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Final States in Hadron-Hadron Collisions and in e^+e^- Annihilation

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Abstract:

A study of final states in hadron-hadron collisions in the transverse position plane (b-plane) is presented. First we discuss the elastic processes. The results here are generalized to describe inclusive reactions. The introduction of a production radius typical to each type of produced particle allows us to relate the $p_\perp$-distributions in inclusive and exclusive two-to-two processes. An argument is presented that the elastic $p_\perp$-distributions should be controlled by these production radii even at large $p_\perp$. We then present evidence for the hypothesis that the characteristics of hadronic production in e^+e^- annihilation are those seen in transverse distribution of hadron-hadron collisions. The e^+e^- inclusive distributions are described in terms of two components, a cluster component which contains the e^+e^- resonances, and a parton component. The decay of the cluster is described by the same production radii as those observed in hadron-hadron collisions. The parton component obeys asymptotically Bjorken scaling.

1. Introduction

In the description of final states near the forward direction in hadron-hadron collisions either of the exclusive two-to-two type or in inclusive processes it is well-known that a characteristic length scale appears. In Regge models it is the slope parameter, $\sqrt{\alpha'}$, and in geometric models it is the interaction radius. With the recent data on particle production at large transverse momenta there is now evidence that one can study the short distance interaction or even point interaction and thereby probe the fine structure of strong interactions.[1,2]

To study these problems phenomenologically in a well defined way it seems an interesting possibility to describe the physical amplitudes in the transverse position plane, where the above mentioned characteristics have a clear meaning.

The transverse position $\vec{b}$ of a particle is defined as the variable conjugate to the transverse c.m. momentum, $\vec{p}_\perp$, in the quantum mechanical sense. The b defined in this way is fundamentally different from the impact parameter , b' , usually considered for the description of small angle data at high energies. The impact parameter, b' , for a two-to-two process is defined at a fixed total c.m. energy, $\sqrt{s}$, by a Fourier-Bessel transform, $A(s,b')$, of the scattering amplitude, $F(s,t)$,[3]

$$A(s,b') = \int_0^\infty \sqrt{-t}\, d\sqrt{-t}\; F(s,t)\, J_0(b'\sqrt{-t}) . \qquad (1)$$

The b' is related to the angular momentum, ℓ , and the c.m. momentum, q , through the relation $\ell \approx bq$. In contrast the b-amplitude cannot be defined at a fixed energy and it is therefore not related in a simple way to the angular momentum. For the description of small angle data at high energies the b' has proved to be very useful[4] and in this kinematical region the b and the b' are similar for any practical purposes.[5] If the physics at large $p_\perp$ is characterized by small distance or point interactions the b' may however for this region not be a convenient quantity because here the interpretation of b' as an impact parameter is invalid. This was the main motivation in Ref.5) for studying instead the amplitude structure in the transverse position plane. Even though it seems to be a somewhat unfortunate name we shall sometimes in the following, like in Refs.5, 6), denote the transverse position as an impact parameter.

In Sect.2 we discuss the choice of variables and derive the b-transform for the process with two final state particles. In Sect.3 we give the connection between simple singularities in the b-plane and the corresponding $p_\perp$-behaviour. We study in Sect.4 the two-body reactions phenomenologically in the b-plane and it is here shown how one is lead to the introduction of an interaction radius. In Sects. 5 and 6 the generalization to inclusive processes is given. It is shown that a unified description of $p_\perp$-distributions at not too large $p_\perp$ in both inclusive and exclusive processes is achieved if we to each type of particle assign a production radius.

The general structure of inclusive $p_\perp$-distributions in hadron-hadron reaction is summarized in Sect.7. The amplitude consists of two components corresponding to two singularities in the b-plane. The radius singularity is at $b^2 = -R^2$, $R \simeq 0.5$ fm. This singularity describes the $p_\perp$-distributions at not too large values of $p_\perp$. The other singularity is at $b = 0$ and it is naturally related to the quark-parton interactions. The correspondence principle of Bjorken and Kogut[7)] is here invoked to explain why processes with 2 final state particles apparently are dominated by the singularity given by the interaction radius.

In Sect.8 we apply these ideas to an analysis of the recently measured[8)] inclusive distributions in e^+e^- annihilation. The hypothesis is here that the characteristics of inclusive distributions in e^+e^- annihilation are those seen in transverse distributions in hadron-hadron collisions. We conclude[9)] that the data can be considered also here as given by two components. The first component which dominates at small p (p = c.m. momentum of the observed particle) can physically be interpreted as describing a cluster or fireball, whose decay is controlled by the same radii as those observed in hadron-hadron collisions. This component include the e^+e^- resonances. The other component is dominating at large p and is similar to the large $p_\perp$ component in hadron-hadron interactions. This is the parton component and it obeys Bjorken scaling for $p \to \infty$.

2. The b-transform

Let us first discuss the choice of variables.

Consider the process $a + b \to c + d$. Denote by $\vec{b}$ the relative transverse position of particle c and d in the c.m. system. Since the transverse position operator does not commute with the transverse momentum operator it follows that it also does not commute with the total energy operator. Hence it is quantum mechanically inconsistent to consider an amplitude $A(s,b)$ as a function of energy and b at the same time. The obvious variable to choose instead of the energy is $p_{\parallel}$, the longitudinal c.m. momentum. Indeed we have the canonical commutation relations

$$[b_i, b_j] = 0 \quad ; \quad [b_i, p_{\perp j}] = i\delta_{ij} \quad ; \quad [b_i, p_{\parallel}] = 0 \quad , \quad i,j = 1,2 \tag{2}$$

where i,j label the two orthogonal transverse directions.

The b-amplitude $A(p_{\parallel}, \vec{b})$ is given by the matrix element of the T-matrix $(S = 1 - iT)$ between suitably defined initial and final states

$$A(p_{\parallel}, \vec{b}) = \langle f | T | i \rangle \quad . \tag{3}$$

Following the previous discussion we choose

$$| f \rangle = | p_{\parallel}, \vec{b} \rangle \quad . \tag{4}$$

The initial state has to specify the longitudinal direction. This is achieved by the condition that the relative transverse momentum, $\vec{k}_{\perp}$, between a and b is zero. We furthermore specify that a and b before the scattering (i.e. at the time $t \to -\infty$) were in some sense close together. This we do by putting $z = 0$, where z is the relative longitudinal position between a and b in the c.m. system. That is

$$| i \rangle = | z = 0 , \vec{k}_{\perp} = \vec{0} \rangle \quad . \tag{5}$$

As a side remark we note that by replacing z by the longitudinal momentum, $k_{\parallel}$, we would in (3) just project out of T the usual scattering amplitude, $F(s,t)$, even with the form (4) of $|f\rangle$.

To obtain the b-transform we express $A(p_{\parallel}, \vec{b})$ in terms of the T-matrix in the momentum representation, $\langle \vec{p} | T | \vec{k} \rangle$, in the following way

$$A(p_{\parallel},\vec{b}) = \langle p_{\parallel},\vec{b}\,|T|\,z=0\,,\,\vec{k}_{\perp}=\vec{0}\,\rangle$$
$$= \sum_{\vec{p}_{\perp},k_{\parallel}} \langle p_{\parallel},\vec{b}\,|\,p_{\parallel},\vec{p}_{\perp}\rangle \langle p_{\parallel},\vec{p}_{\perp}|T|k_{\parallel},\vec{k}_{\perp}=\vec{0}\rangle\langle k_{\parallel},\vec{k}_{\perp}=\vec{0}\,|z=0,\vec{k}_{\perp}=\vec{0}\rangle. \tag{6}$$

As described in Ref.5) this gives the result

$$A(p_{\parallel},\vec{b}) = \int \frac{d^2\vec{p}_{\perp}}{(2\pi)^2}\, e^{-i\vec{b}\vec{p}_{\perp}}\, F(p_{\parallel},\vec{p}_{\perp}) \tag{7}$$

where F is normalized to the differential cross section by

$$\frac{d\sigma}{dt} = \frac{1}{64\pi}\,|F|^2 \tag{8}$$

The possibility of considering a b-transform similar to Eq.(7) was first mentioned by Chang and Raman,[10] who did not, however, apply it in any analysis of data.

The inclusion of spin in the formalism does not present any problems.[5] Here and in the following we do, however, disregard this complication. We therefore neglect any angular dependence in $F(p_{\parallel}, \vec{p}\;)$ and Eq.(7) becomes

$$A(p_{\parallel},b) = \frac{1}{8\pi}\int_0^{\infty} p_{\perp}dp_{\perp}\, J_0(p_{\perp}b)\, F(p_{\parallel},p_{\perp}) \tag{9}$$

where J_0 is the Bessel function of order 0.

