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Broken SU(8) Symmetry and the New Particles.

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Abstract: We study the mass spectra and wave functions for vector and pseudoscalar mesons in broken $SU(8) \supset SU(4)_F \otimes SU(2)_J$, where F stands for flavour and J for usual spin. The connection with the standard mass breaking in $SU(4)_F$ is worked out. We find that even in the presence of strong SU(8) breaking the ideal mixing scheme for the vector mesons can be approximately retained. For the pseudoscalar mesons the mixing of the singlet with the 63-plet representation of SU(8) turns out to be essential and strongly nonideal.

1. Introduction

The discovery of the new resonances, J/ψ, ψ', P_c, χ and X[1] has led to great activity in the study of the SU(4) symmetry group[2]. In these SU(4) symmetry schemes the usual vector mesons ρ, K^*, ω and ϕ together with the $J/\psi(3.1)$ meson are placed in a mixed $\underset{\sim}{15} + \underset{\sim}{1}$ representation of SU(4). It seems that there exists also a 16-plet of pseudoscalar mesons, if the recently discovered $X(2.8)$[3] is the analog of the J/ψ in the pseudoscalar multiplet. It is natural then to ask, whether the 1^- and 0^- multiplets together can be fitted into an $SU(8) \supset SU(4)_F \otimes SU(2)_J$ supermultiplet[4] with F designating the quark flavour (u, d, s, c) and J the quark spin, in analogy to the well-known extension of $SU(3)_F$ to SU(6) symmetry.

We assign the $J^P = 0^-$ and 1^- mesons to the adjoint representation $\underset{\sim}{63}$ of SU(8), obtained from $\underset{\sim}{8} \times \underset{\sim}{\bar{8}} = \underset{\sim}{1} + \underset{\sim}{63}$, where $\underset{\sim}{8}$ is the fundamental quark representation of SU(8). The $SU(4)_F \otimes SU(2)_J$ decomposition is

$$\underset{\sim}{63} = (15,3) + (1,3) + (15,1) , \tag{1.1}$$

with the notation (dim $SU(4)_F$, dim $SU(2)_J$). Thus the $\underset{\sim}{63}$ supermultiplet contains for the vector mesons the SU(4) singlet as well as the 15-plet, but for the pseudoscalar mesons only the 15-plet. Furthermore in these states the isoscalar vector mesons appear ideally mixed, i.e. $\omega = \omega_\sigma \cong \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$, $\phi = \omega_S \cong s\bar{s}$ and $J/\psi = \omega_c \cong c\bar{c}$. In this sense SU(8) symmetry is a good starting point at least for the vector mesons which are known to be near to ideal mixing in broken SU(4)[2]. On the other hand SU(8) symmetry must be badly broken due to the huge mass differences between particles belonging to the same multiplet. It is an interesting question, therefore, whether the ideal mixing scheme can be approximately retained in the presence of strong SU(8) breaking. Another problem is to see whether broken SU(8) can give information about the SU(4) breaking parameters of the 1^- and 0^- multiplets. First steps in this direction have been done recently by Eliezer and Holstein, by Okubo and by Nelson, see refs.4.

In this paper we consider SU(8) breaking for the vector and pseudoscalar mesons. We calculate the mass spectrum and discuss the corresponding wave functions in the case of mixing. These wave functions are of particular importance for the Zweig forbidden decays [5] and have not been studied in detail so far. The outline of the paper is as follows. In section 2 we formulate the symmetry breaking Hamiltonian using a natural extension to SU(8) of the usual SU(4) breaking, and express it by the Casimir operators of various subgroups of SU(8). Section 3 contains the classification of the quarks and of the mesons into multiplets of these subgroups. In section 4 we give the mass relations and study the mixing problem, first for the vector mesons and then for the pseudoscalar mesons. In the latter case we must include the mixing of the $\underset{\sim}{63}$-plet with an SU(8) singlet. Here we always compare, at the SU(4) level, with our previous work [5]. In section 5 we discuss the numerical results and compare with other approaches as, for example, the quark-gluon theory [6] and the S-matrix topological expansion [7]. Some useful relations for SU(8) are collected in the appendices A and B.

2. Spin - Unitary Spin Splitting.

In our previous work [5] on broken $SU(4)_F$ the hamiltonian was written in the form

$$H = m_{oo}\, T_o + m_{33}\, T^3{}_3 + m_{44}\, T^4{}_4, \tag{2.1}$$

with T_o giving the SU(4)-symmetric part, $T^4{}_4$ - the splitting of SU(4) and $T^3{}_3$ - that of SU(3). Combining now unitary spin with ordinary spin and embedding the direct product $SU(4)_F \otimes SU(2)_J$ into a group SU(8), the obvious extension of the regular SU(4) tensor operators $T^3{}_3$ and $T^4{}_4$ is

$$\begin{aligned} T^3{}_3 &\rightarrow T^{(3,\alpha)}{}_{(3,\alpha)} = T^3{}_3 + T^7{}_7 \,, \\ T^4{}_4 &\rightarrow T^{(4,\alpha)}{}_{(4,\alpha)} = T^4{}_4 + T^8{}_8 \,. \end{aligned} \tag{2.2}$$

Here we have used the notation introduced in Appendix A, i.e. each SU(8)-index A = 1,2,...,8 has been written in the form A = (a,α), with a = 1,2,3,4 an $SU(4)_F$-index and α = 1,2 an $SU(2)_J$-index (compare eq.(A.4)). Because of the trace over α, the tensor operators in eq.(2.2) do not take into account spin-spin interactions. In order to uplift the resulting degeneracy between pseudoscalar and vector mesons, we may add, as usual, an interaction of the form $S^{(a,\alpha),(b,\beta)}_{(a,\alpha),(b,\beta)}$, which is essentially $\vec{J}^2 = J(J+1)$, with spin $\vec{J}$ defined in eq.(A.6). The hamiltonian obtained this way

$$H = m_{00} T_0 + m_{33}(T^3{}_3 + T^7{}_7) + m_{44}(T^4{}_4 + T^8{}_8) + m_S S^{(a,\alpha),(b,\beta)}_{(a,\beta),(b,\alpha)} \qquad (2.3)$$

is the same as that considered by Okubo[4]. It predicts the same mass relations as the ordinary simple quark model. More sophisticated mass formulas can be obtained by introducing additional symmetry breaking interactions, the discussion of which we defer to the end of this section.

