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Are the ψ s Really Pure S Wave States?

by

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Abstract

We consider $\psi'(3.7)$ and $\psi''(4.15)$ as mixtures of the 2^3S_1 and 1^3D_1 states in a harmonic oscillator model, in order to explain the series of the leptonic decay widths of (J,ψ), ψ' and ψ''.
The spectrum of P wave states and the radiative width of ψ' are discussed in view of a new heavy meson.
The mixing model is also applied to the excitations of the $\rho(770)$.

1. Introduction

The extremely small width of the vector mesons at resonance energies of 3.1 GeV (J, ψ) and 3.7 GeV (ψ'), and also the broad state at 4.15 GeV (ψ'') [1] gave rise to interpretations in terms of a quark quartet with a new, charmed quark with charge 2/3 e [2].

Whereas in the color scheme [3] or in models with more than one new quark [4] these vector mesons and their decay properties are merely related to each other by internal symmetry, the charm model needs a dynamical correlation between them.

The usual explanation in a $c\bar{c}$ bound state model describes the ψ' as the first and the ψ'' as the second radial excitation of the ground state ψ. Bound state models with any simple potential predict monotonically increasing or monotonically decreasing leptonic decay widths ($\Gamma_{\ell^+\ell^-} \sim |\chi_r(0)|^2/M_r^2$) for the radial excitations.

Much emphasis has been put on asymptotic freedom and quark confinement leading to a nonrelativistic description by a potential linear in the distance R [5], with the property $|\chi(0)|^2$= constant for all radial excitations. If one assumes that the ψ'' belongs to the series of excited states of the ψ, then the decrease of the leptonic width [6] from ψ to ψ', followed by the increase from ψ' to ψ''

	ψ	ψ'	ψ''
$\Gamma_{\ell^+\ell^-}$ [keV]	(4.8 $\pm$ 0.6)	(2.2 $\pm$ 0.3)	$\approx$ 4

Table 1

cannot be explained by the properties of simple radial excitations. For this reason one might speculate that the assignment of ψ' and ψ'' to pure radial excitations is wrong. There can be mixing of the $j^{PC} = 1^{--}$, $\ell = 0$ radial excitations and the $\ell = 2$ orbital excitations present in any dynamical model [7].

Assume that the ψ' and ψ'' are mixtures of the 2^3S_1 and 1^3D_1 states, then the leptonic widths satisfy

$$\frac{M_{\psi'}^2 \Gamma_{\psi'} + M_{\psi''}^2 \Gamma_{\psi''}}{M_{\psi}^2 \Gamma_{\psi}} = \frac{|\chi(0)_{r=1}|^2}{|\chi(0)_{r=0}|^2}$$

The experimental value is ≈ 2. This points towards a potential which increases faster with the distance than the linear confinement potential. Since a model where the $q\bar{q}$ pair is bound by harmonic forces has been successfully used in the spectroscopy of the ordinary mesons it is worthwhile to employ this "Harmonium" picture to the new mesons as well.

In this paper we investigate the ψs, also including the most recently found heavy meson [8], describing the spectrum, leptonic and some radiative decay properties. Though a relativistic covariant field theoretical model for $q\bar{q}$ bound states is available [9], we discuss the mixing in a nonrelativistic model for simplicity. Thus we only can hope to get qualitative results.

2. Harmonium, Mixing and Leptonic Decays

For a harmonic interaction of the form

$$V(R) \sim c + \frac{R^2}{R_o^2} \tag{1}$$

the spectrum is given by

$$M^2 = c' + (n + 3/2)\alpha'^{-1} \tag{2}$$

where n = ℓ + 2r. This spectrum shows degeneracy of all states to the same n, what we do not expect to find in nature. In fact the mass splitting of states with the same n but different j^{PC} (f.i. the $A_2 - A_1 - \delta$ mass splitting) is large in comparison to atomic spectra.

It should be pointed out that our mass spectrum and wave functions do not depend on the quark mass. We do not derive eq. (2) from a Schrödinger equation, but rather use the three dimensional analogue of the relativistic model described in ref. 9).

