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Quark Confinement

by

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With a supplement on recent literature.

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1. Introduction

1.1 The power of the quark model in explaining the hadron spectrum, the general features of deep inelastic electron and neutrino scattering together with the other problems of elementary particle physics is rated so highly by many high energy physicists, that they considered the old Gell-Mann-Zweig quarks as "observed" long before the recent "discovery" of the "charmed quarks" in the new particles. This way of speaking about quarks is of course only a semantic solution to the main puzzle of the quark hypothesis: In spite of a big experimental effort no free particles with fractional charges have been found [1]. It is the aim of this review to be a guide to the theoretical attempts to reconcile this experimental fact with the successful applications of the elementary quark model.

1.2 For a long time there was the general feeling that a fundamentally puzzling discovery was necessary in order to induce essential progress in our understanding of elementary particle physics. "Essential" in this context denotes a step forward comparable to the transition from classical to quantum mechanics. The puzzling quarks, which sometimes behave like free particles but are never observed as isolated particles, the "confined" quarks might represent such a discovery. Therefore, our understanding of elementary particle physics, being close to a phase transition, shows wild fluctuations of ideas for the explanation of the quark puzzle. For example, I mention the tentative introduction of quark operators with a completely unorthodox product definition [2]. In this general review, we are somewhat more conservative. We assume that quantum field theory based on quark fields is a good candidate for hadron dynamics. Therefore we restrict ourselves to confinement models which are developed from phenomenological considerations, or which were made possible by the progress of quantum field theory.

1.3 From a phenomenologist's point of view, quark confinement is more a question of principle and not one of practical relevance. Since the main topic of this summer school is the "new physics" coming from e^+e^- storage rings, one should ask about the relation of our subject to these general themes. I mention two examples for which I adopt the hidden charm scheme [3]:

(a) The small width of the new resonances is explained by Zweig's rule [4]. Since this rule states that quarks - visualized as quark lines - can leave a hadron only combined with an anti-quark (-line), this rule has to be explained by any confinement scheme. Hence, the understanding of Zweig's rule means the understanding of important aspects of confinement and new particles [5].

(b) In the simplest way, confinement of quarks is described with the help of a strong infinite range potential, a "confining potential". The discussion of the J/Ψ -states of the charmonium spectrum [6], derived from such potentials sheds some light on the confinement dynamics.

However, applications are not the main goal of these lectures. I rather try to inform you on some recent theoretical developments in order to provide you with a broader view on possible dynamical explanations of the new phenomena.

1.4 Quark confinement is a challenging question for quantum field theory [7]. Of course, this phenomenon cannot be described in the framework of renormalized perturbation theory. But progress in understanding the structure of quantum field theory, for instance with the help of the renormalization group techniques [8], lead to interesting suggestions for the solution of our problem. Most important, we learned from the close relations between statistical mechanics and quantum field theory [9], that quantum field theory might describe quite different phases of matter than that related to the usual particle interpretation of relativistic field theory. The following examples may illustrate this point: The "dual strings" conjectured from the interpretation of the Veneziano formula might be considered as something like magnetic vortexes in superconductors of the second kind [10]. The "quark bags" can be related to stable classical solutions of non-linear field equations, like solitons first discussed in hydrodynamics [11]. There are models considering hadrons as superfluid droplets in equilibrium with a gaseous phase [12] etc. etc.

1.5 Certainly, the solutions suggested for the problem of quark confinement are still in a very speculative state. I think, in this situation it is not yet appropriate to try a critical assessment of the different models. Hence, in these lectures I shall simply give an introduction to the new concepts related to quark confinement, and I shall illustrate these with help of simple examples. However, I shall try to arrange the different ideas in such a way, that, hopefully, a vague, but coherent picture of quark confinement becomes visible. In this spirit I treat the following topics in the next Sections:

2. The phenomenological approach to quark confinement
3. The standard Lagrangian of hadrondynamics
4. Lagrangian field theory and quark confinement
5. Classical soliton solutions in a simple model. - Physical examples
6. Quantization of extended systems
7. Colour charge screening and quantization on a lattice
8. Short remarks on applications and summary

Even with the modern tools of information [13)], it is nowadays nearly impossible to quote the literature completely. But I hope that the references added to these lectures make these a useful guide to this new field of problems.

2. The phenomenological approach to quark confinement

2.1 The first and most simple idea to explain the absence of free quarks is to assume quarks having a large mass which are bound strongly in hadrons. This was suggested by G. Morpurgo [14)] in the framework of the non-relativistic quark model. In this model it is easy to consider the limiting case of infinitely strong binding by potentials singular at infinite distances, like the linear potential or the harmonic oscillator potential. These interactions do not allow approximately free particle states at large distances at any energy; even the remainder of the free particles, the one particle poles in the bound state wave function disappear. Such potentials are nowadays called "confining potentials". Many phenomenological questions are still discussed in this nonrelativistic framework where sometimes the form of the potential is guessed from field theoretical models, like the linear potential [15)] from the string model with colour charge screening (cf. 7.1e).

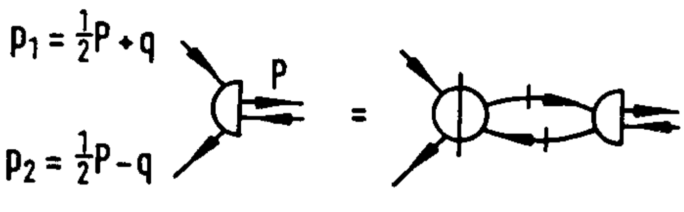
How can one formulate these simple ideas relativistically? The most consistent framework of relativistic quantum mechanics is general field theory, it is well suited to treat phenomenological problems. It has been shown [16)] that a relativistic model with heavy quarks and strong binding, formulated in the language of general field theory, can give the known results of the non-relativistic quark model for the spectrum of the mesons and their properties. The mesons are described here by Bethe-Salpeter amplitudes of the quark fields, and the strong binding is described by a phenomenological B.S.-kernel which compensates the large quark mass. Such a phenomenological discussion in a model with relativistic strong binding rises questions which are typical for the dynamics of quark confinement. For this we shall discuss some of the detailed aspects of the relativistic heavy quark model later. But before that I would like to mention the first attempts [17] of discussing the limit of strict confinement: infinite strong binding within general field theory. There are conjectures [18] of phenomenological interactions singular at large distances which make disappear the one quark pole in the hadron scattering amplitudes. In such models quark production by hadron collision is forbidden. The smoothness of Green's functions following from the absence of quark poles might be related to many phenomena of high energy processes [19]. However, in all these early models, the confining interaction is mostly guessed phenomenologically and there were only vague ideas [20] about their origin. In this respect, and not yet by the reliability or variety of their applications, the Lagrangian models in non-perturbative treatment brought some important progress.

2.2 Now I shall describe briefly the heavy quark model with strong binding [16], formulated in the language of general field theory (GFT). We consider the meson resonances as approximate single-particle states: $\left| {M \atop P}\; {j^{\pi c} \atop j_3}\; {I \atop I_3}\; Y \right\rangle$.

In the quark model they are described as $q\bar{q}$ bound states. The following matrix elements of the quark fields $\psi(x)$, $\bar{\psi}(x)$ between the vacuum $|0\rangle$ and the meson states, the B.S. amplitudes [21]

$$\chi(q,P) = (2\pi)^{3/2}\int dx\, e^{iqx}\langle 0|T\,\psi(\tfrac{x}{2})\bar{\psi}(-\tfrac{x}{2})\,|{M \atop P}\, j^{\pi c}\, q\bar{q}\rangle \tag{2.1}$$

(χ : 4 x 4 matrix in Dirac indices, when the quark label p, n, λ is suppressed) play a role similar to the Schrödinger wave functions in non-relativistic quantum mechanics. However, their meaning is completely different (no "probability interpretation") and given by the interpretation rules of GFT [22]. Structure analysis [23] of GFT tells that there is a Bethe-Salpeter equation for these amplitudes



Analytically this bound-state B.S. equation reads

$$S^{-1}(p_1)\,\chi(q,P)\,\bar{S}^{-1}(p_2) = i\int d^4q'\,\mathcal{K}(q,q';P)\,\chi(q',P) \tag{2.2}$$

There is a normalization condition [24)] for χ following from GFT. In a specific Lagrangian field theory, the B.S. kernel $\mathcal{K}(q,q';P)$ describing the $q\bar{q}$-interaction, and the propagator S(p) are determined by the field equations. Since we use GFT only as a general framework for a model, we have to determine (like in non-relativistic quantum mechanics) $\mathcal{K}$ ($\sim$ potential) and S by considerations guided by phenomenology and simplicity.