We now use Okubo's method to express the SU(8) tensor operators appearing in eq.(2.3) by the generators $X^A{}_B$ of SU(8). First of all we need

$$\begin{aligned} X^4{}_4 + X^8{}_8 &= -Y_{15} \,, \\ X^3{}_3 + X^7{}_7 &= -Y_8 + \tfrac{1}{3} Y_{15} \,, \end{aligned} \qquad (2.4)$$

which follow from (A.10) and (A.11). For the parts which are quadratic in the generators, we obtain

$$\begin{aligned} &2\left[(X\cdot X)^4{}_4 + (X\cdot X)^8{}_8\right] \\ &\quad = \mathcal{C}_2^{(8)} - \mathcal{C}_2^{(6)} + 2\vec{C}^2 + \tfrac{1}{3} Y_{15}{}^2 + 8 Y_{15} \,, \\ &2\left[(X\cdot X)^3{}_3 + (X\cdot X)^7{}_7 + (X\cdot X)^4{}_4 + (X\cdot X)^8{}_8\right] \\ &\quad = \mathcal{C}_2^{(8)} + \tilde{\tilde{\mathcal{C}}}_2^{(4)} - \tilde{\mathcal{C}}_2^{(4)} - 8\left(Y_8 + \tfrac{2}{3} Y_{15}\right). \end{aligned} \qquad (2.5)$$

Here $(X\cdot X)^A{}_B \equiv \sum_{C=1}^{8} X^A{}_C\, X^C{}_B$ and $\vec{C}$ is the spin of the charmed quarks as defined in (A.12). The $\mathcal{C}_2^{(n)}$'s are quadratic Casimir operators of the corresponding groups SU(n). These groups are defined by their generators as follows:

$$SU(8): \quad X^A{}_B \,, \qquad (A,B = 1,2,\dots,8)\;;$$

$$SU(6): \quad \underset{\sim}{X}{}^A{}_B \equiv X^A{}_B + \tfrac{1}{6}(X^4{}_4 + X^8{}_8)\,\delta^A{}_B \,, \qquad (A,B = 1,2,3,5,6,7)\;;$$

$$\widetilde{SU}(4): \quad \tilde{X}^A{}_B \equiv X^A{}_B + \tfrac{1}{4}(X^3{}_3 + X^7{}_7 + X^4{}_4 + X^8{}_8)\,\delta^A{}_B \,,$$
$$(A,B = 1,2,5,6)\;;$$

$$\widetilde{\widetilde{SU}}(4): \quad \tilde{\tilde{X}}^A{}_B \equiv X^A{}_B + \tfrac{1}{4}(X^1{}_1 + X^5{}_5 + X^2{}_2 + X^6{}_6)\,\delta^A{}_B \,,$$
$$(A,B = 3,4,7,8)\,. \tag{2.6}$$

These generators satisfy (A.1)-(A.3) for the respective range of values for A, B. The corresponding Casimir operators are

$$\mathcal{C}_2^{(8)} = \sum_{A,B=1}^{8} X^A{}_B X^B{}_A \,, \qquad \mathcal{C}_2^{(6)} = \sum_{A,B=1,2,3,}^{5,6,7} \underset{\sim}{X}{}^A{}_B \underset{\sim}{X}{}^B{}_A \,,$$

$$\tilde{\mathcal{C}}_2^{(4)} = \sum_{A,B=1,2,5,6} \tilde{X}^A{}_B \tilde{X}^B{}_A \,, \qquad \tilde{\tilde{\mathcal{C}}}_2^{(4)} = \sum_{A,B=3,4,7,8} \tilde{\tilde{X}}^A{}_B \tilde{\tilde{X}}^B{}_A \,. \tag{2.7}$$

According to (A.4), the group SU(6) from eq.(2.6)is nothing else than the usual group $SU(6) \supset SU(3)_F \otimes SU(2)_J$ describing the three quarks u, d and s and their spin. Similarly, $\widetilde{SU}(4) \supset \widetilde{SU}(2)_F \otimes SU(2)_J$ and $\widetilde{\widetilde{SU}}(4) \supset \widetilde{\widetilde{SU}}(2)_F \otimes SU(2)_J$, where

$\widetilde{SU}(2)_F$ and $\widetilde{\widetilde{SU}}(2)_F$ are the SU(2) groups of two flavours, (u,d) and (s,c) respectively.
Using eqs. (2.4) and (2.5) in eq.(2.3) we obtain

$$H = m_0 + m_1\left(\tilde{\mathcal{C}}_2^{(4)} - \tilde{\tilde{\mathcal{C}}}_2^{(4)}\right) + m_2\left(\mathcal{C}_2^{(6)} - 2\vec{C}^2 - \tfrac{1}{3}Y_{15}{}^2\right) \tag{2.8}$$
$$+ m_8 Y_8 + m_{15} Y_{15} + a\, J(J+1)\,.$$

For mesons belonging to a selfconjugate multiplet, the terms linear in Y_8 and Y_{15} do not contribute. Eq.(2.8) leads to the same mass splitting for pseudoscalar and vector mesons, except for an overall shift by 2a of the vector meson mass spectrum. Furthermore the mass relations

$$K-\pi = F-D\,, \qquad K^*-\rho = K-\pi\,, \qquad D^*-\rho = D-\pi\,, \qquad F^*-\rho = F-\pi \tag{2.9}$$

follow and the vector mesons turn out to be ideally mixed:

$$\omega = \rho\,, \qquad 2K^* = \omega+\phi\,, \qquad 2D^* = \omega+\psi, \qquad 2F^* = \phi+\psi. \tag{2.10}$$

As we have already mentioned, in order to obtain more realistic mass formulas we have to introduce additional symmetry breaking interactions both, spin - independent and spin - dependent ones. Such spin - independent interactions are:

$$
\begin{aligned}
&T^{(4,\alpha),(4,\beta)}_{(4,\alpha),(4,\beta)} \sim (F^4{}_4)^2 = Y_{15}{}^2 \,,\\
&T^{(3,\alpha),(3,\beta)}_{(3,\alpha),(3,\beta)} \sim (F^3{}_3)^2 = \left(-Y_8+\tfrac{1}{3}Y_{15}\right)^2,\\
&\sum_{a,b=1}^{4} T^{(a,\alpha),(b,\beta)}_{(b,\alpha),(a,\beta)} \sim \mathcal{C}^{(4)}_2\,,\\
&\sum_{a,b=1}^{3} T^{(a,\alpha),(b,\beta)}_{(b,\alpha),(a,\beta)} \sim \mathcal{C}^{(3)}_2 + \tfrac{1}{3}(F^4{}_4)^2 = \mathcal{C}^{(3)}_2 + \tfrac{1}{3}Y_{15}{}^2\,,\\
&\sum_{a,b=1}^{2} T^{(a,\alpha),(b,\beta)}_{(b,\alpha),(a,\alpha)} \sim \mathcal{C}^{(2)}_2 + \tfrac{1}{2}(F^3{}_3+F^4{}_4)^2 = 2\vec{I}^2 + \tfrac{1}{2}\left(Y_8+\tfrac{2}{3}Y_{15}\right)^2.
\end{aligned}
\tag{2.11}
$$

Here the $\mathcal{C}^{(n)}_2$'s are the quadratic Casimir operators of the usual flavour groups $SU(n)_F$, with $\mathcal{C}^{(2)}_2 = 2I(I+1)$ for isospin. Since the operators $\mathcal{C}^{(4)}_2$ and $\mathcal{C}^{(3)}_2$ split the multiplets of $SU(4)_F$ and $SU(3)_F$, respectively, they destroy ideal mixing for the vector mesons.

