DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 75/57
December 1975

Helicity Content of the Electro-Excitation of Leading Baryon Resonances from Duality and t-Channel Constraints

by

J. G. Körner
Deutsches Elektronen-Synchrotron DESY, Hamburg

I. Bender
Institut für Hochenergiephysik, Universität Heidelberg

A. Actor
Institut für Theoretische Physik, Universität Dortmund

2 HAMBURG 52 · NOTKESTIEG 1

Helicity Content of the Electro-Excitation of Leading Baryon Resonances from Duality and t-Channel Constraints

by

J. G. Körner

Deutsches Elektronen-Synchrotron DESY, Hamburg

I. Bender

Institut für Hochenergiephysik, Universität Heidelberg

A. Actor

Institut für Theoretische Physik, Universität Dortmund

Abstract:

In a previous calculation the helicity structure of the ($q^2 = 0$) photo-excitation of the leading baryon resonances D_{13}, F_{15} and P_{33} was investigated in the context of an explicit dual model for the photoproduction of pseudoscalar mesons. The agreement with experimental results and quark model calculation was good. We extend the investigation to space-like photon momenta with particular emphasis on the q^2-dependence of the relative contributions of the transverse excitations with helicity 1/2 and 3/2 and the longitudinal excitation.

The quark model in its nonrelativistic [1] and relativistic versions [2], or in its modified and extended versions (see e.g. [3]) has been quite successful in accounting for the photo-excitation features of the low-lying baryon resonances [3-5]. In a previous paper it has been shown that some of the observed regularities of the photo-excitation ($q^2 = 0$) helicity structure can also be derived in an explicit dual model from what are essentially SU(3) and t-channel constraints [6]. It is the purpose of this note to extend the analysis of Ref.[6] to space-like photon momenta ($q^2 < 0$) with particular emphasis on the q^2-dependence of the relative contributions of the various helicity couplings.

The content of the dual SU(3) model introduced in Ref.[6,7] is presented schematically in Fig.1. In the s-channel the model contains the two exchange degenerate trajectories $(B_\alpha - B_\gamma, S_\gamma)$ and $(D_\delta, B_\delta - B_\beta)$ (denoted by (S) and (D)), and in the t-channel the EXD natural parity trajectory (n), and the two EXD unnatural parity trajectories (u) and (u^c) with J^P and J^{PC} quantum numbers as indicated in Fig.1.

The dual SU(3) structure was arrived at as usual by first solving the SU(3) (s,t) crossing problem demanding no s- and t-channel exotics and the decoupling of the $\phi = \lambda\bar{\lambda}$. The resulting two independent SU(3) invariant couplings are given by the standard (s,t) duality diagram Fig.2, where the two solutions are variously characterized by their s-channel content as S (for singlet) and D (for decuplet) [8], or equivalently as N (for normal parity) and A (for abnormal parity) [9], or again equivalently as $I_{core} = 0$ and $I_{core} = 1$ in the quark parton language describing the possible two isospin states of the two spectator quarks in the decomposition $3 \times 3 = 6 + \bar{3}$ at the $Y = 2/3$ level. For the (s,u) case there are two independent SU(3) couplings each for the above two s-channel configurations.

The (s,t) and (s,u) SU(3) coupling terms appropriate for the charge coupling of the nucleon have two important properties that allows one to construct a dual description of meson electroproduction which has the correct gauge invariant Born term structure:
(i) The F/D ratio of the $B\bar{B}M$ vertices occurring in the s- and t-channels (see Fig.1) are identical (ii) the appropriate (s,u) SU(3)-invariant coupling is purely (s,u) crossing symmetries. The importance of (i) can be immediately appreciated by noting that gauge invariance forces the appearance of a meson

and nucleon double pole structure in the <u>same</u> amplitude, which then, for consistency reasons, must involve the same $B\bar{B}M$ F/D-ratio in the s- and t-channels. For $q^2 \neq 0$ the singularity at $t = \mu^2$ could no longer be interpreted as a true particle singularity in the case of the (s,u) terms, whereas for $q^2 = 0$ this is possible [6]. Concerning the dual SU(3) structure at the level of the nucleon pole we would like to emphasize that the structure is essentially determined if one wants to incorporate the correct Born term structure. Of course more complicated dual structures [10] could be added in principle for higher J-values. However, in this note we shall be mainly concerned with the implications of the minimal scheme developed in Refs.(6,7).

Returning to Fig.1 one would in general expect all s-channel trajectories to be dual to all t-channel trajectories. However, from the above it is clear that the D-trajectory cannot be dual to a unnatural parity trajectory (u) that starts with the pion since otherwise the $B\bar{B}M$ F/D-structure would be no longer (s,t) crossing invariant (see also Ref.[11]). We have indicated this in Fig.1 by drawing a dotted line between (D) and (u). In Ref.[6] it was shown that this consistency requirement immediately leads to a vanishing E_{1+} photoexcitation of the $P_{33}(1236)$ (see also Ref.[12]). For unnatural parity trajectories starting at higher J-values such a dual connection between (D) and (u) cannot, however, be excluded.

A further very important result of the SU(3)-analysis is the observation that the residue structure of $P_{33}(1236)$, $D_{13}(1520)$ and $F_{15}(1688)$ for excitations on protons is determined by (s,t)- duality alone [6]. The reason for this is that the (s,u) contributions are always proportional to the (s,t) contribution in this case. This observation is at the basis of the following considerations, since it allows one to calculate the s-channel helicity structure of these resonances for excitations on protons from t-channel constraints.

Let us first consider the (D)-sequence with its prominent member, the $P_{33}(1236)$ resonance. In order to obtain information on the multipole structure at a given resonance, one needs to express the residues of the helicity amplitudes $A_{\ell+}$, $B_{\ell+}$ and $S_{\ell+}$ relevant to the (D)-sequence in terms of the residues of a given set of invariant amplitudes. If the parity and spin of the resonance is well defined, this leads to an algebraic 3 x 3 matrix equation given in the appendix by Eq.(A1). We have chosen to express the helicity amplitudes in terms of the residues of the invariant amplitudes A_1, $(t - \mu^2)A_2$ and $(t - \mu^2)A_5$, since these invariants are simple in their t-channel content [13], i.e. to leading order in s these 3 invariants couple only to the (s-channel) helicity flip coupling of the natural parity trajectory and to the unnatural parity trajectory (u).