We give now, along this line, the arguments which lead to a tractable B.S. equation and to a model with many applications:

a) We assume a large quark mass m for practical quark confinement.

b) The propagator we set as $S^{-1}(p) = \gamma p - m$ for reasons of simplicity. This might have the meaning of an approximation for $\langle p^2 \rangle \ll m^2$.

c) We assume convolution type, energy independent kernels for simplicity:

$$\mathcal{K}(q,q';P) = \mathcal{K}(q-q')$$

d) For the spin dependence of the kernel we start with the general ansatz:

$$\mathcal{K} = K^S\, 1\times 1 + K^V \gamma_\mu \times \gamma_\mu + K^T \sigma_{\mu\nu}\times\sigma_{\mu\nu} + K^A \gamma_5\gamma_\mu \times \gamma_5\gamma_\mu + K^P\, \gamma_5\times\gamma_5 \tag{2.3}$$

$$(\Gamma_i \times \Gamma_i)\,\chi = \Gamma_i\, \chi\, \Gamma_i$$

The following phenomenological considerations lead to a restriction of this generality: i) The spectrum derived from the B.S. equation should show the singlet-triplet structure of the non-relativistic quark model. ii) "spin saturation" of the super-strong $q\bar{q}$-interaction implies a kernel which is dominantly of $(-\gamma_5\times\gamma_5)$-type [25)]. (See 2.3b).

e) Wick rotation [26)]: Analytic continuation $q_0 \to i q_4$ is allowed for B.S. equations with kernels which are superpositions of one particle exchanges. The "Wick rotated" equation can be more easily approximated.

f) Bound state mass zero approximation: Since $M \ll m$, it is natural to begin with an equation for M = o, and then use an expansion in $(M/m)^2$. The advantage is that the zero-order Wick-rotated equation has the higher symmetry of $O^{\pi c}(4)$.

g) Smooth potentials: The Fourier transform of a Wick-rotated convolution type B.S.-kernel might be considered as a 4-dimensional potential. In order for the level spacing to be small compared to the quark mass, this potential much be smooth.

h) Oscillator approximation: For low-lying states, smooth potentials in Wick-rotated B.S. equations can be approximated by harmonic forces [27)]. The resulting wave functions are good approximations for space-like relative momenta; their analytic continuation into the time-like region is not justified. This understanding of the relativistic oscillator differs from that of other models not based on field theory [28].

i) The approximate dynamical equation: When we take all these considerations together, we get

$$(\gamma q - im)\chi(\gamma q - im) = -K\gamma_5\chi\gamma_5$$
$$K = \alpha - \beta\Box_q \quad , \quad M = 0$$

as a first approximation to the $M \neq 0$ B.S. equation

$$(\tfrac{i}{2}M\gamma_4 + \gamma q - im)\chi(-\tfrac{i}{2}M\gamma_4 + \gamma q - im) = -(\alpha - \beta\Box_q)\gamma_5\chi\gamma_5 + K_i\Gamma_i\chi\Gamma_i$$
$$K_i\Gamma_i\chi\Gamma_i = O(m^0) \tag{2.4}$$

k) Spectrum and solutions: We can solve Equ. (2.4) in $(M/m)^2$- approximation, and we get for the meson-mass spectrum:

$$M^2 = 4(\alpha^i + m^2 + 2(n+2r+2)\sqrt{\beta^i}) \tag{2.5}$$

The quantum numbers j("spin"), Π ("parity"), C("charge parity") of the states and their multiplicities are related by the following scheme to the O(4)-orbital momentum n = 0, 1, 2, 3, .., and the radial excitation number r = 0, 1, 2, ... :

"spin-singlets": $\quad \Pi = (-1)^{j+1}, \quad C = (-1)^j, \quad j = n, n-2, \ldots$

"spin-triplets":

$$\Pi = (-1)^{j+1}, \quad C = (-1)^{j+1}, \quad j = n+1, n-1, \ldots$$
$$\Pi = (-1)^{j}, \quad C = (-1)^{j}, \quad j = n, n-2, \ldots \quad n \neq 0$$
$$\Pi = (-1)^{j-1}, \quad C = (-1)^{j-1}, \quad j = n-1, n-3, \ldots \quad n \neq 0 \tag{2.6}$$

The mesons of the "leading trajectory" j = n, or j = n $\pm$ 1, n respectively, have the quantum numbers of the non-relativistic quark model. There is singlet-triplet splitting, if the potential parameters α^i, β^i , (i = t(riplet), s(inglet) are modified in order $O(m^0)$. Because of the underlying four-dimensional oscillator, there are more states than in the non-relativistic quark model, namely those with j = n-2, ... etc. Seen intuitively, they correspond to excitations of the "sea of quark pairs and gluons". An example is the case of the vector mesons with $M \approx 1600$ MeV. In the relativistic quark model solving Equ. (2.4) provides us with a list of approximate B.S. amplitudes for all the meson states of the spectrum. As an example we give the amplitude for the radially excited vector mesons

($j^{\pi c} = 1^{--}$, $n = o$, $r = o,1,..$; e_μ polarization vector)

$$\chi_+^V(q,P) = \frac{4\pi}{\sqrt{\beta(r+1)}}\left[\not e\left(1-\frac{\not P}{2m}\right) - \frac{ie\cdot q}{m}\right] e^{-q^2/2\sqrt{\beta}}\, L_r^1\left(\frac{q^2}{\sqrt{\beta}}\right)|q\bar{q}\rangle_V \qquad (2.7)$$

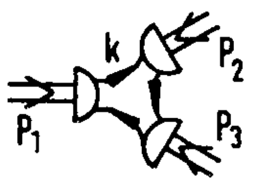
1) Phenomenological applications: This model allows the discussion of the meson spectrum, the strong and leptonic decays of mesons and of their e.m. form factors. It relates the Regge slope to the range parameters of the hadronic decays and the leptonic decay constants, and thus gives an example for the solution of the dynamics of the meson spectrum. For a comprehensive comparison of these results with the experiment we refer to the literature [29].

2.3 The phenomenological relativistic quark model with strongly bound quarks reveals some features as being typical for the confinement dynamics. We want to elaborate a little bit more on this part of the model.

a) In the mass formula Equ. (2.5) the depth of the potential α has to compensate the large quark mass: $\alpha \approx - m^2$. Hence in this very deep potential, we expect many bound states - resonances -, as expressed by the mass formula (2.5) describing linear Regge trajectories, which are infinitely rising if $m^2 \to \infty$. We have to consider this as a typical feature of confinement dynamics. In addition, strong binding in field theory implies apparently more degrees of freedom in resonating states. This is a common feature of confinement models, too.

b) The formulation of our relativistic quark model in the notion of general field theory enables us to make a consistent ansatz for the calculation of mesonic coupling constants, in which all mesons are treated symmetrically [30]. We consider the approximation in which these constants are calculated from the triangle graph with the help of our B.S. amplitudes [31]:

M =



$$= Tr_{SU_3}\cdot(2\pi)^{-9/2}\, i\int d^4k\, Tr\Big[\chi^{(1)}(k,P_1)\cdot\left(\not k - \frac{\not P_1}{2} - m\right)\chi^{(2)}\left(k+\frac{P_3}{2},P_2\right)\left(\not k - \frac{\not P_2 - \not P_3}{2} - m\right)\cdot \chi^{(3)}\left(k-\frac{P_2}{2},P_3\right)\left(\not k+\frac{P_1}{2}-m\right)\Big] \qquad (2.8)$$

+

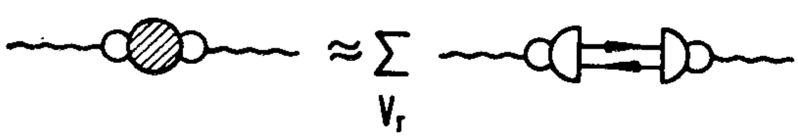
$P_1 + P_2 + P_3 = 0$

The forces between the mesons are then derived from the interaction between quarks. They will increase in general with the quark mass - Equ. (2.7) suggests an increase with m^3. But for the spinor structure of the χ imposed by an $-\gamma_5 \times \gamma_5$ -interaction, all the terms of order m^3, m^2, m in M (Equ. (2.7)) vanish. Thus, in spite of the superstrong forces between quarks, the mesonic forces are of moderate strength. We call this phenomenon the spin saturation of the superstrong binding forces.

