DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 75/26
August 1975

Longitudinal instabilities of a single bunch and the observation of reduced Landau damping in DORIS

by

R. D. Kohaupt

2 HAMBURG 52 · NOTKESTIEG 1

Abstract:

A single electron bunch current in DORIS was limited to 1.5 mA/bunch by longitudinal instabilities.

The shift of the incoherent synchrotron frequency was measured by excitation of the quadrupole mode.*) For 1 mA/bunch at the natural beam dimensions the incoherent shift was found to be sufficient to reduce Landau damping. The observed "strong" longitudinal instability of a single bunch turned out to be a "weak" instability, restrained only by radiation damping.

This weak instability could be removed by "Robinson damping", and 15 mA/bunch were reached.

Introduction
1. Theory
   a) Reduction of Landau damping
   b) Origin of the incoherent frequency shift
2. Measurements and results
   a) Measurements
   b) Results
   c) Cure of longitudinal instability
   d) Bunch lengthening

Introduction

The maximum bunch current for electrons in DORIS were reached[1] when nearly 40 bunches were filled.

When attempting to fill an intense single bunch[2], we observed a strong longitudinal instability which limited the current to about 1.5 mA/bunch.

In coincidence with the instability we picked up signals from our fast loops at the synchrotron frequency. These observations were in contradiction to the measurements of threshold currents of longitudinal instabilities for the

*) The measurements were performed by the DORIS staff as part of the machine development program.)

homogenous filling with 480 bunches, which indicated that no coupling impedance was present in the machine cousing such a strong instability. A "strong" instability, however, can be simulated by a "weak" instability if the Landau damping is reduced by a shift of the incoherent synchrotron frequency due to an inductive wall. This effect was observed in the ISR[3].

When the longitudinal Landau damping is lost in DORIS the longitudinal instabilities are only compensated by radiation damping, which is much smaller than the Landau damping at the natural beam dimensions.

From the measurement of the incoherent frequency shift follows that the Landau damping is also reduced in DORIS.

In section 1 of this report the theoretical background for the reduction of Landau damping is discussed, while in sections 2,3 the data are presented and interpreted.

1. Theory

a) Reduction of Landau damping

For the derivation of the essential equations of Landau damping we follow the analysis of D. Möhl and A.M. Sessler[4]. The equations for motion for the longitudinal coordinate ϕ_i of the i-th particle in the bunch are:

$$\ddot{\phi}_i + \Omega_i^2 \phi_i + A(\phi_i - \bar{\phi}) = (U + iV)\bar{\phi} \qquad (1)$$

where $\bar{\phi}$ notes the center of mass coordinate of the bunch; $\Omega_i/2\pi$ is the unperturbed synchrotron frequency of the i-th particle. The term $(U + iV)\bar{\phi}$ describes an interaction leading to a coherent instability if the interaction parameters U, V exceed critical values.

The term $A(\phi_i - \bar{\phi})$ with real A characterizes a focussing term leading to an incoherent frequency shift, since averaging equ. (1) with respect to all particles this term vanishes. Such a term appears if there are short range longitudinal forces in the machine which decay in the range of a bunch length and which are "attached" to the bunch. Therfore, they influence the motion of a single particle only.

We dropped the radiation effects in equ.(1) but will later come back to this point.

Equ.(1) leads to the following dispersion relation[4,5]

$$1 = (U + A + iV) \int_0^\infty da \frac{a^2 \frac{\partial \rho}{\partial a}}{\Omega^2 (a) + A - \omega^2} \quad \text{with} \int da \, a \, \rho(a) = 1 \qquad (2)$$

which is the determining equation for ω, the frequency of the (stable or unstable) motion of the dipole oscillation $\bar{\phi} \quad e^{i\omega t}$.

Equ.(2) exhibits the effect Landau damping. The function $\rho(a)$ describes the density of particles with the incoherent oscillation amplitude a. Due to nonlinearities, $\Omega$is a function of a. In an electron ring, $\rho(a)$ is Gaussian

$$\rho \sim e^{-\frac{a^2}{2\sigma_S^2}}$$

where σ_S is the longitudinal standard deviation.

The critical values of U, V leading to an instability are defined by those values for which equ.(2) can be solved with real ω.