If the D-trajectory is dual only to the natural parity trajectory, as argued above, the residues of $(t - \mu^2)A_2$ and $(t - \mu^2)A_5$ are uniquely given by the residue of A_1 by observing the t-channel parity constraints of natural parity exchange [13]:

$$(t-\mu^2)A_2 = (t-\mu^2+q^2)t^{-1}A_1 \quad , \tag{1a}$$

$$(t-\mu^2)A_5 = -s\,t^{-1}A_1 \quad . \tag{1b}$$

Note that Eq.(1) leads to the appropriate polynomial t-dependence of the s-channel resonance residues (no ancestors!), if the natural parity trajectory couples evasively. From Eq.(1b) it is apparent that $(t - \mu^2)A_5$ obtains no contribution from natural parity exchange at the leading (parent) level in the s-channel. Thus, to leading order in $\cos\Theta$, we have

$$r_2^n = r_1^n \quad , \tag{2a}$$

$$r_5^n = 0 \quad , \tag{2b}$$

where r_1, r_2 and r_5 denote the leading $\cos\Theta$-power coefficients of the residues of the Dennery amplitudes A_1, $(t - \mu^2)A_2$ and $(t - \mu^2)A_5$.

Using Eqs.(A1), (2a) and (2b) one can then calculate the three relevant helicity amplitudes as

$$\begin{aligned} M_{1+}(q^2) &\propto \left(4(M+m)^2 + q^2(m-3M)/(M-m)\right) G^{J=3/2}(q^2)\,\bar{r}_1^{\,n} \quad , \\ E_{1+}(q^2) &\propto \left(q^2(3m-M)/(M-m)\right) G^{J=3/2}(q^2)\,\bar{r}_1^{\,n} \quad , \\ S_{1+}(q^2) &\propto (6+q^2)(3m-M)\,\left(2M(M-m)\right)^{-1} q_c\, G^{J=3/2}(q^2)\,\bar{r}_1^{\,n} \quad , \end{aligned} \tag{3}$$

where $\sigma = M^2 - m^2$ and q_c is the magnitude of the photon three-momentum in the c.m. frame. $G^{J=3/2}(q^2)$ is a common form factor which we have factored out according to $r_1^n(q^2) = \bar{r}_1^n\, G^{J=3/2}(q^2)$, etc..

In Fig.3 we have plotted the resulting multipole amplitudes E_{1+}, M_{1+} and S_{1+} against q^2, where we have divided out the common form factor $G^{3/2}(q^2)$. The q^2-behaviour of the form factor $G^J(q^2)$ will be discussed later on. At $q^2 = 0$ one notices the vanishing of E_{1+} as already noted earlier. For $q^2 \neq 0$, E_{1+} and the scalar component S_{1+} are non-zero, in contrast to the quark model prediction, where these multipoles are predicted to be zero for all values of q^2. The relative signs of E_{1+}, M_{1+} and S_{1+} agree with the results presented in [14] for $-q^2 \gtrsim 0.6$. However, the ratio E_{1+}/M_{1+} tends to be significantly too large, and the zero of S_{1+} at $q^2 \sim -0.6$ is not in accordance with the data analysis [14].

In view of these difficulties one has to amend the basic model by including also unnatural parity contributions that are dual to the (D)-trajectory. As argued before, these could not, however, couple to the pion, but would have to start at higher mesonic J-values, for example at $J^{PC} = 1^{+-}$ (B-meson). Unfortunately not much is known about the systematics of the contributions of the unnatural parity trajectory to electro-production amplitudes. In most Regge fits the contribution of the EXD (π-B)-trajectory is normalized at the pion pole, so that these additional contributions are not included. If we denote the coupling strengths of these by r_2^B and r_5^B in the notation of Eq.(2) (unnatural parity exchange decouples from A_1, to leading order in s) then the values $r_2^B = 0.12\, r_1^n$ and $r_5^B = 0.44\, r_1^n$ would lead to an excellent agreement with the results of Ref.[14]. If one instead adds $r_2^B = 0$ and $r_5^B = (3m - M)/2(M + m)\, r_1^n$, one would reproduce the quark model result $E_{1+}(q^2) \equiv 0$ and $S_{1+}(q^2) \equiv 0$. Another interesting possibility is to demand asymptotic suppression of the scalar contribution ($\sigma_S/\sigma_T \to q^{-2}$) for all ℓ-values which obtains for $r_2^B = r_5^B = r_1^n$. It is interesting that such a choice leads to a strong asymptotic suppression of the helicity 3/2 contribution viz. $\sigma_{3/2}/\sigma_{1/2} \to q^{-4}$. Hopefully more will become known about the structure of the contribution of the unnatural parity trajectory (π-B) to the electro-production amplitudes in the future so that the addition of such terms could be put on a sounder footing.

The discussion of the helicity structure of the $D_{13}(1520)$ and $F_{15}(1688)$ lying on the S-trajectory is more involved since one has contributions from the couplings of both the natural and unnatural parity trajectory exchanges. The unnatural parity contribution will be normalized by reproducing the correct Born term structure, i.e. for $\gamma p \to n\pi^+$ one has

$$A_1 = \frac{\sqrt{2}\, eg}{s-m^2} F_1(q^2) \quad , \tag{4a}$$

$$(t-\mu^2) A_2 = \frac{\sqrt{2}\, eg}{s-m^2} 2F_1(q^2) \quad , \tag{4b}$$

$$(t-\mu^2) A_5 = \sqrt{2}\, eg \left[\frac{1}{s-m^2} F_1(q^2) - \frac{2}{q^2}\left(F_1(q^2) - F_\pi(q^2)\right)\right] \quad , \tag{4c}$$

where $F_1(q^2)$ and $F_\pi(q^2)$ are the normalized electric nucleon and pion form factors. In Eqs.(4a-4c) we have neglected a possible electric neutron form factor contribution in the u-channel.