The normalized radial wave functions are then given by

$$R_{\ell,r}(R) = \sqrt{\frac{2r!}{\Gamma(\ell+r+3/2)}} \cdot \frac{1}{R_o^{3/2}} \cdot \left(\frac{R}{R_o}\right)^{\ell} \cdot L_r^{\ell+1/2}\left(\frac{R^2}{R_o^2}\right) \cdot e^{-\frac{R^2}{2R_o^2}} \tag{3}$$

and in particular

$$|\chi(0)|^2 = \begin{cases} \dfrac{2\,\Gamma(r+3/2)}{\pi^2\; r!} \cdot R_o^{-3} & \text{for } \ell = 0 \\ 0 & \text{for } \ell \neq 0 \end{cases} \tag{4}$$

In the nonrelativistic limit the electromagnetic decay widths of vector mesons are given by

$$\Gamma_{V \to \ell^+\ell^-} = \frac{16\pi}{3} \alpha^2 Q_V^2 \frac{|\chi(0)|^2}{M_V^2} \tag{5}$$

with $Q_V(\rho^o, \omega, \phi, \psi) := \left(\frac{1}{2}, \frac{1}{18}, \frac{1}{9}, \frac{4}{9}\right) \cdot C$ (6)

where the color factor C is: $C = \begin{cases} 1 & \text{for one quartet} \\ 3 & \text{for three quartets} \end{cases}$

Using this expression for the ground states ψ and $\rho^o(770)$ we find for the radius R_o:

$$R_o(\psi) = 0.22 \text{ fm} \cdot \sqrt[3]{C}$$
$$R_o(\rho^o) = 0.55 \text{ fm} \cdot \sqrt[3]{C} \tag{7}$$

As already mentioned we identify the ψ' and ψ'' with a mixture of the $\ell = 2$, r = 0 and the $\ell = 0$, r = 1 vector states

$$\begin{pmatrix} \psi' \\ \psi'' \end{pmatrix} = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} 1^3D_1 \\ 2^3S_1 \end{pmatrix} \qquad (8)$$

Then the mixing angle is determined by

$$\tan^2\vartheta = \frac{\Gamma_{\psi'} M^2_{\psi'}}{\Gamma_{\psi''} M^2_{\psi''}} \qquad (9)$$

which gives

$$30^\circ \leq |\vartheta| \leq 37^\circ \qquad (10)$$

For $|\vartheta| = 35^\circ$ we obtain with $\frac{|\chi(0)_{r=1}|^2}{|\chi(0)_{r=0}|^2} = \frac{3}{2}$ (eq. (4))

	ψ	ψ'	ψ''
$\Gamma_{e^+e^-}$ [keV]	4.8 ± 0.6 input	1.7 ± 0.3	2.7 ± 0.4

(11)

By introducing the mixing we get a reasonable description of the leptonic widths. As a consequence, in the oscillator model we have a large mass splitting between originally degenerate states.

Is such a rather large mass splitting reasonable? To get a hint for this, we look at the n = 2 excited states of the ρ meson. There we have the g(1686) with $j^{PC} = 3^{--}$, the ρ'(1600) with $j^{PC} = 1^{--}$, and if experimentally confirmed the ρ'(1250) may be identified with the second n = 2 vector meson. We then have the mass spectrum of ρ(770), ρ'(1250) and ρ'(1600) similar to that of ψ, ψ', ψ''.

In the following we give a simple model which describes the spectrum of the excited states of the ρ meson and the ψ as well.

3. "Fine Structure" of the Mass Spectrum

We extend the simple Harmonium model with the mass spectrum eq. (2) by introducing noncentral spin orbit and tensor forces as supposed by Dalitz [10]. We approximate the spin orbit interaction by a constant c_3 and the tensor force $\left[(\sigma_1\cdot\hat{X})\cdot(\sigma_2\hat{X}) - \frac{1}{3}\sigma_1\cdot\sigma_2\right]V_T(R)$ by a potential $V_T(R)\sim 1+(c_5/c_4)R^2R_0^{-2}$.