Above these critical values, the U, V parameters lead to a complex solution ω, which in turn may cause instability, depending an the sign of V.

If A is large as compored to the frequency domain given by $\Omega(a)$ and $\rho(a)$, i.e. to the maximum average frequency spread the integral in (2) beahves like the integral for an electric field of a far distant charge

$$1 = (U + A + iV) \frac{1}{\Omega^2 - \omega^2 + A} \qquad (3)$$

and there is no real solution ω for a finite V, which means that Landau damping is lost.

At this point we come back to the radiation effects. Since we are interested in the stability of the dipole oscillations only we can disregard the diffusion effects[6]. The damping can simply be included by substituting

$$\Omega^2 \rightarrow \Omega^2 + 2\ i\ \Omega\ \delta_r \tag{4}$$

in (2) and (3) where δ_r is the radiation damping rate.

If Landau damping vansihes as in (3) the instability is compensated only by radiation damping and the stability condition is (IVI, $\delta_r << \Omega$)

$$\frac{|V|}{2\Omega} < \delta_r$$

It is convenient to use (2) in the form

$$1 = (\bar{U} + \bar{A} + i\bar{V})\ S(x),\ S(x) = \int_0^\infty d\xi\ \frac{\frac{\partial\rho}{\partial\xi} \cdot \xi}{\xi - x} \tag{4}$$

$$x = \frac{\omega - \Omega\ (0) - \frac{A}{2\Omega\ (0)}}{2\ \Delta\ \Omega}\ , \qquad \bar{U},\ \bar{V},\ \bar{A} = \frac{U,V,A}{2\Omega\ (0)\ \Delta\ \Omega}$$

where $\Delta\ \Omega$ is the frequency spread over a standard deviation. For the nonlinearity of the rf focussing $\Delta\ \Omega$ is given by

$$\Delta\ \Omega \approx -\ \Omega\ (0) \cdot \frac{\sigma_S^2}{16} \qquad \left\{\begin{matrix}\sigma_S \text{ in radian, with respect} \\ \text{to bucket length}\end{matrix}\right\} \tag{5}$$

Dropping radiation damping again, S(x) can be evualted in terms of tabulated functions. For Landau damping, S(x) is only of interest for $x > 0$, since for $x < 0$ S(x) is real, so that no compensation of iV is possible for real x. From (4) we find

$$\bar{V} = L\ (x) = \frac{\pi x e^{-x}}{S(x)^2} \tag{6}$$

S(x) and L(x) are plotted in fig. 1

If no damping is present, the V-parameter leads to a growth rate

$$\delta_g \approx \frac{|V|}{2\Omega\ (0)} \qquad (7)$$

If δ_L denotes the maximum δ_g that can be compensated by Landau damping (Landau damping rate), from (6) follows the relation:

$$\delta_L = \text{Max}\ \{L(x)\} \cdot \Delta\ \Omega \qquad (8)$$

For DORIS parameters:

$$\sigma_S \approx 1,5 \text{ cm}$$

$$\Omega/2\pi \approx 40 \text{ kHz}$$

we obtain $|\Delta\Omega| = 387$ Hz

Since Max $\{L(x)\} \simeq 1$ (see fig.1), the "Landau damping time" is

$$\tau_L = \frac{1}{\delta_L} \approx 2.6 \text{ m sec}$$

The radiation damping time τ_r for phase oscillation at 2 GeV is only

$$\tau_r \approx 13 \text{ m sec}$$

The function L(x) plotted in fig. 1 is asymmetric with respect to the maximum point:

- for a positive shift ΔX with respect to that point L decreases slowly
- for a negative shift L decreases rapidly and is zero for $\Delta x < -2.5$

Because of the definition (4) and (5), an incoherent shift $A > 0$ leads to a slow reduction of Landau damping while an incoherent shift $A < 0$ leads to a rapid reduction and to a total loss of Landau damping for a critical $A_e < 0$.

b) Origin of the incoherent frequency shift

Since $A < 0$ causes a rapid reduction and a total loss even, phase defocussing terms are most dangerous.

Above transition energy, the fields induced by the beam due to an inductive chamber wall reduce the gradient of the accelerating rf voltage, which means phase defocussing[3].

