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WHO IS AFRAID OF ANOMALIES?

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Abstract:

Triangle anomalies are shown to be nonessential for renormalizability of axial vector gauge models. No "compensating" fields are needed to make such models (in particular Weinberg-Salam model) renormalizable.

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It is a general belief that triangle anomalies spoil the renormalizability of gauge theories with γ_5-coupling. To maintain renormalizability one should introduce additional fields "compensating" anomalous terms [1]. In this note we prove that γ^5-anomalies have no influence on the renormalizability of S-matrix, actually there are no anomalies in relevant Green functions. Axial vector gauge theories are renormalizable as well as vector ones.

Formal proof of renormalizability fails in the case of axial gauge theories because to prove "naive" Ward identities one needs invariant regularization procedure and no such procedure was known for axial vector interaction. We shall show that regularization with the help of higher covariant derivatives introduced in the paper [2] and used for Non Abelian gauge theories in [3] can be extended to provide invariant regularization for axial vector gauge theories.

We consider first the simplest and actually the most important Abelian gauge model:

$$\mathcal{L} = -\frac{1}{4} f_{\mu\nu} f_{\mu\nu} + i\bar{\Psi}\gamma_\mu(\partial_\mu - ig\gamma_5 A_\mu)\Psi + \frac{1}{2} m^2 A_\mu A_\mu \tag{1}$$

Although mass term spoils formal gauge invariance the later can be easily restored by some kind of Stueckelberg formalism.

We start with the formulation of the model in explicitly renormalizable gauge and than prove the unitarity of renormalized S-matrix. To make the model explicitly renormalizable we introduce auxiliary Stueckelberg field φ replacing $\mathcal{L}$ by $\tilde{\mathcal{L}}$

$$\tilde{\mathcal{L}} = -\frac{1}{4} f_{\mu\nu} f_{\mu\nu} + i\bar{\Psi}\gamma_\mu\left[\partial_\mu - ig\gamma_5\left(A_\mu + \frac{1}{m}\partial_\mu\varphi\right)\right] + \frac{m^2}{2} A_\mu A_\mu + \frac{1}{2}\partial_\mu\varphi\partial_\mu\varphi \tag{2}$$

Quantization of this Lagrangian leads to the indefinite metric for the field φ all other fields having positive metric. The theory is explicitly renormalizable because the effective propagator

$$\overbrace{\left(A_\mu + \frac{1}{m}\partial_\mu\varphi\right)\left(A_\nu + \frac{1}{m}\partial_\nu\varphi\right)} \sim \frac{g^{\mu\nu} - m^{-2}K^\mu K^\nu}{K^2 - m^2} + \frac{K^\mu K^\nu}{m^2 K^2} = \frac{g^{\mu\nu} - K^\mu K^\nu K^{-2}}{K^2 - m^2} \tag{3}$$

decreases as K^{-2} for large K .

To prove the unitarity of S-matrix in the physical sector one must show that all matrix elements for the transitions from physical states (Ψ, A_μ) to the states containing φ are zero.

To make the following discussion rigorous we introduce intermediate regularization adding to the Lagrangian (2) terms

$$\Delta\mathcal{L}_\Lambda = -\frac{1}{4\Lambda^2} f_{\mu\nu}\Box f_{\mu\nu} + \frac{i}{\Lambda^2}\Box\bar{\Psi}\partial\Psi \tag{4}$$

Introduction of these terms is equivalent to the replacement of the propagators

$$\frac{\hat{p}}{p^2} \rightarrow \frac{\hat{p}}{p^2 - p^4\Lambda^{-2}} = \hat{p}\left\{\frac{1}{p^2} - \frac{1}{p^2-\Lambda^2}\right\}$$

providing ultraviolet convergence of Feynman diagrams. Now we can write regularized Green function generating functional

$$Z = N^{-1}\int exp\left\{i\int\left[\mathcal{L}_\Lambda + j_\mu A_\mu + \bar{\eta}\Psi + \bar{\Psi}\eta + c\varphi\right]d^4x\right\} dA\, d\bar{\Psi}\, d\Psi\, d\varphi \tag{5}$$

where

$$\mathcal{L}_\Lambda = \tilde{\mathcal{L}} + \Delta\mathcal{L}_\Lambda$$

It is convenient to introduce new variables

$$A_\mu \rightarrow A_\mu + m^{-1}\partial_\mu\varphi$$

Then $\tilde{\mathcal{L}}$ will look as follows

$$\tilde{\mathcal{L}} = -\frac{1}{4} f_{\mu\nu}f_{\mu\nu} + i\bar{\Psi}\gamma^\mu(\partial_\mu - ig\gamma_5 A_\mu)\Psi + \frac{m^2}{2}A_\mu^2 - mA_\mu\partial_\mu\varphi \tag{6}$$

Regularized Lagrangian in the exponent (5) can be written in a gauge invariant form by replacing ordinary derivatives in (4) by covariant derivatives

$$\partial_\mu \rightarrow \mathcal{D}_\mu = \partial_\mu - ig\gamma^5\left[\partial_\mu\Box^{-1}(\partial_\nu A_\nu + m^{-1}c(x))\right] \tag{7}$$

$$\Delta\tilde{\mathcal{L}}_\Lambda = -\frac{1}{4\Lambda^2} f_{\mu\nu}\Box f_{\mu\nu} + \frac{i}{\Lambda^2}\mathcal{D}^2\bar{\Psi}\hat{\mathcal{D}}\Psi \tag{8}$$