Concerning the additional spin-dependent interactions we have, for example, the following possibilities:

$$
\begin{aligned}
&S^{(4,\alpha),(b,\beta)}_{(4,\beta),(b,\alpha)} \sim 2\vec{C}\cdot\vec{J}\,, \qquad && S^{(3,\alpha),(b,\beta)}_{(3,\beta),(b,\alpha)} \sim 2\vec{S}\cdot\vec{J}\,, \ldots\\
&S^{(4,\alpha),(4,\beta)}_{(4,\beta),(4,\alpha)} \sim 2\vec{C}\cdot\vec{C}\,, \qquad && S^{(4,\alpha),(3,\beta)}_{(4,\beta),(3,\alpha)} \sim 2\vec{C}\cdot\vec{S}\,, \ldots
\end{aligned}
\tag{2.12}
$$

i.e. $\vec{A}\,\vec{B}$, with $\vec{A}, \vec{B} = \vec{N}, \vec{S}, \vec{C}$ and/or $\vec{J}$. Here $\vec{N} = \vec{p} + \vec{n}$ and $\vec{S}$ are, respectively, the spin of the normal and strange quarks as defined in (A.12). The degeneracy between the pseudoscalar and vector meson mass spectrum can be removed only by interactions containing $\vec{J}$. We choose the linearly independent terms $\vec{J}^2$, $\vec{S}\cdot\vec{J}$ and $\vec{C}\cdot\vec{J}$. Combining these interactions with those from equations (2.8) and (2.11), we write the total hamiltonian (for mesons) in the form

$$\begin{aligned} H = {} & m_0 + m_1\left(\tilde{\mathcal{C}}_2^{(4)} - \tilde{\tilde{\mathcal{C}}}_2^{(4)}\right) + m_2\left(\mathcal{C}_2^{(6)} - 2\vec{C}^2 - \tfrac{1}{3}Y_{15}^{\,2}\right) \\ & + b_4\,\mathcal{C}_2^{(4)} + b_3\left(\mathcal{C}_2^{(3)} + \tfrac{1}{3}Y_{15}^{\,2}\right) + b_2\left[\mathcal{C}_2^{(2)} + \tfrac{1}{2}\left(Y_8 + \tfrac{2}{3}Y_{15}\right)^2\right] \\ & + a_4\,Y_{15}^{\,2} + a_3\left(Y_8 - \tfrac{1}{3}Y_{15}\right)^2 \\ & + a\,\vec{J}^2 + 2b\,\vec{S}\cdot\vec{J} + 2c\,\vec{C}\cdot\vec{J}\,, \end{aligned} \tag{2.13}$$

where the coefficients m_o, m_1, ..., have to be fitted to the masses and wave functions of the pseudoscalar and vector mesons.

3. Classification of particles.

In sect.2 we have expressed the hamiltonian H eq. (2.13) for broken SU(8) in terms of the Casimir operators (2.7) of the various subgroups (2.6). In order to calculate the matrix elements of H we need the classification of the physical particles into irreducible representations of these subgroups. To this end we first establish the transformation properties of the quarks themselves. Under SU(6), the quarks q^A with A = 1, 2, 3, 5, 6, 7 , i.e. u^α, d^α and c^α (α=1,2) belong to the vector representation $\underset{\sim}{6}$, whereas $q^4 = c^1$ and $q^8 = c^2$ are singlets. Similarly, under $\widetilde{SU}(4)$ the quarks (u^α, d^α) belong to a $\underset{\sim}{4}$-plet, whereas s^α and c^α are singlets. The inverse is true for $\widetilde{\widetilde{SU}}(4)$: u^α and d^α are singlets and (s^α, c^α) belong to a $\underset{\approx}{4}$-plet. These transformation properties of the quarks are summarized in table 1 together with the quantum numbers for the SU(4) generators Y_8 and Y_{15} in the GIM[8] quark model. Using table 1 and the quark content of the mesons, given in appendix B, the assignments shown in table 2 follow.

All particles transform irreducibly, except η_8 and η_{15} under $\widetilde{\widetilde{SU}}(4)$. In this case the linear combinations

$$(\eta_8, \eta_{15})_{\tilde{\tilde{1}}} = \sqrt{\tfrac{2}{3}}\,\eta_8 + \sqrt{\tfrac{1}{3}}\,\eta_{15}\,, \qquad (3.1)$$
$$(\eta_8, \eta_{15})_{\widetilde{\widetilde{15}}} = -\sqrt{\tfrac{1}{3}}\,\eta_8 + \sqrt{\tfrac{2}{3}}\,\eta_{15}\,,$$

are the ones which belong, respect-ively, to the singlet and the 15-plet.

To go a step further, we note that the generators of SU(6) commute with those of $\vec{C}$-spin, allowing the decomposition $SU(8) \supset SU(6) \otimes SU(2)_C$. Similarly, we have $SU(8) \supset \widetilde{SU}(4) \otimes SU(2)_S \otimes SU(2)_C$ and $SU(8) \supset \widetilde{\widetilde{SU}}(4) \otimes SU(2)_p \otimes SU(2)_n$. With the values of p-, n-, S- and C-spin also shown in table 2, the following classifications result:

- under $SU(6) \otimes SU(2)_C$:

$$\begin{aligned}
&(35,1): && \pi^{\pm}, \pi^{0}, \rho^{\pm}, \rho^{0}, K^{\pm}, K^{0}, \overline{K^{0}}, K^{*\pm}, K^{*0}, \overline{K^{*0}}, \eta_8, \omega_\sigma, \omega_s\,;\\
&(6,2): && \overline{D^{0}}, D^{-}, F^{-}, \overline{D^{*0}}, D^{*-}, F^{*-}\,;\\
&(\bar{6},2): && D^{0}, D^{+}, F^{+}, D^{*0}, D^{*+}, F^{*+}\,;\\
&(1,3): && \omega_c\,;\\
&(1,1): && \eta_{15}\,;
\end{aligned} \qquad (3.2)$$

- under $\widetilde{SU}(4) \otimes SU(2)_S \otimes SU(2)_C$:

$$\begin{aligned}
&(15,1,1): && \pi^{\pm}, \pi^{0}, \rho^{\pm}, \rho^{0}, \omega_\sigma\,;\\
&(4,2,1): && K^{+}, K^{0}, K^{*+}, K^{*0}\,;\\
&(\bar{4},2,1): && K^{-}, \overline{K^{0}}, K^{*-}, \overline{K^{*0}}\,;\\
&(4,1,2): && D^{-}, \overline{D^{0}}, D^{*-}, \overline{D^{*0}}\,;\\
&(\bar{4},1,2): && D^{+}, D^{0}, D^{*+}, D^{*0}\,;\\
&(1,2,2): && F^{+}, F^{*+}\,;\\
&(1,2,2): && F^{-}, F^{*-}\,;\\
&(1,3,1): && \omega_s\,;\\
&(1,1,3): && \omega_c\,;\\
&(1,1,1): && \eta_8\,;\\
&(1,1,1): && \eta_{15}\,;
\end{aligned} \qquad (3.3)$$

