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2 HAMBURG 52 · NOTKESTIEG 1

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We present numerical estimates for the radiative corrections to the decays $L^{\mp} \to e^{\mp} \nu_e \nu_L$, $\mu^{\mp} \nu_\mu \nu_L$ in the context of a (V $\pm$ A) interaction. The radiative corrections change the Michel parameter appreciably from its bare (V $\pm$ A) value, for the decay $L^{\mp} \to e^{\mp} \nu_e \nu_L$, and have a measurable effect on the electron energy spectrum.

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The "anomalous" μe events[1] and the inclusive muon production in e^+e^- experiments[2] reported by the SLAC - LBL group are being interpreted as coming from the production and decay of a pair of charged heavy leptons, $L^{\mp}$. So far only the leptonic decay modes, namely

$$L^{\mp} \to \begin{matrix} e^{\mp} \nu_e \nu_L \\ \mu^{\mp} \nu_\mu \nu_L \end{matrix} \qquad (1)$$

have been established. Starting from the premise that these events are coming genuinely from charged heavy leptons, there has been a lot of theoretical activity to understand the nature of such heavy leptons [3], the structure of the weak current responsible for their decays, and the nature of the neutrinos being emitted [4] [5]. The tests suggested consist of detailed comparison of the various electron (muon) energy-angle correlations with the data that will be available in the near future.

In particular it has been pointed out [5] that the electron and the muon energy spectra are sensitive to the value of the Michel parameter, ρ . Although the value of ρ does not determine uniquely the nature of the weak interaction responsible for the processes (1) a value of ρ close to 3/4 would suggest a V - A interaction, whereas $\rho = 0$ would favour V + A . On the other hand, it is also known from the study of the μ-decay [6] that the value of ρ is sensitive to the radiative corrections. This then leads to the necessity of including radiative corrections to the processes (1), and comparing the electron (muon) energy spectrum with the ones obtained after incorporating radiative corrections.

The relative importance of the radiative corrections to leptonic processes such as (1) , can be guessed from the order parameter that characterises such corrections. The first order radiative correction to the processes (1) is of order

$$\alpha \left[\ln\left(\frac{m_L}{m_\ell}\right)\right]^2 , \qquad \ell = e, \mu$$

instead of the fine structure constant α. This number is 0.5 for $\ell = e$ and m_L = 2 GeV and 0.06 for $\ell = \mu$ and m_L = 2 GeV, the corresponding number for the μ-decay being 0.2. It is, a priori, obvious that the effect of the radiative correction for the decay $L^{\mp} \to e^{\mp} \nu_e \nu_L$ would be substantially larger than that for the μ-decay, while it will be relatively smaller for the decay $L^{\mp} \to \mu^{\mp} \nu_\mu \nu_L$. The purpose of this note is to present a numerical estimate of the radiative corrections to the Michel parameter ρ, within the context of a V $\pm$ A interaction. We also present the effect of the radiative corrections on the electron energy spectrum relevant for the SLAC - LBL experiment.[2]

We start by recalling that the effective current $\otimes$ current Hamiltonian for the processes (1) leads to the following one parameter Michel formula for the electron (muon) energy distribution:

$$d\Gamma(x) = 4\pi G_{L,R}\left[3(1-x) + 2\rho\left(\tfrac{4}{3}x - 1\right)\right] x^2\, dx \qquad (2)$$

where

$$x = \frac{2E_\ell}{m_L} \qquad \ell = e, \mu$$

$$\rho = \left(\tfrac{3}{4}, 0\right) \qquad \text{for } (V-A,\ V+A)$$

$$G_L = \frac{G_F^2 m_L^5}{384\pi^4} |a_L|^2 \qquad (3)$$

$$G_R = \frac{G_F^2 m_L^5}{384\pi^4} |a_R|^2$$

G_F is the Fermi-coupling constant $= 1.05 \times 10^{-5} (m_p)^{-2}$. a_L and a_R are introduced to let the (V - A), (V + A) interaction a different overall normalisation. The radiative correction to the processes (1) are then implemented by assuming a universal electromagnetic interaction

$$H_I^{e.m.} = \sum_{i=e,\mu,L} e\bar{\Psi}_i \gamma_\lambda \Psi_i A^\lambda .$$