Indeed integration over φ produces $\delta(c + m\partial_\nu A_\nu)$ and therefore this replacement does not change the integrand in (5). On the other hand the expression (8) is manifestly invariant with respect to the gauge transformation

$$\Psi \to e^{ig\alpha(x)\gamma_5}\Psi\,, \quad \bar{\Psi} \to \bar{\Psi} e^{ig\alpha(x)\gamma_5}\,, \quad A_\mu \to A_\mu + \partial_\mu \alpha \tag{9}$$

Now the whole integrand in Eq.(5) apart from source terms, mass term and $m\varphi\partial_\mu A_\mu$, which is actually the gauge fixing term is manifestly gauge invariant. Ward identities can be derived in a usual way. Performing change of variables (9) and simultaneously $\varphi \to \varphi + m\alpha(x)$, where α is infinitesimal, one obtains

$$\int [mc(x) - \partial_\mu j_\mu(x) + ig\bar{\eta}\gamma_5\Psi(x) + ig\bar{\Psi}\gamma_5\eta(x) + m\Box\varphi] \times \tag{10}$$

$$\times \exp\left\{i\int[\mathcal{L}_\Lambda(y) + j_\mu A_\mu(y) + \bar{\eta}\Psi(y) + \bar{\Psi}\eta(y) + c\,\varphi(y)]d^4y\right\} dA\, d\bar{\Psi}\, d\Psi\, d\varphi = 0$$

It follows that

$$\langle T\Box\varphi(x)\varphi(y)\rangle = -i\delta(x-y)\,, \quad \langle T\Box\varphi(x)A_\mu(y)\rangle = im^{-1}\partial_\mu\delta(x-y)$$

$$m\langle T\Box\varphi_z\bar{\Psi}(x)\Psi(y)\rangle = g\{\gamma_5\langle T\Psi(x)\bar{\Psi}(y)\rangle\delta(x-z) + \langle T\Psi(x)\bar{\Psi}(y)\rangle\gamma_5\delta(y-z)\} \tag{11}$$

So the "naive" Ward identities are valid, mass renormalization for fermions is absent and $Z_1 = Z_2$. That means renormalized Lagrangian possesses the same invariance properties as the nonrenormalized one and "naive" identities (10) are true also in renormalized theory. But these identities mean that φ is a free field. Indeed applying to the Fourier transform of (11) operators $\hat{K}, \hat{p}$ and putting $\hat{K} = \hat{p} = 0$, one sees that on shell matrix elements $\bar{\Psi}\Psi \to \varphi$ are equal to zero. In the same way all matrix elements including fields φ vanish on mass shell. Therefore the S-matrix is unitary in the physical $(A_\mu, \bar{\Psi}, \Psi')$ sector.

The same result can be derived by passing from renormalizable gauge (5) to the unitary gauge. Performing in the formula (5) change of variables (9) with $\alpha(x) = \varphi(x)$ one obtains

$$Z_{on\ shell} = N^{-1}\int \exp\left\{i\int[-\tfrac{1}{4}f_{\mu\nu}f_{\mu\nu} + i\bar{\Psi}\gamma_\mu(\partial_\mu - ig\gamma_5 A_\mu)\Psi + \frac{m^2}{2}A_\mu^2 + \right.$$

$$\left. + \tfrac{1}{2}\partial_\mu\varphi\partial_\mu\varphi + \Delta\tilde{\mathcal{L}}_\Lambda + s.t.]d^4x\right\} dA\, d\bar{\Psi}\, d\Psi\, d\varphi \tag{12}$$

clearly showing that φ is a free field. Note however that in the unitary gauge covariant derivatives (7) are no more equal to the ordinary derivatives, and $\Delta\tilde{\mathcal{L}}_\Lambda$ (8) differs from $\Delta\mathcal{L}_\Lambda$ (4).

At first sight our results contradict the well-known fact of the existence of triangle anomaly. However there is no contradiction. We just showed that this anomaly is irrelevant for renormalization of S-matrix. In particular Ward identities (11) for triangle diagram imply only

$$\langle T\partial_\mu A_\mu(x) A_\nu^{tr}(y) A_\rho^{tr}(z)\rangle = 0 \tag{13}$$

and this equality certainly may be satisfied. π_0 -decay still may be governed by the anomaly, but this anomaly has nothing to do with the renormalizability. Note that regularization (8) treats transversal and longitudinal parts of A_μ nonsymmetrically (covariant derivatives (7) depend only on longitudinal components of A_μ). The arguments given in [1] to demonstrate nonrenormalizability of this model used symmetry properties of the triangle diagram and are not applicable to our regularization. We showed explicitly that contrary to the statements [1] S-matrix in a renormalizable gauge is unitary.

All the arguments are transferred directly to spontaneously broken theories. Using the same regularization (of course higher derivatives of Higgs fields should be also introduced according to (7)) one easily proves "naive" Ward identities which are sufficient conditions of unitarity and renormalizability.

The invariant regularization (8) can be easily extended to Non Abelian case. One should just replace covariant derivatives (7) by Yang-Mills covariant derivatives

$$\partial_\mu \to \mathcal{D}_\mu = (\partial_\mu - igTL_\mu) \tag{14}$$

where the chiral field L_μ is defined by

$$\partial_\mu L_\mu = \partial_\mu A_\mu + g[A_\mu, L_\mu], \quad \partial_\mu L_\nu - \partial_\nu L_\mu = g[L_\mu, L_\nu] \tag{15}$$

In complete analogy with the Abelian case in the Landau gauge $\partial_\mu = \mathcal{D}_\mu$, and one can derive "naive" Ward identities proving the absence of anomalies in relevant Green functions.

Our results imply in particular that Weinberg-Salam model for lepton sector is renormalizable independently on hadron sector and no "anomalous compensating" fields are needed.

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