (-\eta_{s} + \eta_{c}) - \mathcal{D}^{\dagger} \overleftrightarrow{\partial}^{\mu} \mathcal{K} \right) F^{*-}_{\mu} + h.c. \right] \\
& - i\sqrt{2} \left(\mathcal{D}^{\dagger} \overleftrightarrow{\partial}^{\mu} \mathcal{D} + F^{+} \overleftrightarrow{\partial}^{\mu} F^{-} \right) \omega_{c\mu} \\
& + i \left(\mathcal{K}^{\dagger} \overleftrightarrow{\partial}^{\mu} \mathcal{K} + \mathcal{D}^{\dagger} \overleftrightarrow{\partial}^{\mu} \mathcal{D} \right) \omega_{\sigma\mu} \Big\} ,
\end{aligned} \tag{3.2}$$

where

$$\mathcal{K}^\dagger = (K^- \ \overline{K^o}), \quad \mathcal{K} = \begin{pmatrix} K^+ \\ K^o \end{pmatrix}, \quad \mathcal{D}^\dagger = (D^o \ D^+), \quad \mathcal{D} = \begin{pmatrix} \overline{D^o} \\ D^- \end{pmatrix}.$$

In (3.2) the vector mesons ω_σ, ω_s and ω_c denote the ideally mixed meson fields (2.5). We see that ω_c couples only to the charmed mesons D and F, so that ω_c is stable against decay into pseudoscalar mesons since with our mass values for D and F these decays are not possible energetically. The same is true for the decay of D^* and F^*. Only through the small mixing of ω_σ and ω_s in Ψ (see table 1a,b) the physical $\Psi(3095)$ can decay into uncharmed mesons K^+K^- and $K^o\overline{K^o}$. The decay rate for $\Psi \rightarrow K\overline{K}$ is obtained from the general formula

$$\Gamma(V \rightarrow P_1 + P_2) = \frac{2}{3} \frac{g^2_{VP_1P_2}}{4\pi} \frac{p_1^3}{m_V^2} , \qquad (3.3)$$

where p_1 is the center-of-mass momentum of the P_1 meson. With (3.2), (3.3) and the expansion coefficients from tables 1a,b it is straight-forward to calculate the decay rates for $\rho \rightarrow \pi\pi$, $K^* \rightarrow K\pi$, $\Phi \rightarrow K\overline{K}$ and $\Psi \rightarrow K\overline{K}$. The first decay was used to determine the overall coupling constant and the following two decays are given to check the consistency of (3.3) and the mixing of the Φ. The result for the two ρ-masses is shown in table 4 together with known experimental data. We see that (3.2) and (3.3) give the correct relation between $\rho \rightarrow \pi\pi$, $K^* \rightarrow K\pi$ and $\Phi \rightarrow K\overline{K}$ if compared with the experimental results. The decay rate for $\Psi \rightarrow K\overline{K}$ seems to be larger by a factor of 10 than the known upper limit [12] of 0.124 KeV. On the other hand we can expect that the coupling constant relations (3.2) are distorted by symmetry breaking effects, which could account for this disagreement.

We notice from (3.2) that the decays $\phi \to K\bar{K}$ and $\psi \to K\bar{K}$ are proportional to the square of α_8, given in (2.4). Therefore from $\Gamma(\psi \to K^+K^-) < 0.124$ KeV we get the following upper limit for $|\alpha_8^{(\psi)}| < 0.00051$. In version 2 ($m_\rho = 0.76009$ GeV) the partial decay width for $\psi \to K^+K^-$ is 3 eV, much below the experimental upper limit. This value is of the same order as the number one expects for $\psi \to K^+K^-$ via a virtual photon.

(b) V → VP.

The coupling of two vector mesons and one pseudoscalar meson is more involved since now the symmetric coupling d_{abc} occurs. Then also the singlet field couples, contrary to the previous case (3.1) where it did not appear due to the antisymmetric nature of the VPP coupling. We have now four independent SU(4) invariant couplings:

$$H_{VVP} = g_{VVP}\, \epsilon^{\varkappa\lambda\mu\nu}\, d_{abc}\, P^a\, \partial_\varkappa V_\lambda^b\, \partial_\mu V_\nu^c \quad , \tag{3.4}$$

$$H_{VVO} = g_{VVO}\, \epsilon^{\varkappa\lambda\mu\nu}\, d_{obc}\, \eta_o\, \partial_\varkappa V_\lambda^b\, \partial_\mu V_\nu^c \quad , \tag{3.5}$$

$$H_{OVP} = 2 g_{OVP}\, \epsilon^{\varkappa\lambda\mu\nu}\, d_{aoc}\, P^a\, \partial_\varkappa \omega_{o\lambda}\, \partial_\mu V_\nu^c \quad , \tag{3.6}$$

$$H_{OOO} = g_{OOO}\, \epsilon^{\varkappa\lambda\mu\nu}\, d_{ooo}\, \eta_o\, \partial_\varkappa \omega_{o\lambda}\, \partial_\mu \omega_{o\nu} \; . \tag{3.7}$$

As usual $d_{obc} = \frac{1}{\sqrt{2}}\delta_{bc}$ and d_{abc} are the symmetric coefficients defined by [11)]

$$[F_a, F_b]_+ = \frac{1}{4}\delta_{ab}\mathbb{1} + d_{abc}F_c \tag{3.8}$$

and a,b,c = 1,2, ..., 15. With only SU(4) symmetry the four coupling constants appearing in (3.4) to (3.7) are independent. In the following we shall assume that

$$g_{OOO} = g_{OVP} = g_{VVO} = g_{VVP} \tag{3.9}$$

so that all four couplings have the form (3.4) with a,b,c going from 0,1, ... 15.

In the following we shall refer to the relations (3.9) as the ideal mixing assumption for the VVP-coupling.

The relations between coupling constants following from (3.4) are again conveniently written in terms of the ideally mixed fields ω_σ, ω_s and ω_c instead of ω_0, ω_8 and ω_{15}.

The result is:

$$\begin{aligned} H_{VVP} &= g_{VVP}\,\epsilon^{\varkappa\lambda\mu\nu}\sum_{a,b,c=0}^{15} d_{abc}\,P^a\,\partial_\varkappa V^b_\lambda\,\partial_\mu V^c_\nu \\ &= g_{VVP}\,\epsilon^{\varkappa\lambda\mu\nu}\Big\{2\vec{\pi}\cdot\partial_\varkappa\vec{\rho}_\lambda\,\partial_\mu\omega_{\sigma\nu} + \vec{\pi}\cdot\partial_\varkappa K^{*\dagger}_\lambda\vec{\tau}\,\partial_\mu K^*_\nu \\ &\quad + \vec{\pi}\cdot\partial_\varkappa D^{*\dagger}_\lambda\vec{\tau}\,\partial_\mu D^*_\nu + \big[K^\dagger\vec{\tau}\,\partial_\varkappa K^*_\lambda\cdot\partial_\mu\vec{\rho}_\nu \\ &\quad + \sqrt{2}\,K^\dagger\partial_\varkappa D^*_\lambda\,\partial_\mu F^{*+}_\nu + K^\dagger\partial_\varkappa K^*_\lambda\,\partial_\mu(\omega_{\sigma\nu} + \sqrt{2}\,\omega_{s\nu}) + h.c.\big] \\ &\quad + \big[D^\dagger\vec{\tau}\,\partial_\varkappa D^*_\lambda\cdot\partial_\mu\vec{\rho}_\nu + \sqrt{2}\,D^\dagger\partial_\varkappa K^*_\lambda\,\partial_\mu F^{*-}_\nu \\ &\quad + D^\dagger\partial_\varkappa D^*_\lambda\,\partial_\mu(\omega_{\sigma\nu} + \sqrt{2}\,\omega_{c\nu}) + h.c.\big] \end{aligned} \tag{3.10}$$

$$+\sqrt{2}\left[F^- \partial_x \mathcal{D}^{*\dagger}_\lambda \partial_\mu \mathcal{K}^*_\nu + F^- \partial_x F^{*+}_\lambda \partial_\mu(\omega_{s\nu} + \omega_{c\nu}) + h.c.\right]$$

$$+\eta_s \sqrt{2}\left[\partial_x \mathcal{K}^{*\dagger}_\lambda \partial_\mu \mathcal{K}^*_\nu + \partial_x F^{*-}_\nu \partial_\mu F^{*+} + \partial_x \omega_{s\lambda} \partial_\mu \omega_{s\nu}\right]$$

$$+\eta_c \sqrt{2}\left[\partial_x \mathcal{D}^{*\dagger}_\lambda \partial_\mu \mathcal{D}^*_\nu + \partial_x F^{*-}_\lambda \partial_\mu F^{*+}_\nu + \partial_x \omega_{c\lambda} \partial_\mu \omega_{c\nu}\right]$$

$$+\eta_\sigma\left[\partial_x \vec{\rho}_\lambda \cdot \partial_\mu \vec{\rho}_\nu + \partial_x \mathcal{K}^{*\dagger}_\lambda \partial_\mu \mathcal{K}^*_\nu + \partial_x \mathcal{D}^{*\dagger}_\lambda \partial_\mu \mathcal{D}^*_\nu + \partial_x \omega_{\sigma\lambda} \partial_\mu \omega_{\sigma\nu}\right]\Big\}.$$

If we relax the constraint (3.9) we have additional terms which are written down in appendix B.

