DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY
DESY 76/24
May 1976

Broken Weyl Symmetry

by

G. Domokos
Deutsches Elektronen-Synchrotron DESY, Hamburg
and
Department of Physics, Johns Hopkins University
Baltimore, MD 21218, USA

2 HAMBURG 52 · NOTKESTIEG 1

To be sure that your preprints are promptly included in the
HIGH ENERGY PHYSICS INDEX ,
send them to the following address (if possible by air mail) :

DESY
Bibliothek
2 Hamburg 52
Notkestieg 1
Germany

Broken Weyl Symmetry [+]

by

G. Domokos

Deutsches Elektronen-Synchrotron DESY, Hamburg

and

Department of Physics, Johns Hopkins University

Baltimore, MD 21218, USA [++]

[+] Invited talk given at the Topical Conference on Spontaneous Symmetry Breaking, Trieste, April 1976. Research supported in part by the U.S. Energy Research and Development Administration, under Contract No. E(11-1) 3285. Report No. COO-3285-29.

[++] Permanent address.

Abstract

We argue that conformal symmetry can be properly understood in the framework of field theories in curved space. In such theories, invariance is required under general coordinate transformations and conformal re-scalings. We examine a gauge model coupled to a Higgs field. In the tree approximation, the vacuum solution exhibits two Higgs phenomena: both the phase (Goldstone boson) and the coordinate dependent part of the radial component of the scalar field can be removed by a Higgs-Kibble transformation. The resulting vacuum solution corresponds to a space of constant curvature and constant vacuum expectation value of the scalar field.

1. Introduction

Theories exhibiting exact conformal symmetry are attractive from the point of view of a theorist; they are outrageous from the point of view of almost all the experimental facts known today.

Why are they attractive?

Because they are more restrictive than theories based on the Poincaré group alone. This is not surprising: any enlargement of the invariance group of a theory is likely to lead to further restrictions on its structure. This statement seems to be true at whatever level one approaches a theory.

i) At a Lagrangian level (considering fields of low spin, i.e. 0, 1/2, 1) conformal symmetry severely restricts the possible couplings between fields. For instance, a minimal coupling between a Dirac field and a gauge field $(\sim e\,\bar{\psi}\gamma^\mu A_\mu \psi)$ is allowed, but Pauli-type couplings $(\sim \bar{\psi}\sigma^{\mu\nu} F_{\mu\nu}\psi)$ are not.

ii) At a more phenomenological level, conformal invariance is known to impose severe (and physically attractive) restrictions on the short distance expansions of operator products or of n -point functions. Hence, the presence of the "larger" symmetry may help in understanding short distance phenomena (deep inelastic scattering of leptons on hadrons, for instance) in terms of fewer parameters than Poincaré invariance alone would give.

iii) There is a reasonable hope that conformally invariant theories are "better behaved" at short distances, see e.g. remarks at the end of this paper.

Why are conformally invariant theories outrageous from an experimental point of view?

The answer is extremely simple. Conformally invariant theories are devoid of any fixed mass (or length) scale. Experimentally, however, we do seem to have rather well-defined energy scales (~ 1 GeV for strong interactions, a length scale of M_W^{-1} or $\sqrt{G_F} \sim 10^{-16}$ cm for weak interactions, etc.). Thus conformal symmetry is necessarily broken.

Now, experience with gauge theories has taught us that the best way of breaking a symmetry while retaining most of the benefits reaped from its presence, is to break the symmetry <u>spontaneously</u>, i.e. to retain the symmetry of the operator structure of the theory, but to look for a <u>ground state</u> of lower symmetry instead.

In the case of conformal symmetry this means in particular that the ground state (the "vacuum") supplies the energy scale of excitations. This intriguing possibility has been particularly emphasized in recent works of Fubini et al. [1], even though it is implicit in earlier works on the subject, see e.g. Salam and Strathdee [2].

In this talk I cannot do justice to all the beautiful work done on conformally invariant quantum field theories. Rather, in order to give an idea about the physics involved, I deliberately take the simplistic point of view that a tree approximation to quantum field theories gives a rough but not entirely unreliable guide to what we may expect to happen.