The b-transform (9) is similar to the b'-transform of Eq.(1) except that the $(\text{energy})^2$, s, is replaced by $p_{\parallel}$ in Eq.(9). This has the important consequence that the integration path in (9) is fully within the physical region. It starts in the forward direction at $p_{\perp} = 0$ and ends at scattering angles of 90° for $p_{\perp} \to \infty$.

3. Singularities of the b-transform

Since the Bessel function of integer order are entire functions with an essential singularity at ∞ the impact parameter transform (9) will be an analytic function of b with singularities controlled by the singular behaviour of the integrand at $p\ = \infty$, i.e. by the character of the asymptotic behaviour of $F(p_{\parallel}, p_{\perp})$ for $p_{\perp} \to \infty$ at given $p_{\parallel}$.

In Section 4 evidence is presented for exponential behaviour in that limit and one finds[5, 11] that the behaviour ($p_{\parallel}$ fixed)

$$F(p_{\parallel}, p_{\perp}) \sim M(p_{\perp})\, p_{\perp}^{-\alpha} e^{-\beta p_{\perp}} \quad , p_{\perp} \to \infty \tag{10}$$

with $M(p_{\perp}) \sim O(1)$ as $p_{\perp} \to \infty$ will give rise to singularities in the b-plane at $b = \pm b_o$ with

$$\operatorname{Im} b_o = \beta \tag{11}$$

Near $b = b_o$ we have the behaviour

$$A(p_{\parallel}, b) \propto (b - b_o)^{\alpha - 3/2} + \text{less singular piece} \tag{12}$$

If further $M(p_{\perp})$ tends to a definite non-zero limit as $p_{\perp} \to \infty$, $\operatorname{Re} b_o = 0$. If, on the other hand, $M(p_{\perp})$ oscillates at ∞, $\operatorname{Re} b_o$ may be $\neq 0$. Thus if

$$M(p_{\perp}) \sim m(p_{\perp})\, e^{i\gamma p_{\perp}} \quad \text{or} \tag{13}$$

$$M(p_{\perp}) \sim m(p_{\perp}) \cos(\gamma p_{\perp} + \delta)$$

with $m(p_{\perp})$ approaching a non-zero constant, then we get

$$\operatorname{Re} b_o = \gamma \quad . \tag{14}$$

4. The b-plane for Two-to-Two Processes

We have now the whole machinery set up for a study of the b-plane and in the following we are going to study the structure of the b-plane for elastic processes.

From Eqs.(8, 9) we see that the b amplitude is studied through data for differential cross sections along lines of fixed $p_{\parallel}$. On Figs.1 and 2 such data are shown for $\pi^{\pm}p$ and pp elastic scattering.

In all cases we see at large $p_\perp$ a clear tendency for the data on these plots to approach straight lines. Combining this observation with the well-known exponential behaviour in t near $t = 0$, or alternatively, with the fact that F is a function of $p_\perp^2$ (i.e. of $|\vec{p}_\perp|$) we are lead, at fixed $p_\parallel$, to a parametrization of the form

$$|F|^2 = a\, e^{-2R\sqrt{p_\perp^2+m^2}} \tag{15}$$

with real parameters a, R and m. It is found that the elastic $\pi^\pm p$ data, except at $p_\parallel \simeq 0$, is well described by the form (15), where the slope parameter $R_{\pi N}$ turns out to be indpendent of $p_\parallel$ and of the charge of the pion. Similarly the pp data are well described by the form (15), where R_{pp} is independent of $p_\parallel$. The values of the R's are

$$R_{\pi N} = 10.6 \pm 0.5 \text{ GeV}^{-1} \;;\; R_{pp} = 8.2 \pm 0.2 \text{ GeV}^{-1} . \tag{16}$$

At $p_\parallel \simeq 0$ there is in the data an extra component which gives oscillations around our form (15).[14)]

In conclusion the data for elastic scattering ($c + d \to c + d$) at large $p_\perp$ appears to contain a leading contribution which has the $p_\perp$ structure of the type

$$F(p_\parallel, p_\perp) \sim e^{-2R_{cd}\, p_\perp} \quad , \; p_\perp \to \infty \quad . \tag{17}$$

From the results described in Sect.3 we conclude that a dominant component of the elastic amplitudes at fixed $p_\parallel$ can be described by a pair of singularities in the b-plane at $b = \pm b_o$, where

$$b_o = i\, R_{cd} \tag{18}$$

and R_{cd} is independent of $p_\parallel$. The precise nature of the singularity cannot be determined from this study but the phenomenological limit on a possible factor of the type $p_\perp^{-\alpha}$ in Eq.(17) is that $|\alpha| \lesssim 1/2$.

The interpretation of R as an interaction radius is based on the observation that the b-amplitude, resulting from a singularity of the type (12) with $\alpha \simeq 0$ and at the position given by Eq.(18), is large in a range of b of the order of R.

5. The Inclusive $p_\perp$ -Distribution

Consider the process

$$a+b \to c + x_1 + x_2 + \cdots + x_n \tag{19}$$

where in the c.m. system the particle c has momentum $\vec{p} = (p_\parallel, \vec{p}_\perp)$ and particle x_i has momentum $\vec{Q}_i = (k_i, \vec{q}_i)$. $p_\parallel$ and k_i are the longitudinal components and $\vec{p}_\perp$ and $\vec{q}_i$ are the transverse components.

The energy is given by (neglect masses)

$$\begin{aligned}\sqrt{s} &= \sqrt{p_\parallel^2 + \vec{p}_\perp^{\,2}} + \sum_{i=1}^{n} \sqrt{k_i^2 + \vec{q}_i^{\,2}} \\ &\simeq \sqrt{p_\parallel^2 + \vec{p}_\perp^{\,2}} + \sum_{i=1}^{n} |k_i|\end{aligned} \tag{20}$$

since on the average, phenomenologically, $|\vec{q}_i| \ll k_i$ (see e.g. Ref.15)).

In the following we want to consider the inclusive process $a + b \to c + X$ only in the transverse plane and it follows from Eq.(20) that energy conservation can be disregarded during integration over the $\vec{q}_i$'s. Then the inclusive cross section is given by (suppressing longitudinal variables)

$$\frac{d\sigma}{d\vec{p}_\perp}(a+b \to c + \underline{X}) = \sum_n \sigma_n(p_\perp) \tag{21}$$

where σ_n in terms of the amplitude F_n for the process (19) is given by

$$\sigma_n(p_\perp) = \int |F_n(\vec{p}_\perp, \vec{q}_1, \vec{q}_2, \cdots, \vec{q}_n)|^2 \, \delta^{(2)}(\vec{p}_\perp + \sum_i \vec{q}_i) \prod_i d^2\vec{q}_i \tag{22}$$

The transverse position $\vec{b}_i$ is defined as the variable conjugate to the transverse momentum $\vec{q}_i$. With a technique similar to that of Sect.2 the amplitude F_n can then be written in terms of its b-transform A_n as[16)]

$$\begin{aligned}F_n(\vec{p}_\perp, \vec{q}_1, \cdots, \vec{q}_n) = \left(\frac{1}{2\pi}\right)^{2n+2} \int d^2b_c \prod_i d^2\vec{b}_i \, e^{-ib_c\vec{p}_\perp} \, e^{-i\sum_i \vec{b}_i\vec{q}_i} \, \times \\ \times \, \delta^{(2)}(\vec{b}_c + \sum_i \vec{b}_i - \vec{B}) \, A_n(\vec{b}_c, \vec{b}_1, \cdots, \vec{b}_n) \, .\end{aligned} \tag{23}$$

The $\vec{b}_c$ is the position of the observed particle, c . The δ-function expresses the fact that the scattering is a function only of differences of $\vec{b}$'s. Due to the translational invariance in the transverse plane $\vec{B}$ is arbitrary and we choose $\vec{B} = \vec{0}$.

Eqs.(22,23) obviously have quite a complicated structure. It is f. ex. clear that the $p_\perp$ -behaviour of F_n in principle depends on not only $\vec{b}_c$ but on all the $\vec{b}_i$'s too, simply because, as expressed by Eq.(23), the scattering is a function only of differences of b's. In Ref.6) it is however shown that in the case where one on the average has many particles in the final state, then the averaging over all the unobserved particles gives the result that the $p_\perp$ -dependence depends only on the b-structure of the observed particle, c.