Because of saturation, the $O(m^0)$ terms in Equ. (2.5) are now important for the calculation of $\mathcal{M}$. Therefore we cannot determine the mesonic couplings from the Regge slope alone, but they depend also on some other potential parameters. Thus we can use only the relative coupling strength of particles along a Regge slope as a test of our model. For example, we get $\Gamma(\rho \to 2\pi)$ = 146 MeV (input), $\Gamma(f(1270) \to 2\pi)$ = 140 MeV (Exp. 130 MeV), $\Gamma(g(1620) \to 2\pi)$ = 76 MeV (Exp. 64 MeV).

This discussion should give some impression of the problems posed by confinement dynamics for the meson decays. "Spin saturation of quark forces" may a possible way to solve these. There are similar ideas in an attempt to derive Zweig's rule in the framework of Wilson's confinement scheme [5].

c) Large series of resonances is a characteristic feature of a relativistic model of strongly bound quarks. On the other hand, at small distances we expect partonlike behaviour of the quarks. It is one of the main problems of confinement dynamics to reconcile these two aspects of the quark model. Our phenomenological model can also give a hint for the solution of this problem. For this we consider the reaction $e^+e^- \to$ hadrons. The cross-section of this process is related to the Green's function of the e.m. current $\langle 0|T j_\mu(x) j_\nu(y)|0\rangle$; $j_\mu(x) = Z\bar{\Psi}(x)\gamma_\mu Q \Psi(x)$. In our model of strongly bound heavy quarks this Green's function should be approximated by the intermediate bound states with the correct quantum numbers, i.e. the vector mesons states [32].



(2.9)

On the assumption that the individual vector mesons dominate locally, the cross-section is given by

$$\sigma_{tot}(e^+e^- \to hadr.) = \frac{16\pi^2\alpha^2}{s^2} \sum_{V_r} g_{V_r}^2 \frac{M_V \Gamma_V}{(s-M_V^2)^2 + M_V^2\Gamma_V^2} \qquad (2.10)$$

With the mass spectrum Equ. (2.5) and the photon-vector-meson couplings derived from the B.S. amplitudes Equ. (2.7):

$$\langle 0 | j_\mu(0) | V \rangle = g_V e_\mu(V) = (2\pi)^{-11/2} \, Z \, \mathrm{Trace}\, Q \gamma_\mu \int d^4q \, \chi(q,P)$$

$$g_V = (-1)^r Z \frac{4}{\pi} \sqrt{r+1} \sqrt{\beta} \langle Q_V \rangle \qquad \text{for radial excitations}$$

$$g_V = 0 \qquad \text{otherwise} \tag{2.11}$$

We calculate a smoothed cross-section

$$\sigma_{tot}(e^+e^- \to \text{hadr.}) = \frac{\pi\alpha^2}{s} Z^2 \sum_V \langle Q_V \rangle^2 \left(1 + \frac{1.6\ \mathrm{GeV}^2}{s}\right) \tag{2.12}$$

It shows partonlike behaviour for large energies $E = \sqrt{s}$ [33].

3. The "Standard Lagrangean" of Hadron Dynamics

3.1 In the preceding Section 2, we tried to show how a phenomenological quark model, which takes into account the experimental absence of free quarks together with the principle of relativistic quantum mechanics, may bring about a first impression on the problems of the dynamics of quark confinement. In the remaining sections we want to consider this problem embedded in a more fundamental approach to hadron dynamics. Of course, a Lagrangean field theory of hadron dynamics is not yet really established. But the quark field theory with a non-abelian gauge invariant, renormalizable interaction incorporates a large part of the experimental and theoretical experience in elementary particle physics, hence it is an educated guess, that hadron dynamics might be based on a "Standard Lagrangian" [34] similar to that of the successful QED:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{q} \not{D} q + \bar{q} M q \tag{3.1}$$

Here $F_{\mu\nu}(x)$ denotes the "gluon-field", the analogue of the electromagnetic field; the quark fields $q(x), \bar{q}(x)$ take the place of the electron field in QED; D_μ ; $\not{D} = D_\mu \gamma^\mu$ corresponds to the gauge invariant derivative $\partial_\mu + ieA_\mu$; M is the bare mass-matrix.

3.2 The intricacies of $\mathcal{L}$ are hidden in the details suppressed by the notation. I shall sketch these details as a reminder of the ideas and experimental discoveries which are focussed in the simple expression (3.1);

a) The quark field q(x) is coloured and flavoured: $q(x) \equiv q_{fc}(x)$

b) The "Flavour" f = $p, n, \lambda, \ldots c$ (?) denote the quark degrees of freedom, derived from the SU(3) structure of the hadron spectrum and confirmed by the properties of the valence quarks in deep inelastic e-ν-scattering etc. The discovery of the new particles adds new flavours like charm [3] c, Han-Nambu colour [35] (?) etc. to the quark fields. There is a general prejudice that the flavour degrees of freedom don't play an essential rôle in the dynamics of quark confinement.

c) The "Colour" C = red, white and blue is introduced [36] to solve the statistics problem of the baryon spectrum in the quark model. It is used to satisfy the condition that the sum of the electric charges Q of all hadron and lepton fields should vanish, a condition necessary to avoid the socalled "triangle anomalies" [37]. Colour is welcome to improve the calculation [36] of $\Gamma(\pi^0 \to 2\gamma)$ and to give larger values of R = $\sigma(e^+e^- \to hadr.)/\sigma(e^+e^- \to \mu^+\mu^-) = \sum_{f,c} Q^2_{f,c}$. The postulate that free particle states are colour singlets, guarantees that hadrons have conventional charges. Therefore the coupling of the interaction to the colour degrees of freedom is essential for quark confinement.

d) The "Gluons" [38] were introduced in order to mediate the interaction between quarks within the framework of renormalizable field theory. They form a coloured octet of vector fields $A^c_\mu(x)$:

$$C = \bar{r}w, \bar{r}b, \bar{w}r, \bar{w}b, \bar{b}w, \bar{b}r, (r\bar{r}-\bar{b}b), (\bar{r}r+\bar{b}b-2\bar{w}w)$$

which are singlets with respect to flavour.
As coloured objects, gluons are confined. Since gluons carry no electric or Cabbibo charge, their "observation" in deep inelastic electron or neutrino scattering is even more indirect than the "observation" of quarks. The energy sum rule:

$$1-\epsilon = 9\left[\int dx\, F_2^{\gamma N}(x) - \frac{1}{6}\int dx\, F_2^{\nu N}(x)\right] \approx 1/2$$

ϵ = percentage of 4-momentum P_μ carried by gluons. [39]
F(x) Scaling structure functions of deep inelastic scattering is considered as an indication of the reality of gluons.

Clearly more direct experimental evidence of these basic fields is of the utmost importance for the justification of hadron dynamics based on the standard Lagrangian.

The standard Lagrangian is constructed from the quark and gluon fields, taking into account the following symmetry and dynamical principles:

e) The postulate of "renormalizability" of a Lagrangian field theory, reduces possible Lagrangians to a very restricted class. Renormalizable field theories represent the only complete and consistent relativistic dynamical system. Therefore it is theoretically rational to postulate renormalizability. However, the physical meaning of this assumption is unclear.

The renormalization group techniques [8] allows the characterization of the asymptotic behaviour of renormalizable field theories for large momenta (small distances), and small momenta (large distances). In particular, a relation for Green's functions in momentum space: $\tau(q_1, \cdots, q_n)$ of the type

$$\tau(\lambda q_1, \ldots \lambda q_n; g, m) = \lambda^{\dim \tau} e^{-n \int_g^{g(\lambda)} \frac{\gamma(\lambda')}{\beta(\lambda')} d\lambda'} \tau(q_1, \ldots q_n; g(\lambda), m) \tag{3.2}$$

$$\frac{dg(\lambda)}{d \log \lambda} = \beta(g) \tag{3.3}$$

follows from the solution of the Callan-Symanzik equation. It relates a dilatation of the momenta by a factor λ to a change of the coupling constant from g to $g(\lambda)$. The function $\beta(g)$, according to which Equ. (3.3) determines the λ-dependence of the coupling constant, and $\gamma(g)$ responsible for the dynamical dimensions of the fields, might in principle be calculated from the Lagrangian. From a practical point of view, it is more important that already the qualitative features of $\beta(g)$ have serious implications on the general structure of the theory. Thus zeros of $\beta(g)$ imply the existence of limits of the "running" coupling constant $g(\lambda) : g(\lambda) \to g_0$ for $\lambda \to \infty$, or 0, if $\beta(g_0) = 0$ and $\beta'(g_0) < 0$, or $\beta'(g_0) > 0$ respectively. In these cases the theory is called ultraviolet or infrared asymptotically stable.