As far as the geometrical aspects of the problem are concerned, I want to advertise a somewhat unconventional (for a particle physicist, at least ...) point of view. I wish to claim that the proper framework in which conformal symmetry is to be investigated is a field theory in curved space, i.e. gravity cannot be neglected, except perhaps at some later stage of the calculations. (I shall try to justify this claim by exhibiting a - somewhat bizarre - form of a Higgs phenomenon which can be understood only if one considers theories in curved space.)

2. The geometrical aspects of conformal symmetry

I adopt Weyl's original point of view [3] (which was also the starting point of Kastrup in his works [4]) about conformal transformations. A conformal transformation is a local rescaling of the units of length (or, equivalently, of the units of energy, since we use natural units, $\hbar = c = 1$.).

If at any point, P , of space one is given an infinitesimal vector $dx^{\mu}(P)$ (representing, classically, a "measuring rod" of laboratory size), then we change the units locally, by

$$dx^{\mu}(P) \longrightarrow s(P)\,dx^{\mu}(P). \qquad (\ s = \text{real})$$

(In what follows, the point P is characterized by its coordinates, so that I write s(x), instead of s(P), etc.)

Hence, the length squared of dx^{μ} becomes

$$g_{\mu\nu}(x)\,dx^{\mu}dx^{\nu} \longrightarrow s^2(x)\,g_{\mu\nu}(x)\,dx^{\mu}dx^{\nu}. \qquad (1)$$

Now, since we generally deal with functions of points (fields, currents, etc.) and not with functions of infinitesimal displacements, one reinterprets the transformation - via eq. (1) - as a rescaling of the metric tensor, rather than that of the infinitesimal vector.

In infinitesimal form $\left(s(x) \approx 1 + \Lambda(x) \right)$,

$$\delta g_{\mu\nu}(x) = 2\Lambda(x) g_{\mu\nu}(x), \tag{2}$$

from which

$$\delta g^{\mu\nu}(x) = -2\Lambda(x) g^{\mu\nu}(x),$$
$$\delta\sqrt{-g} = 4\Lambda(x)\sqrt{-g} \tag{3}$$

follow immediately.

We shall call these transformations Weyl transformations or conformal rescalings. Weyl transformations do not affect the coordinates.

It is intuitively evident that if $g_{\mu\nu}(x)$ is the metric of some general Riemann space, then so is $s^2(x)\, g_{\mu\nu}(x)$ (for "sufficiently well-behaved" s(x)) and, moreoever, it is a space of the same signature as $g_{\mu\nu}(x)$. Hence, Weyl transformations form an infinite parameter Abelian group, with composition law

$$\delta_1 \delta_2\, g_{\mu\nu} = \delta_2 \delta_1\, g_{\mu\nu} = 2\left(\Lambda_1(x) + \Lambda_2(x) \right).$$

This group is henceforth denoted by W.

What is commonly called the group of conformal transformations of Minkowski space is an entirely different object. It is the group of coordinate transformations which induces a conformal rescaling on the Minkowski metric.

In infinitesimal form $(x^{\mu'} = x^\mu + \epsilon^\mu)$, we have, of course,

$$\delta g_{\varrho\sigma} = -(\partial_\nu g_{\varrho\sigma}\epsilon^\nu + g_{\varrho\nu}\partial_\sigma\epsilon^\nu + g_{\nu\sigma}\partial_\varrho\epsilon^\nu).$$

However, $\partial_\nu g_{\varrho\sigma} = 0$ (Minkowski space) and $\delta g_{\varrho\sigma}$ must be proportional to $g_{\varrho\sigma}$ (conformal rescaling), hence, the Killing condition reads in our case:

$$\partial_\sigma\epsilon_\varrho + \partial_\varrho\epsilon_\sigma = \frac{1}{2} g_{\varrho\sigma}(\partial_\alpha\epsilon^\alpha). \qquad (4)$$

It is well known that the solution of (4) is (with $g_{\varrho\sigma} \to \eta_{\varrho\sigma} = \mathrm{diag}(+1,-1,-1,-1)$:

$$\epsilon_\alpha = \mathcal{S}_{\alpha\beta}x^\beta + T_\alpha + Q_\alpha(x^\varrho x_\varrho) - 2x_\alpha(x^\varrho Q_\varrho), \qquad (5)$$
$$+ Dx_\alpha,$$

which corresponds to infinitesimal Poincaré transformations $(\mathcal{S}_{\alpha\beta}, T_\alpha)$ "conformal accelerations" (Q_α) and dilatations (D). This is a 15-parameter, non-Abelian group, isomorphic to $SO(4,2)$.