We now apply (3.10) to various decay modes using the general formula for the decay width of $V_1 \to V_2 + P$:

$$\Gamma(V_1 \to V_2 + P) = \frac{1}{3}\,\frac{g^2_{V_1 V_2 P}}{4\pi}\,p^3 \,, \qquad (3.11)$$

where p is the center-of-mass momentum of the P meson. The kinematically allowed decays are $\phi \to \rho\pi$, $\psi \to \rho\pi$, $\psi \to K^*\bar{K}$, $\psi \to \omega\eta$, $\psi \to \omega\eta'$, $\psi \to \phi\eta$ and $\psi \to \phi\eta'$.

All these decays can occur only via mixing. In particular $\phi \to \rho\pi$ and $\psi \to \rho\pi$ are proportional to the corresponding $(\alpha_\sigma)^2$, whereas $\psi \to K^*\bar{K}$ is proportional to $(\alpha_\sigma^{(\psi)} + \sqrt{2}\,\alpha_s^{(\psi)})^2$ and $\psi \to \omega\eta$ is $\sim (\alpha_\sigma^{(\psi)}\alpha_\sigma^{(\omega)}\alpha_\sigma^{(\eta)} + \sqrt{2}\,\alpha_s^{(\psi)}\alpha_s^{(\omega)}\alpha_s^{(\eta)} + \sqrt{2}\,\alpha_c^{(\psi)}\alpha_c^{(\omega)}\alpha_c^{(\eta)})^2$.

The results are in table 5a,b again for version 1 and 2 concerning the input of the ρ – mass and also changing $\eta' \longrightarrow$ E (which means to use table 2b for the pseudoscalar wavefunctions). In version 1 the partial decay widths of the ψ for both, η' and E choice, come out much too large. In particular $\Gamma(\psi \to \rho\pi)$ is near 50 MeV which is three orders of magnitude larger than the total width of the ψ and presumably a factor 10^5 too large compared to the measured $\Gamma(\psi \to \rho\pi)$. [13] We consider it unreasonable to account for such a large discrepancy by symmetry violations of the coupling constants. One way to overcome this discrepancy in our model is to reduce the admixture of noncharmed quarks in ψ, in particular to reduce $\alpha_{\sigma}^{(\psi)}$. In the last section we have seen that this can be achieved by changing m_{ρ} to 0.76009 GeV (version 2). Indeed the results for this version come out satisfactory (see table 5a,b) . $\Gamma(\psi \to \rho\pi)$ is now 1.73 eV. There is a little difference between the choices η' versus E in both versions. All other decay channels of the VP-type are very small in version 2. In particular for $\psi \to K^{*}\bar{K}$ a very strong cancellation occurs. As a general conclusion of the decays $V \to VP$ we note that version 2, which corresponds to $m_{\rho} = 0.76009$ GeV is clearly preferred over version 1 , which corresponds to a larger ρ-mass. The two-body decays of the vector mesons considered so far should serve as examples what can be expected for the decay width of ψ into particular channels. There are many more decay channels which could be considered along similar lines, if the multiplets involved have been calculated. Good candidates would be the scalar mesons 0^{+} $(\delta, \kappa, \varepsilon, \varepsilon', \psi_s, D_s, F_s)$ or the axial vector mesons related to A_1 or B. The spectra and wave functions have not been calculated yet. But we have computed the tensor meson multiplet A_2, K^{**}, f, f' and ψ_T in section 2. With this we can compute the decay width for $\psi \to K^{**+}K^{-}$.

(c) $V \to TP$

The SU(4) structure of the VTP vertex is identical to the VPP and is given by the antisymmetric coupling (3.1). To determine the coupling constant we use the known decay $A_2^+ \to \rho^+ \pi^o$. Other known decays, which we use for a consistency check are $K^{**+} \to \rho^+ K^o$, $K^{**+} \to \omega K^+$ and $K^{**+} \to K^{*+} \pi^o$. They are calculated from the formula

$$\Gamma(T \to V + P) = \frac{g^2_{TVP}}{2\pi} p^5 . \tag{3.12}$$

Then $\psi \to K^{**+} K^-$ is obtained from

$$\Gamma(V \to T + P) = \frac{g^2_{TVP}}{12\pi} \frac{m^2_\psi}{m^2_{A_2}} p^5 \tag{3.13}$$

if the decay $A_2 \to \rho \pi$ is used for normalization. The results are presented in table 6 for the two ρ-mass versions.
We see that in version 1 the decay $\psi \to K^{**+} K^-$ is rather large. But for the smaller ρ-mass it becomes negligible. All other decays are in agreement with the experimental data, showing that the breaking of the coupling constants inside the SU(3) octuplet is small.

The calculation of the ψ decay is particularly simple if no mixing for the final state particles occurs and the final state couples only to ω_σ and not to ω_s. As we have seen this situation occurs for $\psi \to \rho^+ \pi^-$. The partial width for this decay channel was directly proportional to $(\alpha_\sigma^{(\psi)})^2$ times the $\omega\rho\pi$ coupling constant squared. Similarly for $\psi \to \bar{p} p$ the decay width is:

$$\Gamma(\psi \to \bar{p}p) = \frac{g^2_{\omega\bar{p}p}}{4\pi}\ \frac{2}{3}\ \frac{m_\psi^2 + 2m_p^2}{m_\psi^2}\ p\left(\frac{\alpha_\sigma^{(\psi)}}{\alpha_\sigma^{(\omega)}}\right)^2 \tag{3.14}$$

The coupling constant is fairly known from low-energy proton-proton scattering analysis with one-boson-exchange potentials , from which one obtains $g^2_{\omega\bar{p}p}/4\pi = 10$ [14]. With this we calculate the decay width for $\psi \to \bar{p}p$, in version 1, $\Gamma(\psi \to \bar{p}p)$ = **6.85** MeV and, in version 2, $\Gamma(\psi \to \bar{p}p)$ = **47.8** eV. The result in version 2 agreees fairly well with the experimental result $\Gamma(\psi \to \bar{p}p) = (180 \pm {}^{130}_{70})\ eV$ [12]. The experimental value 180 eV corresponds to $|\alpha_\sigma^{(\psi)}/\alpha_\sigma^{(\omega)}|$ = 0.00012. In the same way one can calculate $\Gamma(\psi \to B^+\pi^-)$ from $\Gamma(B^+ \to \omega\pi^+)$ according to

$$\Gamma(\psi \to B^+\pi^-) = \left(\frac{\alpha_\sigma^{(\psi)}}{\alpha_\sigma^{(\omega)}}\right)^2 \frac{p_B\, m_B^2}{p_\omega\, m_\psi^2}\ \Gamma(B^+ \to \omega\pi^+)\,. \tag{3.15}$$

The result is $\Gamma(\psi \to B^+\pi^-)$ = 49.4 KeV for version 1 and 344 eV in version 2. The input is $\Gamma(B^+ \to \omega\pi^+)$ = 120 MeV [9]. In (3.19) p_B is the center-of-mass momentum in the decay $\psi \to B\pi$ and p_ω is the momentum in the decay $B \to \omega\pi$.