Relations of type (3.2) hold only for non-exceptional Euclidean momenta. This has the practical consequence that conclusions from asymptotic stability apply mainly to averages over physically observed quantities [40].

f) Asymptotic Freedom: Field theories, which approach for large momenta the free field theory, i.e. for which $g(\lambda) \to 0$ for $\lambda \to \infty$; are called

asymptotically free.
In asymptotic free theories, Bjorken scaling for $\sigma(e^+e^- \rightarrow \text{hadrons})$ [41] and for deep inelastic e, ν -scattering [41] can be proven - if formulated appropriately. Hence the experimental discovery of scaling (!) and the success of the parton model suggests the postulate of asymptotic freedom [41b].

g) Non-abelian gauge invariance. The only renormalizable, asymptotic free theories are the non-abelian gauge theories [41] with Lagrangians like Equ. (2.1), i.e. an interaction between fermions and gauge vector mesons minimally induced by the gauge invariant derivative D. Assuming the colour SU(3) as gauge group, the coupling becomes explicitely

$$F_{\mu\nu}^{c} = \partial_\mu A_\nu^{c} - \partial_\nu A_\mu^{c} - g f^{cc'c''} A_\mu^{c'} A_\nu^{c''}$$

$$D_\mu = \partial_\mu + i g A_\mu^{c} \lambda^{c}/2$$

(λ^c the Gell-Mann matrices, $f^{cc'c''}$ the SU(3)-antisymmetric octet tensor).
We have motivated this assumption under (c).

h) The SU(N)-symmetry in the flavour degrees (SU(3), SU(4)) of freedom might be broken by the mass term $\bar{q}Mq$ in $\mathcal{L}$. In the limit $M \rightarrow 0$ the standard Lagrangian is SU(N) symmetric. This implies the conservation laws of charge, hypercharge, isospin etc., as well as the existence of the corresponding conserved or partially conserved currents and axial currents, on which current algebra is based. M is not expected to be related to the mass of free quarks. On the contrary, it is conjectured that the correct standard Lagrangian leads to a theory which is infrared instable: $g(\lambda) \rightarrow \infty$ for $\lambda \rightarrow 0$, and that the strong interaction at large distances assures quark confinement.

i) This conjecture is sometimes called: "Infrared Slavery". [42]

It is this aspect of the dynamics based on the standard Lagrangian, on which we have to elaborate further.

4. Lagrangian Field Theory and Quark Confinement

4.1 Assuming now the standard Lagrangian Equ. (3.1) as a basis for hadron dynamics; how should one organize the "solution" of the quantized field equations derived from it? The renormalization group technique mentioned in 3.2.e., suggests to arrange the solution in three parts:

a) On the one hand we have to consider the small distance limit. This should explain the properties of the partons, like their symmetry properties expressed by sum rules for their quantum numbers (f.e. $R = \sum Q_i^2$, etc.) together with their dynamical behaviour observed in the scaling laws of deep inelastic e and ν scattering etc.

b) On the other hand we have to discuss the large distance limit. Here one has to consider the hadronic stable particles and resonances, their exact and approximate symmetries, like isospin, SU(3), spontaneously broken chiral symmetry, etc. Further we have to explain dual dynamics, combining the dynamics of the meson spectrum and the peripheral part of reaction dynamics. Last but not least we have to understand the screening of the charges of colour symmetry, i.e. quark confinement.

c) The remaining problem is the connection of the two limiting cases. In a sense, this is the real problem. Here the Lagrangian approach to hadron dynamics should show its full power in expressing the dynamical parameters of the scaling region as well as those of the hadron spectrum by the renormalized coupling constant g and the mass parameters related to the normalization point. This is also the "general problem of quark confinement", of which the answer to the question: Are there particles with the quantum numbers of the partons? is concerned with a particular aspect of the relation between the large and small distance limit.

4.2 Since the Standard Lagrangian describes an asymptotically free field theory: $g(\lambda) \to 0$ for $\lambda \to \infty$ (cf. 3.2.f), the problems related to the small distance limit can in principle be handled relatively easily. Equation (3.2) relates the Green's function at large momenta, to those of a theory with a small coupling constant. However, for small coupling constants we may apply conventional perturbation theory. Therefore we may get along this line reliable asymptotic expressions of Green's functions describing deep inelastic e, ν -scattering and e^+e^- annihilation[41]. into hadrons. However, because our main problem here is quark confinement related to the large distance limit, we have to refer to the literature and to other lectures of this school[43] for further discussion of this topic.

4.3 It is guessed that the field theory or the standard Lagrangian is infrared instable: $g(\lambda) \to \infty$, for $\lambda \to 0$ (cf. 3.2.i). Therefore, the infrared limit is related by Equ. (3.2) to theories with large coupling constants. The standard Lagrangian with large g may describe quite different phases of hadronic matter compared to the approximate free particle states at small g. But we understand strong coupling theories much less than weak coupling perturbation theory. Hence our knowledge of the phenomen appearing in the region of strong coupling is much more speculative. The following may illustrate such a line of speculations:

(a) Strong coupling may induce dynamical spontaneous symmetry breaking [41a], which is related to a shift in the vacuum fluctuations $\langle \bar{\Psi}(x)\Psi(x)\rangle \neq 0$. This process is visualized as an analogue of the formation of Cooper pairs in superconductivity.

(b) In order to make spontaneous symmetry breaking more obvious, the standard Lagrangian is supplemented by a scalar Higg's field $\phi(x)$, which via the Higgs mechanism, produces spontaneous symmetry breaking (cf. 5.1.d). The modified $\mathcal{L}$ becomes, according to some speculative calculations [44],

$$\mathcal{L} = -\frac{1}{4} Z_3(\phi) F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\,\partial_\mu\phi - V(\phi) + i\bar{q}\not{D}q + \bar{q}Mq \tag{4.1}$$

This Higgs Yang Mills Lagrangian [45] is renormalizable, but now longer asymptotically free. But as already mentioned, this $\mathcal{L}$ has to be considered only as an approximate model for describing the long distance - strong coupling-phenomena induced by the somewhat obscure mechanism of dynamical symmetry breaking. It should not be considered in the small distance limit.

(c) This Lagrangian (4.1) describes the relativistic version of the Landau-Ginsburg theory of superconductors of the second kind [46] (for $Z(\phi) = 1$, $V(\phi) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$), where $\phi(x)$ is a phenomenological field describing Cooper-pairs. Hence one expects the existence of stable classical solutions of the field equations derived from (4.1) representing the magnetic vertex lines of the super conductor. One suspects [10] that these strings are the objects on which the Veneziano dual dynamics is based (cf. 5.2.c). But quite generally, there is the wide spread hope that the new phenomena appearing for large coupling constants might be described with the help of stable classical solutions, of which the main features survive the quantization procedure. It is expected that the quarks are confined in such strongly bound structures ("SLAC-Bag" [47] "MIT Bag" [48] etc. (cf. 5.3).

(d) There is another approach to the strong coupling limit of the standard Lagrangian which is based on a lattice approximation to the space time continuum [49]. Also this method seems to indicate the existence of stringlike structures [50].

As already emphasized, these conjectures on the large distance limit of the standard Lagrangian field theory are highly speculative and mainly guided by the desire to explain successfully the phenomena of high energy physics. But one should consider them seriously as possibilities and one might study them in more detail in models, which do not have the full complexity of the standard $\mathcal{L}$. In this spirit we shall consider stable classical solutions, their quantization, and quark confinement in stringlike structures in very simplified versions.

4.4 With so little knowledge on the large distance limit, there is little to say about the connection of the both limits of the standard Lagrangian. (For a phenomenological comment cf. 2.3c). However, in order to give a first impression on the problems showing up here, we may have a look on the recently discussed relation [51] between the 2-dimensional sine Gordon equation [52]

$$\partial^\mu \partial_\mu \phi + (\mu^2/\beta) : \sin \beta \phi : = 0 \tag{4.2}$$

and the Thirring model: [53]

$$(-i \gamma^\mu \partial_\mu - m_0) \psi = g : (\gamma^\mu \bar{\psi} \gamma_\mu \psi) : \psi \tag{4.3}$$

It is claimed [54] that in the quantized form the two models are equivalent, if one relates

$$\frac{\beta^2}{4\pi} = \frac{1}{1 + g/\pi} \tag{4.4}$$

In this equivalence the spinor particle of the Thirring model is described by the stable classical soliton solution of (4.2):

$$\phi_{so}(x) = \frac{4}{g} \operatorname{tang}^{-1} e^{\mu x} \tag{4.5}$$

This non-perturbative solution of (4.2) (leading term in $1/\beta$) allows a good description of the excitations, form factors etc. of this particle in the limit of small coupling constants of the sine Gordon theory, i.e. for strong coupling in the Thirring model. For further study we have to refer to the literature.