There is a connection between the transformations (2) and (5). A simple way of stating the connection was given by Zumino [5]; for a more general analysis see e.g. Fulton *et al.*, ref. 6.

Zumino shows that if a (classical or "effective") action in curved space is invariant under both general coordinate transformations and under W, then the flat-space limit of the action is invariant under (5).

(I am not going to repeat Zumino's proof here. The trick is to show that the effect of Q_α and D, which do not leave $\eta_{\mu\nu}$ invariant, can always be compensated by an appropriate Weyl transformation).

In what follows, we shall be exclusively concerned with Weyl transformations. Needless to say, many of the results can be translated into the "coordinate transformation language" in the limit of flat space.

Adopting the point of view advocated here, one is actually led to consider the so-called Weyl geometry [6)] (instead of the familiar Riemann geometry) as a basis of the theory of gravitation. However, in order to stick to familiar ground, I shall not go into the details of Weyl geometry. I have a good excuse for this shortcut: everything I have to say, can be told in the language of Riemann geometry with an extra invariance group (the Weyl group) added as an icing on the cake.

The representation theory of W looks deceptively simple. (The main trouble, from the physical point of view, is that W has to be represented on a set of quantized gravitational and matter fields, and no one quite knows how to do this.) Since W is Abelian, one expects its irreducible representations to be one-dimensional. Hence, if $\phi_{(A)}(x)$ is a physically meaningful (classical) field, and (A) stands for a set of spinor indices with respect to a local spinor frame, then ϕ should transform under (2) as

$$\delta \phi_{(A)}(x) = w \, \Lambda(x) \, \phi_{(A)}(x) , \tag{6}$$

where the real number w is the "dimension" (or weight) of ϕ .

The weight is almost always equal to the canonical dimension, with the important exception of gauge fields.

Gauge fields corresponding to (local) internal symmetries carry weight zero. (The covariant metric tensor carries weight = + 2.) This is easy to see. The gauge-invariant action of the gauge field itself is of the form:

$$W = -\frac{1}{4} \int d^4x \sqrt{-g} \; g^{\mu\nu} g^{\rho\sigma} \; \vec{F}_{\mu\rho} \bullet \vec{F}_{\nu\sigma} \quad , \tag{7}$$

where the dot stands for the scalar product in internal symmetry space with respect to the standard Killing metric. The field tensor itself is the totally antisymmetric derivative,

$$\vec{F}_{\mu\nu} = \vec{\nabla}_\mu \times \vec{A}_\nu ,$$

where $\vec{\nabla}_\mu$ is the gauge and Riemann covariant derivative, $\vec{A}_\mu$ is the vector potential.

Just by counting weights, one verifies that $\vec{A}_\mu$, and hence $\vec{F}_{\mu\nu}$, has to be assigned weight zero. The consistency of the assignment can then be verified by constructing gauge invariant couplings to "matter"-fields, e.g. to a spinor field.

3. A gauge model: Higgs phenomena in Weyl-invariant theories

In order to illustrate how spontaneous symmetry breaking (SSB) works in W-invariant theories, I consider a Salam-Ward-Weinberg type model in curved space, i.e. together with gravity. For the sake of simplicity, I take the gauge group to be $U(1)$, although the formalism and the results can be readily generalized to more complicated gauge groups.

In flat space the action reads:

$$W = \int d^4x \, \{ \tfrac{1}{2} \eta^{\mu\nu} \phi^* (\overleftarrow{\partial}_\mu + if A_\mu)(\overrightarrow{\partial}_\nu - if A_\nu) \phi \\ - \frac{\lambda}{4!} (\phi^* \phi)^2 - \tfrac{1}{4} F_{\mu\nu} F_{\rho\sigma} \eta^{\mu\rho} \eta^{\nu\sigma} + \mathcal{L}(\psi) \} , \qquad (8)$$

where ϕ is a massless complex Higgs field, A_μ is the vector potential and $\mathcal{L}(\psi)$ is the massless, gauge-invariant Lagrangian density of - say - quarks and leptons. The "charge" carried by the Higgs field is f.