To obtain this result one has to infer a smoothness assumption on the impact parameter amplitude. Since hadrons are assumed to be extended objects with spatial extension of the order of 1 fm, we also assume that A_n is significantly different from zero only in a limited region in $\vec{b}$-space with extension $\simeq$ 1 fm. We furthermore assume that n is big enough so that A_n , as a function of $\vec{b}_i$, is approximately constant over a range of the order of 1 fm/n. From this assumption one can show that[6)]

$$\sigma_n(p_\perp) \simeq \frac{1}{(2\pi)^2 n} \int d^2\vec{b}_2 \cdots d^2\vec{b}_n \left| \int_0^\infty b_c db_c J_0(p_\perp b_c) A_n(b_c, b_1 = -\sum_{i=2}^{n} \vec{b}_i, \vec{b}_2, \vec{b}_3, \ldots, \vec{b}_n) \right|^2 \quad (24)$$

and we see explicitly that the $p_\perp$ -behaviour is now completely determined by the singularities in the b_c-plane.

6. Unified b-plane Structure for Exclusive and Inclusive Processes.

In order to unify the description of $p_\perp$ -distributions of inclusive and exclusive two-to-two processes we summarize what are the b's pertaining to inclusive and exclusive scattering. The b appearing in Eq.(9) is the relative distance between the two final state particles and we have (see Fig.3)

$$\begin{aligned} a+b &\rightarrow c+\underline{X} &&: \quad b = b_c \\ a+b &\rightarrow c+d &&: \quad b = b_c - b_d \end{aligned} \quad (25)$$

From here it seems plausible that the R_{cd} of Eq.(18) should be composed of two terms

$$R_{cd} = R_c + R_d \tag{26}$$

where R_c and R_d are some characteristic lengths (production radii) associated with particles c and d. We here already implicitly made the assumption that $R_{c,d}$ are independent of the particles a and b in the initial state.

The result (26) is obtained if we associate singularities to the position b_j at $b_j = \pm i R_j$. From Eqs.(25) we then get the singularity b_o of Eqs.(18,26). We also get singularities at $b = \pm b_o'$ with $b_o' = i(R_c - R_d)$. If $R_c \simeq R_d$ this interaction will however be very central (almost point-like) and we expect such terms to show up at much larger values of $p_\perp$ than the contribution given by the singularity b_o.

It is now clear how, within this scheme, one is able to relate the $p_\perp$-distributions of inclusive and exclusive processes. From Eqs.(25) the impact parameter, b , for the process $a + b \to c + X$ is given by the impact parameter b_c of the observed particle, c . b_c has singularities at $\pm i R_c$ and these produce a $p_\perp$-spectrum of the form

$$\frac{d\sigma}{d\vec{p}_\perp}(a+b \to c+\underline{\overline{X}}) \sim e^{-2R_c p_\perp} \tag{27}$$

In the whole discussion of inclusive distributions we have considered only the transverse variables. In addition to the transverse position b (or the transverse momentum $p_\perp$) we choose the longitudinal momentum , $p_\parallel$, and the missing mass, M , which for each process (19) is given by

$$M = \sum_{i=1}^{n} \sqrt{k_i^2 + \vec{q}_i^{\,2}} \quad , \tag{28}$$

as the variables specifying the inclusive amplitude. This we do because b commutes with $p_\parallel$ and M and not, for example, with the more conventional variables $\sqrt{s}$ and $x = 2p_\parallel/\sqrt{s}$ or the scattering angle. R_c is therefore in principle a function of $p_{\parallel}$ and M. We have, however, in the analysis of elastic data seen that R_c is independent of $p_{\parallel}$ and since we now assume that R_c is independent of the other particles we must have that R_c is also independent of M .

From Eqs.(17,26,27) we can relate the inclusive to the exclusive $p_{\perp}$-distributions. It is clear from the observed $p_{\perp}$-distributions at ISR[1] that the form (27) only works in a limited $p_{\perp}$-range. On Fig.4 we compare the radii as found by fits to the ISR data in this $p_{\perp}$-range with the pion and nucleon radii as calculated from Eqs.(16,26):

$$R_{\pi} = 3.25 \pm 0.25 \text{ GeV}^{-1} \; ; \; R_p = 2.05 \pm 0.05 \text{ GeV}^{-1} \tag{29}$$

On Fig.5 the inclusive data at $\sqrt{s} = 52.3$ are compared with the fits of the form (27). The fact that we look at data at a fixed energy instead of at a fixed missing mass is unimportant since in practice here $\sqrt{s} \simeq M$.

The inclusive kaon slope R_K is over the ISR energies consistent with

$$R_K \simeq 2.5 \text{ GeV}^{-1} \; . \tag{30}$$

This taken together with the R_p of Eq.(29) gives us a value for $R_{Kp} = R_K + R_p$. This is in Ref.6) confronted with the 90° cross section for the process $K^+p \rightarrow K^+p$. The data for this process are not by themselves good enough for a determination of a slope but the comparison is satisfactory.

In conclusion the comparison of slopes in inclusive and exclusive $p_{\perp}$-distributions works well over the whole range of ISR energies thus confirming the simple factorization property of the contributions from the radius singularity expressed by Eqs.(17,26,27). We furthermore consider this result as strong evidence for the physical significance of the production radius R_c associated with each type of produced particle, c.

7. General Two-Component Structure of Inclusive $p_{\perp}$ - Distribution.

As mentioned earlier the description of the inclusive data in the whole $p_{\perp}$-range of the ISR experiments requires an extra component in addition to the radius contribution of Eq.(27). In Ref.1) is presented a parametrization of the inclusive data which at $p_{\parallel} = 0$ can be written as (here we use that $\sqrt{s} \simeq M$)

$$E\frac{d\sigma}{d^3p}\bigg|_{p_{\parallel}=0} = C(M)\,e^{-2Rp_\perp} + \frac{f(x)}{(p_\perp^2+m^2)^N} \quad ; \quad x = \frac{2p_\perp}{M} \qquad (31)$$

The second term of Eq.(31) is dominating at large values of $p_\perp$ and M. The particular form of this term is discussed theoretically in the constituent interchange model of Ref.2). For the production of pions, kaons and nucleons the parameter m is in the range $1.0 \lesssim m \lesssim 1.2$ GeV and f(x) is in a limited range of x given by

$$f(x) \simeq (1-x)^\gamma \approx e^{-\gamma x} \qquad (32)$$

with $\gamma \simeq 12$. The power N is $N \simeq 8$.

Whereas the first term of Eq.(31) in the b-plane is described by a singularity at $b^2 = -R^2$ the second term is described by a singularity at $b = 0$, see Sect.3, and the natural interpretation is, like in Ref.2), that we here see the effect of the point parton interactions.

The parton interaction is by Eqs.(31,32) seen to descrease monotonically with decreasing M at fixed value of $p_\perp$. Application of the correspondence principle of Bjorken and Kogut[7)] then implies that the exclusive two-to-two $p_\perp$ - distributions (low missing mass) should be dominated by the radius singularity over a much larger $p_\perp$-range than the inclusive $p_\perp$-distributions. This is in agreement with the results discussed in the previous sections, where no significant trace of an extra component at large $p_\perp$ was observed in elastic data in addition to the pure radius contribution of Eq.(17).

8. Application to e^+e^- Inclusive Distributions

In carrying over the general ideas presented in the previous sections to e^+e^- annihilation there is one obvious change. Since e^+e^- annihilation is thought to proceed via one photon exchange we are lacking the preferred direction which we had in hadron-hadron collisions. If there is any similarity between the two cases one would therefore expect the inclusive p-distributions (p = total c.m. momentum of the observed particle) in e^+e^- annihilation to be similar to the $p_\perp$-distributions in hadron-hadron collisions.

To explain the inclusive e^+e^- data several models have been proposed. They fall roughly into two classes. The quark parton picture and the cluster or thermodynamical picture. The cluster models give p-distributions of the type of the first term of Eq.(31) and the parton models give contributions like the second term of Eq.(31), where now $p_\perp$ is replaced by p. In comparison with data none of these models work: either they fail at large or at small p [17]. In the following we argue[9] that the data at presently measured energies contain both components and that the inclusive p-distributions in e^+e^- annihilation have the form of Eq.(31) with, however, $p_\perp$ replaced by p.

First we show that the invariant e^+e^- inclusive distributions contain at low c.m. energies, $\sqrt{s}$, indeed a large cluster component with the same radii for π, K and p as found in hadron-hadron collisions. This is illustrated on Fig.6 by the DASP data[8] on the $\Psi(3.1)$ resonance of inclusive production of π's, K's and $\bar{p}$'s. The data are excellently described by the cluster component

$$E_h \frac{d\sigma_{CLUSTER}}{d^3p}(3.1\ \mathrm{GeV}) = C\, e^{-2R_h p} \tag{33}$$

and we emphasize with the same radii as found in hadron-hadron collisions, Eqs.(29,30).