4. Radiative Decays.

In this section we calculate the radiative decays of the vector mesons $V \to P\gamma$ and the two-photon decay of the pseudoscalar mesons, $P \to 2\gamma$. The radiative decay of the ψ is particularly interesting, since the photon has also a $c\bar{c}$ component and as we have seen the pseudoscalar mesons η and $\eta'(E)$ have appreciable $c\bar{c}$ admixture too, so that $\psi \to P\gamma$ for these mesons might be large. To deduce the SU(4) relations for the matrix elements of the electromagnetic current between the V- and P-states, we first use U- and W-spin invariance. As was shown in I the electromagnetic current J is an U- and W-spin scalar. From this the following relations are obtained for $\langle P | J | V \rangle$:

$$\langle \pi^+ | J | \rho^+ \rangle = \langle K^+ | J | K^{*+} \rangle = \langle D^+ | J | D^{*+} \rangle = \langle F^+ | J | F^{*+} \rangle ,$$

$$\langle K^0 | J | K^{*0} \rangle = \frac{1}{2} \left(3 \langle \eta_8 | J | \omega_8 \rangle - \langle \pi^0 | J | \rho^0 \rangle \right) ,$$

$$\langle D^0 | J | D^{*0} \rangle = \frac{1}{2} \left(\langle \eta_8 | J | \omega_8 \rangle + 4 \langle \eta_{15} | J | \omega_{15} \rangle - 3 \langle \pi^0 | J | \rho^0 \rangle \right) ,$$

$$\langle \eta_8 | J | \rho^0 \rangle = \langle \pi^0 | J | \omega_8 \rangle = \frac{\sqrt{3}}{2} \left(\langle \pi^0 | J | \rho^0 \rangle - \langle \eta_8 | J | \omega_8 \rangle \right) , \tag{4.1}$$

$$\langle \eta_8 | J | \omega_{15} \rangle = \langle \eta_{15} | J | \omega_8 \rangle = \frac{1}{\sqrt{3}} \langle \eta_{15} | J | \rho^0 \rangle = \frac{1}{\sqrt{3}} \langle \pi^0 | J | \omega_{15} \rangle =$$
$$= \frac{1}{2\sqrt{2}} \left(\langle \eta_8 | J | \omega_8 \rangle + 2 \langle \eta_{15} | J | \omega_{15} \rangle - 3 \langle \pi^0 | J | \rho^0 \rangle \right) ,$$

$$\langle \eta_0 | J | \rho^0 \rangle = \sqrt{3} \langle \eta_0 | J | \omega_8 \rangle = -\sqrt{\frac{3}{2}} \langle \eta_0 | J | \omega_{15} \rangle ,$$

$$\langle \pi^0 | J | \omega_0 \rangle = \sqrt{3} \langle \eta_8 | J | \omega_0 \rangle = -\sqrt{\frac{3}{2}} \langle \eta_{15} | J | \omega_0 \rangle .$$

These relations are valid for any power of J. Therefore they are also correct including higher orders of electromagnetic interactions. Two more relations are derived from the property that J is a linear combination of an isoscalar and an isovector term, i.e.

$$\langle \pi^+ | J | \rho^+ \rangle = \langle \pi^0 | J | \rho^0 \rangle = - \langle \eta_8 | J | \omega_8 \rangle \tag{4.2}$$

A further relation can be obtained from the general structure of the matrix elements of J, given in I(4.5):

$$J = e_o + 3e_1 \, (\vec{W}^2 - \vec{U}^2) - 3fe_1 Q \tag{4.3}$$

Because of charge conjugation invariance only the first two terms in (4.3) contribute to $V \to P + \gamma$.
From (4.3) we get an additional relation

$$\langle D^o | J | D^{*o} \rangle = 4 \langle D^+ | J | D^{*+} \rangle \tag{4.4}$$

This way all matrix elements $\langle P | J | V \rangle$ with P and V belonging to the fifteenplet are expressed by $\langle \pi^o | J | \rho^o \rangle$.
To calculate all $V \to P + \gamma$ transition matrix elements including singlet -fifteenplet mixing we need further relations for $\langle \eta_o | J | \omega_o \rangle$, $\langle \eta_o | J | \rho_o \rangle$ and $\langle \pi^o | J | \omega_o \rangle$.
Formulas which relate these matrix elements to $\langle \pi^o | J | \rho^o \rangle$ cannot be deduced from the general properties of J, as U- and W-spin and I-spin decomposition. But we know from I(4.1) that

$$J \sim \rho^o + \frac{1}{\sqrt{3}} \omega_8 - \sqrt{\frac{2}{3}} \omega_{15} + \frac{\sqrt{2}}{3} \omega_o \tag{4.5}$$

Then we can express the matrix elements of J by matrix elements of ρ^o, ω_8, ω_{15} and ω_o . For the latter we can use our relations for the VVP couplings, considered in section 3. Then from SU(4) invariance and ideal mixing considered there we have:

$$\langle \eta_0 | J | \rho^0 \rangle = \langle \pi^0 | J | \omega_0 \rangle = \frac{3}{\sqrt{2}} \langle \pi^0 | J | \rho^0 \rangle \,, \qquad (4.6)$$
$$\langle \eta_0 | J | \omega_0 \rangle = \langle \pi^0 | J | \rho^0 \rangle \,.$$

This way all matrix elements $\langle P | J | V \rangle$ are expressible by $\langle \pi^0 | J | \rho^0 \rangle$. The radiative widths are calculated from the general formula

$$\Gamma(V \to P + \gamma) = \frac{1}{3} \frac{g^2_{VP\gamma}}{4\pi} p^3 \,, \qquad (4.7)$$

where p is the center-of-mass momentum of the P meson.
The results are shown in table 7a,b again for the two ρ-mass versions 1 and 2 and with η' or E . The decay widths are normalized to $\Gamma(\omega \to \pi^0 + \gamma)$ = 870 keV [9]. We see that with η' (0.9576) the radiative partial decay widths of ψ are rather large, up to 4.6 MeV for $\psi \to \eta' \gamma$. Of course, we must expect some breaking effects of the coupling constants. For example , if we modify the general formulas by the factor $(m_\omega / m_\psi)^2$, the width for $\psi \to \eta' \gamma$ is reduced to 295 KeV. The same reductions occur for $\psi \to \eta \gamma$ and $\psi \to \psi_P \gamma$.
In case the η' is a 0^- particle and belongs to the same SU(4) multiplet as π, K, η we must conclude that an appreciable fraction of the ψ decay goes into $\eta' \gamma$ and $\psi_P \gamma$.
A further test for the mixing of η and η' in this context are the decays $\rho^0 \to \eta \gamma$, $\eta' \to \rho^0 \gamma$ and $\eta' \to \omega \gamma$.
The corresponding widths have in both versions 1 and 2 the values 55 KeV, 105 KeV and 12 KeV, respectively.
Unfortunately only an upper limit for $\Gamma(\eta' \to \rho^0 \gamma) < 274$ KeV is

known[9]. It seems that the known experimental upper limit for $\Gamma(\psi \to \eta\gamma)$ around 1 KeV is already in disagreement with our value even if the SU(4) breaking factor $(m_\omega / m_\psi)^2$ is included. The situation is much improved with the E assignment as partner of η (see table 7b). Here $\psi \to \eta\gamma$ comes out around 3 KeV which is in fair agreement with the experimental upper limit if breaking effects are included. $\psi \to E\gamma$ is still larger. $\psi \to \psi_P\gamma$ is much reduced because less phase space is available with $m_{\psi_P} \approx 3.0$ GeV. So also in this version an appreciable fraction of the ψ decay should go into hadrons and one single photon.

We also considered the two-photon decay of the pseudoscalar mesons $\eta \to 2\gamma$, $\eta' \to 2\gamma$ and $\psi_P \to 2\gamma$ on one side, and $\eta \to 2\gamma$, $E \to 2\gamma$ and $\psi_P \to 2\gamma$ for the other assignment. As input we use the known width for $\pi^o \to 2\gamma$: $\Gamma(\pi^o \to 2\gamma) = 7.71$ eV [9]. According to U- and W-spin invariance we have the following relations:

$$\langle 2\gamma | \pi^o \rangle = \sqrt{3}\, \langle 2\gamma | \eta_8 \rangle = -\sqrt{\frac{3}{2}}\, \langle 2\gamma | \eta_{15} \rangle . \qquad (4.8)$$

Using the representation (4.5) for the electromagnetic current and the relations (4.1), (4.2) and (4.6) for the $\langle P | J | V \rangle$ matrix elements we obtain one further relation for the SU(4) singlet component η_o:

$$\langle 2\gamma | \eta_o \rangle = \frac{5\sqrt{2}}{3}\, \langle 2\gamma | \pi^o \rangle . \qquad (4.9)$$

With these relations we calculated the decay widths as exhibited in table 8, again for the two choices of the ρ-mass, which differ only very little. We see that in both assignments, η' and E, the values for $\Gamma(\eta \to 2\gamma)$ are in good agreement with a new experimental value [15]. Up to this point all our calculations for radiative decays, $V \to P\gamma$ and $P \to 2\gamma$ are based on full SU(4) symmetric couplings. This is equivalent to the assumption that the VVP couplings are in their SU(4) symmetric form as in (3.10) and the coupling of the photon to the vector mesons ρ^0, ω, ϕ and Ψ is given by (4.5).