We can find essentially two objects contributing to the inductivity of the wall:

(i) the inductive part of a resistive wall
(ii) accelerating cavities and cavity-like objects
From (1) we have

$$\Delta f_{inc} = \frac{A}{4\pi\ \Omega} \tag{9a}$$

(i) Following F. Sacherer[7] the shift of the incoherent freqency in the case of a resistive wall is

$$\Delta f_{inc} = 0.01 \times \frac{\bar{I}_B\ R}{U_c\ q} \ \frac{L}{2\pi\ b} \left\{\frac{L}{\sigma_S}\right\}^{5/2} f_S \tag{9b}$$

$f_S = \Omega/2\pi$

I_B ≙ average current/bunch

R ≙ impedance per square at revolution frequency

L ≙

b ≙ chamber radius

U_c ≙ mean peak voltage

q ≙ harmonie number

For DORIS (f_S = 40 kHz, $\bar{I}_B$ = 1 mA, R = 2 · $10^{-3}\Omega$, L = 288 m, b = 7 cm, $U_c \approx$ 2 MV, q = 480)

we obtain

$$\Delta f\ inc \approx 43\ \text{Hz} \quad \text{at} \quad 1\ \text{mA/bunch}$$

(ii) When the higher modes of an accelerating cavity or of a cavity-like object are excited by a passing bunch the head of the bunch loses energy while the tail of the bunch gains a non-negligible part[8]. Thus apart from an average energy loss of the bunch, the excitation of the higher cavity modes has the effect of phase defocussing. The incoherent frequency shift is given by the relation

$$\Delta f_{inc} = \frac{1}{2} f_S \frac{U}{q\omega_o U_c} \qquad (10)$$

ω_o = 2π x revolution frequency

U = time gradient of the induced voltage at the bunch center

For DORIS we have 12 cavities per ring and from ref.(9) we find for $\sigma_S = 1.5$ cm, $\bar{I}_B = 1$ mA/bunch

$$\Delta f_{inc} \simeq 1 \text{ kHz} \quad \text{at} \quad 1 \text{ mA/bunch},$$

which is large enough to be detected.

2. Measurements and results

a) measurements

Altgough the frequency of the coherent dipole oscillation is not affected by a shift of the incoherent synchrotron frequency, the frequencies of the higher bunch shape modes are changed[10]. For the lowest shape mode, the quadrupole mode, we obtain[10,11]

$$\Delta f_{inc} = 2 \Delta f_Q,$$

where

$$\Delta f_Q = 2 f_S - f_{S,Q},$$

$f_{S,Q}$ being the resonance frequency of the quadrupole mode. Therefore, the measurement of Δf_Q as a function of the bunch current is a direct measurement of the incoherent frequency shift.

The excitation of the quadrupole mode is possible if the phase focussing force is modulated in amplitude near $2 f_S$[12].

For DORIS the klystron output can be phase modulated up to a modulation frequency of 300 kHz.

The phase modulation at 2 $f_S \simeq 80$ kHz can be transformed in an amplitude modulation of the accelerating voltage if the accelerating cavities are detuned from resonance, so that the phase modulated r f input becomes modulated in amplitude at the slope of the frequency responce of the cavities. Operating the machine in this special state of the cavity tune, f_S and Δf_Q were measured as a function of bunch current.

The beam response was picked up by a fast loop, demodulated and given on a spectrum analyser.

b) <u>results</u>
The results are shown in fig. 2, where Δf_Q is plotted as a function of bunch current.

The result is rather surprising. At 1 mA/bunch the incoherent shift $\Delta f_{inc} = 2\Delta f_Q$ has reached almost 3 kHz.

As far as the origin of this shift is concerned, the measurement favours the explanation that the accelerating cavities and the cavity-like objects in the ring produce the effect.

As far as the Landau damping is concerned, this damping should be lost, since at 1 mA/bunch we obtain (4) and (9):

$$|\Delta x| = \frac{\pi \, \Delta \, f \; inc}{\Delta \, \Omega} > 2.5$$

According to the arguments of section 1 a this means the loss of Landau damping.