- under $\widetilde{\widetilde{SU}}(4) \otimes SU(2)_p \otimes SU(2)_n$:

$$
\begin{aligned}
&(15,1,1): && F^{\pm},\ F^{*\pm},\ -\sqrt{\tfrac{1}{3}}\eta_8+\sqrt{\tfrac{2}{3}}\eta_{15},\ \omega_s,\ \omega_c\ ;\\
&(4,2,1): && K^-,\ D^0,\ K^{*-},\ D^{*0}\ ;\\
&(\bar{4},2,1): && K^+,\ \overline{D^0},\ K^{*+},\ \overline{D^{*0}}\ ;\\
&(4,1,2): && \overline{K^0},\ D^+,\ \overline{K^{*0}},\ D^{*+}\ ;\\
&(\bar{4},1,2): && K^0,\ D^-,\ K^{*0},\ D^{*-}\ ;\\
&(1,2,2): && \pi^+,\ \rho^+\ ;\\
&(1,2,2): && \pi^-,\ \rho^-\ ;\\
&(1,3,1): && \sqrt{\tfrac{1}{2}}(\omega_\sigma+\rho^0)\ ;\\
&(1,1,3): && \sqrt{\tfrac{1}{2}}(\omega_\sigma-\rho^0)\ ;\\
&(1,1,1): && \pi^0\ ;\\
&(1,1,1): && \sqrt{\tfrac{2}{3}}\eta_8+\sqrt{\tfrac{1}{3}}\eta_{15}\ .
\end{aligned}
\tag{3.4}
$$

The numbers in parentheses refer to the dimension of the representation of the corresponding subgroups.

4. Mass relations and the mixing problem

Here we consider the particle spectra following from the hamiltonian eq.(2.13) and the classification of particles given in section 3. The eigenvalues of the various Casimir operators are

$$
\begin{aligned}
\mathcal{C}_2^{(6)} &= 35/6 \quad \text{for } \underset{\sim}{6},\underset{\sim}{\bar{6}}\ , & \mathcal{C}_2^{(4)} &= 15/4 \quad \text{for } \underset{\sim}{4},\underset{\sim}{\bar{4}}\ ,\\
&= 12 \quad \text{for } \underset{\sim}{35}\ ; & &= 8 \quad \text{for } \underset{\sim}{15}\ ;\\
\mathcal{C}_2^{(3)} &= 8/3 \quad \text{for } \underset{\sim}{3},\underset{\sim}{\bar{3}}\ , & \mathcal{C}_2^{(2)} &= 3/2 \quad \text{for } \underset{\sim}{2}\ (I=1/2)\ ,\\
&= 6 \quad \text{for } \underset{\sim}{8}\ ; & &= 4 \quad \text{for } \underset{\sim}{3}\ (I=1)\ ;
\end{aligned}
\tag{4.1}
$$

and zero for the singlet representations. Terms like $2\vec{S}\cdot\vec{J}$ are easiest evaluated using $2\vec{S}\cdot\vec{J} = \vec{J}^2 + \vec{S}^2 - (\vec{N}+\vec{C})^2$. The masses are then given by:

$$\begin{aligned}
\pi &= m_0 + 8m_1 + 12m_2 + 8b_4 + 6b_3 + 4b_2 ,\\
K &= m_0 + 12m_2 + 8b_4 + 6b_3 + 2b_2 + a_3 ,\\
D &= m_0 + 4m_2 + 8b_4 + 3b_3 + 2b_2 + a_4 ,\\
F &= m_0 - 8m_1 + 4m_2 + 8b_4 + 3b_3 + a_4 + a_3 ,\\
\\
P_{8,8} &= m_0 - \tfrac{8}{3}m_1 + 12m_2 + 8b_4 + 6b_3 ,\\
P_{15,15} &= m_0 - \tfrac{16}{3}m_1 + 8b_4 ,\\
P_{8,15} &= \tfrac{8\sqrt{2}}{3}m_1 ,
\end{aligned} \tag{4.2}$$

($P_{i,j} = \langle \eta_i | H | \eta_j \rangle$) for the pseudoscalar mesons, and

$$\begin{aligned}
\rho &= \pi + 2a ,\\
K^* &= K + 2a + 2b ,\\
D^* &= D + 2a + 2c ,\\
F^* &= F + 2a + 2b + 2c ,\\
V_{8,8} &= P_{8,8} + 2a + \tfrac{8}{3}b ,\\
V_{15,15} &= P_{15,15} + 2a + \tfrac{1}{3}b + 3c ,\\
V_{0,0} &= m_0 + 8m_2 + 2a + b + c ,\\
V_{8,15} &= P_{8,15} - \tfrac{2\sqrt{2}}{3}b ,\\
V_{0,8} &= \tfrac{8\sqrt{6}}{3}m_1 - \tfrac{2\sqrt{6}}{3}b ,\\
V_{0,15} &= \tfrac{8\sqrt{3}}{3}m_1 + 4\sqrt{3}m_2 + \tfrac{1}{\sqrt{3}}b - \sqrt{3}c ,
\end{aligned} \tag{4.3}$$

($V_{i,j} = \langle \omega_i | H | \omega_j \rangle$) for the vector mesons.

From eqs. (4.2) we have

$$F - D = K - \pi, \tag{4.4}$$

and from eqs. (4.3) it then follows ·

$$F^* - D^* = K^* - \rho. \tag{4.5}$$

Relations between pseudoscalar and vector mesons, like those given in eq.(2.9), are possible only if $b = 0$ and/or $c = 0$.

Of particular interest is the mixing of the isoscalar vector mesons ω, ϕ and J/ψ. The (symmetric) mass matrix in the $(8,15,0)$-representation can be read off from eq. (4.3). Expressing part of the parameters by the masses of ρ, K^* and D^*, we have

$$M_V = \begin{array}{c} 8 \\ 15 \\ 0 \end{array} \begin{pmatrix} \frac{1}{3}(4K^*-\rho) - \frac{4}{3}(b_2+a_3) & -\frac{\sqrt{2}}{3}(K^*-\rho) + \frac{\sqrt{2}}{3}(-2b_2+a_3) & -\sqrt{\frac{2}{3}}(K^*-\rho) + \sqrt{\frac{2}{3}}(-2b_2+a_3) \\ & \frac{1}{6}(K^*+9D^*-4\rho) - \frac{3}{2}b_3 - \frac{2}{3}b_2 - \frac{3}{2}a_4 - \frac{1}{6}a_3 & -\frac{1}{2\sqrt{3}}(3D^*-K^*-2\rho) - \frac{1}{\sqrt{3}}\left(\frac{9}{2}b_3 + 2b_2 - \frac{3}{2}a_4 + \frac{1}{2}a_3\right) \\ & & \frac{1}{2}(K^*+D^*) - 8b_4 - \frac{9}{2}b_3 - 2b_2 - \frac{1}{2}a_4 - \frac{1}{2}a_3 \end{pmatrix} . \tag{4.6}$$

We now require the matrix (4.6) to be equivalent with our mass mixing matrix for broken SU(4) [5], written in the form

$$M_V = \begin{array}{c} 8 \\ 15 \\ 0 \end{array} \begin{pmatrix} \frac{1}{3}(4K^*-\rho) & -\frac{\sqrt{2}}{3}(K^*-\rho) & -\sqrt{\frac{2}{3}}(K^*-\rho) + \Delta A \\ & \frac{1}{6}(K^*+9D^*-4\rho) & -\frac{1}{2\sqrt{3}}(3D^*-K^*-2\rho) + \Delta B \\ & & \frac{1}{2}(K^*+D^*) + \Delta M_0 \end{pmatrix} , \tag{4.7}$$

where ΔA, ΔB and ΔM_0 measure the deviations from ideal mixing. This requirement implies

$$b_2 = 0, \quad a_3 = 0, \quad a_4 + b_3 = 0, \tag{4.8}$$

which has the consequence

$$\begin{aligned} \Delta A &= 0, \\ \Delta B &= -2\sqrt{3}\, b_3, \\ \Delta M_0 &= -4(b_3+2b_4) \ . \end{aligned} \tag{4.9}$$

Thus our ansatz for SU(8) mass breaking and the requirement that concerning SU(4) the mass breaking transforms according to the regular representation, eq .(2.1), have the consequence that $\Delta A = 0$. This is one of the constraints for ideal mixing.