5. Classical Models

5.1 In order to give an impression on the new type of bound state structures described by stable solutions of classical field equations (cf. 4.3.c), we discuss briefly a simple $1+1$ -dimensional example [55]. This model is the starting point of the so called "SLAC-BAG" describing strongly bound quarks [47]. In spite of being simple and explicitely solvable, it shows many characteristic features which allow the illustration of new concepts used in the discussion of quark confinement models.

The fields of this model consist of a colourless quark field $\Psi(x)$ of only one flavour, a Higg's field $\phi(x)$, but no gluons. The model is described by

a) The field equations

$$(-\partial_t^2+\partial_x^2)\phi + m^2\phi - \lambda\phi^3 = -g\Psi^+\beta\Psi \tag{5.1}$$

$$\begin{gathered} i(\gamma^0\partial_t+\gamma^1\partial_x)\Psi = g\phi\Psi \\ \beta=\gamma^0=\begin{pmatrix}0&1\\1&0\end{pmatrix};\quad \gamma^1=\begin{pmatrix}0&1\\-1&0\end{pmatrix} \end{gathered} \tag{5.2}$$

b) These are derived from the Lagrange density

$$\begin{aligned}\mathcal{L} = {} & \tfrac{1}{2}(\partial_\mu\phi)^2+\tfrac{1}{2}m^2\phi^2-\tfrac{1}{4}\lambda\phi^4 \\ & + i\bar\Psi\gamma^\mu\partial_\mu\Psi + g\phi\bar\Psi\Psi \quad .\end{aligned} \tag{5.3}$$

or the Hamiltonian $\qquad \pi(x) = \partial_t\phi(x)$

$$\begin{aligned} H = \int d^1x\, \Big\{ & \tfrac{1}{2}\big(\pi^2(x)+(\tfrac{d}{dx}\phi)^2\big)+\tfrac{\lambda}{4}\big(\phi^2(x)-\tfrac{m^2}{\lambda}\big)^2 \\ & + \Psi^+\big(\alpha\tfrac{\partial}{\partial x}+g\beta\phi\big)\Psi(x) \Big\}\end{aligned} \tag{5.4}$$

c) There are the trivial classical solutions of Equs. (5.1,2)

$$\phi^{\pm}_{cl.} = \pm\frac{m}{\sqrt{\lambda}} \quad , \quad \Psi = 0 \tag{5.4}$$

corresponding to the minimum of the "potential energy" $V(\phi)$:

$V(\phi) = \frac{\lambda}{4}\int(\phi^2(x) - \frac{m^2}{\lambda})^2 d^1x$, normalized to zero.

d) The energy degeneracy of the classical ground state solutions (5.4) implies spontaneous symmetry breaking: None of the two ground state solutions $\phi_{cl.}^+$ and $\phi_{cl.}^-$ has the symmetry $\phi(x) \to -\phi(x)$, $\psi \to -\psi$ of the field equations (5.1,2).

e) Expanding around the stable vacuum solutions (5.4):
$\phi(x) = \varphi(x) \pm \frac{m}{\sqrt{\lambda}}$ leads to

$$(-\Box - 2m^2)\varphi \pm 3m\sqrt{\lambda}\,\varphi^2 - \lambda\varphi^3 = -g\psi^+\beta\psi \tag{5.5}$$

$$(-i\not{\partial} \pm g\frac{m}{\sqrt{\lambda}})\psi + g\,\varphi(x)\,\psi = 0 \tag{5.6}$$

thus determining the classical mass parameters of the quark field m_q and of the Higgs field μ :

$$m_q = g\,\frac{m}{\sqrt{\lambda}} \quad , \quad \mu = \sqrt{2}\,m \ . \tag{5.7}$$

f) "Kink solution": There is another stationary solution, with $\psi = 0$, of

$$\left(\frac{d^2}{dx^2} + m^2 - \lambda\phi^2\right)\phi(x) = 0 \tag{5.8}$$

namely

$$\phi_{KI}(x) = \frac{m}{\sqrt{\lambda}} \tanh \frac{mx}{\sqrt{2}} \tag{5.9}$$

It has some characteristic features of the classical solutions, of the type we are always talking about:

(i) $\phi_{KI}(x)$ is classically stable (cf. 5.1g,6).

(ii) $\phi_{KI}(x)$ connects the two degenerate vacuum solutions:

$\phi_{KI}(x) \to \pm\frac{m}{\sqrt{\lambda}}$ for $x \to \pm\infty$. This illustrates the relation between spontaneous symmetry breaking and stable classical solutions.

(iii) The total energy of $\phi_{KI}(x)$ according to (5.4) is finite

$$E(\phi_{KI}) = \frac{\sqrt{8}}{3}\,\frac{m^3}{\lambda} \tag{5.10}$$

the energy density is concentrated in a finite region around x = o.
(iv) The kink solution $\Phi_{KI}(x)$ is singular for $\lambda \to 0$, i.e. "non-perturbative".

It has been suggested [56] to generally call solutions of non-linear field equations with such properties "solitons" (cf. Equ. (4.4)).

g) "Topological stability" [57]: The kink solution $\Phi_{KI}(x)$ is stable, because it is the lowest energy solution with the conserved topological number $\mathcal{K} \neq 1$:

$$\mathcal{K} = \frac{\sqrt{\lambda}}{2m}\left(\Phi(\infty) - \Phi(-\infty)\right) \tag{5.11}$$

Note the relation between spontaneous symmetry breaking and the existence of such "topological quantum numbers" (cf. 5.1f (ii)). According to the definition (5.11) $\mathcal{K}$ is conserved, because the spatial asymptotic behaviour of solutions $\Phi(x,t)$ of the field equation (5.1) remain fixed. The topological property of the 2-dimensional spacetime allows to relate $\mathcal{K}$ to a conserved current:

$$\begin{aligned} B^{\mu}(x,t) &= \frac{\sqrt{\lambda}}{2m}\,\epsilon^{\mu\nu}\partial_{\nu}\Phi(x,t) \\ \mathcal{K} &= \int B_0(x,t)\,d^1x \\ \epsilon^{\mu\nu} &= -\epsilon^{\nu\mu}, \quad \epsilon^{01} = 1 \end{aligned} \tag{5.12}$$

f) "Kink with trapped quark": There is a simultaneous solution of Equs. (5.1) and (5.2) with :

$$\begin{aligned} \Phi(x) &= \Phi_{KI}(x) = \frac{m}{\sqrt{\lambda}}\tanh\frac{mx}{\sqrt{2}} \\ \Psi_{tr}(x) &= C\cdot\left[\cosh\frac{xm}{\sqrt{2}}\right]^{-\sqrt{2/\lambda}\,g}\begin{pmatrix}1\\-i\end{pmatrix} \end{aligned} \tag{5.13}$$

and with a classical energy according to Equ. (5.4) of

$$E(\Phi_{KI},\Psi_{tr}) = E(\Phi_{KI}) = \frac{\sqrt{8}}{3}\frac{m^3}{\lambda} = \frac{\sqrt{8}}{3}\frac{m^2}{g\sqrt{\lambda}}m_q \,. \tag{5.14}$$

The trapping of the quark does not add to the kink energy. Anyhow, for an appropriate choice of the parameters, the trapped quark may have a very small mass compared to the "free" quark (cf. Equ. (5.7)). Thus the state "kink with trapped quark" described by the classical solutions (5.13) represents an example of a "field theoretical bound state" with strong binding. Such type of states may describe hadrons with strongly bound or confined quarks. In contrast to our phenomenological model (cf. 2.2), strong binding is derived from an invariant, local, renormalizable Lagrangian Equ. (5.3).