The loss of Landau damping seems to be in excellent agreement with the observations. When the longitudinal instabilities in DORIS were studied for a homogenous filling with 480 bunches, mean-current longitudinal instabilities appeared at an average current of 30 mA. This corresponds to a small bunch current of 0.06 mA/bunch where Landau damping is not affected. When the instability appeared, the beam performed "stable" synchrotron oscillations with an amplitude depending on the current. The mechanism for this is simple: the coherent depole motion leads - due to nonlinearrities- to an increase of the bunch size. So the Landau damping increases until it balances the instability.

This increase of Landau damping is possible only within the Landau-region (see fig. 1). Outside the Landau-region an increase of the bunch shape cannot help to increase the damping (this is not stirctly true, since the increased shape is not Gaussian). Therefore, if the bunch current exceeds the threshold value defined by radiation damping only, the instability must lead to a sudden loss of current above the treshold.

This effect was observed in DORIS for the single electron bunch above 1.5 mA/bunch.

The reduction of Landau damping was established also by measuring the total half width of the dipole mode. At very low bunch currents (0.02 mA/bunch) with a homogeneous filling of the 480 buckets this width was found to be between 200 Hz and 300 Hz, depending on the rf parameters. This is in good agreement with the theoretical value of Landau damping.

For the single bunch, the width was measured as a function of bunch current. The result is plotted in fig. 4.

Above 0.5 mA/bunch the width approaches a constant value which is compatible with radiation damping. It should be emphasized, however, that the width is not only determined by damping, but also by the growth rate of the instability. Therefore, above 1.5 mA/bunch the width decreases rapidly.

C cure for longitudinal instability

Since the strong longitudinal instability turned out to be a "weak" one we tried to compensate this instability by "Robinson Damping".

If the cavities are detuned to lower frequencies with respect to the rf, this damping provides a maximum damping rate

$$1/\tau_R \approx \frac{\bar{I}_B \cdot Z}{U_c}\,\Omega \qquad (12)$$

τ_R ≙ robinson damping time

Z ≙ shunt impedance of the cavities

For DORIS the damping time at 1mA/bunch is nearly 1 msec per cavity group (4 cavities per transmitter) which is sufficent to compensate the "weak" instability.

When that special state of Robinson damping was achived for DORIS, the instability in fact was removed and 1.5 mA/bunch were reached.

d) Bunch length

When the Robinson damping was working and we reached currents of 10 ... 15 mA/bunch, the incoherent shift Δf_{inc} was measured again (fig. 3). At 10 mA/bunch we found $\Delta f_{inc} \approx 11$ kHz at a synchrotron frequency $f_S = 22$ kHz. From the potential model(7) then follows a bunch lengthening of a factor of two due to the inductive chamber.

References

1. DORIS, PRESENT STATUS AND FUTURE PLANS
   Proceedings IX International Conference on High Energy Accelerators (1974)

2. H. Gerke et al.:"Calorimetric measurements of parasitic mode losses at different rf-structure in DORIS" Internal Report, DESY H3-75/2 (1975)

3. S. Hansen et al.: "Effects of psace charge and reactive wall impedances on bunched beams" 1975 Particle Accelerator Conference, Accelerator Engeneering and Technology, Washington 1975

4. D. Möhl, A.M. Sessler: "The use of rf-knockout for Determination of the Characteristics of the Transverse Coherent Instability of an intense beam" Proceedings of the 8th International Conference on High Energy Accelerators, CERN 1971

5. H.G. Hereward: "Landau damping by non-linearitis" CERN/MPS/DL 69-11, 1969

6. H.G. Hereward: "Damping of Bunch-Shape Oscillations by Synchrotron Radiation" Internal Report, DESY H2-75/2, 1975

7. F. Sacherer: "Bunch lengthening" PEP Note-45, 1973

8. A. Papiernik, M. Chatard-Moulin, B. Jecko:"Fast beam Cavity Interaction and its effect on bunch shape in storage rings" Proceedings of the IXth International Conference on High Energy Accelerators, Stanford, 1974