From the experimental measured leptonic decays of these vector mesons we know that the $V-\gamma$ couplings deviate from their SU(4) symmetric values:

$$f_\rho^{-1} : f_\omega^{-1} : f_\phi^{-1} : f_\Psi^{-1} = 1 : \frac{1}{3} : -\frac{\sqrt{2}}{3} : \frac{2\sqrt{2}}{3} \quad , \tag{4.10}$$

where f_V^{-1} is defined as usual $\langle \gamma | V \rangle = e\, m_V^2\, f_V^{-1}$. From the known experimental leptonic decay widths [$\Gamma(\rho \to e^+e^-) = 6.45$ KeV, $\Gamma(\omega \to e^+e^-) = 0.76$ KeV, $\Gamma(\phi \to e^+e^-) = 1.34$ KeV and $\Gamma(\Psi \to e^+e^-) = 4.8$ KeV [9) 13)]] we have instead of (4.10):

$$f_\rho^{-1} : f_\omega^{-1} : f_\phi^{-1} : f_\Psi^{-1} = 1 : 0.34 : -0.40 : 0.43 \tag{4.11}$$

To see the influence of the symmetry violations implied by (4.11) we recalculated the decays $V \to P\gamma$ with the usual vector dominance model . The results are also shown in table 7a and 7b. We see that the radiative rates are reduced, the reduction factors are 0.68 for ϕ decays and 0.20 for ψ decays. With this reduction the radiative decays of the ψ in the version, where the E is the partner of the η , are consistent with the known experimental information on $\psi \to \eta \gamma$[12] and that the total width of the ψ is only 69 KeV.[13]

5. Conclusions

In this paper we tried to understand the extreme smallness of the two-body decays of the new ψ (3.1) resonance in the framework of broken SU(4) with SU(4) invariant three-meson couplings.
We saw that the wave functions of those vector mesons, which are mixtures of SU(4) singlet and 15-plet, depend very sensitively on the input masses, particularly the ρ-mass. With the breaking parameter α as obtained for the vector mesons, the spectra of pseudoscalar and tensor mesons were determined.
The tensor mesons come out near to ideal mixing and ψ_T, the tensor analog of the ψ, has a mass around 3.8 GeV, above the mass of the ψ (3.7). With the pseudoscalar mesons there is a problem, since two candidates are available as partners of the η , the η' (0.958) and the E (1.416). In the η' version the ψ_P , the pseudoscalar analog of the ψ , has a mass around 2.7 GeV and the wave functions are far from being ideally mixed. This has the consequence that the radiative decays $\psi \to P \gamma$ ($P = \eta, \eta', \psi_P$) come out rather large, inconsistent with the experimentally known total width of the ψ . Therefore with the assumptions concerning symmetry breaking made in this work a model based on charm and

with the $\eta'(0.958)$ as a member of the pseudoscalar multiplet is in trouble. In the same model with E (1.416) belonging to the pseudoscalar multiplet the radiative widths of $\psi \to P\gamma$ ($P = \eta, E, \psi_P$) come out much smaller, not inconsistent with presently available bounds for $\psi \to \eta\gamma$ and with the total width of the ψ . The width for $\psi \to E\gamma$ is presumably still toolarge. Regarding the strong decays the ρ-mass can be chosen within reasonable limits in such a way that the admixture of noncharmed quarks in the ψ is of the order of 10^{-4}. This is sufficient to reduce the strong two-body decays of the ψ to a level consistent with available experimental data.

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Appendix A

Here we give the connection between the physical fields and the fields transforming irreducibly under SU(4). For definiteness let us consider the pseudoscalar mesons first.
In terms of the quark fields q^i =(u,d, s,c) and the antiquark fields $\bar{q}_i = (q^i)^\dagger$ =($\bar{u}$, $\bar{d}$, $\bar{s}$, $\bar{c}$) we define sixteen meson fields $P^i{}_j$ by

$$P^i{}_j = q^i \bar{q}_j \qquad (i,j = 1,2,3,4). \tag{A.1}$$

Since the $P^i{}_j$'s define an hermitian 4x4 - matrix $(\hat{P})^i{}_j$, we may expand $\hat{P}$ in terms of a complete set of sixteen hermitian 4x4-matrices $\lambda_0, \lambda_1, \ldots, \lambda_{15}$ according to

$$P^i{}_j = \frac{1}{\sqrt{2}} \sum_{a=0}^{15} P_a (\lambda_a)^i{}_j . \tag{A.2}$$

The "coefficients" $P_a = \frac{1}{\sqrt{2}} Sp(\hat{P}\lambda_a)$ are the fields used in (3.1) and (3.4) - (3.7) to define the corresponding couplings.

The matrices λ_a have the following properties:

$$\lambda_a^\dagger = \lambda_a \qquad (a = 0,1, \ldots, 15) \tag{A.3}$$

$$Sp\,\lambda_a = \begin{cases} 0 & a = 1,2,\ldots,15 \\ 2\sqrt{2} & a = 0 \end{cases} \tag{A.4}$$

$$Sp(\lambda_a \lambda_b) = 2\,\delta_{ab} \qquad (a,b = 0,1,\ldots,15) \tag{A.5}$$

$$\left[\tfrac{1}{2}\lambda_a, \tfrac{1}{2}\lambda_b\right]_- = \sum_{c=0}^{15} i f_{abc}\, \tfrac{1}{2}\lambda_c \qquad (a,b = 0,1, \ldots 15) \tag{A.6}$$

$$\left[\tfrac{1}{2}\lambda_a, \tfrac{1}{2}\lambda_b\right]_+ = \sum_{c=0}^{15} i\, d_{abc}\, \tfrac{1}{2}\lambda_c \qquad (a,b=0,1,\dots,15) \quad \text{(A.7)}$$

where the representation independent constants f_{abc} (with $f_{ab0}=0$) are completely antisymmetric and the d_{abc}'s (with $d_{ab0}=\frac{1}{\sqrt{2}}\delta_{ab}$) are completely symmetric in all their indices. Explicit values of f_{abc} and d_{abc} can be found, for example, in ref. [11]. A convenient realization (choice) of the matrices λ_a (with $\lambda_0 = \frac{1}{\sqrt{2}}\mathbb{1}$), to which we adhere in this paper, was given in ref. [16].

From the above relations and with the charge assignment (2/3, - 1/3, -1/3, 2/3) for the quarks (u, d, s, c), we obtain:

$$\begin{aligned}
\pi^+ &= u\bar{d} = P^1{}_2 = \tfrac{1}{\sqrt{2}}(P_1 - iP_2)\,,\\
\pi^- &= d\bar{u} = P^2{}_1 = \tfrac{1}{\sqrt{2}}(P_1 + iP_2)\,,\\
\pi^0 &= \tfrac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) = \tfrac{1}{\sqrt{2}}(P^1{}_1 - P^2{}_2) = P_3\,,\\
K^+ &= u\bar{s} = P^1{}_3 = \tfrac{1}{\sqrt{2}}(P_4 - iP_5)\,,\\
K^- &= s\bar{u} = P^3{}_1 = \tfrac{1}{\sqrt{2}}(P_4 + iP_5)\,,\\
K^0 &= d\bar{s} = P^2{}_3 = \tfrac{1}{\sqrt{2}}(P_6 - iP_7)\,,\\
\overline{K^0} &= s\bar{d} = P^3{}_2 = \tfrac{1}{\sqrt{2}}(P_6 + iP_7)\,,\\
\overline{D^0} &= u\bar{c} = P^1{}_4 = \tfrac{1}{\sqrt{2}}(P_9 - iP_{10})\,,\\
D^0 &= c\bar{u} = P^4{}_1 = \tfrac{1}{\sqrt{2}}(P_9 + iP_{10})\,,\\
D^- &= d\bar{c} = P^2{}_4 = \tfrac{1}{\sqrt{2}}(P_{11} - iP_{12})\,, \qquad \text{(A.8)}\\
D^+ &= c\bar{d} = P^4{}_2 = \tfrac{1}{\sqrt{2}}(P_{11} + iP_{12})\,,\\
F^- &= s\bar{c} = P^3{}_4 = \tfrac{1}{\sqrt{2}}(P_{13} - iP_{14})\,,\\
F^+ &= c\bar{s} = P^4{}_3 = \tfrac{1}{\sqrt{2}}(P_{13} + iP_{14})\,,\\
\eta_8 &= \tfrac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) = \tfrac{1}{\sqrt{6}}(P^1{}_1 + P^2{}_2 - 2P^3{}_3) = P_8\,,\\
\eta_{15} &= \tfrac{1}{\sqrt{12}}(u\bar{u} + d\bar{d} + s\bar{s} - 3c\bar{c}) = \tfrac{1}{\sqrt{12}}(P^1{}_1 + P^2{}_2 + P^3{}_3 - 3P^4{}_4) = P_{15}\,,\\
\eta_0 &= \tfrac{1}{2}(u\bar{u} + d\bar{d} + s\bar{s} + c\bar{c}) = \tfrac{1}{2}(P^1{}_1 + P^2{}_2 + P^3{}_3 + P^4{}_4) = P_0\,.
\end{aligned}$$