The SPEAR group now has data in the energy range between 3 and 7.4 GeV[8]. Unfortunately they have made no particle separation and we have to choose an average radius $\langle R\rangle$, $R_p \leq \langle R\rangle \leq R_\pi$, in order to describe the charged particle inclusive distributions. With the choise $\langle R\rangle = 2.5\ \mathrm{GeV}^{-1}$, see Eqs.(29,30), we compare the experimental data on Fig.7 with the cluster component

$$E \frac{d\sigma_{CLUSTER}}{d^3p} = C(\sqrt{s})\, e^{-2\langle R\rangle p} \tag{34}$$

The strength, $C(\sqrt{s})$, is obtained by normalization to the data at $p = 0.3$ GeV. It is seen that the low p data are nicely described by (34) even at the highest energy, $\sqrt{s} = 7.4$ GeV. The function $C(\sqrt{s})$ is shown in Fig.8 and compared with σ_{tot}, from where we obtain

$$C(\sqrt{s}) \simeq (10\ \mathrm{GeV}^{-2})\, \sigma_{tot}(\sqrt{s}) \ . \tag{35}$$

It is clearly seen from the peak in $C(\sqrt{s})$ near 4.1 GeV that the cluster component includes the resonances and this is just what one would expect if our physical picture is correct, where the cluster component describes the isotropic decay of fireballs.

The remaining part of the cross section should be the parton component $E\, d\sigma_{PARTON}/d^3p$ and similar to the second term of Eq.(31). It is an interesting question to see whether this contribution obeys Bjorken scaling[18] for $p \to \infty$. If this is the case we have N = 2 and on Fig.9 we plot $(p^2 + m^2)^2 x^{-2}\, E\, d\sigma_{PARTON}/d^3p$ as a function of the scaling variable $x = 2p/\sqrt{s}$. The significance of the factor x^{-2} is discussed in Ref.9). The mass parameter m is chosen as m = 1.2 GeV, i.e. in the middle of the range required by the various particles at ISR, see Sect.7. We see that the plotted quantity does indeed scale (within appreciable errors at large x) and it is furthermore well approximated by an exponential, cf. Eq.(32). This means that

$$s\frac{d\sigma_{PARTON}}{dx} \simeq D\left(\frac{p^2}{p^2+m^2}\right)^2 \frac{e^{-ax}}{x} = D\,\frac{x^3}{\left(x^2+\frac{4m^2}{s}\right)^2} e^{-ax} \tag{36}$$

where $D = 1.83 \times 10^4$ nb GeV^2 and $a = 6.05$. Here it is explicitly seen that the parton component asymptotically obeys Bjorken scaling where the approach to scaling is set by the mass m.

To summarize the main points of this section we have seen that the e^+e^- inclusive p-distributions can be described by a sum of two terms, a cluster component and a parton component. The full line on Fig.7 represents the sum of these two contributions as given by Eqs.(34,36). As seen by Eqs.(34-36) the parton component becomes more and more important as energy increases and eventually one is left with the pure scaling parton component. For the further discussion of the interesting physical quantities $\langle E_{ch}\rangle$ and $\langle n_{ch}\rangle$, the average energy per charged particle and the average multiplicity, within this scheme we refer to Ref.9).

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Figure Captions

Fig.1: Differential cross sections for $\pi^{\pm}p \to \pi^{\pm}p$ at given values of $p_{\parallel}$. Data from Ref.12). Curves are parametrization of data of the form (15). The lower $p_{\perp}$ scale is used for all given values of $p_{\parallel}$.

Fig.2: Differential cross sections for $pp \to pp$ at given values of $p_{\parallel}$. Data from Ref.13). The straight lines all correspond to exponential behaviour $e^{-8.28\, p_{\perp}}$. The lower $p_{\perp}$ scale is used for all given values of $p_{\parallel}$.

Fig.3: Impact parameter for inclusive and exclusive processes.

Fig.4: The π, K and N radii. The bands give R_{π} and R_{p} as calculated by Eqs.(16,26) from πN and pp elastic scattering. The points are the radii as estimated from the ISR 90° spectra for π^{+}(o), π^{-}(•), K^{+}(□), K^{-}(■), p(△) and $\bar{p}$(▲).

Fig.5 The π, K, p and $\bar{p}$ spectra at s = 53 GeV and $\theta_{cm} = 90°$ compared with the contribution from the radius singularity, Eq.(27).

Fig.6: Invariant cross sections for inclusive $e^{+}e^{-}$ production of $\pi^{\pm}$, $K^{\pm}$ and $\bar{p}$ at s = 3.1 GeV. The full line represent the cluster contribution, Eq.(33), with the slopes given by Eqs.(29,30). Data from DASP[8)].

Fig.7: Invariant cross sections for inclusive charged particle production. The dashed line gives the cluster contribution, Eq.(34), the dot-dashed line the parton contribution, Eq.(36), and the full line the sum of these two contributions. Data from SPEAR[8)].

Fig.8: The strength, $C(\sqrt{s})$, of the cluster component, Eq.(34), compared with $\sigma_{tot}(\sqrt{s})$. Data from SPEAR[8)].

Fig.9: The scaling parton cross section. Data from the energies $\sqrt{s}$ = 3.8, 4.2, 4.8, 6.2, 7.4 GeV.