9. J. Le Duff:"Energy Loss Due to Single Bunch Passage Through the Cavities" Internal Report, DESY PET 74/4 , 1974

10. F. Sacherer: "Methods for COMPUTING Bunched-Beam Instabilities" CERN/ST-BR/72-5

11. F. Sacherer: "A Longitudinal Stability Criterion For Bunched Beams" CERN/MPS/Int.Br/73-3, 1973

12. M.A. Allen et al.: "Some observations on Bunch Lengthening at SPEAR" SPEAR-171, 1974

Ackknowledgement

The author is grateful to A. Hofmann (CERN) for stimulating discussions

<u>figure captions</u>

fig. 1: Plot of the Dispersion funchions L(X), S(X) and Landau region

fig. 2: Plot of the measured ΔfQ versus bunch current without Robinson damping dt f_S = 37 kHz

fig. 3: Plot of the measured ΔfQ versus bunch current with Robinson damping at f_S = 22 kHz

fig. 4: Plot of the measured total half width of the dipole mode-frequency versus bunch current

$$S(x) = \begin{cases} \left[\left(1 - X\,\overline{E}_i(X)\,e^{-X}\right)^2 + \left(\pi X e^{-X}\right)^2 \right]^{1/2} & \text{for } X > 0 \\ 1 + |X|\,E_i(-|X|)\,e^{|X|} & \text{for } X < 0 \end{cases}$$