Instead of the fields η_8, η_{15} and η_0 we can equivalently use the fields η_σ, η_s and η_c defined by

$$\begin{aligned}
\eta_\sigma &= \tfrac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) = \tfrac{1}{\sqrt{2}}(P^1{}_1 + P^2{}_2) = \tfrac{1}{\sqrt{3}}P_8 + \tfrac{1}{\sqrt{6}}P_{15} + \tfrac{1}{\sqrt{2}}P_0 \,, \\
\eta_s &= s\bar{s} = P^3{}_3 = -\sqrt{\tfrac{2}{3}}\,P_8 + \tfrac{1}{\sqrt{12}}P_{15} + \tfrac{1}{2}P_0 \,, \\
\eta_c &= c\bar{c} = P^4{}_4 = -\tfrac{\sqrt{3}}{2}P_{15} + \tfrac{1}{2}P_0 \,.
\end{aligned} \tag{A.9}$$

Since $P_a = \eta_a$ for a = 8,15,0, these relations give $\eta_\sigma, \eta_s, \eta_c$ in terms of $\eta_8, \eta_{15}, \eta_0$. The invers relations are:

$$\begin{aligned}
\eta_8 &= \tfrac{1}{\sqrt{3}}\eta_\sigma - \sqrt{\tfrac{2}{3}}\,\eta_s \,, \\
\eta_{15} &= \tfrac{1}{\sqrt{6}}\eta_\sigma + \tfrac{1}{\sqrt{12}}\eta_s - \tfrac{\sqrt{3}}{2}\eta_c \,, \\
\eta_0 &= \tfrac{1}{\sqrt{2}}\eta_\sigma + \tfrac{1}{2}\eta_s + \tfrac{1}{2}\eta_c \,.
\end{aligned} \tag{A.10}$$

Finally, we note that the meson fields (A.1) do not transform irreducibly under SU(4); but rather decompose into a singlet and a fifteenplet, according to the identity

$$P^i{}_j = \tfrac{1}{4}(q^k\bar{q}_k)\delta^i_j + \left(q^i\bar{q}_j - \tfrac{1}{4}(q^k\bar{q}_k)\delta^i_j\right). \tag{A.11}$$

The mixing of the singlet ($\eta_0 = P_0 = \frac{1}{2}q^k\bar{q}_k$) and the fifteenplet ($P_1, P_2, \ldots, P_{15}$) implied by (A.11) is known as ideal mixing, and $\eta_\sigma, \eta_s, \eta_c$ - as the ideally mixed fields.
All relations obtained so far are valid for any meson multiplet. Thus, for the vector and tensor meson multiplets we have only to make the following formal substitutions:

$$\begin{aligned}
(\pi, K, \eta_8, \eta_{15}, D, F; \eta_0) &\to (\rho, K^*, \omega_8, \omega_{15}, D^*, F^*; \omega_0) \qquad \text{(A.12)} \\
&\to (A_2, K^{**}, \omega_{T8}, \omega_{T15}, D^{**}, F^{**}; \omega_{T0}).
\end{aligned}$$

Appendix B

In this appendix we give the explicit expressions of the couplings appearing in (3.4) - (3.6):

$$
\begin{aligned}
H_{VVP} = g_{VVP}\,\epsilon^{\varkappa\lambda\mu\nu} \Big\{ & \vec{\pi}\cdot\partial_\varkappa \vec{\rho}_\lambda\, \partial_\mu \Big(\tfrac{2}{\sqrt{3}}\omega_{8\nu} + \sqrt{\tfrac{2}{3}}\,\omega_{15\nu} \Big) \\
& + \vec{\pi}\cdot\partial_\varkappa K^{*\dagger}_\lambda \vec{\tau}\,\partial_\mu K^*_\nu + \vec{\pi}\cdot\partial_\varkappa D^{*\dagger}_\lambda \vec{\tau}\,\partial_\mu D^*_\nu \\
& + \Big[K^\dagger \vec{\tau}\,\partial_\varkappa K^*_\lambda \cdot \partial_\mu \vec{\rho}_\nu + \sqrt{2}\, K^\dagger \partial_\varkappa D^*_\lambda\, \partial_\mu F^{*+}_\nu \\
& + K^\dagger \partial_\varkappa K^*_\lambda\, \partial_\mu \Big(-\tfrac{1}{\sqrt{3}}\omega_{8\nu} + \sqrt{\tfrac{2}{3}}\,\omega_{15\nu} \Big) + h.c. \Big] \\
& + \Big[D^\dagger \vec{\tau}\,\partial_\varkappa D^*_\lambda \cdot \partial_\mu \vec{\rho}_\nu + \sqrt{2}\, D^\dagger \partial_\varkappa K^*_\lambda\, \partial_\mu F^{*-}_\nu \\
& - D^\dagger \partial_\varkappa D^*_\lambda\, \partial_\mu \Big(-\tfrac{1}{\sqrt{3}}\omega_{8\nu} + \sqrt{\tfrac{2}{3}}\,\omega_{15\nu} \Big) + h.c. \Big] \qquad\qquad (B.1) \\
& + \Big[\sqrt{2}\, F^- \partial_\varkappa D^{*\dagger}_\lambda\, \partial_\mu K^*_\nu - F^- \partial_\varkappa F^{*+}_\lambda\, \partial_\mu \Big(\tfrac{2}{\sqrt{3}}\omega_{8\nu} + \sqrt{\tfrac{2}{3}}\,\omega_{15\nu} \Big) + h.c \Big] \\
& + \eta_8 \Big[\tfrac{1}{\sqrt{3}}\, \partial_\varkappa \vec{\rho}_\lambda \cdot \partial_\mu \vec{\rho}_\nu - \tfrac{1}{\sqrt{3}}\, \partial_\varkappa K^{*\dagger}_\lambda\, \partial_\mu K^*_\nu + \tfrac{1}{\sqrt{3}}\, \partial_\varkappa D^{*\dagger}_\lambda\, \partial_\mu D^*_\nu \\
& - \tfrac{2}{\sqrt{3}}\, \partial_\varkappa F^{*-}_\lambda\, \partial_\mu F^{*+}_\nu + \partial_\varkappa \omega_{8\lambda}\, \partial_\mu \Big(-\tfrac{1}{\sqrt{3}}\omega_{8\nu} + \sqrt{\tfrac{2}{3}}\,\omega_{15\nu} \Big) \Big] \\
& + \eta_{15} \Big[\tfrac{1}{\sqrt{6}}\, \partial_\varkappa \vec{\rho}_\lambda \cdot \partial_\mu \vec{\rho}_\nu + \sqrt{\tfrac{2}{3}}\, \partial_\varkappa K^{*\dagger}_\lambda\, \partial_\mu K^*_\nu - \sqrt{\tfrac{2}{3}}\, \partial_\varkappa D^{*\dagger}_\lambda\, \partial_\mu D^*_\nu \\
& - \sqrt{\tfrac{2}{3}}\, \partial_\varkappa F^{*-}_\lambda\, \partial_\mu F^{*+}_\nu + \tfrac{1}{\sqrt{6}}\, \partial_\varkappa \omega_{8\lambda}\, \partial_\mu \omega_{8\nu} \\
& - \sqrt{\tfrac{2}{3}}\, \partial_\varkappa \omega_{15\lambda}\, \partial_\mu \omega_{15\nu} \Big] \Big\} ,
\end{aligned}
$$