It is well known that the vector mesons ω, ϕ and J/Ψ are almost ideally mixed i.e. $\omega \approx \omega_\sigma$, $\phi \approx \omega_s$ and $J/\Psi \approx \omega_c$. Therefore it is more convenient to consider the matrix (4.7) in the (σ,s,c)-representation, where it is given by

$$M_V = \begin{array}{c} \sigma \\ s \\ c \end{array} \left(\begin{array}{c:c:c} \rho + 2(\Delta_1+\Delta_2-\Delta_3) & \sqrt{2}\,\Delta_1 & \sqrt{2}\,\Delta_2 \\ & 2K^* - \rho + (\Delta_1 - \Delta_2 + \Delta_3) & \Delta_3 \\ & & 2D^* - \rho + (-\Delta_1 + \Delta_2 + \Delta_3) \end{array} \right) . \tag{4.10}$$

The connection between the parameters Δ_1, Δ_2 and Δ_3 describing the deviations from ideal mixing, and the equivalent set of parameters ΔA, ΔB and ΔM_0 appearing in (4.7) is given by

$$\begin{aligned} \Delta A &= 2\sqrt{2}\,(\Delta_2 - \Delta_3)\ , \\ \Delta B &= \frac{1}{\sqrt{3}}\,(3\Delta_1 - \Delta_2 - 2\Delta_3) \\ \Delta M_0 &= 2\,(\Delta_1 + \Delta_2) \end{aligned} \tag{4.11}$$

For ideal mixing $\Delta_1 = \Delta_2 = \Delta_3 = 0$ and eq.(4.10) then yields the masses given in eq.(2.10). In our case we get from (4.9)

$$\Delta_1 = -2b_3 - 2b_4 \quad , \qquad \Delta_2 = \Delta_3 = -2b_4 \ . \tag{4.12}$$

The three parameters b_3, b_4 and D^* are then determined from the eigenvalues ω, ϕ and J/Ψ of the matrix (4.10).

We now come to the pseudoscalar mesons. Since in this case the SU(4) singlet does not belong to the $\underset{\sim}{63}$ representation of SU(8), nothing can be inferred about the matrix elements $P_{0,8}$ $P_{0,15}$ and $P_{0,0}$. If we write the mass mixing matrix in a similar form as for the vector mesons, eq. (4.7), we have

$$M_P = \begin{array}{c} 8 \\ 15 \\ 0 \end{array} \left(\begin{array}{c|c|c} \frac{1}{3}(4K-\pi) & -\frac{\sqrt{2}}{3}(K-\pi) & -\sqrt{\frac{2}{3}}(K-\pi) + \Delta A_P \\ & \frac{1}{6}(K+9D-4\pi) & -\frac{1}{2\sqrt{3}}(3D-K-2\pi) + \Delta B_P \\ & & \frac{1}{2}(K+D) + \Delta M_{0P} \end{array} \right) , \tag{4.13}$$

with arbitrary parameters ΔA_p, ΔB_p and ΔM_{op}. In addition to this there is also no correlation between the mass parameters of the pseudoscalar mesons and the vector mesons in the (8,15)-submatrix, since according to eqs. (4.3) the π-, K- and D-masses can be fixed independently of the ρ-, K^*- and D^*masses, as long as b, c $\neq$ 0.

5. Discussion.

In the following we shall investigate whether the mass mixing matrix M_V as given by eq . (4.10) with the SU(8) constraint (4.12) can be fitted to the observed masses of ρ, K^*, ω, ϕ and J/ψ and whether the resulting wave functions can account for the Zweig forbidden decays of the ϕ and J/ψ.

The three tensor invariants of the matrix (4.10), if expressed by the eigenvalues of M_V (squared masses of ω, ϕ and J/ψ), yield three equations for the unknowns $D^* = m_{D^*}^2$, b_3 and b_4. By eliminating D^* and b_3 we obtain a quadratic equation in b_4. With the standard mass values (in GeV) $m_\omega = 0.783$, $m_\phi = 1.02$, $m_{J/\psi} = 3.095$, $m_{K^*} = 0.894$ and $m_\rho = 0.770$[9,10] the equation for b_4 has no real solution. On the other hand the experimental masses have errors, the error being largest for the ρ mass: $m_\rho = 0.77 \pm 0.01$. Therefore we fix the rather well determined masses of ω, ϕ, J/ψ and K^* at the values, already used in our earlier papers[5], ($m_\omega = 0.7827$, $m_\phi = 1.0197$, $m_{J/\psi} = 3.095$ and $m_{K^*} = 0.89435$) and vary only the ρ mass. It is remarkable that real solutions for b_4 are obtained for $m_\rho \lesssim 0.7609$, which still lies inside the experimental error bars. These changes of m_ρ have no appreciable effect on the masses of the charmed mesons which come out around $m_{D^*} = 2.26$ and $m_{F^*} = 2.30$.(*)

Much more sensitive to the variation of the ρ mass are the wave functions of ω, ϕ and J/ψ. To first order in the deviations from ideal mixing (Δ_1, Δ_2, Δ_3) the wave functions in the (σ,s,c)-basis are given by

$$
\begin{aligned}
|\omega\rangle &\simeq \left(1 \,,\; \frac{2\sqrt{2}(b_3+b_4)}{\phi-\omega} \,,\; \frac{2\sqrt{2}\, b_4}{\psi-\omega} \right), \\
|\phi\rangle &\simeq \left(\frac{-2\sqrt{2}(b_3+b_4)}{\phi-\omega} \,,\; 1 \,,\; \frac{2\, b_4}{\psi-\phi} \right), \\
|J/\psi\rangle &\simeq \left(\frac{-2\sqrt{2}\, b_4}{\psi-\omega} \,,\; \frac{-2\, b_4}{\psi-\phi} \,,\; 1 \right).
\end{aligned}
\tag{5.1}
$$

In our earlier work[5] we saw that, in order to have the decay rate for $J/\psi \to \rho\pi$ as small as experimentally observed,[9] the component

*In fact for given m_ρ two solutions for b_4, b_3, D^* and F^* are obtained. For $b_4 \to 0$, as required by the J/ψ decays (see below), the two solutions merge.