$$L(x) = \frac{\pi X e^{-X}}{S^2(x)}$$

$$X = \frac{\omega - \Omega(0) - A/2\Omega}{2\,\Delta\,\Omega}$$

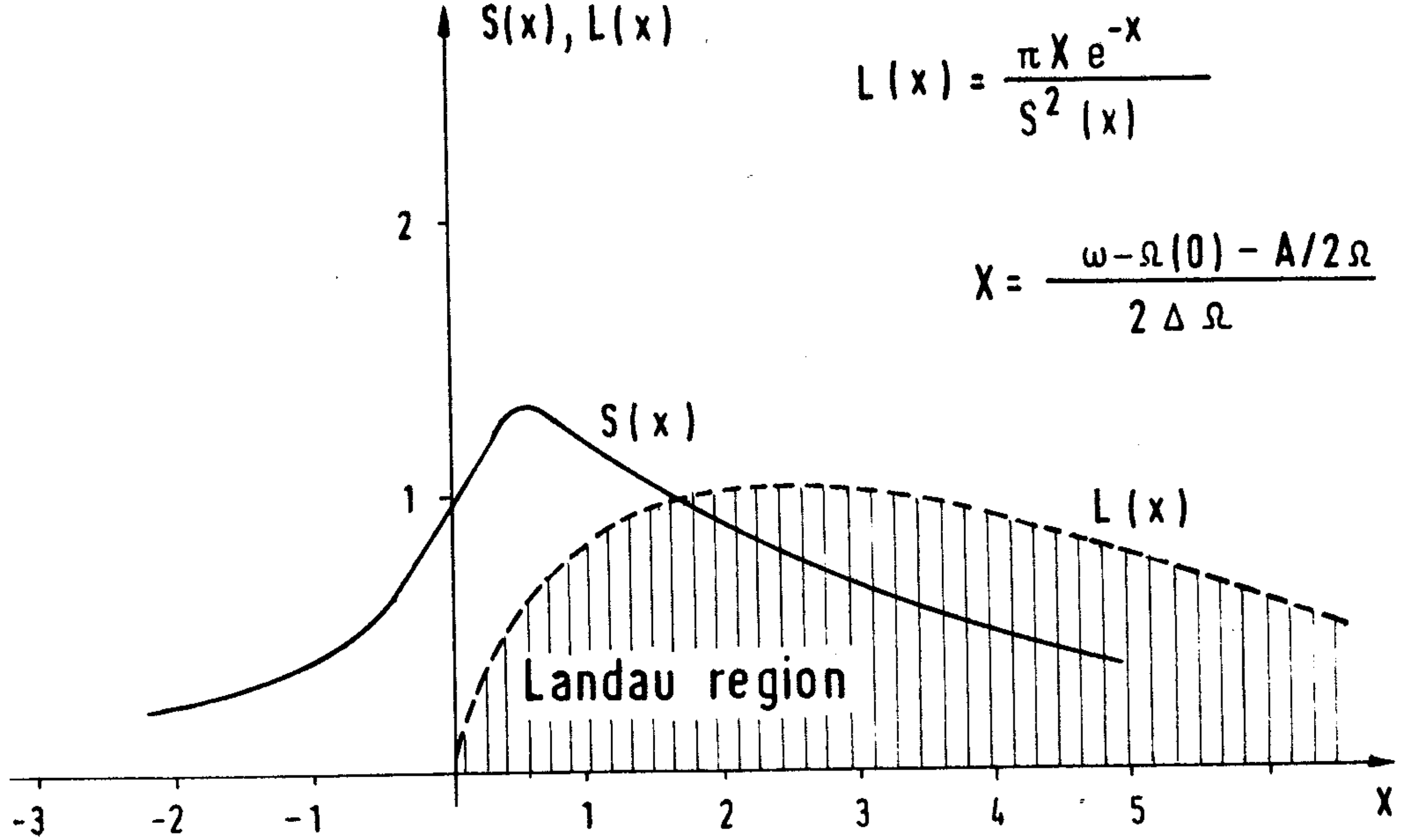
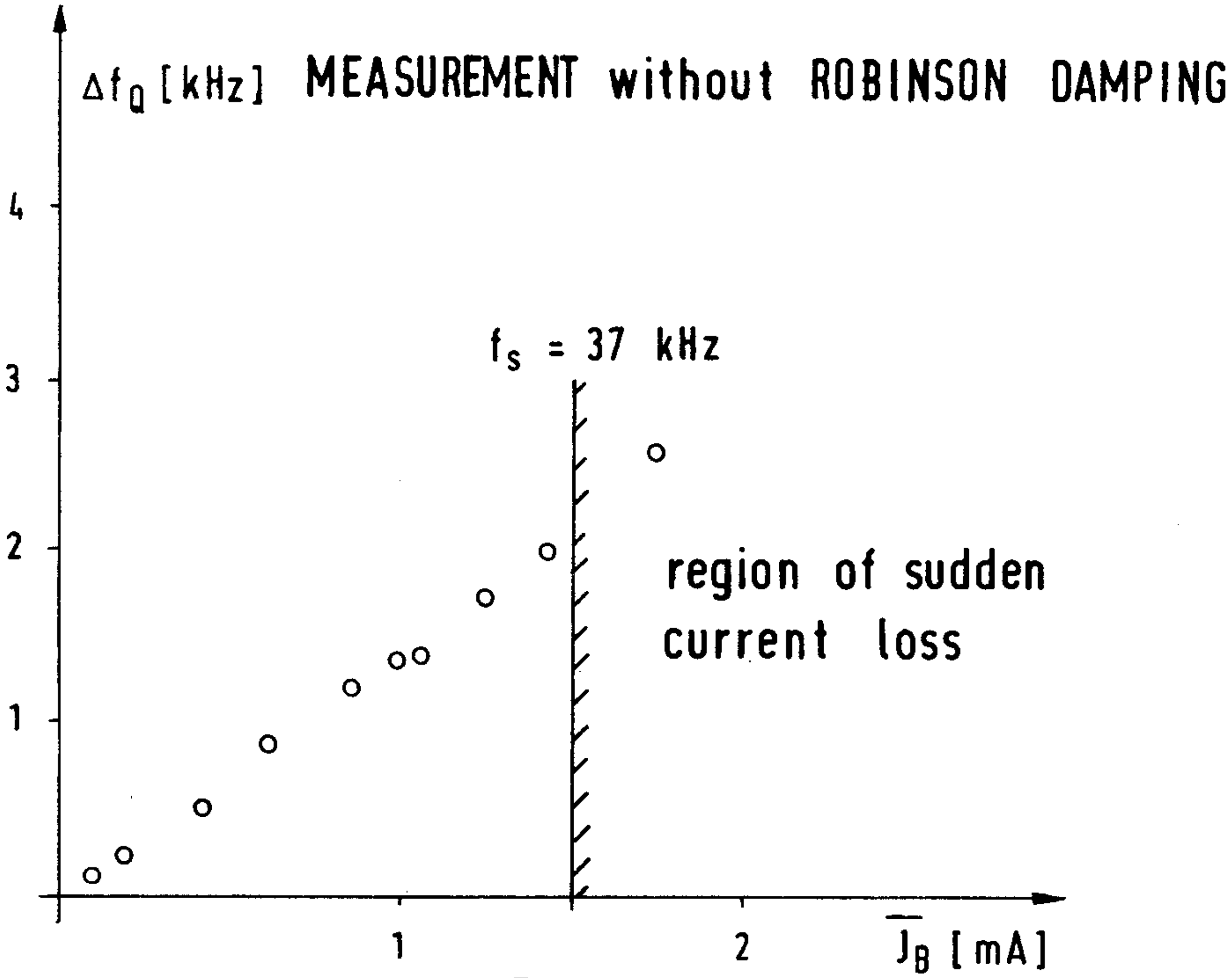
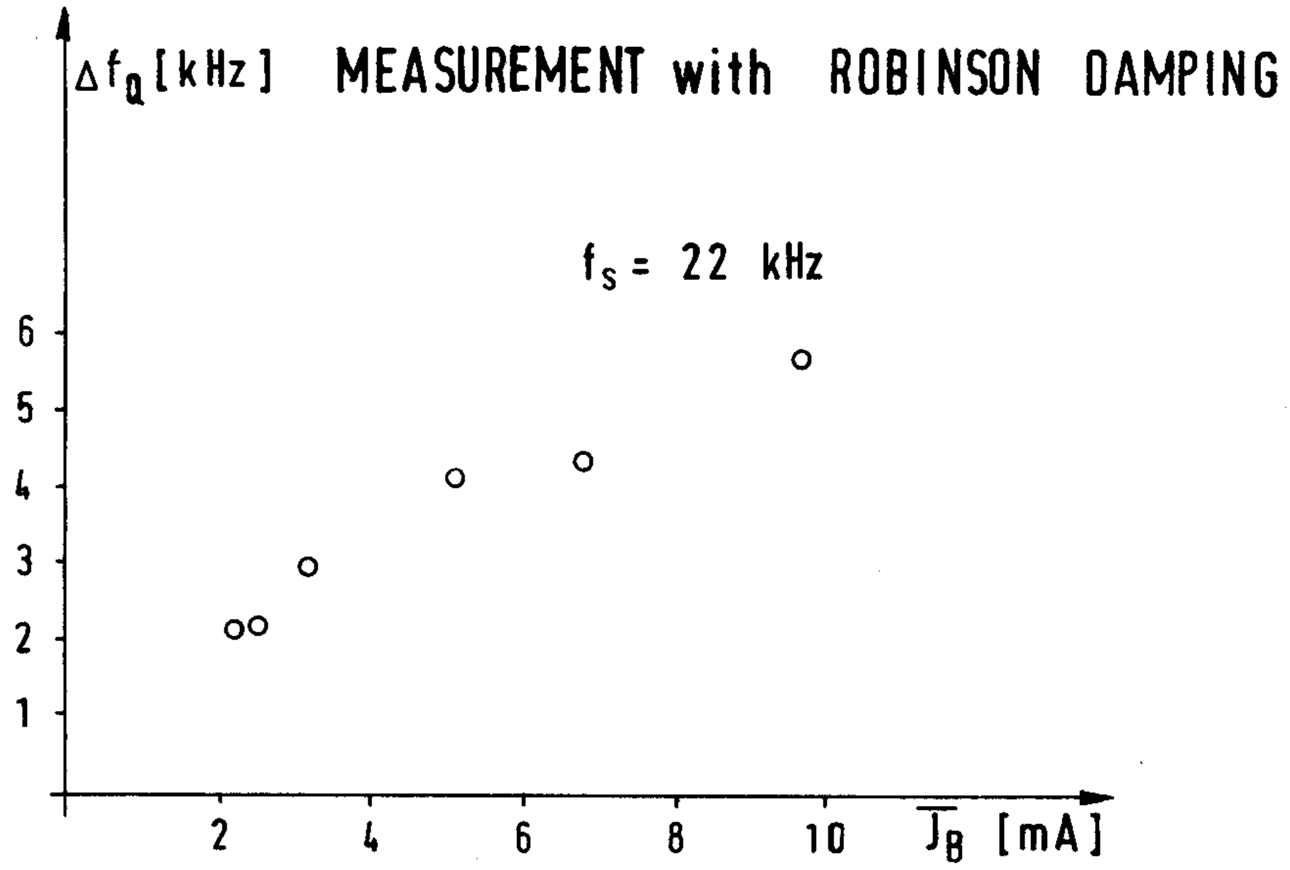