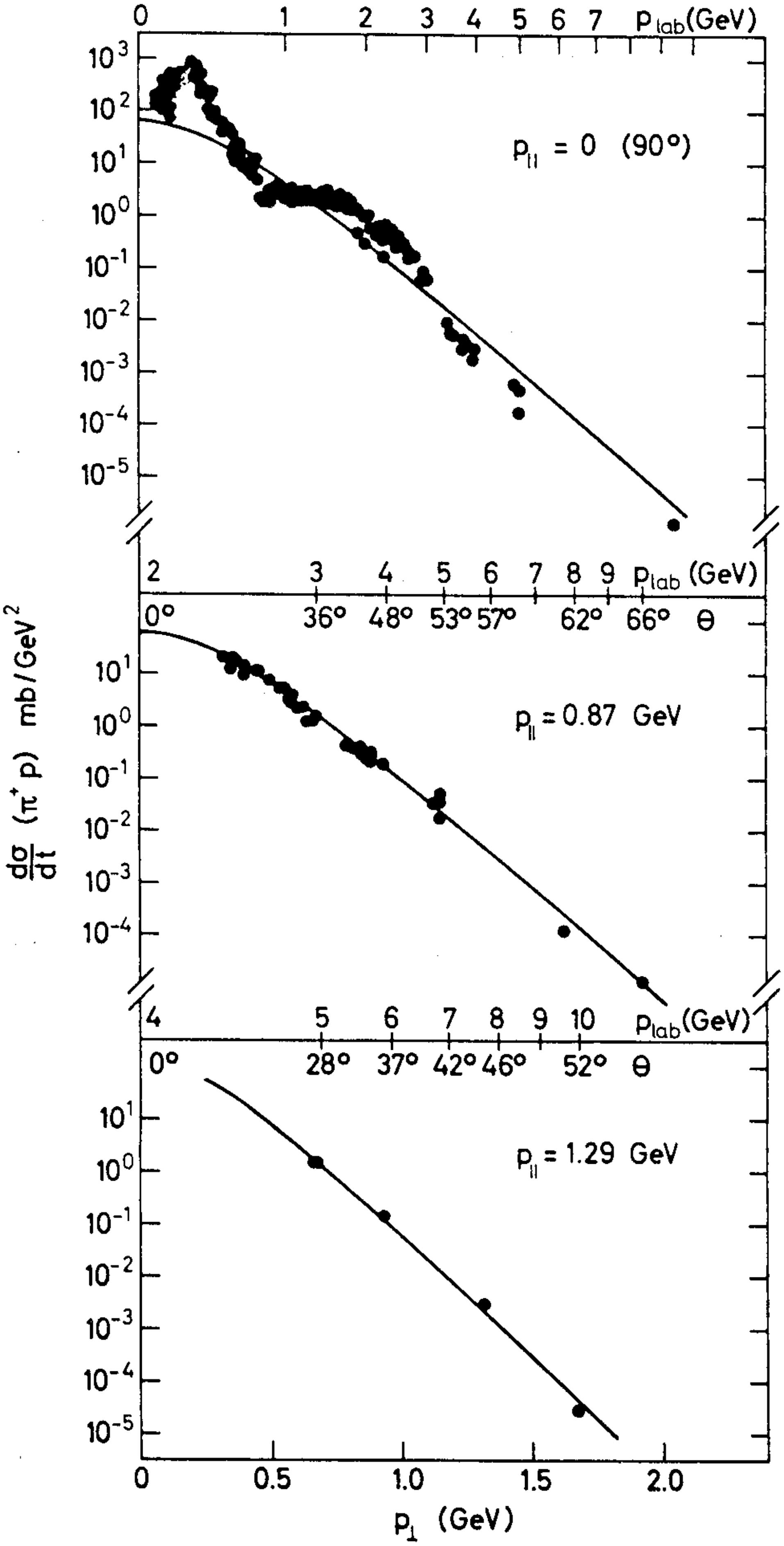
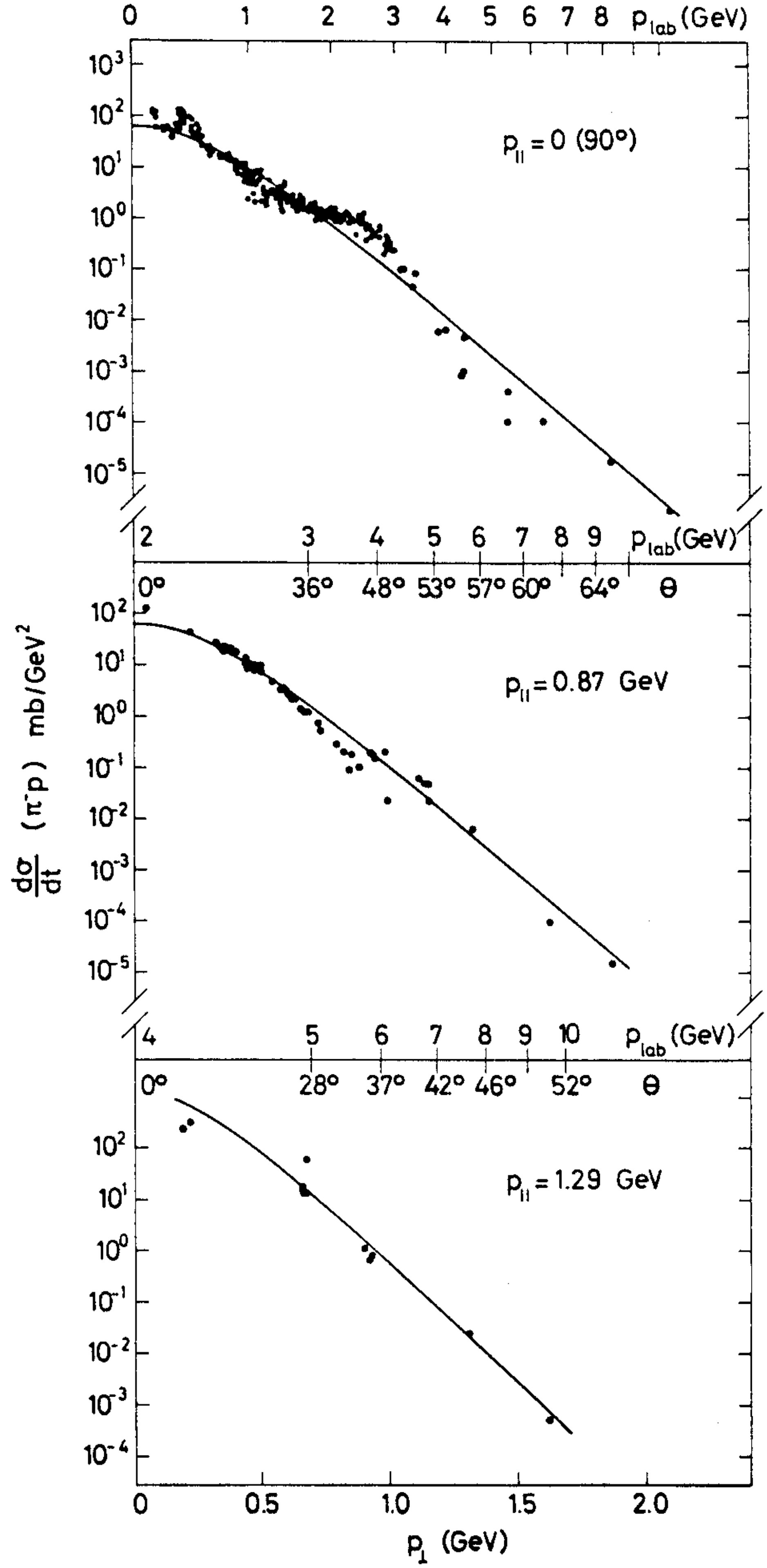

0 1 2 3 4 5 6 7 p_{lab}(GeV)
10^3 10^2 10^1 10^0 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5}
$p_{\parallel}$ = 0 (90°)
2 3 4 5 6 7 8 9 p_{lab} (GeV)
0° 36° 48° 53° 57° 62° 66° Θ
10^1 10^0 10^{-1} 10^{-2} 10^{-3} 10^{-4}
$p_{\parallel}$ = 0.87 GeV
$\frac{d\sigma}{dt}$ (π^+p) mb/GeV^2
4 5 6 7 8 9 10 p_{lab}(GeV)
0° 28° 37° 42° 46° 52° Θ
10^1 10^0 10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5}
$p_{\parallel}$ = 1.29 GeV
0 0.5 1.0 1.5 2.0
$p_{\perp}$ (GeV)

Fig. 1a



0 1 2 3 4 5 6 7 8 p_{lab} (GeV)
$p_{\parallel} = 0$ (90°)
2 3 4 5 6 7 8 9 p_{lab} (GeV)
0° 36° 48° 53° 57° 60° 64° θ
$p_{\parallel} = 0.87$ GeV
4 5 6 7 8 9 10 p_{lab} (GeV)
0° 28° 37° 42° 46° 52° θ
$p_{\parallel} = 1.29$ GeV
$\frac{d\sigma}{dt}$ ($\pi^- p$) mb/GeV^2
0 0.5 1.0 1.5 2.0
$p_\perp$ (GeV)