In Ref.[6] it was shown that the electric form factor of the proton is adequately described by a dual current model GVDM-type form factor of the form

$$F_1(q^2) = \left[(1-q^2/m_\rho^2)(1-q^2/(m_\rho^2+s_o))\right]^{-1} \quad , \tag{5}$$

where s_o is the inverse Regge slope $s_o = 1/\alpha'$. Similarly one has for the pion form factor

$$F_\pi(q^2) = (1-q^2/m_\rho^2)^{-1} \quad . \tag{6}$$

The two form factors Eq.(5) and Eq.(6) satisfy the identity

$$\frac{\alpha'}{2-\alpha_o} F_1(q^2) = \frac{1}{q^2}\left(F_1(q^2) - F_\pi(q^2)\right) \quad , \tag{7}$$

which allows us to rewrite Eq.(4c) as

$$(t-\mu^2) A_5 = \sqrt{2}\, eg\, F_1(q^2) \left(\frac{1}{s-m^2} - \frac{2\alpha'}{2-\alpha_o}\right) \quad . \tag{8}$$

(α_o is the intercept of the vector-tensor trajectory: $\alpha_o = 1 - \alpha' m_\rho^2$)

The minimal unnatural parity exchange contribution is thus fixed by the Born term structure (4b) and (4c). The natural parity contribution is the same as in Eq.(2). Adding these to the s-channel Born terms in Eq.(4) one obtains

$$\gamma_1 \propto \left(1 + \frac{a_1^S}{\alpha'}(J - 1/2)\right) G^J(q^2) \quad , \tag{9a}$$

$$\gamma_2 \propto \left(2 + \frac{a_1^S}{\alpha'}(J - 1/2)\right) G^J(q^2) \quad , \tag{9b}$$

$$\gamma_5 \propto \left(1 - \frac{2}{2-\alpha_0}(J - 1/2)\right) G^J(q^2) \quad . \tag{9c}$$

The factor $(J - \frac{1}{2})$ appearing in (9) is related to the occurrence of J-dependent residues in dual functions. The parameter a_1^S fixes the strength of the contribution of natural parity exchange.

Using the projection formula in the appendix we find for the helicity amplitudes

$$\begin{aligned}
\frac{1}{\ell} A_{\ell+1,-} \propto \Big[&-6\omega\left(\frac{m}{M} - \frac{a_1^S}{\alpha'}\frac{\ell}{2}\right) \\
&- q^2\left(\frac{6}{M} + \left(-\frac{\omega}{2-\alpha_0} + \frac{a_1^S}{\alpha'}\frac{3M+5m}{4}\right)\ell\right)\Big] G^J(q^2) \,, \\
B_{\ell+1,-} \propto \Big[&6\omega\left(2 + \frac{a_1^S}{\alpha'}\ell\right) - q^2\left(\frac{2\omega}{2-\alpha_0} + \frac{a_1^S}{\alpha'}\frac{\omega}{2}\right)\ell\Big] G^J(q^2) \,, \\
\frac{\ell+1}{\ell} q_c^{-1} S_{\ell+1,-} \propto \Big[&-6\left(2 + \left(-\frac{\omega}{M}\frac{1}{2-\alpha_0} + \frac{a_1^S}{\alpha'}\frac{M+3m}{4M}\right)\ell\right) \\
&+ q^2\left(\frac{1}{2-\alpha_0}\frac{\omega}{M} - \frac{a_1^S}{\alpha'}\frac{M+3m}{4M}\right)\ell\Big] G^J(q^2) \,.
\end{aligned} \tag{10}$$

As is apparent from Eq.(10) the size and the sign of the parameter a_1^S crucially determines the relative behaviour of the three helicity amplitudes. In an explicit dual model there are various ways in which the strength of a_1^S and thereby the relative contribution of natural and unnatural parity contributions can be determined [6]. A normalization in the t-channel at the ρ-pole gives $a_1^S/\alpha' \simeq 0.8$, where the sign is determined relative to π-exchange from SU(6). From a Regge fit to forward high energy scattering one obtains $a_1^S/\alpha \simeq 0.4$. Normalizing to the helicity 3/2 amplitude of the D_{13} excited off protons one has $a_1^S/\alpha' \simeq 0.5$ and finally, normalizing to the $P_{33}(1236)$-excitation one obtains $a_1^S/\alpha' \simeq 1.6$. For latter normalization one uses the fact that the relativ contribution of the natural parity exchange to resonances on the S- and D-trajectories is fixed by requiring a decoupling of the f-ω trajectory from the (s-channel) helicity flip coupling, i.e. $(F/D)_t = 1/3$.

One notices that all the above estimates give a substantial suppression of the helicity 1/2 amplitude $A_{\ell+1,-}$ at $q^2 = 0$ through a cancellation effect of natural parity and unnatural parity contributions. Similarly in the quark model the suppression of the helicity 1/2 coupling arises from a cancellation of an orbital and spin coupling term, the relative contributions of which are fixed by a parameter related to the strength of the oscillator potential [2,4,5,15].

In Figs.4a and 4b we have plotted our predictions for the relative size of the three helicity couplings for D_{13} and F_{15} excited off protons, where we have chosen as representative values of a_1^S/α' the two values 0.4 and 0.8. We have multiplied the couplings with normalization factors that facilitate the comparison with the three cross sections measured in single arm electro-production experiment [16]:

$$\sigma_{1/2} \propto \sum_{\ell=0}^{\infty} (\ell+1)\,|A_{\ell+1,-}|^2 \quad ,$$
$$\sigma_{3/2} \propto \sum_{\ell=0}^{\infty} \tfrac{1}{4}\ell(\ell+1)(\ell+2)\,|B_{\ell+1,-}|^2 \quad , \quad \text{i.e. } \sigma_T = \sigma_{1/2} + \sigma_{3/2} \; , \tag{11}$$
$$\sigma_L \propto \sum_{\ell=0}^{\infty} (\ell+1)^3\,|S_{\ell+1,-}|^2 (-q^2)\, q_c^{-2} \quad .$$

One can see that, similar to the quark model [5,15], the helicity 1/2 coupling rises much quicker with $-q^2$ than the helicity 3/2 coupling. However the effect is less dramatic than in the quark model. Whereas in the quark model of Ravndal [5] $\sigma_{1/2}$ equals $\sigma_{3/2}$ already at $-q^2 \simeq 0.3$ for the D_{13} resonance (see Fig.5), equality of the two couplings in our model is reached only at $q^2 \simeq 2.6$ and $q^2 \simeq 1.4$ for the two above values of a_1^S/α' (see Fig.4). In Fig.5 we have plotted the $\frac{1}{2} - \frac{3}{2}$ asymmetry

$$A_{1/2;3/2} = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \tag{12}$$

for $D_{13}(1520)$ and $F_{15}(1688)$ using $\alpha_1^S/\alpha' = 0.8$, and for comparison, the quark model prediction [5] and the experimental asymmetries according to the latest analysis of Ref.[14]. For this value of α_1^S/α' equality of the two cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ is reached somewhat earlier than what is indicated by the data, although a better agreement would obtain for smaller values of a_1^S/α' . Once can check that for all values $0.4 \leq a_1^S/\alpha' \leq 1.6$ the q^2-variation of the ratio $\sigma_{1/2}/\sigma_{3/2}$ close to $q^2 = 0$ is not as rapid as predicted by the quark model and is thus in better agreement with the data.