5.2 The 1 + 1-dimensional models are very helpful for getting acquainted with the properties of soliton type solutions of non-linear field equations. But of course, physical models should be described in 3 + 1 dimensions. There are several attempts to describe hadrons by such type of solutions, called bags and strings, in which quarks are strongly bound or confined. We shall mention now in a cursory manner some of the more popular ones. Several example will also be treated in the lecture by Prof. Y. Nambu.

a) The dynamical structure of the "SLAC-BAG" [47] is derived from the 2-dimensional model discussed above. This model is considered in 3 + 1 dimensions. It is argued that

$$\begin{aligned} \Phi(\vec{x}) &= \Phi_{KI}(r-R) \\ \Psi(\vec{x}) &= \Psi_{tr}(r-R) \end{aligned} \tag{5.15}$$

with $r = |\vec{x}|$ and $\begin{pmatrix} 1 \\ -i\frac{\vec{\sigma}\cdot\vec{x}}{r} \end{pmatrix}$ substituted for $\begin{pmatrix} 1 \\ -i \end{pmatrix}$ in 5.13

represent good approximation of the 3 + 1-dimensional equations (5.1) and (5.2), if $g \gg \sqrt{\lambda} \gg 1$. Since we have seen that the interaction of the quarks with the Higgs-field $\Phi(x)$ does not perturb the structure of the kink solution, one may put N quarks in the corresponding 3-dimensional bag without disturbing significantly its structure.

The radius R of the bag is determined by minimalizing the energy, it results:

$$R_N = N^{1/3} R_0 \;, \quad E_N = \frac{2}{2R_0} N^{2/3} \;, \quad R_0 = \frac{\lambda^{1/6}}{2m} \tag{5.16}$$

The stability of the SLAC-Bag is no longer topological, but dynamical. Comparing the energy of N free quarks: $Ngm/\sqrt{\lambda}$ with E_N, we see that for small λ and large g the SLAC-bag is a field theoretical bound state with strong binding: "It is for a quark energetically favourable to dig a hole from one classical Higgs vacuum to the other. The kinetic energy necessary for that guarantees the finite size of the hole" [47]. This mechanism does not explain why the bound states are colour neutral. For this an effective coupling to the gluons is considered. This can be arranged in such a form that according to rough estimates the coloured states have a much higher energy than the colourless ones - a mechanism suggested first by Y. Nambu [58]. The SLAC-Bag describes only approximate confinement of coloured objects.

b) The MIT-Bag [48]: Different types of bag structures can be produced by assuming different types of "potentials" $V(\phi)$ for the Higgs field. Using a potential with a metastable vacuum first suggested by Vinciarelli [59] and applying an appropriate limit procedure, it can be shown [60] that the socalled "MIT-bag" is a limit of a field theoretical bag. In this model the classical fields of quarks and gluons are confined in the inner of the bag. It is described by the Lagrangian

$$L = \int_{\text{Bag}} d\vec{x} \left\{ -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{q} \not{\partial} q - B \right\} \tag{5.17}$$

where the volume term B is the only remainder from the pressure of the Higgs field from the outside of the bag. The field equations derived from (5.17) are

$$\left.\begin{aligned} D^{\mu}_{cc'} F^{c'}_{\mu\nu}(x) &= -g \bar{q} \lambda^c \gamma_\nu q(x) \\ i \not{D} q(x) &= 0 \end{aligned}\right\} \text{ in the bag} \tag{5.18}$$

$$q = \bar{q} = F_{\mu\nu} = 0 \qquad \text{outside.}$$

Since it is assumed that the coupling of the gluons inside the bag is very weak, the MIT-bag model realizes the ideas of infrared slavery and asymptotic freedom in an almost exaggerated form. As a limiting case of a field theoretical bag, its surface is determined dynamically, expressed by the following boundary conditions on the surface of the bag:

$$\begin{gathered} n^\mu F_{\mu\nu}(x) = 0 \quad , \qquad i \not{n} q(x) = q(x) \\ -\tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} + \tfrac{1}{2} n_\mu \partial^\mu \bar{q} q - B = 0 \end{gathered} \tag{5.19}$$

n_μ : normal vector of the surface.

It follows from field equations and boundary conditions, that the colour charges of the bags are zero; therefore we have exact confinement of coloured objects.

c) Dual strings: The similarity of the Higgs Yang-Mills Lagrangian to the Ginzburg-Landau Lagrangian for the phenomenological description of superconductivity (cf. 4.3b,c) lead Nielsen and Oleson [(61)] to suggest, that there might be classical solutions which correspond to the vertex lines in superconductors of the second kind. In order to get finite vertex lines, the "gluon magnetic" flux has to be absorbed by the quarks as magnetic monopoles with respect to gluon coupling. The question of the stability of such solutions is open [62)]. There is the conjecture that non-abelian gauge theories allow topologically stable solutions, in which quarks are trapped. This conjecture is supported by the existence of a solution of this kind for a single monopol [63)]. (But this does not trap quarks [64)]).

Anyhow, stringlike soliton solutions are very much preferred for physical reasons: First, because vertices would provide a physical model for dual strings [65)]. Second, one-dimensional systems may show more easily colour charge screening [66)], (cf. 7.), and hence confinement. Under this aspect several forms of Lagrangians of type Equ. (4.1) are discussed in the literature [67)].

6. Quantization of extended Systems

We pursue the idea that the main phenomen in the large distance, strong coupling limit is the formation of bags or strings to which quarks are confined. These structures might be described by classical soliton solutions of non-linear field equations. It is assumed, of course, that such types of stable classical solutions survive, together with their main features the quantization procedure. However, in view of the long and incomplete story of the quantization of extended relativistic systems, this hypothesis is difficult to check.

6.1 In order to get a feeling for the problems involved here, we consider our simple field equation (cf. 5.1; $\psi \equiv 0$)

$$(-\partial_t^2 + \partial_x^2 + m^2 - \lambda\phi^2)\,\phi(x,t) = 0 \tag{6.1}$$

with the classical "kink" solution

$$\phi_{KI}(x) = \frac{m}{\sqrt{\lambda}} \tanh \frac{xm}{\sqrt{2}} \tag{6.2}$$

a) For a first step towards quantization, we consider the oszillations around this solution. Therefore we expand

$$\phi(x,t) = \phi_{KI}(x) + \sum_n e^{i\omega_n t}\varphi_n(x) \tag{6.3}$$

which leads to the linearized equation

$$\left(-\frac{d^2}{dx^2} - m^2 + 3\lambda\phi_{KI}^2(x)\right)\varphi(x) = \omega_n^2\,\varphi_n(x) \tag{6.5}$$

These equations can also be solved explicitely [68], which makes our simple model so illustrative. We get for

$$\begin{aligned}
&\omega_0 = 0\ , \quad \varphi_0(x) = \left[\cosh\frac{mx}{\sqrt{2}}\right]^{-2} \sim \frac{d}{dx}\phi_{KI}(x)\\
&\omega_1 = \frac{\sqrt{3}}{2}\mu\ , \quad \varphi_1(x) = \sinh\frac{mx}{\sqrt{2}}\ \varphi_0(x)\\
&\omega_2 = \mu\ , \quad \varphi_2(x) = \ldots\\
&\omega_k = \sqrt{\mu^2+k^2}\ , \quad \varphi_k(x) = e^{ikx}\left[3\tanh\frac{xm}{\sqrt{2}} + \left(\frac{k}{\mu}\right)^2 - 1 - 3i\frac{k}{\mu}\tanh\frac{xm}{\sqrt{2}}\right]\\
&\qquad\qquad \to e^{\pm i\delta(k)}e^{ikx} \quad \text{for } x \to \pm\infty
\end{aligned} \tag{6.6}$$

These solutions have the following meaning:

$\omega_0 = 0$ represents the translation mode of the kink.

ω_1, ω_2 are classical excitation frequencies of the kink.

$\omega_k = \sqrt{\mu^2+k^2}$ are the frequencies of the scattering modes. It gives the correct energy momentum relation of the asymptotically free mesons.

b) The zero frequency translation mode is classically unstable. Therefore it must be separated from the other modes with the help of a canonical transformation, as has been extensively discussed by N.H. Christ and T.D. Lee [56]. The discussion of this transformation is rather involved, we sketch here only the main features. We complete the expansion (6.3) around the static kink solution to a canonical transformation between the canonical field coordinates $\phi(x), \dot{\phi}(x)$ at t = 0, and the new coordinates Z ("kink position"), P ("total momentum") $\pi(\bar{k}), q(\bar{k}), \bar{k}=1,2,k$ (the canonical coordinates of the modes 6.6):

$$\begin{aligned} \phi(x) &= \phi_{KI}(x-z) + \int \varphi_{\bar{k}}(x-z)\, q(\bar{k})\, d\mu(\bar{k}) \\ \dot{\phi}(x) &= -\frac{1}{M_{KI}} P \frac{d\phi_{KI}(x-z)}{dx} + \int \varphi_{\bar{k}}(x-z)\, \pi(\bar{k})\, d\mu(\bar{k}) + \dots \end{aligned} \tag{6.7}$$

$M_{KI} = \frac{(\sqrt{2}\,m)^3}{3\lambda}$ is the classical rest energy of the kink.