$\langle\sigma|J/\psi\rangle$ must be extremely small, of the order $1\cdot 10^{-4}$, implying $b_4 \simeq -3.2 \times 10^{-4}$. This can be achieved by adjusting the ρ mass appropriately. Then

$$\langle 8|J/\psi\rangle = \frac{1}{\sqrt{3}}\left(\langle\sigma|J/\psi\rangle - \sqrt{2}\langle s|J/\psi\rangle\right) \simeq 10^{-2}\, b_4 \tag{5.2}$$

is very small and consequently J/ψ behaves to a good approximation as an SU(3) singlet. This explains the negligible decay rates for $J/\psi \to K\bar{K}$, $K^*\overline{K^*}$, $K^{**}\overline{K^{**}}$, $K^{**}\bar{K}$[9]. Actually it is well known that the admixture of uncharmed quarks in the J/ψ must have a non-negligible SU(3) octet component in order to reproduce the ratio of the decay rates for $J/\psi \to K^{*+}K^-$ and $J/\psi \to \rho^+\pi^-$. Our value (5.2) for the octet component is however much too small to fit this ratio. We obtain 0.88 as compared to (0.36±0.17) experimentally.[9] This is not necessarily a drawback of our SU(8) model since it does not include electromagnetic mass shifts. On the other hand the experimental values for $J/\psi \to \rho\pi$, $K^*\bar{K}$ contain also the contribution of $J/\psi \to \gamma \to \rho\pi$, $K^*\bar{K}$, which must be subtracted before comparing with our model.

With the wave functions (5.1) we can calculate also the ratio $\Gamma(J/\psi \to \Lambda\bar{\Lambda})/\Gamma(J/\psi \to \bar{p}p)$. We obtain 0.96, in agreement with the experimental value (0.76±0.53).[9]

The coefficient

$$\langle s|\omega\rangle = -\langle\sigma|\phi\rangle \simeq \frac{2\sqrt{2}\,(b_3+b_4)}{\phi-\omega} \tag{5.3}$$

determines the ω-ϕ mixing in the (σ,s)-sector. The diagonalization of M_V yields $b_4 \simeq -3.39 \times 10^{-4}$ and $b_3+b_4 \simeq -0.00878$, implying $\langle\sigma|\phi\rangle \simeq 0.058$. This value can be tested by comparing the decay rates $\Gamma(\omega \to 3\pi) = (9.0 \pm 0.7)$ MeV and $\Gamma(\phi \to 3\pi) = (0.66 \pm 0.07)$ MeV.[10]

Describing these decays by the usual $\rho\pi$ dominance model including finite width corrections one gets[11] $g_{\omega\rho\pi} = (17.5 \pm 0.8)\ \mathrm{GeV}^{-1}$ and $g_{\phi\rho\pi} = (0.90 \pm 0.06)\ \mathrm{GeV}^{-1}$. Therefore $\langle\sigma|\phi\rangle = g_{\phi\rho\pi}/g_{\omega\rho\pi} = 0.051 \pm 0.006$, in good agreement with our value obtained above. We emphasize that with an appropriate ρ mass we can achieve correct values for $\langle\sigma|J/\psi\rangle$ and $\langle\sigma|\phi\rangle$ which determine the Zweig forbidden decays of ϕ and J/ψ.

It is interesting to compare the matrix M_V obtained on the basis of broken SU(8) symmetry with other approaches as, for example, the quark-gluon field theory[6] and the topological expansion of the S-matrix[7]. Both approaches lead, for each spin multiplet, to mass mixing matrices of the form (4.10), but with the constraints $\Delta_1 = \Delta_2 = \Delta_3 \equiv G$, i.e. $b_3 = 0$, instead of (4.12). From our discussion above it is clear that such mass matrices with a constant G cannot produce acceptable solutions for both, the mass spectrum and the wave functions. To overcome this difficulty in these approaches G is considered as a mass dependent parameter. The mass dependence of G for large masses is derived from asymptotic freedom[12] in the quark-gluon theory and from asymptotic planarity[7] in the S-matrix topological expansion.

We note that, as a result of our mixing analysis for the vector mesons, the parameter $b_3 + b_4 = -0.00878$, which determines the mixing in the (σ,s)-submatrix, is larger than $b_4 = -0.00034$, which determines the admixture of charmed quarks, by roughly a factor 25. This might suggest to consider the limiting case $b_4 = 0$, i.e. $\Delta_2 = \Delta_3 = 0$, as a first approximation. Then diagonalisation of (4.10) and elimination of the parameter D^* leads to the well-known Schwinger formula[13] of the masses of ω, ϕ, ρ

and K^*

$$(3\phi - 4K^* + \rho)(4K^* - \rho - 3\omega) = 8\,(K^* - \rho)^2. \tag{5.4}$$

This formula is very well satisfied, showing that the vector mesons approximately satisfy the constraint $\Delta_2 = \Delta_3 = 0$. The D^* mass is then given by

$$2D^* = \Psi + \rho + \frac{1}{3}(\omega + \phi - 2K^*)\,, \tag{5.5.}$$

yielding $m_{D^*} = 2.26$ GeV and, via eq.(4.5), $m_{F^*} = 2.30$ GeV.

In the case that the pseudoscalar mesons would fulfill also the constraints $\Delta_2 = \Delta_3 = 0$, the corresponding mass formula (5.4) for the π, K, η and η' masses is obtained. However this formula is not satisfied, showing that the pseudoscalar mesons are far from the limiting case $\Delta_2 = \Delta_3 = 0$. A better limiting case is described by the constraints $P_{0,8} = P_{0,15} = 0$ in eq. (4.13):

$$\Delta A_{\bar{p}} = \sqrt{\frac{2}{3}}\,(K - \pi)\,,$$
$$\Delta B_p = \frac{1}{2\sqrt{3}}(3D - K - 2\pi)\,, \tag{5.6}$$

which yield the mass formula of Bjorken and Glashow[14]:

$$(3\eta' - 4K + \pi)(4K - \pi - 3\eta) = 2\,(K - \pi)^2, \tag{5.7}$$

together with

$$D = \frac{2}{3}\left(\eta + \eta' + \pi - \frac{3}{2}K\right). \tag{5.8}$$

Whereas eq . (5.7) is well satisfied (with $m_{\eta'} = 0.958$ GeV), eq .(5.8) gives a much too small D mass ($m_D = 0.77$ GeV). In addition, this limiting case has the problem of large $c\bar{c}$ admixtures in η and η' and correspondingly large admixtures of uncharmed quarks in the SU(4) singlet state ψ_P. Therefore this limiting case is not acceptable too. Even the more realistic case with the D mass calculated from $D = D^* - \rho + \pi$ (corresponding to $c = 0$ in eqs.(4.3)) and the parameters ΔA_P, ΔB_P and ΔM_{oP} determined from the eigenvalues $\eta(0.549)$, $\eta'(0.958)$ and $\Psi_P(2.85)$ of the matrix (4.13) does not lead to acceptable wave functions for η, η' and ψ_P. The problem can be solved only by considering D as a free parameter to be fixed by some input concerning the wave functions. From our previous work[5] we know that a satisfactory fit to the various decay rates is obtained for $m_D \gtrsim 2.05$ GeV, corresponding to $c = 0.17$. It is thus essential for the pseudoscalar multiplet to exploit the freedom given by $c \neq 0$ in eqs.(4.3). As a consequence the mass spectra of pseudoscalar and vector mesons cannot differ by an overall shift only.