The generalization of this action to curved space is straight-forward. Basically, the recipe is:

$$\eta^{\mu\nu} \longrightarrow g^{\mu\nu}(x)$$

$$d^4x \longrightarrow d^4x \sqrt{-g}$$

$$\partial_\mu \longrightarrow \nabla_\mu \qquad \text{(covariant derivative).}$$

One finds then that a piece of the action (8), namely

$$W' \equiv \int d^4x \sqrt{-g} \, \{ \tfrac{1}{2} f^2 \phi^* \phi \, g^{\mu\nu} A_\mu A_\nu - \tfrac{\lambda}{4!} (\phi^*\phi)^2 - \tfrac{i}{2} f \phi^* \overleftrightarrow{\partial}_\mu \phi \, g^{\mu\nu} A_\nu - \tfrac{1}{4} F_{\mu\nu} F_{\rho\sigma} g^{\mu\rho} g^{\nu\sigma} + \mathcal{L}(\psi) \}$$

is automatically Weyl invariant, see Zumino loc. cit. and the previous Section. One can even add coupling terms between ϕ and ψ without destroying W-invariance, say

$$W'' = H \int d^4x \sqrt{-g} \, (\bar{\psi} \phi \psi + \text{conj.})$$

where H is a dimensionless coupling constant. (Terms of this type generate Fermion masses in standard, flat-space gauge theories when the gauge symmetry is spontaneously broken. We shall presently see that they do the same job here too.) The assignment of weights is, of course,

$$w_A = 0, \quad w_\phi = -1, \quad w_\psi = -3/2 .$$

We are in trouble, however, with the kinetic part of the Higgs field,

$$T = \tfrac{1}{2} \int d^4x \sqrt{-g} \, g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi$$

Indeed, under (2), T behaves as

$$\delta T = \tfrac{1}{2} \int d^4x \sqrt{-g} \, \phi^* \phi \, \Box \Lambda \tag{9}$$

where $\Box$ is the covariant D'Alembertian, $\Box\Lambda = (-g)^{-1/2} \partial_\mu [(-g)^{1/2} \times g^{\mu\nu} \partial_\nu \Lambda]$. In order to make T Weyl-invariant, one has to add a piece to the action in such a way that its change cancels (9). It has been discovered long ago by Gürsey [7)] and Penrose [8)] that the compensating term is necessarily proportional to the scalar curvature, R, viz.

$$-\frac{1}{12}\int d^4x\,\sqrt{-g}\,\phi^*\phi\,R$$

Thus the total, Weyl-invariant action is

$$W=\int d^4x\,\sqrt{-g}\left\{\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi-\frac{1}{12}\phi^*\phi\,R\right\}+W'+W'' \qquad (10)$$

Two properties of (10) are worth noticing. First, the term $\sim\phi^*\phi R$ looks like a mass term in the Higgs Lagrangian. (Thus, together with $\sim\lambda(\phi^*\phi)^2$ it may be expected to generate a non-vanishing vacuum expectation value (VEV) for ϕ [9)].) Second, the same term is "almost" the Hilbert-Palatini Lagrangian for the gravitational field. (The difference is, of course, that the gravitational coupling constant, $(16\pi G)^{-1}$, is replaced by a field, $(\phi^*\phi)/12$. We find therefore that a W-invariant extension of a Salam-Ward-Weinberg gauge theory with (hopefully!) SSB, is possible only if one includes a theory of gravitation as well.

However, the theory of gravitation is not the orthodox Einstein theory; rather, it is the "conformal relativity" advocated by Brans and Dicke, Hoyle, Sciama and others.

Now, we are interested in solutions exhibiting SSB.

We assume that only ϕ (and possibly, the gravitational field) have non-vanishing VEV. We work in the tree approximation throughout.

First of all, we notice that the "phase" of ϕ (the Goldstone boson) can be removed by a gauge transformation. Indeed, if we introduce a polar decomposition of ϕ , viz.

$$\phi(x) = \varphi(x)\, U(x) \tag{11}$$

where φ is positive, U is unitary (in our example, $U = \exp i\Theta$), then the gauge transformation, $A_\mu \to A_\mu - if\, \partial_\mu \log U$, absorbs U into the longitudinal part of the vector potential. This is just the familiar Higgs phenomenon.