By considering the previously degenerate states only we obtain a mass formula, which reads

$$M^2 = c_1 + c_2\cdot n \quad \text{for} \quad |n, r, \ell; j=\ell\rangle$$

and

$$M^2 = c_1 + c_2\cdot n + c_3\begin{pmatrix} -(j+1) & 0 \\ 0 & j \end{pmatrix} + \tag{12}$$
$$+ \frac{c_4}{2j+1}\begin{pmatrix} j+1 & 0 \\ 0 & j \end{pmatrix} + \frac{c_5}{2j+1}\begin{pmatrix} (j+1)(n+3/2) & \sqrt{j(j+1)2r(n+j+2)} \\ \sqrt{j(j+1)2r(n+j+2)} & j(n+3/2) \end{pmatrix}.$$

$$\text{for} \begin{pmatrix} |n, r'=r-1, \ell'=\ell+2; j=\ell'-1\rangle \\ |n, r, \ell, j=\ell+1\rangle \end{pmatrix} \qquad \text{with} \quad n=\ell+2r, \quad \ell, r\geq 0 .$$

The eigenvalues are obtained by diagonalizing eq. (12) with the Rotation

$$R(\vartheta) = \begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix}$$

where the angle ϑ is defined in accordance with eq. (8). This five parameter mass formula contains splitting and mixing and we may hope to get a crude approximation of the real situation within each multiplet and also between multiplets belonging to different main quantum numbers n.

Light Mesons

Now we apply the mass formula eq. (12) to the excitations of the ρ meson. The five parameters c_i were determined by a best fit to the masses of ρ, δ, A_2, $\rho'(1250)$, $\rho'(1600)$ and g(1686).

We found two solutions shown in the term scheme fig. 1, which differ in the mixing angle of the 1^3D_1 and 2^3S_1 states and in the masses of the 2^{--} and 1^{++} mesons. The main feature of these solutions is the possible identification of the 2^{--} meson either with the $F_1(1540)$ or the $X_1(1440)$. The coincidence, also with respect to the A_1 meson, is better in the second case.

Another consequence of eq. (12) is the mixing between the 1^3D_1 and the 2^3S_1 states. From the mixing angle $\vartheta = -22.5^\circ$ we would predict a ratio of the electromagnetic widths of the $\rho'(1250)$ and the $\rho'(1600)$

$$\frac{\Gamma_{e^+e^-}(\rho'(1250))}{\Gamma_{e^+e^-}(\rho'(1600))} = \tan^2\vartheta \, \frac{M^2(1600)}{M^2(1250)} = 0{,}28 \qquad (13)$$

and from $|\chi(0)_{r=1}|^2 / |\chi(0)_{r=0}|^2$ we get

$$\Gamma_{e^+e^-}(\rho'(1600)) = \Gamma_{e^+e^-}(\rho(770)) \, \frac{M^2(770)}{M^2(1600)} \cdot \frac{3}{2} \cdot \cos^2\vartheta \approx 2 \text{ keV} \qquad (14)$$

while the experimental value is [11)]

$$\Gamma_{e^+e^-}\left(\rho'(1600)\right) = (1.5 - 2.5)\ \text{keV}$$

Heavy Mesons

We found from the leptonic widths that the range R_o of the wave functions of the ψ s is smaller than that of the light mesons, eq. (7). Therefore we expect different parameters c_i for the ψs. The limits given in (10) indicate a different mixing angle, too. We therefore take the masses of ψ, the 1^3D_1 and 2^3S_1 mixings, ψ' and ψ'', and the angle $|\vartheta| = 35^o$ as input.
Then the masses of the other heavy mesons, in particular those with $j^{PC} = 2^{++}, 1^{++}, 0^{++}$ depend on one parameter which we chose to be c_5/c_4. In the figures 2a and 2b we show the spectrum of the P wave states. There is evidence [8)] for a meson around M = 3.5 GeV [12)]. We therefore may tentatively choose the four solutions indicated by the dotted lines in fig. 2(a,b), which lead to the masses in table 2.
All solutions have in common that the region around 3.5 GeV has nearly degenerate 0^{++} and 1^{++} states, whereas the 2^{++} state lies close to the ψ' around 3.65 GeV.

A comparison between the parameters of the heavy and the light mesons shows that the inverse Regge slopes α'^{-1}, which are essentially given by c_2, are approximately in the ratio $\alpha'^{-1}(\psi)/\alpha'^{-1}(\rho) \sim R_o^2(\rho)/R_o^2(\psi)$. A correlation between the Regge slopes and the leptonic widths of vector mesons has been discussed originally in ref. 13.