This transformation has to be considered as an expansion in powers of λ. The Hamiltonian becomes now, up to zero order in λ

$$\begin{aligned} H &= \frac{1}{2}\int dx \left\{ \dot{\phi}^2 + \left(\frac{d}{dx}\phi\right)^2 + \frac{1}{2}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2 \right\} \\ &= M_{KI} + \frac{1}{2}\int \left(\pi^2(\bar{k}) + \omega^2(\bar{k})\, q^2(\bar{k})\right) d\mu(\bar{k}) \end{aligned} \tag{6.8}$$

which agrees with the classical interpretation of the modes discussed above. Note that in this order of approximation, the recoil of the kink is neglected in the meson-kink-scattering. Therefore it is relatively easy to achieve relativistic covariance by boosting the kink solution and the expansion (6.7) [56].

c) The quantum mechanical kink state $|KI\rangle$ is characterized by the absence of excited modes:

$$\left(\sqrt{\omega_{\bar{k}}}\, q(\bar{k}) + i\pi(\bar{k})/\sqrt{\omega_{\bar{k}}}\right)|KI\rangle = 0 \tag{6.9}$$

Formally implementation of the canonical transformation by a "dressing" operator, leads to an approximate representation of $|KI\rangle$ by a coherent state:

$$|KI\rangle \sim \; : \exp\left\{-i\int dx\, \phi_{KI}(x) \overleftrightarrow{\partial} \phi(x)\right\} : |0\rangle \tag{6.10}$$

It follows from (6.9) that the vertex $\langle KI|\phi(x)|KI\rangle$ is essentially given by the classical kink solution [69]

6.2 We want to add to this sketchy introduction to the quantization problem further references and some general remarks:

a) Because of the topological stability, the transition amplitudes between the vacuum $|0\rangle$ and $|KI\rangle$ of polynomials of $\phi(x)$ vanish: $\langle 0|\phi(x_1)\dots\phi(x_n)|KI\rangle = 0$ This rises the question of the appropriate fields, which describe the transition between the vacuum sector and the kink sector (cf. 7.1.d).

b) Coherent states like (6.10) as trial states in a variational method were used by several groups as a starting point of quantization [70].

c) The characteristic properties of $|KI\rangle$: $\langle KI|\phi(x)|KI\rangle \sim \phi_{KI}(x)$; $\langle 0|\phi(x_1)\dots\phi(x_n)|KI\rangle = 0$ were used by Goldstone and Jackiw [69] to derive from a Green's function formulation the classical kink solution for the vertex.

d) Besides these different attempts of canonical quantization [71] the 2-dimensional ϕ^4 -model, the Sine-Gordon-model, etc. were also studied by the WKB-methods [72]. In the context with quantization, the conjecture on the relation between the massive Thirring model and the Sine-Gordon model should be mentioned again [51] (cf. 4.3).

7. Colour Charge Screening

In hadron dynamics based on the standard Lagrangian $\mathcal{L}$ (cf. 3.1), the idea of quark confinement is generalized to the conjecture of the non-existence of all kinds of free coloured particles. Since $\mathcal{L}$ is invariant under SU(3)-colour gauge transformations, there are conserved currents of colour $j_\mu^c(x)$. However, it is the meaning of the conjectured "colour charge screening", that all physical states have colour charge

$$Q^c = \int j_0^c \, d\vec{x} = 0$$

The phenomenon of charge screening was discovered by J. Schwinger [73] in his investigation of two-dimensional, massless QED. A. Casher, J. Kogut and L. Susskind [66] proposed it as a model of quark confinement. It was presented very clearly by L. Susskind at the previous "International Summer School in Theoretical Physics" at Bonn. Only for rounding off our introduction to the ideas on quark confinement, we want to add some short remarks on colour charge screening.

7.1 The Dirac-Maxwell equations for a massless Dirac field in two dimensions are

$$i\gamma^{\mu}\partial_{\mu}\Psi(x) = -e\gamma^{\mu}A_{\mu}(x)\Psi(x)$$

$$= \frac{1}{2}\lim_{\substack{\epsilon\to 0\\ \epsilon^2<0}} \gamma^{\mu}\left[A_{\mu}(x+\epsilon)\Psi(x) + \Psi(x)A_{\mu}(x-\epsilon)\right] \tag{7.1}$$

$$\partial^{\nu}F_{\mu\nu}(x) = e\, j_{\mu}(x)$$

$$= e \lim_{\epsilon\to 0}\left\{\overline{\Psi}(x+\epsilon)\gamma_{\mu}\Psi(x) - f^{-1}(\epsilon)\langle 0|\overline{\Psi}(x+\epsilon)\gamma_{\mu}\Psi(x)|0\rangle \cdot \cdot (1-e\epsilon^{\mu}A_{\mu}(x))\right\} \tag{7.2}$$

For the definition of two-dimensional QED one has to complete these equations by the known commutation and anticommutation relations. It is well-known, that a covariant representation of QED by local fields requires the introduction of a Hilbert space with indefinite metric. However, those vectors of this space, which have negative or zero norm, do not have a gauge-invariant, physical meaning. However, there are special gauges of the fields, which allow the restriction to physical states only, whereby the fields loose locality and relativistic covariance. The following operator solutions of Equ. (7.1), (7.2), expressed by a free massive field are of this type [74]:

$$\Psi_a(x) = N_1 :\exp i\sqrt{\pi}\,\gamma_5\Phi(x): \chi_a \tag{7.3}$$

$$j_{\mu}(x) = -\frac{1}{\sqrt{\pi}}\epsilon_{\mu\nu}\partial^{\nu}\Phi(x)$$

$$F_{\mu\nu}(x) = \partial_{\mu}A_{\nu}(x) - \partial_{\nu}A_{\mu}(x) \tag{7.4}$$

$$A_{\mu}(x) = -\frac{\sqrt{\pi}}{e}\epsilon_{\mu\nu}\partial^{\nu}\Phi(x) \tag{7.5}$$

The numerical 2-spinors χ_a are phases $|\chi_a| = 1$, the free field satisfies

$$-\partial_{\nu}\partial^{\nu}\Phi(x) = \mu^2\Phi(x) \quad , \qquad \mu = e/\sqrt{\pi}$$

$$[\Phi(x), \Phi(x')] = i\,\Delta(x-x') \tag{7.6}$$

It is easy to verify that (7.3) is really a solution of the Dirac-Maxwell equations (7.1), (7.2). On the other hand, it was shown [75] by a detailed mathematical analysis, that all irreducible solutions of these properly quantized equations are essentially equivalent by operator gauge transformations to the special solutions given by Eqs. (7.3) and (7.4). In particular, this means that the gauge invariant observables expressed by the solution (7.3) describe completely and uniquely the physical content of two-dimensional, "massless" QED. As examples of such observables we mention the electric field tensor $F_{\mu\nu}(x)$ and the bilocal operator T(x,y) [74]:

$$F_{\mu\nu}(x) = -\frac{e}{\sqrt{\pi}}\,\epsilon_{\mu\nu}\,\phi(x)$$

$$\begin{aligned} T(x_1,x_2) &= \Psi(x_1)\exp\Big\{ie\int_{x_1}^{x_2} A^{\nu}(t)\,dt_{\nu}\Big\}\overline{\Psi}(x_2) \\ &= N(x_1-x_2) : \exp i\sqrt{\pi}\Big\{-\int_{x_1}^{x_2}\epsilon^{\mu\nu}\partial_\nu\phi(t)\,dt_\mu + \gamma_5^1\phi(x_1) - \gamma_5^2\phi(x_2)\Big\} : \end{aligned} \tag{7.7}$$

Let us now draw some conclusions from this analysis, which illustrate some of the main features of charge screening:

a) The physical states consist of free massive pseudoscalar particles ("mesons").

b) The charge of these particles is zero: $Q = \int_{-\infty}^{+\infty} j_0(x,t)\,dx = \int_{-\infty}^{+\infty}\partial_x\phi(x,t)\,dx = 0$