We end up with the following effective action for the VEV:

$$W_{eff} = \int d^4x \sqrt{-g}\, \left\{ \tfrac{1}{2} g^{\mu\nu} \partial_\mu \varphi\, \partial_\nu \varphi - \tfrac{1}{12} \varphi^2 R - \tfrac{\lambda}{4!} \varphi^4 \right\} \tag{12}$$

Variation of (12) with respect to the field quantities gives the equations obeyed by the VEV. The result is the following:

1) Gravity. (Variation with respect to $g^{\mu\nu}$).

$$\begin{aligned} \frac{\varphi^2}{6} G_{\mu\nu} = \partial_\mu \varphi\, \partial_\nu \varphi - g_{\mu\nu} \left(\tfrac{1}{2} g^{\rho\sigma} \partial_\rho \varphi\, \partial_\sigma \varphi - \tfrac{\lambda}{4!} \varphi^4 \right) \\ + \tfrac{1}{6} \left(g_{\mu\nu} \Box \varphi^2 \right) - \tfrac{1}{12} \{ \nabla_\mu , \nabla_\nu \} \varphi^2 \end{aligned} \tag{13}$$

where $G_{\mu\nu} \equiv R_{\mu\nu} - 1/2 g_{\mu\nu} R$ is the Einstein tensor. The r.h.s. of (13) is just the conformal energy-momentum tensor of the field φ , see refs. 7,8.

2) The scalar field equation. (Variation with respect to φ)

$$\Box\varphi + \frac{1}{6}\varphi R - \frac{\lambda}{6}\varphi^3 = 0 \tag{14}$$

The trouble is that (14) is already contained in (13), hence φ cannot be regarded as an independent degree of freedom. Indeed, by taking the trace of (13), one gets

$$\varphi\left(\frac{1}{6}\varphi R + \Box\varphi - \frac{\lambda}{6}\varphi^3\right) = 0$$

which either means $\varphi \equiv 0$ (<u>no SSB</u>) or that (14) is a consequence of (13). The point is, of course, that we have not exploited the W-invariance of (12). Just as gauge invariance of the full action permitted the removal of the Goldstone boson (Higgs phenomenon), the W-invariance of (12) permits the removal of the coordinate dependence of φ . Indeed, since φ is a "radial component" in internal space, it is nonnegative, cf. eq. (12). Hence, we can write $(\varphi \not\equiv 0)$:

$$\varphi(x) = \chi \exp -\frac{1}{2}\varrho(x) \tag{15}$$

where χ is a <u>constant</u> of dimension $(\text{length})^{-1}$ and ϱ is real, $-\infty < \varrho < \infty$. (Since ϕ is a <u>classical</u> field, the transformations (12) and (15) are well defined.) In terms of ϱ , eq. (12) becomes:

$$W_{eff} = \chi^2 \int d^4x \sqrt{-g}\, e^{-\rho} \Big\{ \frac{1}{8} g^{\mu\nu} \partial_\mu \varphi \, \partial_\nu \rho - \frac{1}{12} R - \frac{\lambda \chi^2}{4!} e^{-\rho} \Big\}$$

Upon performing the finite Weyl transformation,

$$g'_{\mu\nu}(x) = e^{-\rho(x)} g_{\mu\nu}(x), \quad \varphi'(x) = e^{\frac{1}{2}\rho} \varphi(x) \tag{16}$$

we find $\varphi'(x) = \chi = const$. The scalar curvature changes as follows:

$$R = e^{-\rho} \left(R' + 3 \Box' \rho + \frac{3}{2} g'^{\mu\nu} \partial_\mu \rho \, \partial_\nu \rho \right) \tag{17}$$

(Here the primes mean that the D'Alembertian and the scalar curvature are calculated with the metric $g'_{\mu\nu}$).

As a result of (16), the effective action becomes:

$$W_{eff} = -\frac{\chi^2}{12} \int d^4x \sqrt{-g'} \left[R' + \frac{\lambda}{2} \chi^2 \right] \tag{18}$$

Hence the VEV, φ' is just a constant, the coordinate dependence is now absorbed into the metric. (To be precise, this "second Higg-Kibble transformation" absorbs the coordinate dependence of φ into the determinant of $g_{\mu\nu}$, see ref. 7.). Now the gravitational equations are:

$$\frac{\chi^2}{12} \left(G'_{\mu\nu} - \frac{1}{4} g'_{\mu\nu} \lambda \chi^2 \right) = 0 \tag{19}$$

If the vacuum is non-trivial $(\chi \neq 0)$, then we have an Einstein equation with a cosmological constant. The solution of (19) is a space of constant curvature, for which the Riemann tensor is characterized by one number, say K. The Einstein tensor is

$$G'_{\mu\nu} = 3 K g'_{\mu\nu} \tag{20}$$

and (19) gives

$$K = \frac{\lambda}{12} \chi^2 \tag{21}$$

There is just one relation between K and χ, the equations for the VEV do not determine both quantities. We may take the point of view that the vacuum determines the scale of φ (χ is of dimension $(\text{length})^{-1}$) and then the presence of a non-trivial VEV generates a curved metric.