Returning to Fig.4 one notes that the scalar amplitudes S_{2-} and S_{3-} tend to be too large in the range $-q^2 \approx 0 - 3\ \text{GeV}^2$. Similar to the quark model of Ref.[5] they have a zero for large $-q^2$ ($-q^2 \sim 10 - 20$) but dominate the transverse parts asymptotically. One could reverse the line of arguments as was done for the P_{33}-resonance and use the known suppression of scalar excitations in the s-channel to gain additional information on additional unnatural parity contributions in the t-channel. Again these additional contributions would have to decouple from the pion pole. For example, if one adds such contributions with strengths $r_2^B = a_1^S/\alpha'\ (J - \frac{1}{2})G^J(q^2)$ and $r_5^B = ((a_1^S/\alpha') + 2/(2 - \alpha_o))G^J(q^2)$ one would have a suppression of the scalar components with $\sigma_S/\sigma_T \to q^{-2}$ and $\sigma_{3/2}/\sigma_{1/2} \to q^{-4}$ for all values of ℓ without changing the small $\sigma_{1/2}/\sigma_{3/2}$ ratio at $q^2 = 0$ significantly.

Up to this point we have only been concerned with the relative contributions of the three helicity amplitudes for a particular resonance. In order to obtain information of the q^2-dependence of the form factors $G^J(q^2)$ one has to turn to an explicit dual model. In the dual current model of meson electro-production described in Ref.[6] the J-dependent (normalized) transition form factors $G^J(q^2)$ are given by

$$G^J(q^2) = \prod_{n=0}^{c+J-\frac{1}{2}} \left(1 - q^2/(m_g^2 + n s_o)\right)^{-1} \tag{13}$$

for the leading resonances. At large $-q^2$ $G^J(q^2)$ behaves as

$$G^J(q^2) \to (-q^2)^{-c-J+\frac{1}{2}} \quad . \tag{14}$$

The connection of the asymptotic q^2-power and the spin of the excited resonance given in (13) is a necessary requirement for the Drell-Yan threshold relation to hold and also follows from dimensional arguments [17]. In Ref.[18] it was shown that form factors of the type (13) are well suited to describe the q^2-dependence of the excitation of leading isobars. The constant c entering in (13) and (14) determines the J-independent part of the asymptotic q^2-power and is set to c = 2 for a canonical form factor behaviour.

The calculation of the relative coupling strengths of resonances lying on the same EXD trajectory as e.g. the D_{13} and the F_{15} has been treated in detail in Ref.[6] with satisfactory results and will not be repeated here.

A final remark concerns the threshold and pseudothreshold constraint structure of the multipole amplitudes (3) or helicity amplitudes (10). Since the Dennery amplitudes are constraint free for all q^2 by construction it is clear that the multipole and helicity amplitudes calculated according to (A1) obey the usual threshold and pseudothreshold constraints (see e.g.[18]. Although threshold ($q^2 = (M + m)^2$) and pseudothreshold ($q^2 = (M - m)^2$) are not close to the space-like q^2-region, the constraints on the multipole amplitudes at these points are still an influencing factor on the q^2-behaviour of the multipoles or $q^2 \leq 0$. This point has been discussed in more detail in Ref.[18].

We close by remarking that the duality scheme in its predictions for the excitation structure of leading baryon resonances is perhaps less compelling and by far less economic than the quark model. For example, in explicit dual models involving baryons one has the well-known difficulties with parity-doubling. Also the information of such models on the daughter structure is unreliable. Further, an analysis of the isospin structure of the electro-magnetic excitation would necessitate a study of constraints coming from the u-channel. It is however quite intriguing that both models, starting from very different physical ideas, lead to similar results for some of the excitation features of the low lying baryon resonances. The duality scheme is of course more closely related to the more conventional calculational schemes that seek to exploit the mutual s- and t-channel interdependence through a study of dispersion relations in the form of FESR or fixed t dispersion relations. One may hope that part of the insight gained from studying explicit dual models may be used as a guiding principle in the construction of dispersion relation solutions.

ACKNOWLEDGEMENT:

One of the authors (J.G.K.) expresses his gratitude to Prof. G. v. Gehlen for some helpful correspondence and very informative discussions.

APPENDIX

For a resonance of a given spin and parity one can derive a (3 x 3)-matrix connecting the residues of the helicity amplitudes $A_{\ell+}$, $B_{\ell+}$ and $S_{\ell+}$ to the leading $\cos\Theta$-power coefficients r_1, r_2 and r_5 of the residues of the Dennery amplitudes A_1, $(t-\mu^2)A_2$ and $(t-\mu^2)A_5$ by comparing the coefficients of the leading $\cos\Theta$-power [19,20]. For the abnormal parity sequence $\frac{1}{2}^+ \to \frac{3}{2}^+, \frac{5}{2}^-, \frac{7}{2}^+ \ldots$ one obtains

$$\begin{bmatrix} A_{\ell+} \\ \ell B_{\ell+} \\ (\ell+1) S_{\ell+}/q_c \end{bmatrix} = \frac{1}{(\ell+1)\tau_{\ell+1}} \frac{M}{\sigma} \sqrt{\frac{\mathcal{E}_2}{\mathcal{E}_1}} \; \underset{\approx}{M} \begin{bmatrix} r_1 \\ r_2 \\ r_5 \end{bmatrix} \tag{A1}$$