4. Radiative Widths

A central problem in charmonium type models is the observed smallness of the radiative decay width of ψ' into P wave states. In order to test the Harmonium model we calculate these radiative widths in dipole approximation. We use the formula

$$\Gamma = \frac{4}{3} \alpha Q_c^2 k^3 |X_{fi}|^2 \tag{15}$$

With the help of the oscillator wave functions (eq. (3)) and with $1^3D_1 - 2^3S_1$ - mixing we obtain for the decays of ψ' into 0^{++}, 1^{++}, 2^{++} P wave states:

$$\Gamma = \Gamma_0 \cdot \begin{Bmatrix} \frac{1}{9}\left[-\sin\vartheta - \sqrt{5}\cos\vartheta\right]^2 & & \psi' \to 0^{++} \\ \frac{1}{3}\left[-\sin\vartheta + \frac{\sqrt{5}}{2}\cos\vartheta\right]^2 & \text{for} & \psi' \to 1^{++} \\ \frac{5}{9}\left[-\sin\vartheta - \frac{\sqrt{5}}{10}\cos\vartheta\right]^2 & & \psi' \to 2^{++} \end{Bmatrix} \tag{16}$$

with

$$\Gamma_0 = \frac{4}{3} \alpha Q_c^2 k^3 R_0^2 \quad , \quad Q_c^2 = \frac{4}{9} .$$

k is the photon momentum and the radius R_0 is determined from the leptonic decay width and given in eq. (7). If we calculate Γ_0 with k = 0.2 GeV we obtain

$$\Gamma_0 \approx 44 \text{ keV} \cdot \sqrt[3]{C^2} \tag{17}$$

For the solutions from fig. 2(a,b) we find in the one quartet model the partial widths of ψ' into γ + (P wave state):

		$M(0^{++})$ [GeV]	$M(1^{++})$ [GeV]	$\Gamma(\psi' \to \gamma + 0^{++})$ [keV]	$\Gamma(\psi' \to \gamma + 1^{++})$ [keV]
1	$\vartheta = +35^{\circ}$	3.5	3.53	28	1
2		3.53	3.5	15	2
3	$\vartheta = -35^{\circ}$	3.5	3.42	8	90
4		3.42	3.5	22	33

Table 2

In case three and four one should see two states experimentally, while in cases one and two one should only see the one state $j^{PC} = 0^{++}$. The 2^{++} decay mode is suppressed in all cases mainly because of its mass very close to the ψ'. We have to favour the first two cases, last not least because of the experimental upper limit for the radiative decay width $\Gamma(\psi' \to \gamma + X)$ [14] which in its most restrictive form even might rule out this.

Summary

We have described the decrease and increase in the leptonic decay widths of (J,ψ), ψ' and ψ'' by a phenomenological mixing model. As a consequence we obtained a one parameter representation of the spectrum of the excitations of the (J,ψ). These considerations get a new impact by the discovery of a new heavy meson. From its mass and our estimate of the radiative decay width of ψ' we would favour a $j^{PC} = 0^{++}$ assignment.

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Figure Captions

Fig. 1. Comparison of the mixing model (spin orbit and tensor force) with experiment. (____ denotes established resonances. For illustration pure spin orbit splitting is also shown.) The parameters c_i in the mass formula eq. (12) are:

c_i: [GeV^2]	c_1	c_2	c_3	c_4	c_5	c_5/c_4
$\vartheta = +19^\circ$	0.36	0.99	0.22	0.34	-0.20	-0.60
$\vartheta = -22.5^\circ$	0.45	0.82	0.24	-0.64	0.22	-0.35

Fig. 2(a,b). Mass spectrum of the $c\bar{c}$ states with $j^{PC} = 0^{++}, 1^{++}, 2^{++}$ as functions of $-c_5/c_4$, according to eq. (12). The dotted lines denote the restrictions with one state at 3.5 GeV. The parameters c_i are then:

	c_i: [GeV^2]	c_1	c_2	c_3	c_4	c_5	c_5/c_4
1	$\vartheta = +35^\circ$	8.89	3.61	0.29	2.92	-1.15	-0.39
2		8.65	3.61	0.35	3.46	-1.15	-0.33
3	$\vartheta = -35^\circ$	8.88	2.84	0.68	-1.66	1.15	-0.69
4		9.41	2.84	0.54	-2.85	1.15	-0.40