We close this section by establishing the connection with our earlier papers[5]. There the mixing problem for vector and pseudoscalar mesons has been discussed in terms of the parameters α_V and α_P introduced by Okubo[2] and which roughly measure the ratio of SU(4) breaking to SU(3) breaking. When expressed by the particle masses, these parameters are given by:

$$\alpha_V = \frac{3D^* - K^* - 2\rho}{2\sqrt{2}\,(K^* - \rho)}, \tag{5.9}$$

$$\alpha_P = \frac{3D - K - 2\pi}{2\sqrt{2}\,(K - \pi)} . \tag{5.10}$$

Using eqs.(4.3) we see that the parameters α_V and α_P are independent as long as b and/or c have nonvanishing values. Thus the equality $\alpha_V = \alpha_P$ following from SU(8) symmetry and often assumed in SU(4) calculations, remains true also in the case of SU(8) symmetry breaking as long as $b = c = 0$, i.e. as long as the vector and pseudoscalar mass spectra differ only by an overall shift given by 2a.

Other parameters used in our earlier calculation were β_V and β_P, which appeared in the matrix elements $V_{0,15}$ and $P_{0,15}$, respectively. Here we can say something only about β_V. It is related so the parameter b_3 by

$$\beta_V = 1 + 12\, b_3 \left(3D^* - K^* - 2\rho\right)^{-1} . \tag{5.11}$$

Usually $\beta_V = 1$ is assumed, which amounts to neglecting the additional splitting of different SU(3) multiplets, as given by b_3 in eq.(2.13). With the values of D^* and b_3 obtained in this paper we get $\alpha_V = 21.2810$ and $\beta_V = 0.9924$, in close agreement with the values previously found[5].

Appendix A

Here we give the generators and Casimir operators of the group SU(8) and its various subgroups. In Okubo's notation the generators of SU(8) are given by a set of 64 operators $X^A{}_B$ (A, B = 1, 2, ..., 8) satisfying

$$[X^A{}_B, X^C{}_D]_- = \delta^A{}_D X^C{}_B - \delta^C{}_B X^A{}_D \,, \tag{A.1}$$

$$X^A{}_A = 0 \,, \tag{A.2}$$

$$(X^A{}_B)^\dagger = X^B{}_A \,. \tag{A.3}$$

Owing to the decomposition SU(8) $\supset$ SU(4)$_F$ $\otimes$ SU(2)$_J$, we may split each SU(8)-index A = 1, 2, ..., 8 into an SU(4)$_F$-index a = 1,2,3,4 (for u, d, s, c quarks) and an SU(2)$_J$-index α = 1,2 (for ordinary spin up or down). With the correspondence A = (a,α)

$$\begin{array}{llll} 1 = (1,1), & 2 = (2,1), & 3 = (3,1), & 4 = (4,1), \\ 5 = (1,2), & 6 = (2,2), & 7 = (3,2), & 8 = (4,2), \end{array} \tag{A.4}$$

the generators of SU(4)$_F$ are given by

$$F^a{}_b = X^{(a,\alpha)}{}_{(b,\alpha)} = \sum_{n=1}^{15} F_n (\lambda_n)^a{}_b \,, \qquad F^a{}_a = 0 \,, \tag{A.5}$$

and those of SU(2)$_J$ by

$$J^\alpha{}_\beta = X^{(a,\alpha)}{}_{(a,\beta)} = \sum_{k=1}^{3} J_k (\sigma_k)^\alpha{}_\beta \,, \qquad J^\alpha{}_\alpha = 0 \,. \tag{A.6}$$

Here the λ_n's are a set of standard hermitian 4 x 4-matrices. The generators F_n and J_K, defined by (A.5) and (A.6) respectively, obey the commutation relations

$$[F_m, F_n]_- = i f_{mnl} F_l \,, \tag{A.7}$$

$$[J_i, J_j]_- = i \epsilon_{ijk} F_k \,, \tag{A.8}$$

where f_{mnl} (ϵ_{ijk}) are the wellknown structure constants of SU(4) (SU(2)).

The generators of the various SU(2)-subgroups of SU(4)$_F$ are:

$$I_+ = F^2{}_1 = X^2{}_1 + X^6{}_5 \,,$$

$$I_3 = \tfrac{1}{2}(F^1{}_1 - F^2{}_2) = \tfrac{1}{2}(X^1{}_1 + X^5{}_5 - X^2{}_2\, X^6{}_6) \,;$$

$$U_+ = F^3{}_2 = X^3{}_2 + X^7{}_6 \,,$$

$$U_3 = \tfrac{1}{2}(F^2{}_2 - F^3{}_3) = \tfrac{1}{2}(X^2{}_2 + X^6{}_6 - X^3{}_3 - X^7{}_7) \,;$$

$$V_+ = F^3{}_1 = X^3{}_1 + X^7{}_5 \,,$$
$$V_3 = \tfrac{1}{2}(F^1{}_1 - F^3{}_3) = \tfrac{1}{2}(X^1{}_1 + X^5{}_5 - X^3{}_3 - X^7{}_7) \,;$$
$$W_+ = F^4{}_1 = X^4{}_1 + X^8{}_5 \,,$$
$$W_3 = \tfrac{1}{2}(F^1{}_1 - F^4{}_4) = \tfrac{1}{2}(X^1{}_1 + X^5{}_5 - X^4{}_4 - X^8{}_8) \,; \qquad \text{(A.9)}$$
$$X_+ = F^4{}_2 = X^4{}_2 + X^8{}_6 \,,$$
$$X_3 = \tfrac{1}{2}(F^2{}_2 - F^4{}_4) = \tfrac{1}{2}(X^2{}_2 + X^6{}_6 - X^4{}_4 - X^8{}_8) \,;$$
$$Z_+ = F^4{}_3 = X^4{}_3 + X^8{}_7 \,,$$
$$Z_3 = \tfrac{1}{2}(F^3{}_3 - F^4{}_4) = \tfrac{1}{2}(X^3{}_3 + X^7{}_7 - X^4{}_4 - X^8{}_8) \,.$$

Note that $\vec{I}$, $\vec{U}$ and $\vec{V}$ are nothing else than the familiar isospin, U-spin and V-spin already known from SU(3). Furthermore we have

$$F_8 = -\frac{1}{2\sqrt{3}}(3F^3{}_3 + F^4{}_4) = -\frac{1}{2\sqrt{3}}\left[3(X^3{}_3 + X^7{}_7) + X^4{}_4 + X^8{}_8\right] \,,$$
$$F_{15} = -\sqrt{\tfrac{2}{3}}\, F^4{}_4 = -\sqrt{\tfrac{2}{3}}\,(X^4{}_4 + X^8{}_8) \,, \qquad \text{(A.10)}$$

which are related to hypercharge Y_8 and hypercharm Y_{15} by

$$Y_8 = \frac{2}{\sqrt{3}} F_8 \,, \qquad Y_{15} = \sqrt{\tfrac{3}{2}}\, F_{15} \,. \qquad \text{(A.11)}$$