There are no charged fermions.

c) The fermion fields $\Psi_a(x)$ (Equ. (7.3)) do not anti-commute at space-like distances (the $\Psi_a(x)$ commute!), hence these fields are non-local Fermi fields. However, the observables $F_{\mu\nu}(x)$ and T($x_1 y_2$) commute for totally space-like distances, thus assuring causality.

d) This special structure of two-dimensional QED becomes only transparent by the consistent use of gauge invariant observables and state vectors. In this spirit, one should consider only gauge invariant field amplitudes (B.S. amplitudes). We get by direct calculation [74]:

$$\langle 0|\,T(x_1,x_2)\,|{}^{\mu}_{p}\rangle = \tilde{N}\gamma^{\mu} z_{\mu}\left(\epsilon^{\nu\rho} z_\nu p_\rho - \gamma_5(pz)\right)\frac{\sin pz}{(pz)}\,\frac{e^{ipX}}{z^2}$$

$$X = \tfrac{1}{2}(x_1+x_2)\quad,\quad z = x_1 - x_2 \tag{7.8}$$

Due to the non-local structure of $T(x_1,x_2)$, the general properties of the gauge invariant field amplitudes differ considerably from those of the conventional B.S. amplitudes, on which we have based our phenomenological approach to quark confinement (cf. 2.2). At this stage, it is not yet clear how relevant this two-dimensional model is for phenomenological physics. However, it gives interesting indications for the further development of realistic quark models with strong binding.

e) Gauge invariant states describe necessarily charged particles together with their electric field. With this in mind, one may get some insight into the dynamical mechanism of charge screening by a classical consideration [76]. The electric field $E(x)$, $F_{\mu\nu}(x) = \epsilon_{\mu\nu} E(x)$ of a static point charge satisfies the equation (cf. Equ. (7.2)):

$$\frac{dE}{dx} = e\,\delta(x-y) \quad ; \quad E(x) = e\,\Theta(x-y) \tag{7.9}$$

The constant field on the right side of the charge point is responsible for the infinite energy of this state. In contrast to this, a charge dipole

$$j_0(x) = e\,(\delta(x-y_1) - \delta(x-y_2))$$ with the field

$$E(x) = e\,(\Theta(x-y_1) - \Theta(x-y_2))$$ has a finite energy,

which is proportional to the distance of the two charges. This may explain why in gauge invariant two-dimensional QED, all states with finite energy have charge zero. There acts a linear confining potential (cf. 2.1) between charged particles!
It is evident that this argument relies heavily on the restriction to 1+1-dimensions.

f) A.Casher, J. Kogut, and L. Susskind [77] discussed the following 2-point function in two-dim. QED:

$$\langle 0|T\,S(x)S(0)|0\rangle = (2\pi^2x^2)^{-1}\exp\{-4\pi i(\Delta_F^{(2)}(x^2,0) - \Delta_F^{(2)}(x^2,\mu^2))\}$$

$$S(x) = \overline{\Psi}(x)\Psi(x)$$

From this expression they gain an intuitive picture for the process $e^+e^- \rightarrow$ hadrons: In the first moment the pair, created by the annihilation of the pointlike (scalar!) photon, propagates freely. Then the field between them strongly polarizes the vacuum. Finally the propagating dipole field catches the original pair, and the polarization charge annihilates it. Thus the strong polarizability of the vacuum prevents the appearance of free charges [78].

g) We discussed the mechanism of charge screening only for a single type of charges. There are generalizations [79] to more charges, which come closer to the real problem of the screening of the colour charges, related to the non-abelian gauge group SU(3).

h) These ideas on colour charge screening derived from 1+1-dimensional QED further the preference for string-like soliton solutions (cf. 5.3c) describing the strong coupling limit of the 3-1-dimensional standard Lagrangian. These strings might have the right spacial configuration to allow the charge screening mechanism.

7.2 Does colour charge screening happen also in the long distance limit of the 3+1 dimensional standard Lagrangian? In order to investigate this question, one should keep in mind the following results of the preceeding discussion:
(i) One has to consider the standard $\mathcal{L}$ with strong coupling (cf. 4.3).
(ii) charge screening becomes transparent only if one regards strictly the requirements of gauge invariance (cf. 7.1). It is known that field theories with strong coupling might be approached with help of a lattice approximation to space [80] (and time). Hence it is natural to consider gauge invariance for a lattice field theory. It was first suggested by K. Wilson [49] to study quark confinement in such a framework. He developped a theory of quantized fields on a lattice, which possess local gauge groups [80], using the Feynman path integral method of quantum mechanics [81]. A Hamiltonian formulation of Wilson's lattice gauge theory was given by J. Kogut and L. Susskind [82]. These investigations seem to indicate the following result: String like states in which quark-antiquark pairs are linked by gauge fields form energetically preferred configurations. In the case of non-abelian gauge groups, the crudest strong coupling approximation gives that the energy of such a state is proportional to the length of the string (cf. 7.1e). Thus the lattice gauge theories support our general view on quark confinement. They may even be applied to derive the Zweig's rule [5]. On the other hand, many problems of these theories remain to be solved. In particular, there is the open question of the systematic transition to the continuum. Of course, these short remarks are meant only as a reference to those important investigations concerned with quark confinement.

8. Conclusions

These lectures had the aim to give an introduction to the problem of quark confinement and to the different models and notions invented for the solution of this fundamental puzzle of the quark theory of hadrons. Seen from a point of uncritical, synthetizing imagination, the following picture emerges:

"Similar to QED, hadron dynamics is described by a renormalizable field theory in which coloured and flavoured quark fields are coupled to gluon fields according to the principles of local, non-abelian $SU_c(3)$ gauge invariance. This theory is asymptotically free, thus reproducing essentially the results of the quark parton model. In the large distance limit the theory is instable. The strong interaction causes, via vacuum polarization, the formation of physical strings, consisting of confined $q\bar{q}$-pairs linked by gluon lines. Colour charges are completely screened in this theory. The quantum mechanics of field strings provides the starting point for the dual dynamics of mesons". [83)]

At this stage, this picture may have the quality of a fairy-tale. In these lectures we summarized the facts on which it is based, and we illustrated the theoretical concepts from which it is built with the help of very (too?) simple examples. Even so not correct in any detail, this picture may provide a general pattern of hadron dynamics, which is worthwhile to be pursued further.

There remains the question of the relation of confinement models to actual experimental results. The phenomenological approach to quark confinement with help of a model of strongly bound massive quarks allows the interpretation of data of the meson spectrum, their leptonic and hadronic decay properties. The more theoretical models like the SLAC- and MIT-bag also produce their first figures [84)] on hadron properties. But much more has to be done for a real theoretical and quantitative understanding of high energy phenomenology. As example I mention just one more problem, the problem of spallation of bags or strings, which provides the description of the hadronic decays of mesons and baryons.

I hope that all these considerations made clear how stimulating the problem of quark confinement already has been for the development of theoretical elementary particle physics.

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Supplement: Recent Literature

The HEP Retrieval System at DESY was used for a survey of the scientific publications on the subject of the lectures on "Quark confinement" at the International Summer Institute in Theoretical Physics, DESY, Hamburg 15 - 26 September 1975. The basis of the survey are papers listed under the keywords "QUARK CONFINEMENT", or "BAG", or "QUARK, MASSIVE" until 26 March, 1976.

The papers are listed according to the topics treated in the preceding lecture notes. With the help of a proper arrangement and the addition of keywords in brackets, we tried to give hints on the significance of the different contributions. Perhaps our tendency to mention most of the references supplied by the computer does not sufficiently take into account the scientific quality of the different papers. Anyhow, we consider this supplement as an experiment exploring the usefulness of computerized information systems for a full evaluation of the published literature, in the best tradition of science.

1. PHENOMENOLOGICAL USE OF CONFINING POTENTIALS (SECT. 1.3).

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2. PHENOMENOLOGICAL, FIELD THEORETICAL (MASSIVE) QUARK MODEL (SECT. 2).

a. General formulation (Sect. 2.1).
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3. INFRARED SLAVERY. DYNAMICAL SPONTANEOUS SYMMETRY BREAKING (SECT. 4.3).

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4. EXAMPLES OF FIELD THEORETICAL BAGS.

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5. SLAC-BAG (SECT. 5.2.a).

W.A. Bardeen et al., Ref. 47.
S.D. Drell, SLAC-Pub. 1683 (Erice-Lectures).
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