The result can be translated into the language of Fubini et al.,[1)] of course. Indeed, a space of constant curvature is essentially unique, once K is given. One may coordinatize the space by means of the "standard" coordinate system, in which the metric is of the form:

$$g'_{\mu\nu} = \frac{\eta_{\mu\nu}}{(1+\frac{1}{4}Kx^2)^2} \tag{22}$$

Fubini et al. insists on having $g'_{\mu\nu} = \eta_{\mu\nu}$. This can be achieved by means of a finite Weyl transformation applied on (22). As a result, the field φ picks up a coordinate dependence, viz.

$$\varphi(x) = \frac{12K}{\lambda} (1 + \frac{1}{4} K x^2)^{-1} \tag{23}$$

which just gives the solution found in ref. 1. for the nonlinear equation (14) restricted to flat space. (The authors of ref. 1. use $a = K^{-1/2}$ - the radius of curvature - instead of K.)

4. Questions of interpretation

At first sight, the interpretation of the results presented here seems to be quite obvious. We found that the entire coordinate dependence of a Higgs field can be transformed away. Therefore, if W is spontaneously broken, it is only the constant, χ , which has a direct physical meaning. The difference between the physical pictures suggested in ref. 1 and here arises from the different "conformal frames" used. One may argue that this difference is insignificant (just as quantum electrodynamics can be described e.g. either in a Lorentz or in a Coulomb gauge.) However, this is not necessarily the case. In a full quantum theory, the vacua corresponding to "Fubini's world" and to "our world" may well turn out to be inequivalent ones. I stick to the "conformal frame" in which φ = const, because i) in this frame the geometrical aspects of the problem appear in a transparent way, ii) there is evidently no problem with the Principle of Equivalence - at least at a classical level. (For instance, if one seeks an eikonal solution to the spinor equation, the term W'' in the action gives rise to a constant Fermion mass, hence, the corresponding "classical" path is a geodesic.)
It is clear now that one has to identify: $\chi^2/12 = (16\pi G)^{-1}$, where G is Newton's constant, see eq. (18). Now, $\lambda\chi^2/2$ is the cosmological constant. Astronomical observations tolerate a cosmological constant, say $\sim 10^{-57}$ cm^{-2}, which gives a fantastically small value for λ , roughly $\lambda \sim 10^{-122}$. There is nothing fundamentally wrong with this, although one

becomes somewhat suspicious on seeing such "funny" numbers. (A possible way out may have been suggested by Zel'dovich [10]. He conjectures that a cosmological constant is <u>induced</u> by vacuum fluctuations of physical fields and he gets a tolerable order of magnitude.) Whichever point of view is taken, the interpretation given here survives as long as λ is not <u>exactly</u> zero. Now, in "our" conformal frame we find that if $\chi \neq 0$, "the vacuum is filled by something", which, not surprisingly, generates a nonvanishing curvature. Depending on the sign of λ, the "vacuum" corresponds to a spatially closed geometry which expands forever $(\lambda < 0)$ or to a spatially open geometry which oscillates in time $(\lambda > 0)$. This can be best seen by introducing Robertson-Walker coordinates, in which the infinitesimal distance corresponding to the metric (22) becomes:

$$ds^2 = dt^2 - a^2 \cosh^2 t/a \left[d\chi^2 + \sin^2\chi \left(d\Theta^2 + \sin^2\Theta \, d\phi^2 \right) \right]$$

$$(4+1 \text{ De Sitter}, \ \lambda < 0)$$

and

$$ds^2 = dt^2 - a^2 \cos^2 t/a \left[d\chi^2 + \sinh^2\chi \left(d\Theta^2 + \sin^2\Theta \, d\phi^2 \right) \right]$$

$$(3+2 \text{ De Sitter}, \ \lambda > 0)$$

respectively. Here $a^2 = |K|^{-1}$. (It is to be noted, however, that in the case of $\lambda > 0$, these coordinates do not cover the entire space.)