$$H_{VVO} = \frac{1}{\sqrt{2}} g_{VVO} \, \epsilon^{\kappa\lambda\mu\nu} \, \eta_0 \Big\{ \partial_\kappa \vec{\rho}_\lambda \cdot \partial_\mu \vec{\rho}_\nu$$

$$+ 2 \partial_\kappa \mathcal{K}^{*\dagger}_\lambda \partial_\mu \mathcal{K}^*_\nu + 2 \partial_\kappa \mathcal{D}^{*\dagger}_\lambda \partial_\mu \mathcal{D}^*_\nu \tag{B.2}$$

$$+ 2 \partial_\kappa F^{*+}_\lambda \partial_\mu F^{*-}_\nu + \partial_\kappa \omega_{8\lambda} \partial_\mu \omega_{8\nu}$$

$$+ \partial_\kappa \omega_{15\lambda} \partial_\mu \omega_{15\nu} \Big\} \; ,$$

$$H_{OVP} = \sqrt{2} \, g_{OVP} \, \epsilon^{\kappa\lambda\mu\nu} \, \partial_\kappa \omega_{0\lambda} \Big\{ \vec{\pi} \cdot \partial_\mu \vec{\rho}_\nu$$

$$+ \left[\mathcal{K}^\dagger \partial_\mu \mathcal{K}^*_\nu + \mathcal{D}^\dagger \partial_\mu \mathcal{D}^*_\nu + F^- \partial_\mu F^{*+}_\nu + h.c. \right] \tag{B.3}$$

$$+ \eta_8 \, \partial_\mu \omega_{8\nu} + \eta_{15} \, \partial_\mu \omega_{15\nu} \Big\} \; .$$

Table and Figure Captions:

Table 1: Predicted masses of charmed vector mesons D^* and F^* and wave functions for mixed states ω, ϕ, ψ . Concerning the input masses we have, wherever required, averaged over the masses of particles in an isospin multiplet. (1) and (2) denote two versions concerning the input mass m_ρ. α_1 and α_2 are the corresponding breaking parameters as defined in I (3.6). All masses are in GeV.

Table 2a: The same as table 1 for pseudoscalar mesons with input π, K, η and $\eta'(0.958)$ and α_1 and α_2 respectively.

Table 2b: The same as table 2a with E (1.416) instead of $\eta'(0.958)$ as input.

Table 3: The same as table 1 for tensor mesons with input A_2, K^{**}, f', f and α_1 and α_2 respectively.

Table 4: Predictions of the decay width for $\psi \to K^+K^-$ for the two ρ-mass versions. The other decays $K^{*+} \to K^+\pi^o$ and $\phi \to K^+K^-$ are to check SU(3) breaking and the wave functions of the ϕ. The decay width $\rho^o \to \pi^+\pi^-$ serves as normalization input.

Table 5a: Predictions of the decay widths for $\psi \to VP$ with $\eta'(0.958)$ as partner of η, normalized to the width of $\phi \to \rho^+\pi^-$.

Table 5b: Same as table 5a with η' replaced by E.

Table 6: Prediction of the decay width for $\psi \to K^{**+}K^-$ normalized

to the decay width of $A_2^+ \to \rho^+ \pi^o$. The other channels are to check SU(3) breaking.

Table 7a: Radiative decay widths $\Psi \to P\gamma$ with $\eta'(0.958)$ as partner of η and with $\Gamma(\omega \to \pi^o\gamma)$ used as input.

Table 7b: Radiative decay widths $\Psi \to P\gamma$ with E(1.416) as partner of η and with $\Gamma(\omega \to \pi^o\gamma)$ used as input.

Table 8: Two-photon decay widths of η, η' and Ψ_P, and η, E and Ψ_P based on wave functions in table 2a and table 2b, respectively. As input we used $\Gamma(\pi^o \to 2\gamma)$ = 7.71 eV.

Fig. 1: Mixing parameters $\alpha_\sigma^{(\Psi)}$ and $\alpha_s^{(\Psi)}$ as a function of the ρ-mass in the vicinity of m_ρ = 0.76 GeV. The arrow denotes the ρ-mass chosen in version 2.

Fig. 2: Pseudoscalar, vector and tensor meson masses for charmed mesons D and F and new mesons Ψ as a function of mass breaking parameter α . Ψ_P is the new I = 0 pseudoscalar state together with η, η', whereas Ψ_{PE} is the compagnon of η and E. Otherwise notations of particles as in tables 1-3.

Table 1

1)

$m_{\omega} = 0.7827$, $m_{\phi} = 1.0197$, $m_{\psi} = 3.095$, $m_{K*} = 0.89435$

$m_{\varrho} = 0.7674$

$\alpha_1 = 21.42010$, $m_{D*} = 2.218$ $m_{F*} = 2.265$

	α_σ	α_s	α_c
ω	0.99747	-0.06646	-0.02507
ϕ	0.06590	0.99758	-0.02228
ψ	0.02649	0.02057	0.99944

2)

$m_{\omega} = 0.7827$, $m_{\phi} = 1.0197$, $m_{\psi} = 3.095$, $m_{K*} = 0.89435$

$m_{\varrho} = 0.76009$

$\alpha_2 = 21.17838$, $m_{D*} = 2.255$ $m_{F*} = 2.304$

	α_σ	α_s	α_c
ω	0.99836	-0.05727	-0.00008
ϕ	0.05727	0.99836	0.00004
ψ	0.00007	-0.00005	1.00000

Table 2a

1)

$m_{\pi} = 0.13803$, $m_K = 0.4957$, $m_{\eta} = 0.5488$, $m_{\eta'} = 0.9576$

$\alpha_1 = 21.42010$

$m_{\psi_P} = 2.744$, $m_D = 2.162$, $m_F = 2.213$

	α_σ	α_s	α_c
η'	0.67672	0.69468	0.24386
η	0.71016	-0.70328	-0.03269
ψ_P	-0.19421	-0.15106	0.96926

2)

$m_{\pi} = 0.13803$, $m_K = 0.4957$, $m_{\eta} = 0.5488$, $m_{\eta'} = 0.9576$

$\alpha_2 = 21.17838$

$m_{\psi_P} = 2.729$, $m_D = 2.150$, $m_F = 2.202$

	α_σ	α_s	α_c
η'	0.67679	0.69467	0.24370
η	0.71016	-0.70329	-0.03252
ψ_P	-0.19398	-0.15106	0.9631

Table 2b

1)

$m_{\pi} = 0.13803$, $m_K = 0.4957$, $m_{\eta} = 0.5488$, $m_E = 1.416$

$\alpha_1 = 21.42010$

$m_{\psi_P} = 3.013$, $m_D = 2.162$, $m_F = 2.213$

	α_σ	α_s	α_c
E	0.74825	0.66206	-0.04240
η	-0.66246	0.74908	0.00574
ψ_P	0.03556	0.02379	0.99908

2)

$m_{\pi} = 0.13803$, $m_K = 0.4957$, $m_{\eta} = 0.5488$, $m_E = 1.416$

$\alpha_2 = 21.17838$

$m_{\psi_P} = 2.998$, $m_D = 2.150$, $m_F = 2.202$

	α_σ	α_s	α_c
E	0.74819	0.66201	-0.04420
η	-0.66245	0.74908	0.00591
ψ_P	0.03702	0.02485	0.99901

Table 3

1)

$m_{A_2} = 1.3100$, $m_{K^{**}} = 1.4200$, $m_{f'} = 1.5161$, $m_f = 1.2672$

$\alpha_1 = 21.42010$

$m_{\psi_T} = 3.830$, $m_{D^{**}} = 2.807$, $m_{F^{**}} = 2.860$

	α_σ	α_s	α_c
f	0.99654	0.07672	-0.03201
f'	-0.07743	0.99677	-0.02141
ψ_T	0.03027	0.02382	0.99926

2)