with

$$\underset{\approx}{M} = \begin{bmatrix} -2\omega^2\mathcal{E}_1 & \left(2\omega^2\mathcal{E}_1 + \frac{1}{2}\beta\nu + \frac{1}{2}q^2(M-3m)\right) & -\frac{1}{2}q^2\nu \\ 0 & -\beta\nu & -q^2\nu \\ -2\omega\mathcal{E}_1 & \left(2\omega\mathcal{E}_1 + (\sigma+q^2)(M-3m)/4M\right) & -\nu(\sigma+q^2)/2M \end{bmatrix},$$

where M and m are the resonance and nucleon mass and τ_ℓ is the leading order coefficient of the Legendre polynomial of order ℓ, i.e. $\tau_\ell = (2\ell)!\, 2^{-\ell}(\ell!)^{-2}$. We have used the abbreviations

$$\begin{aligned} \omega &= (M-m) & \mathcal{E}_1 &= M+m-q_{c0} \\ \nu &= -(M+m) & \mathcal{E}_2 &= M+m-k_{c0} \\ \sigma &= M^2-m^2 & \beta &= \sigma - \tfrac{1}{2}q^2 \end{aligned},$$

where q_{co} and k_{co} are the virtual photon and pion energy in the c.m. system

$$q_{co} = (s+q^2)/2M \quad ,$$
$$k_{co} = (s+\mu^2)/2M \quad .$$

For the normal parity sequence $\frac{1}{2}^+ \to \frac{3}{2}^-, \frac{5}{2}^+, \frac{7}{2}^- \ldots\ldots$ the corresponding relation is given through McDowell symmetry by replacing $A_{\ell+} \to -A_{\ell+1,-}$, $B_{\ell+} \to B_{\ell+1,-}$, $S_{\ell+} \to S_{\ell+1,-}$ and $M \to -M$. In our convention we have $J = \ell + 1/2$ for both sequences. Our multipoles are normalized as in [19].

The multipole amplitudes $M_{\ell+}$ and $E_{\ell+}$ are related to the helicity amplitudes by

$$M_{\ell+} = (2A_{\ell+} - (\ell+2)B_{\ell+})/2(\ell+1) \quad ,$$
$$E_{\ell+} = (2A_{\ell+} + \ell B_{\ell+})/2(\ell+1) \quad , \qquad \text{(A2)}$$

and

$$M_{\ell+1,-} = (2A_{\ell+1,-} + \ell B_{\ell+1,-})/2(\ell+1) \quad , \qquad \text{(A3)}$$
$$E_{\ell+1,-} = (-2A_{\ell+1,-} + (\ell+2)B_{\ell+1,-})/2(\ell+1) \quad .$$

For large $M = \sqrt{s}$ and $q^2 = 0$ one can read off from (A1) a special case of a class of relations discussed by B. Schrempp [11] concerning the multipole content of high energy photoproduction amplitudes. One knows from the Stichel relations that, to leading order in s, reaction amplitudes with photons polarized parallel to the reaction plane ($\|$) obtain contributions only from the amplitude combination $(A_1 - t\,A_2)$ (i.e. unnatural parity exchange, see (1a)), and reaction amplitudes with photons polarized perpendicular to the reaction plane ($\perp$) only from the invariant amplitude A_1 (i.e. natural parity exchange) (see e.g. [13]). Taking the limit $M \to \infty$ in (A1) one finds $A_{\ell+}/B_{\ell+} \to \pm\ell/2$ and $A_{\ell+1,-}/B_{\ell+1,-} \to \mp\,\ell/2$ for the two possible cases of abnormal and normal parity transitions (upper sign: ($\|$); lower sign: ($\perp$)). If one expresses these statements in terms of multipole amplitudes one can state the following: At high energies and in the forward direction leading resonances are excited only electrically by parallel polarized photons and only magnetically by perpendicular polarized photons.

References

1. R. P. Feynman, M. Kislinger and F. Ravndal; Phys. Rev. D3, 2706 (1971)
R. G. Lipes; Phys. Rev. D5, 2849 (1972)

2. L. A. Copley, G. Karl and E. Obryk; Phys. Letters 29B, 117 (1969); Phys. Rev. D4, 2844 (1971)

3. J. L. Rosner; Physics Reports 11C, 189 (1974)

4. R. L. Walker, in Vol. Liverpool Conference on Electron and Photon Interactions at High Energies, Ed. W. Braben, Daresbury Nuclear Physics Laboratory (1969)

5. F. Ravndal; Phys. Rev. D4, 1466 (1971)

6. I. Bender, J. G. Körner, V. Linke and M. G. Schmidt; Nuovo Cimento 16A, 377 (1973)

7. I. Bender, J. G. Körner, V. Linke and M. G. Schmidt; Lett. al Nuovo Cimento 3, 441 (1972)

8. C. Lovelace; Proc. Roy. Soc. Lond. A318, 321 (1970)

9. M. Chaichian, S. Kitakado, S. Pallua, B. Renner and J. De Azcarraga; Nucl. Phys. B51, 221 (1973)
B. Flume-Gorczyca and S. Kitakado; Nuovo Cimento 28A, 321 (1975)

10. J. L. Rosner; Phys. Rev. D7, 172 (1973)
T. Eguchi; Nucl. Phys. B60, 277 (1973); ibid B70, 390 (1974)

11. B. Schrempp; Thesis Universität Hamburg (1970)

12. G. v. Gehlen, in Bonn Conference on Electron and Photon Interactions at High Energies, Eds. H. Rollnik and W. Pfeil, North Holland Publishing Company (1974)