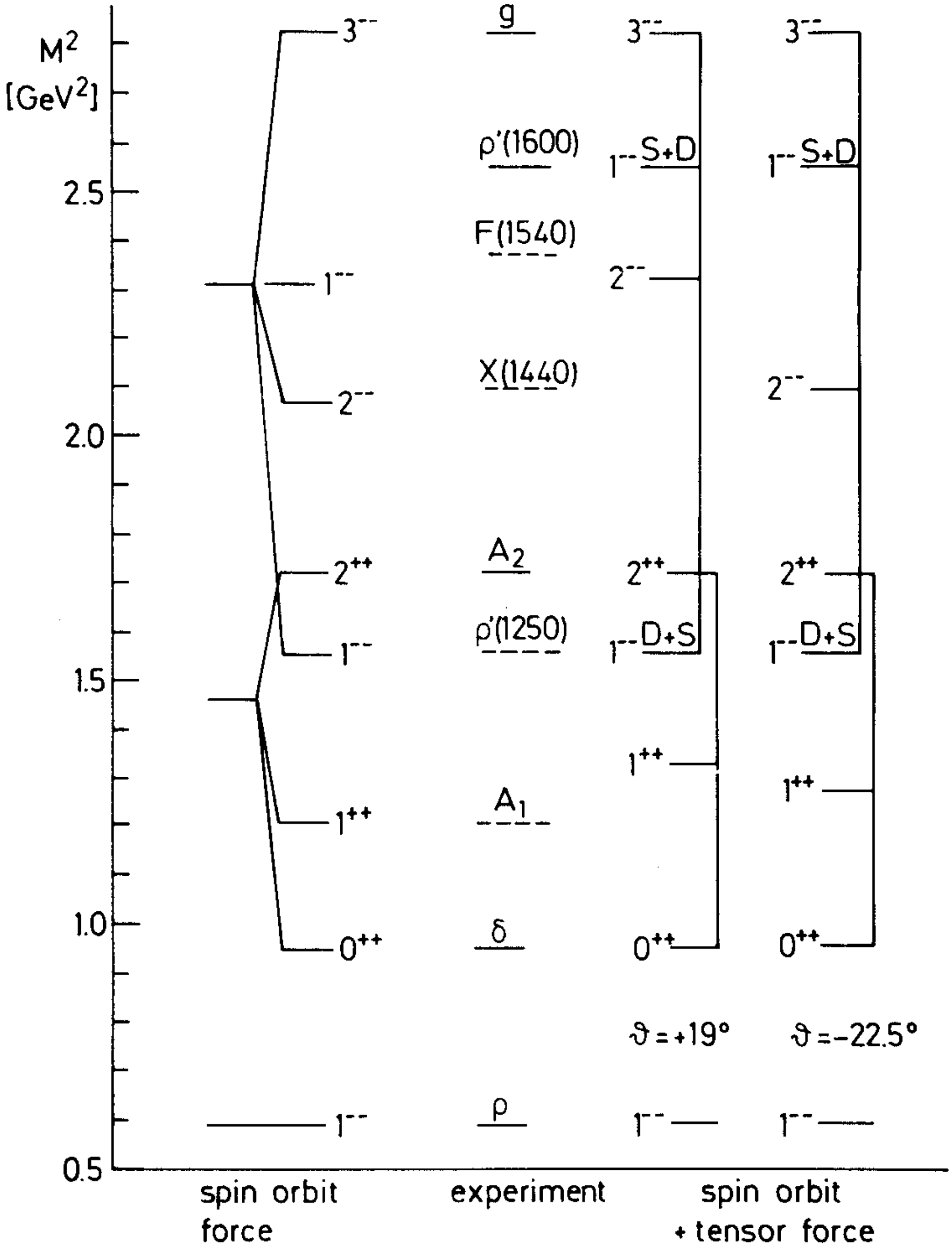