Fig. 1b

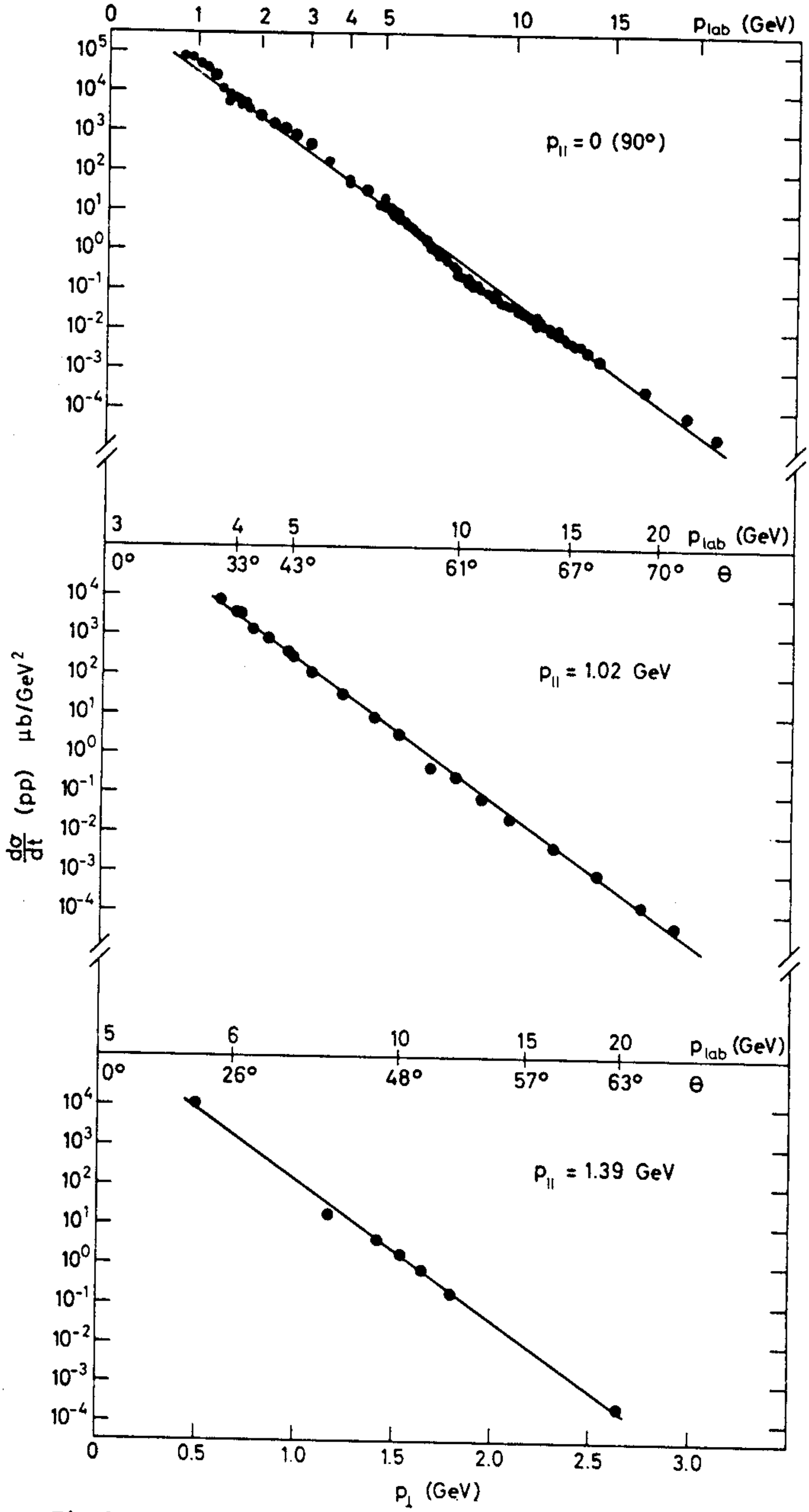


Fig.2

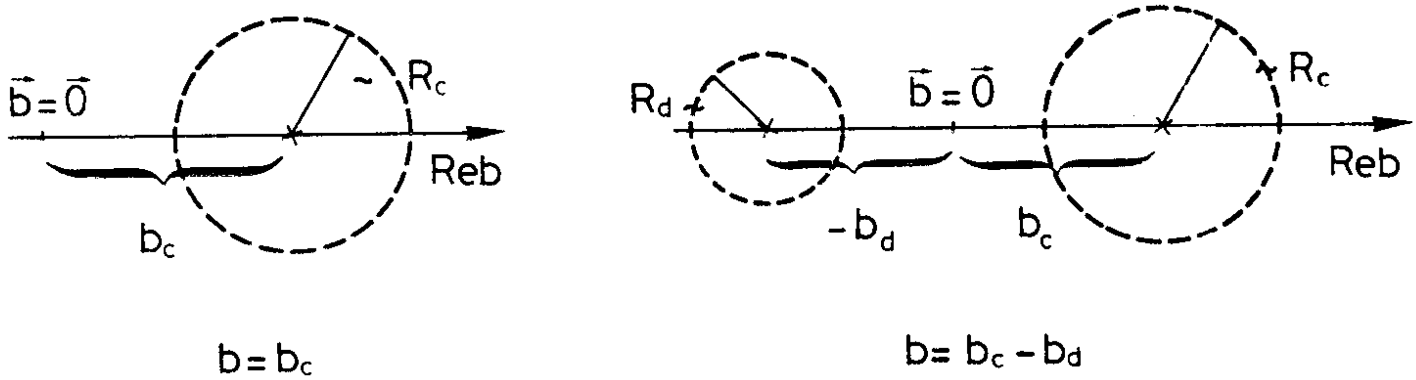


Fig.3

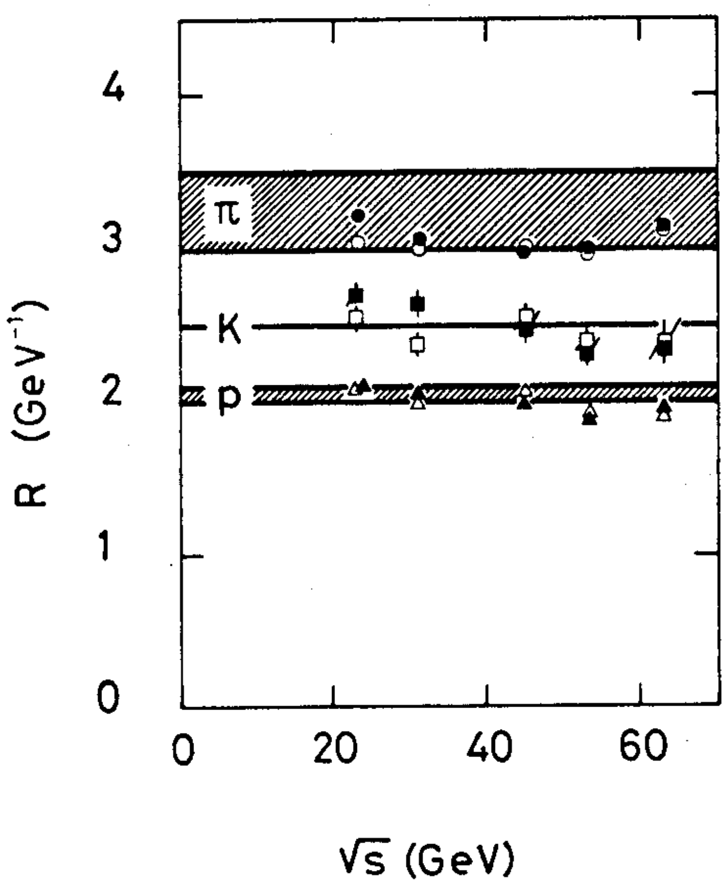


Fig.4

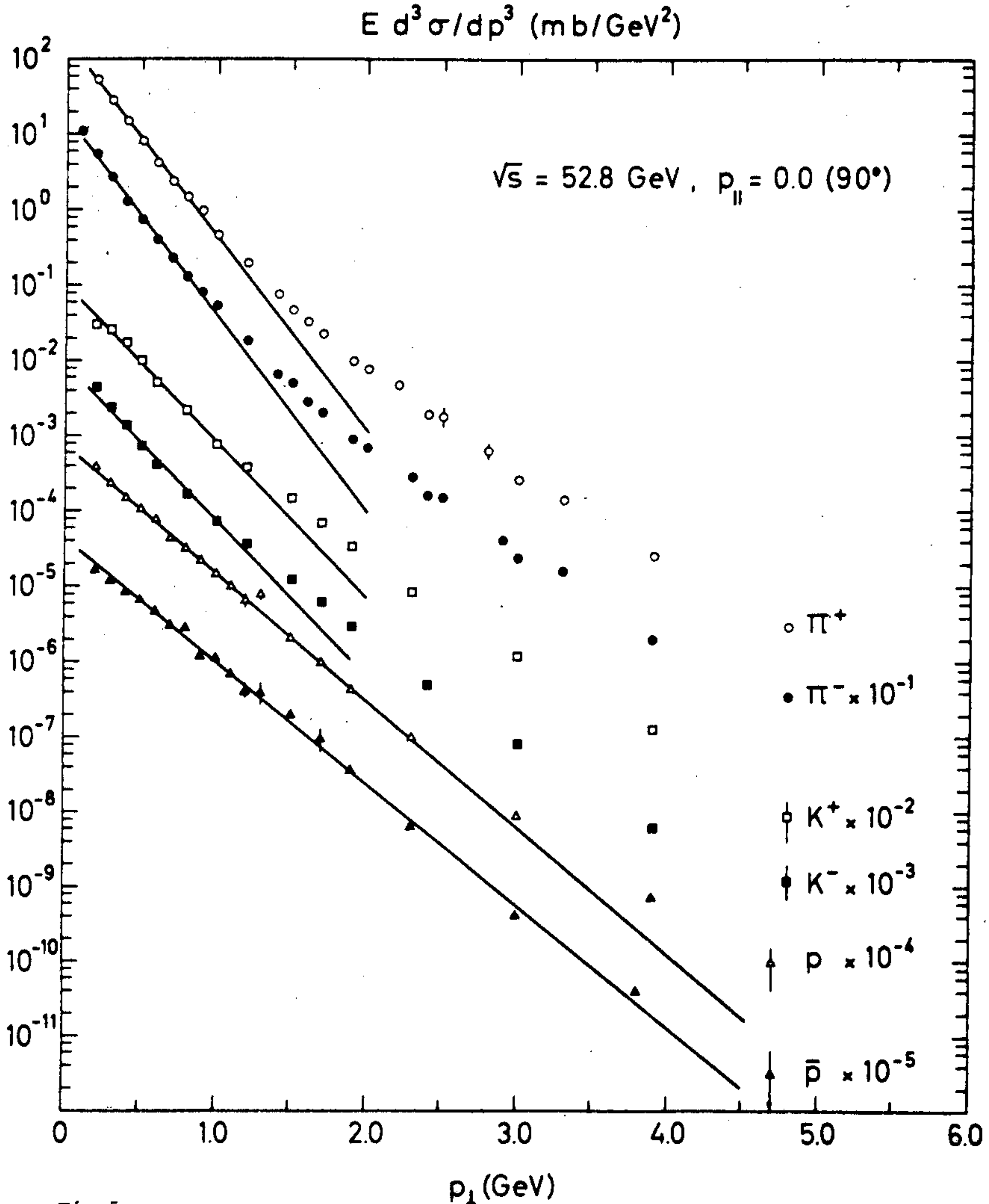
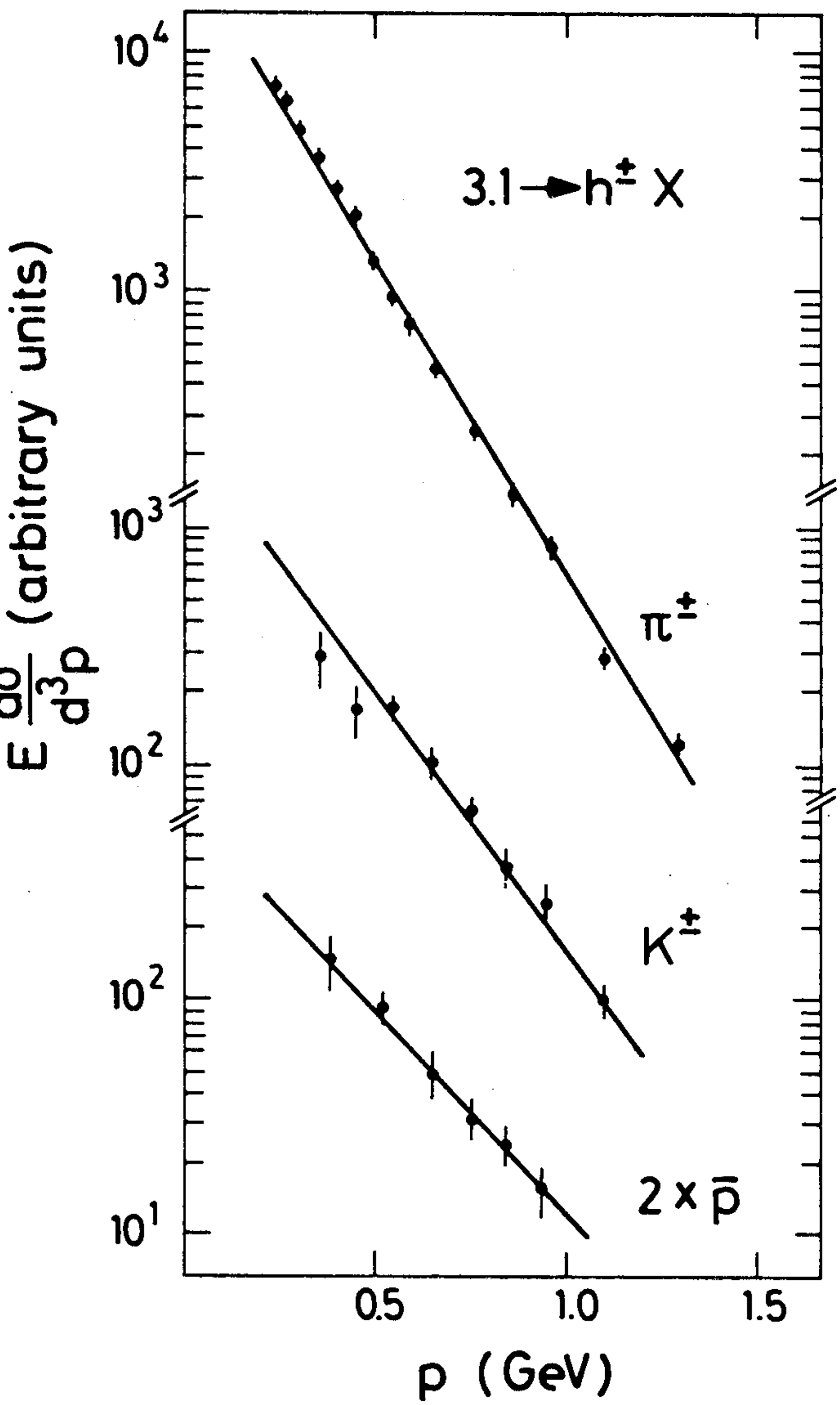
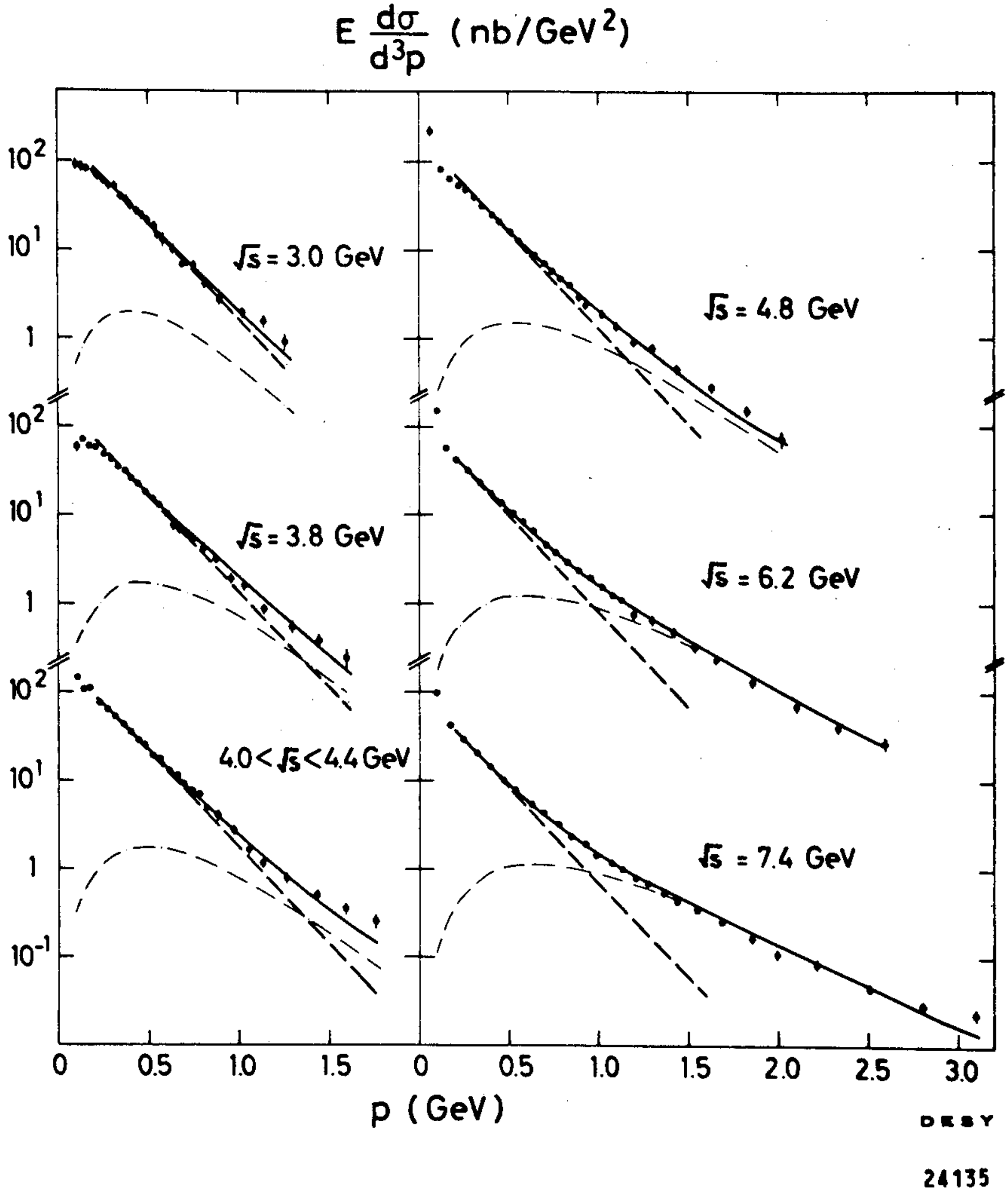

E d³σ/dp³ (mb/GeV²)
√s = 52.8 GeV , p∥ = 0.0 (90°)
π⁺
π⁻ x 10⁻¹
K⁺ x 10⁻²
K⁻ x 10⁻³
p x 10⁻⁴
p̄ x 10⁻⁵
p⊥ (GeV)

Fig.5



$E\frac{d\sigma}{d^3p}$ (arbitrary units)
10^4
10^3
10^3
10^2
10^2
10^1
$3.1 \rightarrow h^\pm X$
$\pi^\pm$
$K^\pm$
$2 \times \bar{p}$
0.5
1.0
1.5
p (GeV)