13. A. Actor, I. Bender and J. G. Körner; Nuovo Cimento 24A, 369 (1974)

14. R.C.E. Devenish and D. H. Lyth; Nucl. Phys. B93, 109 (1975)

15. F. E. Close and F. J. Gilman; Phys. Letters 38B, 541 (1972)

16. F. J. Gilman; Physics Reports 4C, 95 (1972)

17. R. A. Brandt and G. Preparata; Phys. Rev. Lett. 21, 1530 (1970)

18. R.C.E. Devenish, T. E. Eisenschitz and J. G. Körner; DESY 75/48 (1975)

19. H. F. Jones; Nuovo Cimento 40, 1018 (1965)

20. G. v. Gehlen; Nucl. Phys. B9, 17 (1969)

Figure Captions

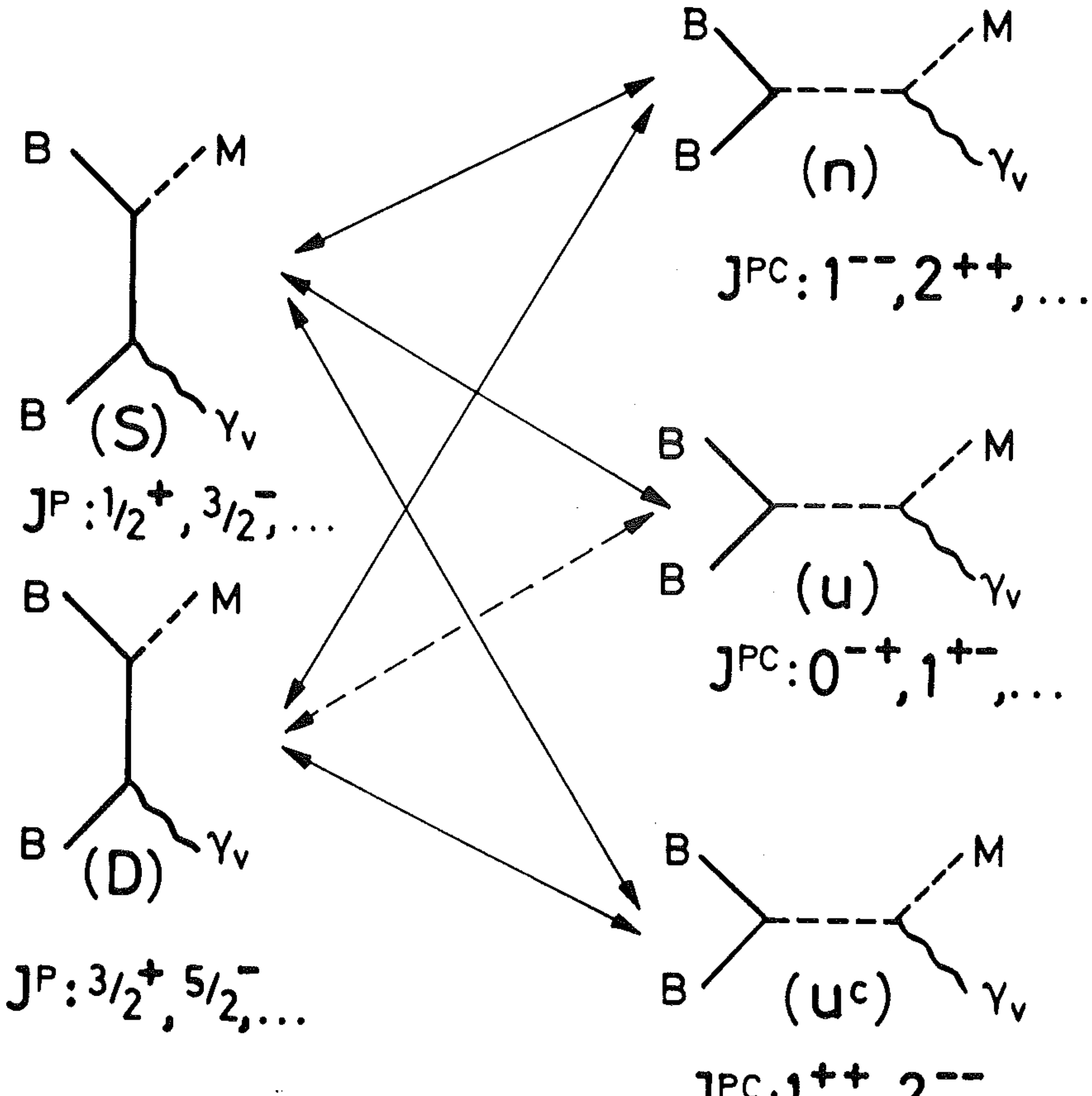
Fig. 1 Schematic representation of the duality scheme for the electro-production amplitudes.

Fig. 2 (s,t) duality diagram for meson electro-production

Fig. 3 Relative contributions of E_{1+}, M_{1+} and S_{1+} to the excitation of $P_{33}(1236)$.

Fig. 4 Relative contributions of normal parity helicity amplitudes to (a) $D_{13}(1520)$ and (b) $F_{15}(1688)$ off protons. The helicity amplitudes are drawn in a scale which facilitates comparison with the cross sections Eq.(11). Solid lines: $a_1^S/\alpha' = 0.8$; dotted lines: $a_1^S/\alpha' = 0.4$.

Fig. 5 Asymmetries $A_{1/2};_{3/2}$ for the excitations of D_{13} and F_{15} off protons. Solid lines: this calculation (KBA); dotted lines: quark model prediction of Ravndal [5](R); shaded band: results of the data analysis of Devenish and Lyth [14] (DL).



B
M
B
(S)
γ_v
$J^P: 1/2^+, 3/2^-, \ldots$
B
M
B
(D)
γ_v
$J^P: 3/2^+, 5/2^-, \ldots$
B
M
B
(n)
γ_v
$J^{PC}: 1^{--}, 2^{++}, \ldots$
B
M
B
(u)
γ_v
$J^{PC}: 0^{-+}, 1^{+-}, \ldots$
B
M
B
(u^c)
γ_v
$J^{PC}: 1^{++}, 2^{--}, \ldots$