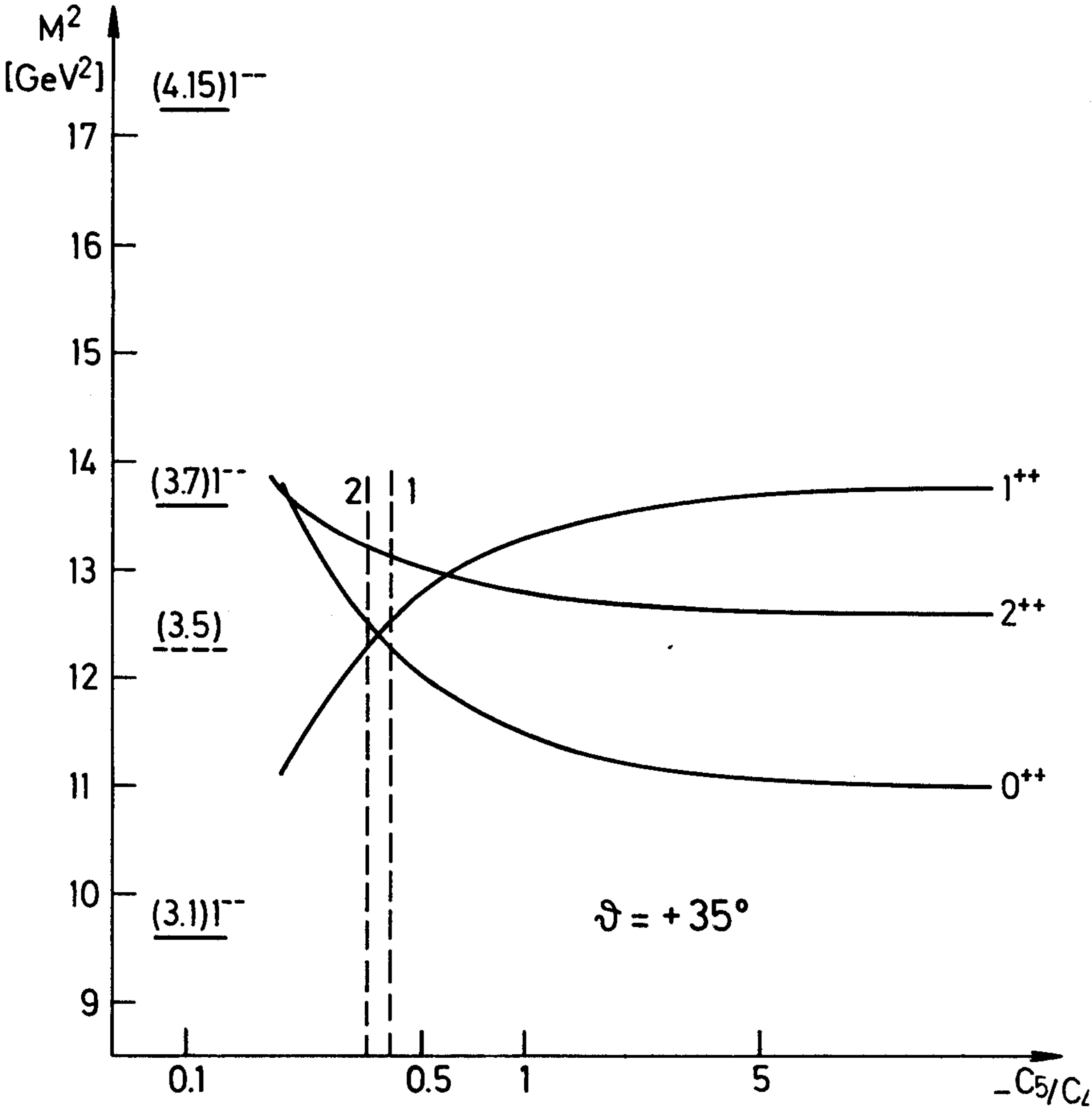