There is some potential trouble with this interpretation. One easily convinces oneself that all the mass parameters arising in such a theory are proportional to $\chi = (3/4\pi G)^{-1/2}$, <u>which is of the order of the Planck mass, $\sim 10^{19}$ GeV</u>. In other words, how can one get down to mass scales governing, say, strong interactions? [11]

(As far as I can understand matters, Fubini <u>et al.</u> face an equally serious problem. They want to interpret K as a "strong interaction scale", say $K \sim 1\ \mathrm{GeV}^2$. In the geometrical language used here, their trouble - as emphasized by De Alfaro and Furlan at this Conference - stems from the fact that De Sitter space is not asymptotically flat. Hence, one does not get momentum conservation unless one resorts to additional physical postulates [1].)

In our interpretation, at any rate, there is no translation invariance - and there should not be - because the "vacuum is curved". (Of course, De Sitter space has a 10-parameter group, which, however, contracts to the Poincaré group in the limit $K \to 0$. <u>If</u> one assumes that χ is furnished by the ground state - and thus it is fixed - then $K \to 0$ means $\lambda \to 0$).

The <u>morale</u> is, I think, that as long as one breaks Weyl symmetry spontaneously, one is given just <u>one</u> length (or mass) scale. Therefore, whatever point of view is taken about the physical meaning of the results, one is <u>either</u> facing the problem of "getting down" from the Planck mass to - say - 1 GeV, <u>or</u> one has to explain why do the angles of a triangle of sides $\sim$ 1 cm (or 1 Ångström) add up so accurately to 180 degrees.

A final remark is in order. While we worked in the tree approximation throughout, there are indications that a fully quantized version of such a theory - with <u>unbroken</u> Weyl symmetry - is quite well-behaved and may even be renormalizable [12]. It remains to be seen whether <u>spontaneously</u> broken

Weyl symmetry retains the "good" short distance properties, while bringing at the same time the model closer to physical reality.

Acknowledgements

Most of the new material presented in this talk arose from collaboration and discussions with M.M. Janson and S. Kövesi-Domokos; this applies, in particular to the suggestion of a "second Higgs mechanism" [13]. I wish to thank Profs. S. Fubini and G. Furlan for discussions on their point of view concerning the breaking of conformal invariance, and to Prof. K. Symanzik for his critical remarks.
My thanks are due to Prof. H. Joos for the hospitality extended to me at DESY where this work was completed. Financial support by the Alexander von Humboldt Stiftung - in the form of a senior U.S. scientist award - is gratefully acknowledged.

Footnotes and References

1) S. Fubini, CERN preprint, TH-2129 (1976).
   V. De Alfaro, S. Fubini and G. Furlan, CERN preprint, TH 2125 (1976)

2) A. Salam and J. Strathdee, Phys. Rev. 184, 1760 (1969)

3) H. Weyl, Raum-Zeit-Materie. 6th edition, Springer, Berlin (1970). References to Weyl's original papers are found in this book.

4) H.A. Kastrup, Ann. der Physik, 7, 388 (1962)

5) B. Zumino, in Brandeis University Summer Institute (Lectures on Elementary Particles and Quantum Field Theory, Ed. S. Deser *et al.*), 2, 438. M.I.T. press, Cambridge, MA. (1970)

6) H. Weyl, *loc. cit.* and T. Fulton, F. Rohrlich and L. Witten, Rev. Mod. Phys. 34, 442 (1962)

7) F. Gürsey, Ann. Phys. (N.Y.) 24, 211 (1963)

8) R. Penrose, Proc. Roy. Soc. A284, 204 (1965)

9) G. Domokos, M.M. Janson and S. Kövesi-Domokos, Nature 257, 203 (1975)

10) Ya. B. Zel'dovich, Sov. Phys. Uspekhi, 95, 209 (1968) and references quoted therein

11) There have been, however, speculations that in a unified theory of weak, electromagnetic and strong interactions one indeed needs mass scales of the order of the Planck mass; see H. Georgi, H. Quinn and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974)

12) R. Kallosh, Phys. Lett. 55B, 173 (1975)
    F. Englert, C. Truffin and R. Gastmans, Bruxelles preprint (1976)

13) G. Domokos and M.M. Janson, in preparation.