In an analogous way we may define:

$$SU(2)_p : \quad p_+ = X^5{}_1 \,, \quad p_3 = \tfrac{1}{2}(X^1{}_1 - X^5{}_5) \,;$$
$$SU(2)_n : \quad n_+ = X^6{}_2 \,, \quad n_3 = \tfrac{1}{2}(X^2{}_2 - X^6{}_6) \,; \qquad \text{(A.12)}$$
$$SU(2)_S : \quad S_+ = X^7{}_3 \,, \quad S_3 = \tfrac{1}{2}(X^3{}_3 - X^7{}_7) \,;$$
$$SU(2)_C : \quad C_+ = X^8{}_4 \,, \quad C_3 = \tfrac{1}{2}(X^4{}_4 - X^8{}_8) \,.$$

Here $\vec{p}$, $\vec{n}$, $\vec{S}$ and $\vec{C}$ are the ordinary spin of u, d, s and c quarks, respectively. They commute among themselves and may add up to the spin $\vec{N} = \vec{p} + \vec{n}$ of the "normal" quarks (i.e. the spin of u and d quarks), or to the total spin $\vec{J} = \vec{N} + \vec{S} + \vec{C}$ defined in (A.6).

Appendix B

In this appendix we give the quark content of pseudoscalar and vector mesons in SU(8). The quarks, coming in four flavours (u, d, s and c), belong to the representation $\underset{\sim}{4}$ of $SU(4)_F$ and to the representation 1/2 of $SU(2)_J$. Combining unitary spin with ordinary spin according to the direct product $SU(4)_F \otimes SU(2)_J \subset SU(8)$, each quark field $q^{a\alpha}$ carries two indices: an

$SU(4)_F$-index a = 1,2,3,4 for u,d,s and c quarks, respectively, and an $SU(2)_J$-index α = 1,2 for spin up (↑) or down (↓). The antiquarks are described by $\bar{q}_{a\alpha} = (q^{a\alpha})^+$, with $\bar{q}_{a1} = -\bar{q}_a(\downarrow)$ and $\bar{q}_{a2} = \bar{q}_a(\uparrow)$, however.

The mesons M are built from a quark-antiquark pair:

$$M^{a\alpha}{}_{b\beta} = q^{a\alpha}\,\bar{q}_{b\beta} \qquad (a,b = 1,2,3,4\,;\quad \alpha,\beta = 1,2)\,. \tag{B.1}$$

The pseudoscalar mesons are then given by

$$P^a{}_b = \frac{1}{\sqrt{2}}\,M^{a\alpha}{}_{b\alpha}\,, \tag{B.2}$$

and the vector mesons by

$$V^a{}_b(j) = \frac{1}{\sqrt{2}}\,M^{a\alpha}{}_{b\beta}\,(\sigma_j)^\beta{}_\alpha \qquad (j = 1,2,3)\,, \tag{B.3}$$

where σ_j are the usual Pauli matrices. From (B.3) we get for the vector mesons having ordinary spin projection $J_z = \pm 1$, O:

$$V^a{}_b(+1) = M^{a1}{}_{b2}\,,\quad V^a{}_b(-1) = M^{a2}{}_{b1}\,,\quad V^a{}_b(0) = \frac{1}{\sqrt{2}}\left(M^{a1}{}_{b1} - M^{a2}{}_{b2}\right). \tag{B.4}$$

The connection between the fields $P^a{}_b$ ($V^a{}_b$) and the fields π, K, D, F, η_8, η_{15}, η_0 (ρ, K*, D*, F*, ω_8, ω_{15}, ω_0) is the same as in usual SU(4).

Under SU(8) the 64 fields (B.1) decompose into a singlet $M^{a\alpha}{}_{a\alpha}$ = $= 2\sqrt{2}\,P_0$ and a $\underset{\sim}{63}$-plet. The latter, if decomposed with respect to $SU(4)_F \otimes SU(2)_J$, contains a pseudoscalar $\underset{\sim}{15}$-plet (but no singlet!), and an ideally mixed vector singlet and vector $\underset{\sim}{15}$-plet.

Table 1: Spin and unitary spin of the quarks

	p	n	N	S	C	J	SU(6)	$\widetilde{SU}(4)$	$\approx{SU}(4)$	SU(3)	$SU(2)_I$	Y_8	Y_{15}
u	$\frac{1}{2}$	0	$\frac{1}{2}$	0	0	$\frac{1}{2}$	6	4	1	3	2	1/3	1/4
d	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	$\frac{1}{2}$	6	4	1	3	2	1/3	1/4
s	0	0	0	$\frac{1}{2}$	0	$\frac{1}{2}$	6	1	4	3	1	-2/3	1/4
c	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1	1	4	1	1	0	-3/4

Table 2: Spin and unitary spin of pseudoscalar and vector mesons.

	p	n	N	S	C	J	SU(6)	$\widetilde{SU}(4)$	$\widetilde{\widetilde{SU}}(4)$	SU(3)	$SU(2)_I$	Y_8	Y_{15}
$\pi^{\pm,0}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	0	0	35	15	1	8	3	0	0
$\rho^{\pm,0}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0	0	1	35	15	1	8	3	0	0
K^+	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	0	35	4	$\overline{4}$	8	2	1	0
K^0	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	0	35	4	$\overline{4}$	8	2	1	0
K^{*+}	$\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	1	35	4	$\overline{4}$	8	2	1	0
K^{*0}	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1	35	4	$\overline{4}$	8	2	1	0
$\overline{D^0}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	0	6	4	$\overline{4}$	$\overline{3}$	2	1/3	1
D^-	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	0	6	4	$\overline{4}$	$\overline{3}$	2	1/3	1
$\overline{D^{*0}}$	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{2}$	1	6	4	$\overline{4}$	$\overline{3}$	2	1/3	1
D^{*-}	$\frac{1}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{2}$	1	6	4	$\overline{4}$	$\overline{3}$	2	1/3	1
F^-	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	6	1	15	$\overline{3}$	1	-2/3	1
F^{*-}	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$	1	6	1	15	$\overline{3}$	1	-2/3	1
η_8			0	0	0	0	35	1		8	1	0	0
η_{15}			0	0	0	0	1	1		1	1	0	0
$(\eta_8,\eta_{15})_{\tilde{\tilde{1}}}$	$\frac{1}{2}$	$\frac{1}{2}$				0			1				
$(\eta_8,\eta_{15})_{\tilde{\tilde{15}}}$	0	0				0			15				
ω_σ	$\frac{1}{2}$	$\frac{1}{2}$	1	0	0	1	35	15	1	$1\oplus 8$	1	0	0
ω_s	0	0	0	1	0	1	35	1	15	$1\oplus 8$	1	0	0
ω_c	0	0	0	0	1	1	1	1	15	$1\oplus 8$	1	0	0

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