Fig.6



E dσ/d³p (nb/GeV²)
√s = 3.0 GeV
√s = 3.8 GeV
4.0 < √s < 4.4 GeV
√s = 4.8 GeV
√s = 6.2 GeV
√s = 7.4 GeV
10²
10¹
1
10⁻¹
0
0.5
1.0
1.5
2.0
2.5
3.0
p (GeV)
DESY
24135

Fig.7

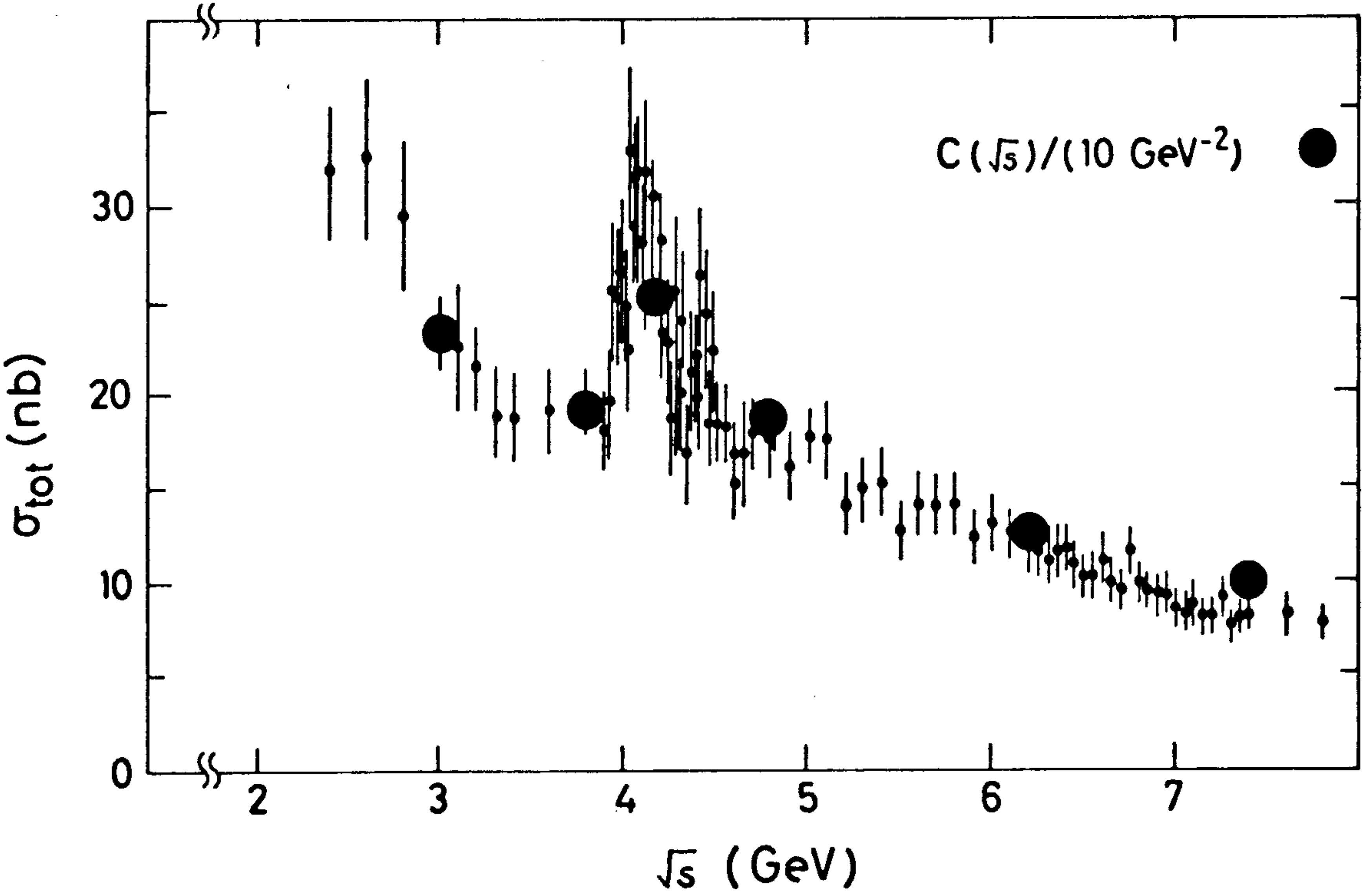


Fig.8

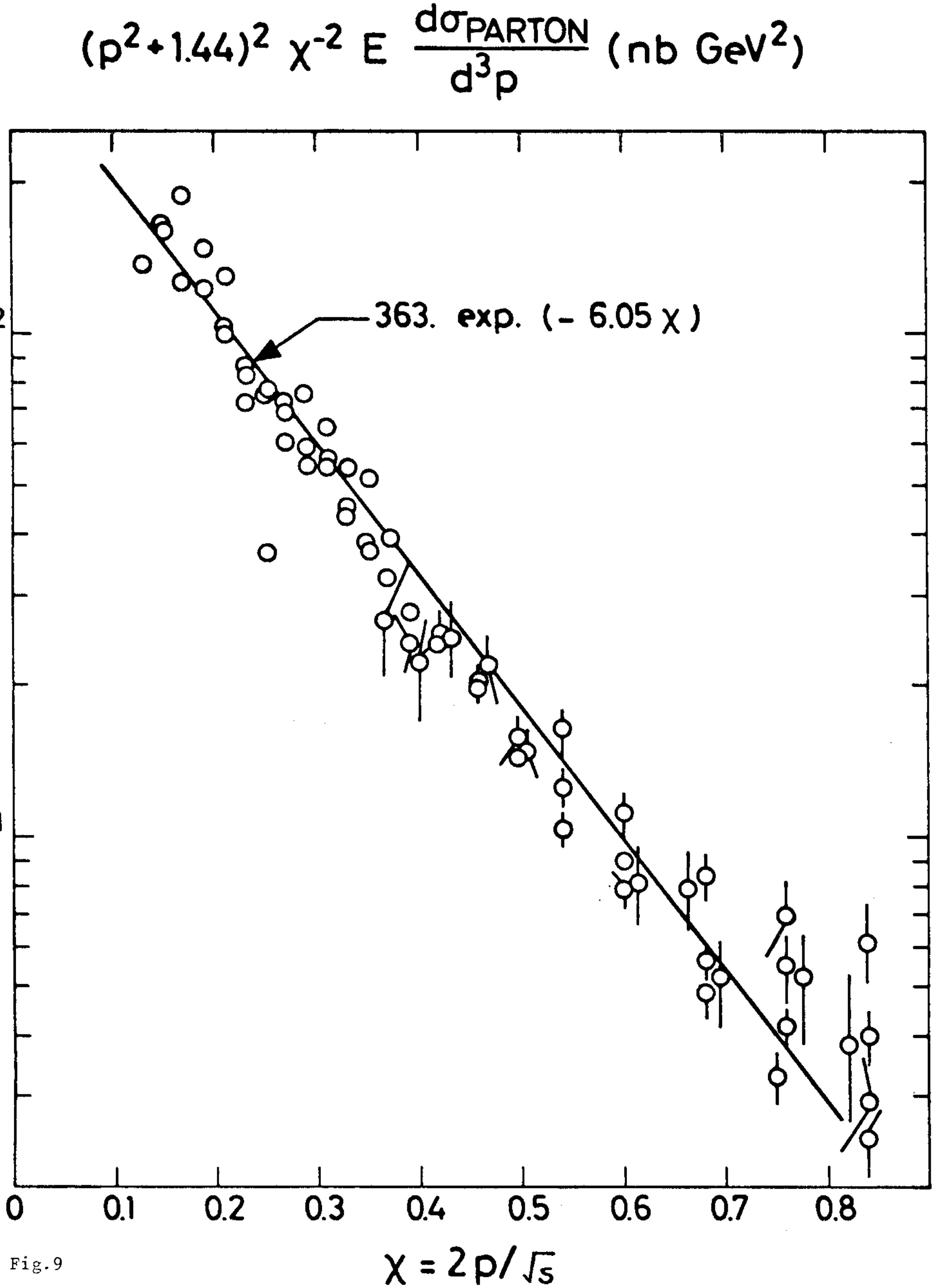

$(p^2+1.44)^2 \chi^{-2} E \frac{d\sigma_{PARTON}}{d^3p}$ (nb GeV^2)
10^2
10^1
363. exp. (- 6.05 χ)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
$\chi = 2p/\sqrt{s}$

Fig.9