$m_{A_2} = 1.3100$, $m_{K^{**}} = 1.4200$, $m_{f'} = 1.5161$, $m_f = 1.2672$

$\alpha_2 = 21.17838$

$m_{\psi_T} = 3.812$, $m_{D^{**}} = 2.795$, $m_{F^{**}} = 2.848$

	α_σ	α_s	α_c
f	0.99654	0.07672	-0.03194
f'	-0.07743	0.99677	-0.02141
ψ_T	0.03020	0.02379	0.99926

Table 4

$V \longrightarrow P_1P_2$	$\Gamma(V \to P_1P_2)$ (1)	$\Gamma(V \to P_1P_2)$ (2)	$\Gamma(V \to P_1P_2)$ (exp)
$\rho^0 \to \pi^+\pi^-$	150.4 MeV	150.4 MeV	(150.4 ± 2.9)MeV
$K^{*+} \to K^+\pi^0$	14.8 KeV	14.8 KeV	(16.6 ± 0.3) KeV
$\phi \to K^+K^-$	1.73 MeV	1.76 MeV	(1.94 ± 0.19) MeV
$\psi \to K^+K^-$	1.07 KeV	3 eV	$<$ 124 eV

Table 5a

$V_1 \to V_2P$	$\Gamma(V_1 \to V_2P)$ (1)	$\Gamma(V_1 \to V_2P)$ (2)	$\Gamma(V_1 \to V_2P)$ (exp)
$\phi \to \rho^+ \pi^-$	223.0 KeV	223 KeV	(223 $\pm$ 6) KeV
$\psi \to \rho^+ \pi^-$	17.31 MeV	173 eV	(138 $\pm$ 85) eV
$\psi \to K^{*+} K^-$	16.17 MeV	< 1 eV	
$\psi \to \omega \eta'$	0.26 MeV	3.8eV	
$\psi \to \omega \eta$	1.97 MeV	15 eV	
$\psi \to \phi \eta'$	0.64 MeV	3.8 eV	
$\psi \to \phi \eta$	1.89 MeV	17 eV	

Table 5b

$V_1 \to V_2P$	$\Gamma\,(V_1 \to V_2P)$ (1)	$\Gamma\,(V_1 \to V_2P)$ (2)	$\Gamma\,(V_1 \to V_2P)$ (exp)
$\phi \to \rho^+\pi^-$	223 KeV	223 KeV	(223 ± 6) KeV
$\psi \to \rho^+\pi^-$	17.31 Mev	173 eV	(138 ± 85) eV
$\psi \to K^{*+}K^-$	16.17 MeV	<1 eV	
$\psi \to \omega E$	0.98 MeV	12 eV	
$\psi \to \omega\eta$	2.01 MeV	15 eV	
$\psi \to \phi E$	0.82 MeV	5 eV	
$\psi \to \phi\eta$	1.93 MeV	17 eV	

Table 6

$T \rightarrow VP$	$\Gamma(T \rightarrow VP)$ (1)	$\Gamma(T \rightarrow VP)$ (2)	$\Gamma(T \rightarrow VP)$ (exp)
$A_2^+ \rightarrow \rho^+ \pi^0$	35.7 MeV	35.7 MeV	(35.7 ± 4.6) MeV
$f' \rightarrow K^{*+} K^-$	1.17 MeV	1.17 MeV	$<$ 7 MeV
$K^{**+} \rightarrow \rho^+ K^0$	4.87 MeV	4.87 MeV	(4.6 ± 1.8) MeV
$K^{**+} \rightarrow \omega K^+$	2.26 MeV	2.49 MeV	(4.4 ± 2.3) MeV
$K^{**+} \rightarrow K^{*+} \pi^0$	8.79 MeV	8.79 MeV	(14.8 ± 2.8) MeV
$\psi \rightarrow K^{**+} K^-$	95.3 KeV	280 eV	

Table 7a

$V \to P\gamma$	$\Gamma(V \to P\gamma)$ (1)	$\Gamma(V \to P\gamma)$ (2)	$\Gamma(V \to P\gamma)$ (2),(VDM)	$\Gamma(V \to P\gamma)$ (exp)
$\omega \to \pi^0 \gamma$	870 KeV	870 KeV	870 KeV	(870 ± 86) KeV
$\phi \to \pi^0 \gamma$	8.72 KeV	6.57 KeV	4.46 KeV	$<$ 14.7 KeV
$\psi \to \pi^0 \gamma$	41.31 KeV	$<$ 1 eV	$<$ 1 eV	$<$ 380 eV
$\omega \to \eta \gamma$	5.23 KeV	5.51 KeV	5.51 KeV	$<$ 50 KeV
$\phi \to \eta' \gamma$	0.72 KeV	0.70 KeV	0.48 KeV	
$\phi \to \eta \gamma$	176.52 KeV	175.70 KeV	119.2 KeV	(125 ± 43) KeV
$\psi \to \eta' \gamma$	4.52 MeV	4.61 MeV	921 KeV	
$\psi \to \eta \gamma$	190.22 KeV	100.91 KeV	20.16 KeV	(0.1 - 2.0) KeV
$\psi \to \psi_P \gamma$	0.97 MeV	1.09 MeV	217.8 KeV	

Table 7b

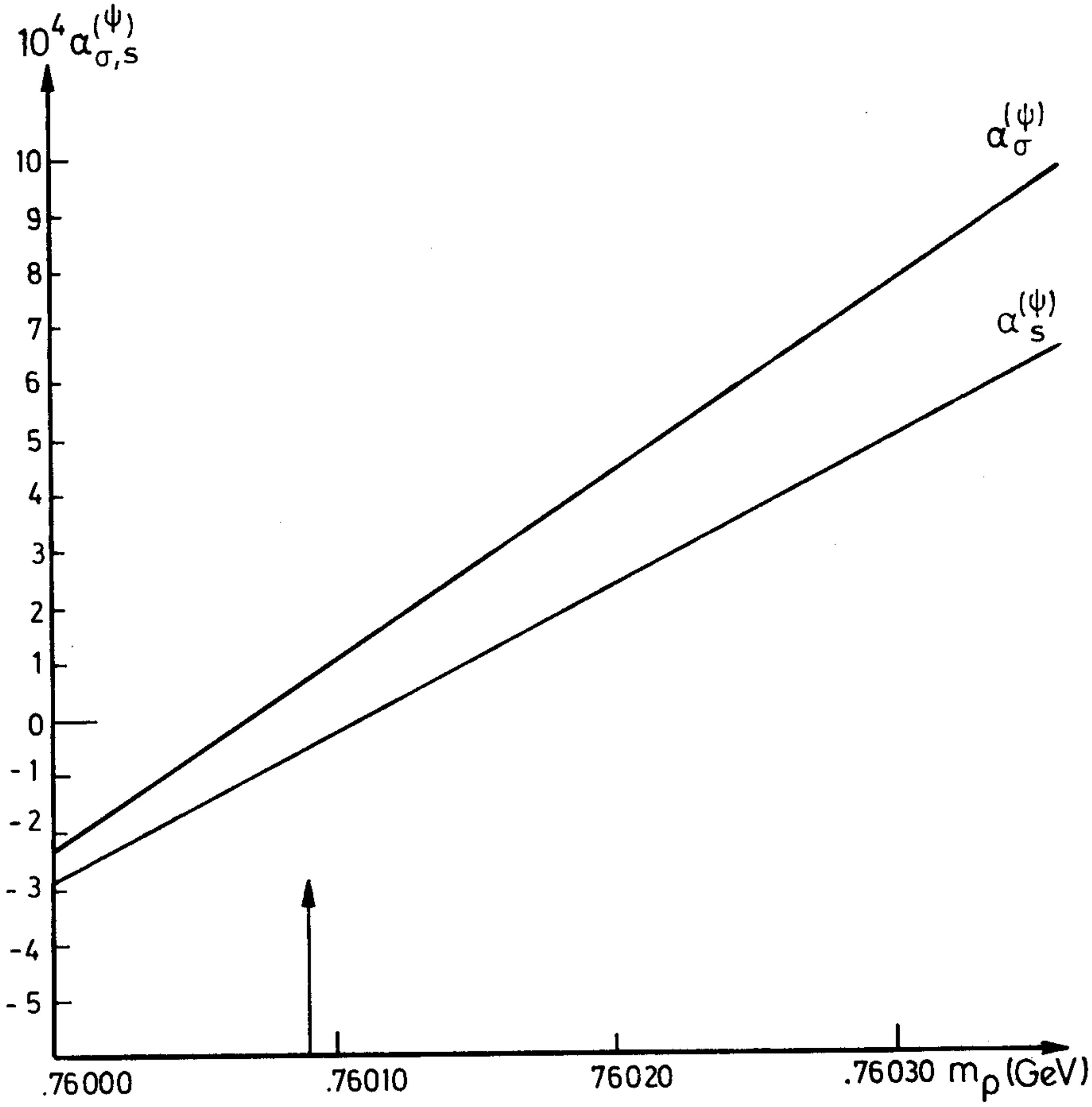
$V \to P\gamma$	$\Gamma(V \to P\gamma)$ (1)	$\Gamma(V \to P\gamma)$ (2)	$\Gamma(V \to P\gamma)$ (2),(VDM)	$\Gamma(V \to P\gamma)$ (exp)
$\omega \to \pi^0\gamma$	870 KeV	870 KeV	870 KeV	(870 ± 86) KeV
$\phi \to \pi^0\gamma$	8.72 KeV	6.57 KeV	4.46 KeV	< 14.7 KeV
$\psi \to \pi^0\gamma$	41.31 KeV	< 1 eV	< 0.2 KeV	< 380 eV
$\omega \to \eta\gamma$	4.41 KeV	4.62 KeV	4.62 KeV	< 50 KeV
$\phi \to \eta\gamma$	199.59 KeV	197.93 KeV	134.33 KeV	(125 ± 43)KeV
$\psi \to E\gamma$	101.73 KeV	101.28 KeV	20.24 KeV	
$\psi \to \eta\gamma$	3.86 KeV	3.34 KeV	0.67 KeV	(0.1 - 2.0)KeV
$\psi \to \psi_P\gamma$	14.90 KeV	24.80 KeV	4.96 KeV	