Fig. 1



Δf_Q [kHz] MEASUREMENT without ROBINSON DAMPING
4
3
2
1
f_s = 37 kHz
region of sudden current loss
1
2
J_B [mA]

Fig. 2



Δf_Q [kHz] MEASUREMENT with ROBINSON DAMPING
f_s = 22 kHz
6
5
4
3
2
1
2
4
6
8
10
J_B [mA]

Fig. 3

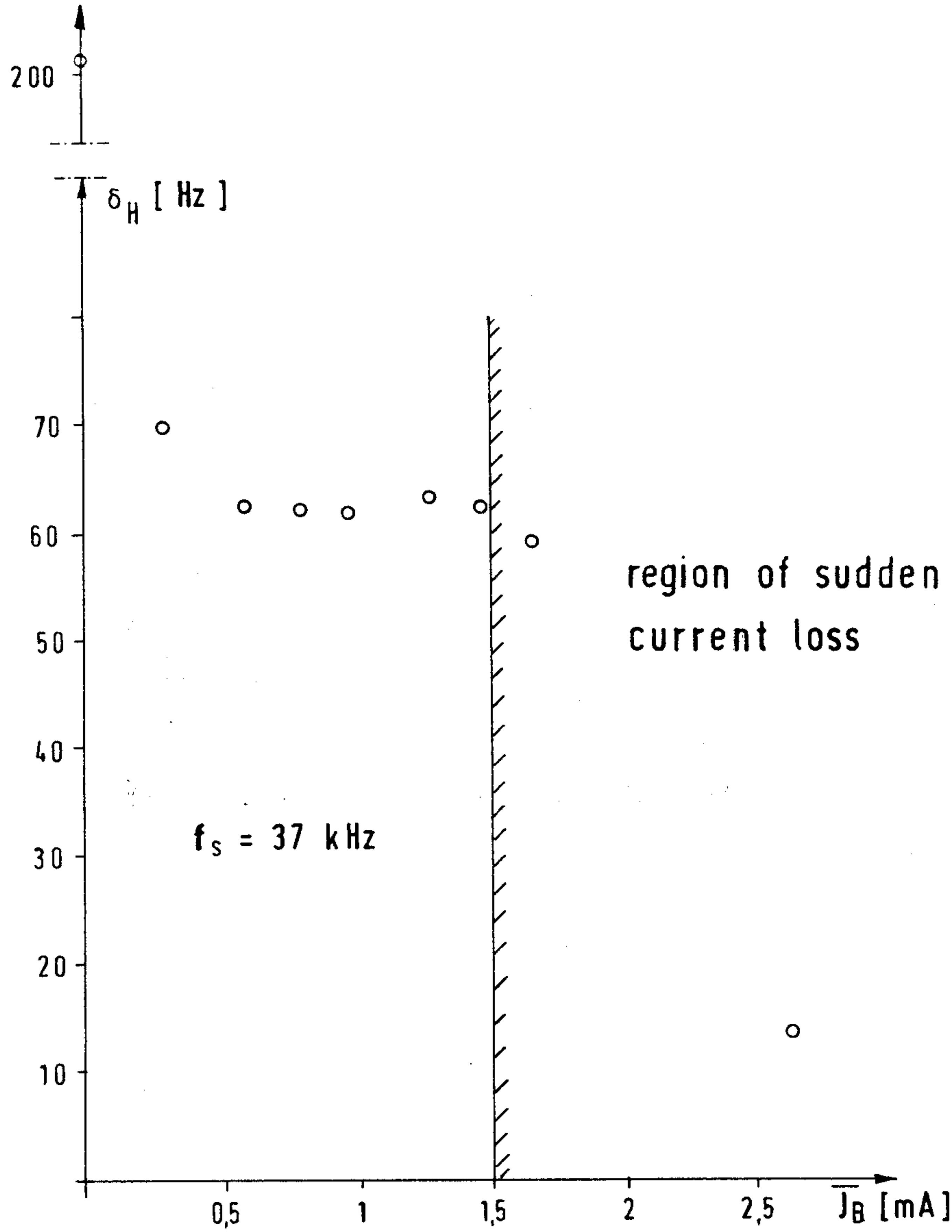

200
δH [Hz]
70
60
50
40
30
20
10
region of sudden current loss
fs = 37 kHz
0,5
1
1,5
2
2,5
JB [mA]

Fig. 4