Fig.1

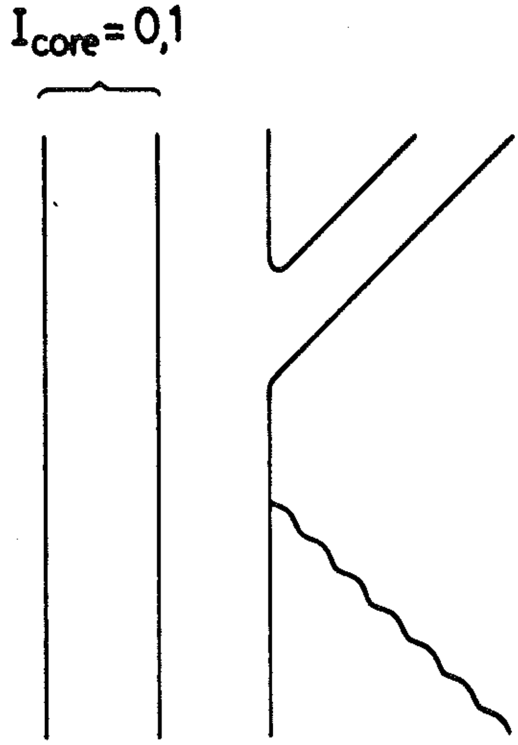
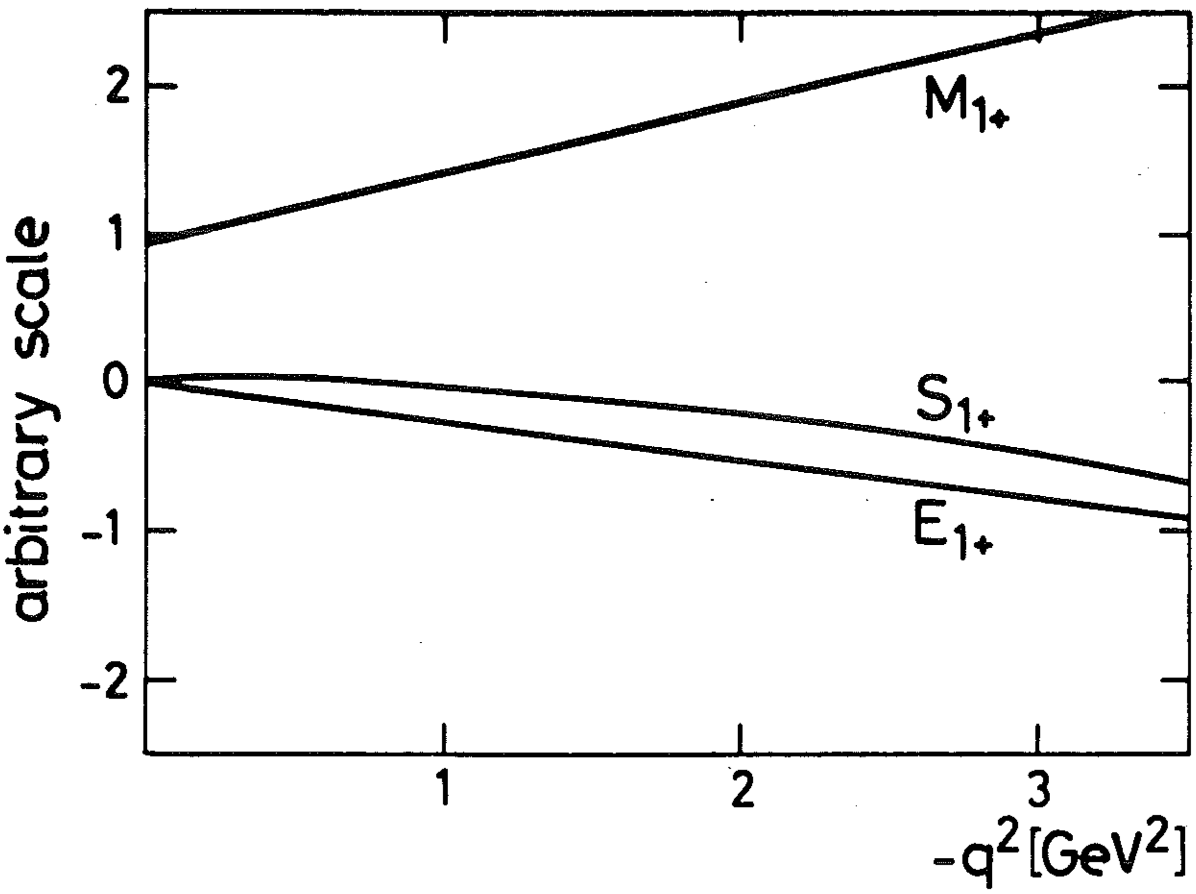
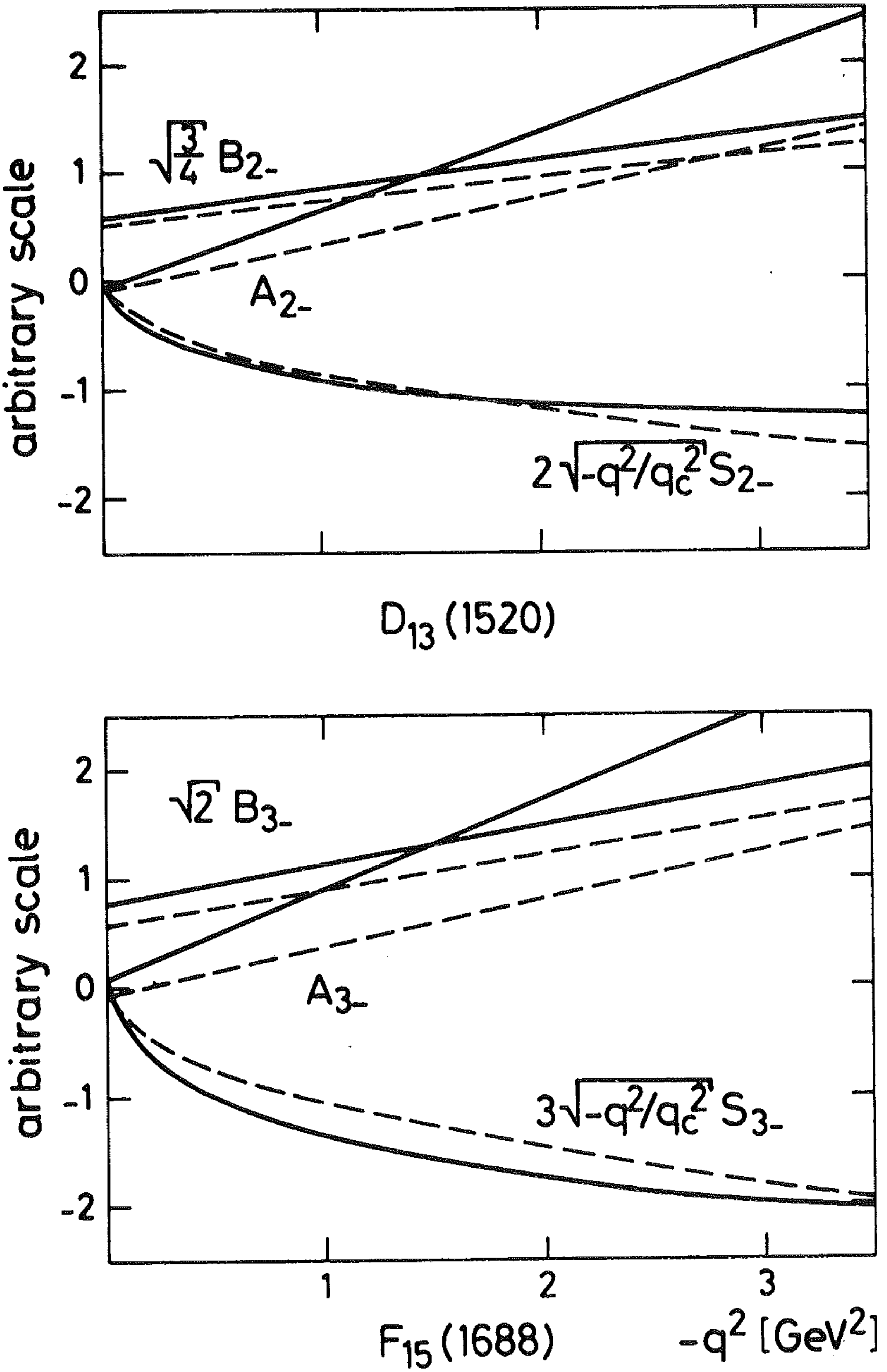