Table 8

$P \rightarrow 2\gamma$	$\Gamma(P \rightarrow 2\gamma)$ (1)	$\Gamma(P \rightarrow 2\gamma)$ (2)	$\Gamma(P \rightarrow 2\gamma)$ (exp)
$\eta \rightarrow 2\gamma$	0.324 KeV	0.324 KeV	(0.374 $\pm$ 0.060) KeV
$\eta' \rightarrow 2\gamma$	10.10 KeV	10.10 KeV	$<$ 19 KeV
$\psi_P \rightarrow 2\gamma$	133.0 KeV	128.8 KeV	
$\eta \rightarrow 2\gamma$	0.284 KeV	0.284 KeV	(0.374 $\pm$ 0.060) KeV
$E \rightarrow 2\gamma$	19.48 KeV	19.39 KeV	
$\psi_P \rightarrow 2\gamma$	327.6 KeV	323.7 KeV	

References:

1) J.J. Aubert et al., Phys. Rev. Letters 33 (1974) 1404
2) J. E. Augustin et al. , Phys. Rev. Letters 33 (1974) 1406
3) C. Bacci et al., Phys. Rev. Letters 33 (1974) 1408
4) W. Braunschweig et al., Phys. Letters 53B (1974) 393, 491
L. Criegee et al., Phys. Letters 53B (1974) 489
5) J.E. Augustin et al. SLAC - PUB - 1520 (LBL - 3621)
6) L. Criegee et al., Phys. Letters 53 B (1974) 489
7) A. Kazi, G. Kramer and D.H. Schiller, Desy report, Desy 75/10 In the following this paper will be referred to as I.
8) Similar results have been reported recently by K. Kajantie, C. Montonen, M. Roos and N. Törnquist, University of Helsinki, preprint 3- 75 (1975)
9) Particle Data Group, Phys. Letters 50B (1974), Number 1
10) A. M. Boyarski et al., SLAC - PUB - 1523 (LBL - 3632)
11) Z. Maki, T. Masakawa and I. Umemura, Progr. Theor. Phys. 47 (1972) 1682,
12) DASP - collaboration, private communication.
13) Spear - detector result, communicated by S. Brodsky
14) For a review see G. Schierholz, Desy report Desy 74/53 (1974)
15) A. Browman et al., Phys. Rev. Letters 32 (1974) 1067
16) D. Amati, H. Bacry, J. Nuyts and J. Prentki, Nuovo Cimento, 34 (1964) 1732



$10^4 \alpha_{\sigma,s}^{(\psi)}$
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
$\alpha_\sigma^{(\psi)}$
$\alpha_s^{(\psi)}$
.76000
.76010
76020
.76030
m_ρ (GeV)

Fig.1

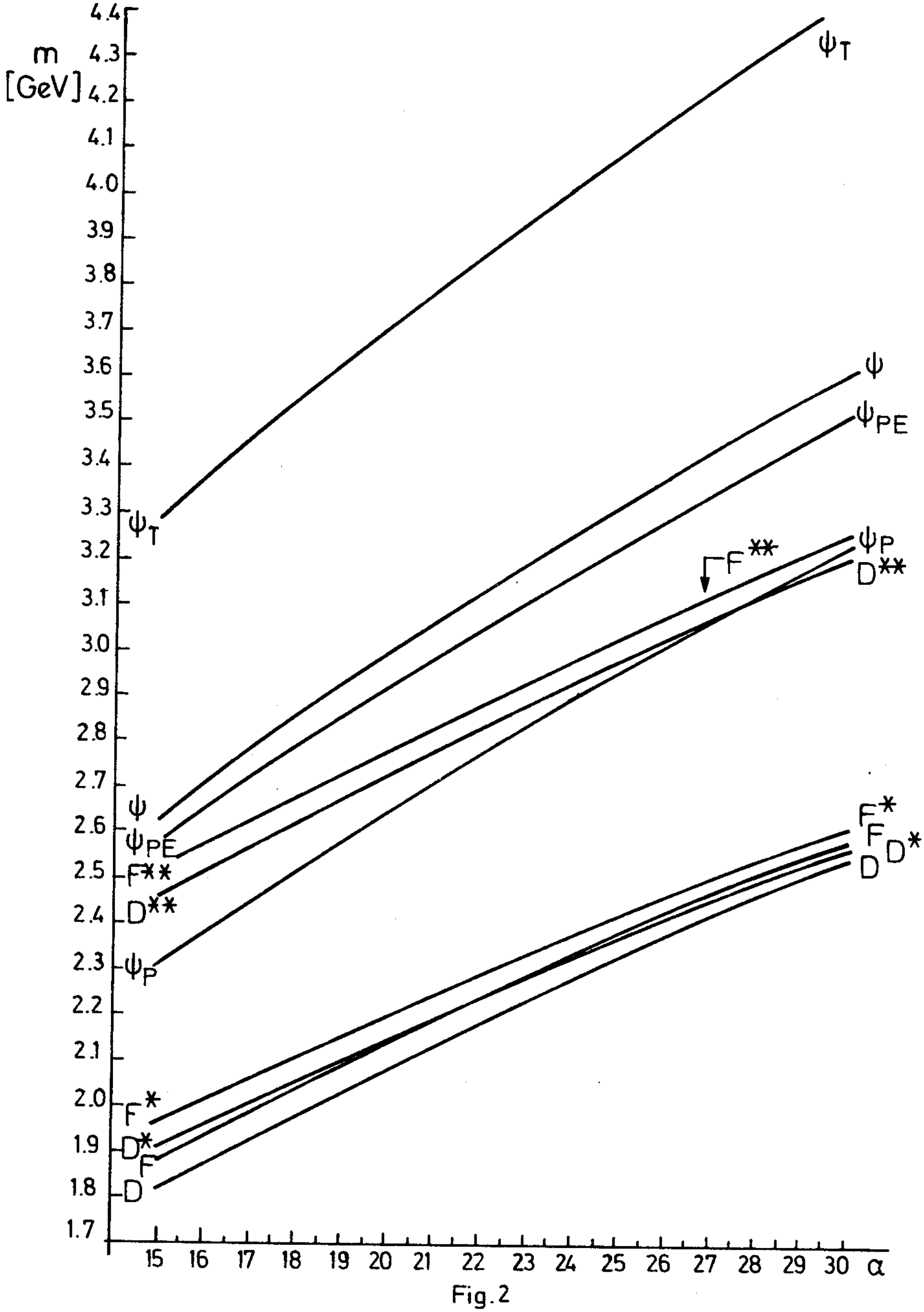

m
[GeV]
4.4
4.3
4.2
4.1
4.0
3.9
3.8
3.7
3.6
3.5
3.4
3.3
3.2
3.1
3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
ψ_T
ψ
ψ_PE
ψ_P
F**
D**
F*
F
D*
D
15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 α

Fig. 2