M2
[GeV2]
2.5
2.0
1.5
1.0
0.5
3--
1--
2--
2++
1--
1++
0++
1--
g
ρ'(1600)
F(1540)
X(1440)
A2
ρ'(1250)
A1
δ
ρ
3--
1-- S+D
2--
2++
1-- D+S
1++
0++
ϑ = +19°
1--
3--
1-- S+D
2--
2++
1-- D+S
1++
0++
ϑ = -22.5°
1--
spin orbit
force
experiment
spin orbit
+ tensor force

Fig. 1



M^2 [GeV^2]
17
16
15
14
13
12
11
10
9
(4.15)1^{--}
(3.7)1^{--}
(3.5)
(3.1)1^{--}
2
1
1^{++}
2^{++}
0^{++}
$\vartheta = +35°$
0.1
0.5
1
5
$-C_5/C_4$

Fig. 2a

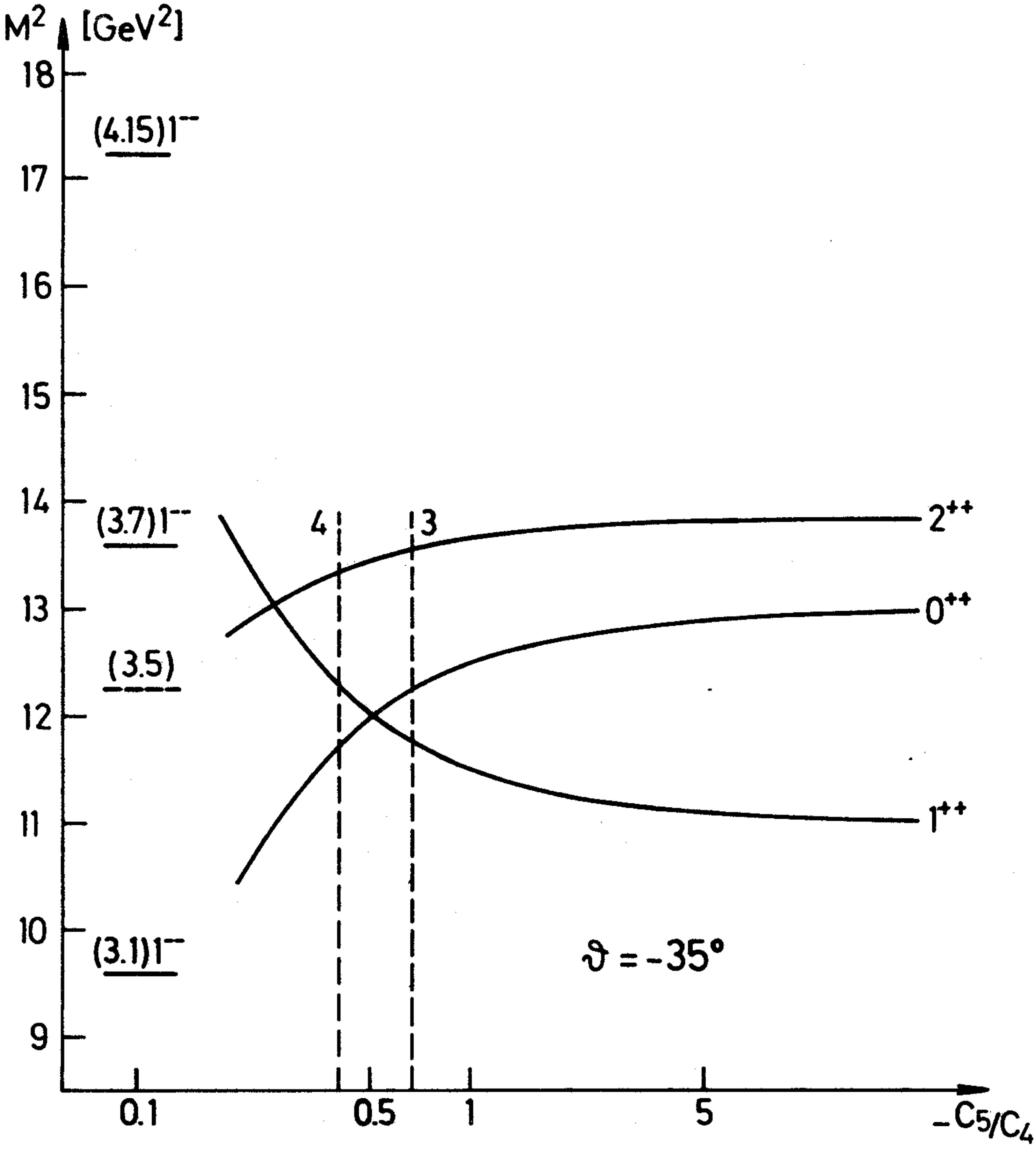

M²
[GeV²]
18
17
16
15
14
13
12
11
10
9
(4.15)1⁻⁻
(3.7)1⁻⁻
(3.5)
(3.1)1⁻⁻
4
3
2⁺⁺
0⁺⁺
1⁺⁺
ϑ = -35°
0.1
0.5
1
5
-C5/C4

Fig. 2b