Fig. 2



P_{33} (1236)

Fig. 3



arbitrary scale
2
1
0
-1
-2
$\sqrt{\frac{3}{4}}\, B_{2-}$
A_{2-}
$2\sqrt{-q^2/q_c^2}\, S_{2-}$
D_{13} (1520)
$\sqrt{2}\, B_{3-}$
A_{3-}
$3\sqrt{-q^2/q_c^2}\, S_{3-}$
1
2
3
F_{15} (1688)
$-q^2$ [GeV^2]

Fig. 4

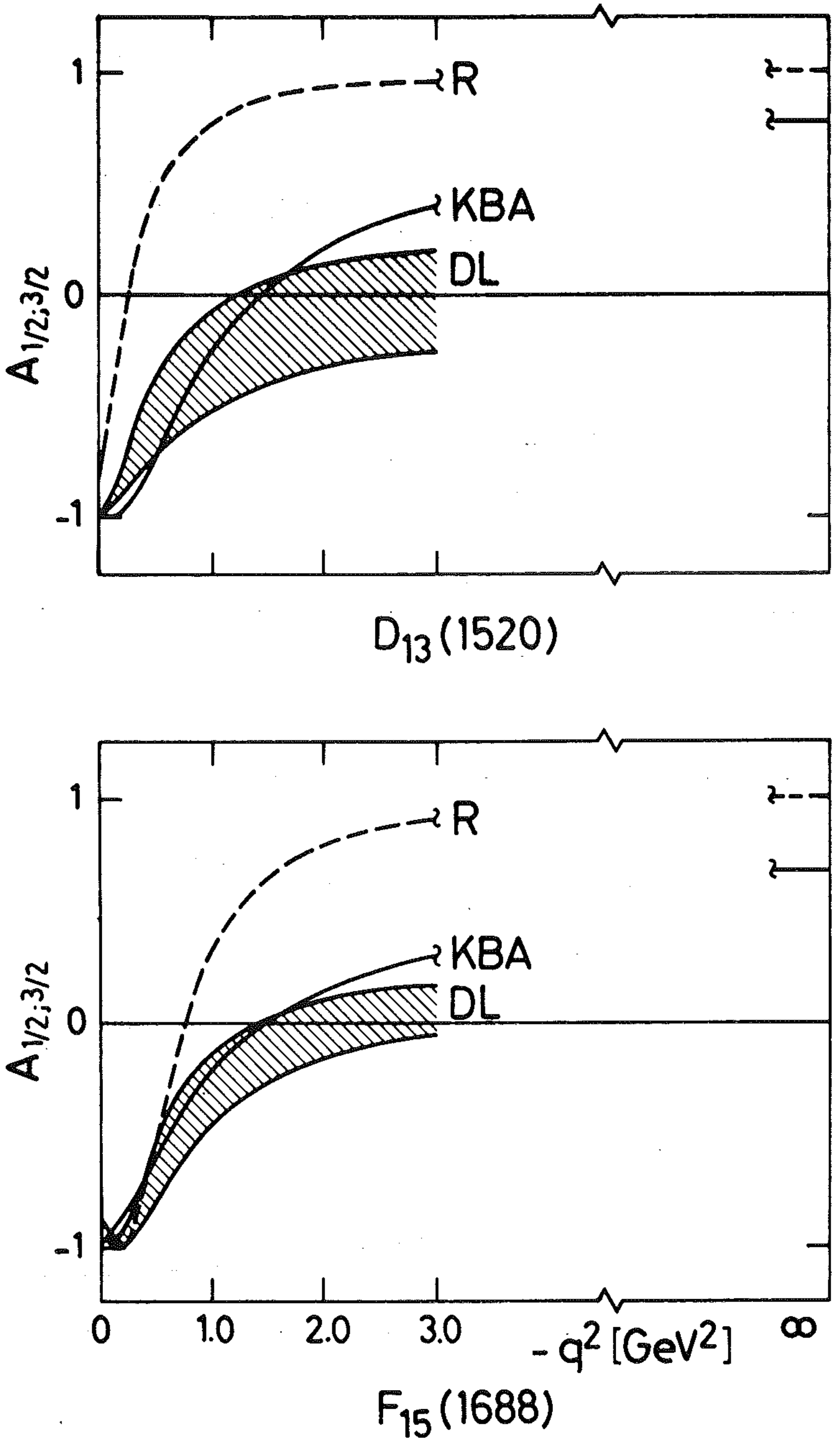

1
0
-1
$A_{1/2;3/2}$
R
KBA
DL
D_{13} (1520)
1
0
-1
$A_{1/2;3/2}$
R
KBA
DL
0
1.0
2.0
3.0
$-q^2$ [GeV^2]
∞
F_{15} (1688)

Fig. 5