The diagrams that contribute to the first order in α (i.e. to order $G_F^2\alpha$ in the decay probability) are shown in Fig. 1 (a) (Virtual Photon Contribution) and Fig. 1 (b) (Bremsstrahlung contribution). The decay distribution including the radiative correction for the μ-decay was calculated in reference (6) by assuming an effective current $\otimes$ current interactions. We shall rely on their calculations and present here (after some straightforward algebra) the relevant formulae for the V $\pm$ A interactions. The electron (muon) energy spectrum, including the radiative corrections, then reads as

$$\frac{d\Gamma(x)}{dx} = 4\pi G_{L,R}\, x^2\left[3(1-x) + 2\rho\left(\tfrac{4}{3}x - 1\right) + \frac{\alpha}{4\pi} f_{L,R}(x)\right] \qquad (4)$$

where

$$f_L(x) = (6-4x)R(x) + (6-6x)\ln x + \frac{1-x}{3x^2}\left[(5+17x-34x^2)(\omega_\ell + \ln x) - 22x + 34x^2\right]$$

$$(5)$$

$$f_R(x) = 12(1-x)R(x) + 12\ln x + 36(1-x)(\omega_\Lambda - \omega_\ell) + \frac{1-x}{x^2}\left[2(1+4x-17x^2)(\omega_\ell + \ln x) - 12x + 57x^2\right]$$

$$R(x) = 2\sum_{n=1}^{\infty}\frac{x^n}{n^2} - \frac{1}{3}\pi^2 - 2 + \omega_\ell\left(\frac{3}{2} + 2\ln\frac{1-x}{x}\right) - \ln x\left(2\ln x - 1\right) + \left(3\ln x - 1 - \frac{1}{x}\right)\ln(1-x) , \quad (6)$$

$$\omega_\ell = \ln\left(\frac{m_\ell}{m_L}\right) \qquad (\ell = e, \mu)$$

$$\omega_\Lambda = \ln\left(\frac{m_\Lambda}{m_L}\right)$$

where m_Λ is a cut-off, which appears for the V + A interaction and will be replaced by the mass of W - boson in a renormalisable theory. We recall the finiteness of the radiative correction for the (V-A) case in a current ⊗ current theory; a consequence of (V - A) being invariant under Fierz transformation.

In table (1), we present the values of the functions $f_{L,R}(x)$ appearing in Eq. (5). Columns (3) and (5) give the contribution of the radiative correction, as a fraction (%) of the bare decay probability, for the decay $L^\mp \to e^\mp \nu_e \nu_L$ and $L^\mp \to \mu^\mp \nu_\mu \nu_L$, respectively, for the (V - A) case. The same quantities for the (V + A) case are given in columns (7) and (9). We observe that the radiative correction for both the (V - A) and (V + A) cases is substantial for the decay $L^\mp \to e^\mp \nu_e \nu_L$, at the lower and the upper end of the electron energy spectrum. In Fig. (2) we plot the electron energy spectrum for both the (V ± A) cases.

The shape of the radiative correction suggests that over a large range of x, $d\Gamma/dx$ could still be approximated by a linear formula (2) with an effective Michel parameter ρ_{eff}. To illustrate this point we have plotted in Fig. (3) the quantity †

$$S(x) = \frac{2}{1 + \frac{\alpha}{2\pi} F_L(0)}\left[3 - 2x + \frac{\alpha}{2\pi} f_L(x)\right] \quad (7)$$

corresponding to the decay distribution of $L^- \to e^- \bar{\nu}_e \nu_L$ for the V - A case. As is clear from Fig. (3), $S(x)$ is well described by a straight line in the range $0.3 \leqslant x \leqslant 0.95$. Fitting this line to the Michel formula

$$S_1(x) = 12(1-x) + 8\rho_{eff}\left(\frac{4}{3}x - 1\right) \quad (8)$$

one finds ρ_{eff} = 0.66. This shows that the Michel parameter for $L^\mp \to e^\mp \nu_e \nu_L$ changes by 12% from its uncorrected V - A value in this range. The same fit to $L^\mp \to e^\mp \nu_e \nu_L$ with V + A interaction leads to a value of ρ_{eff} = -0.13 to be contrasted with ρ = 0 without radiative correction. Alternatively, one can determine ρ_{eff} by fitting $S(x)$ with $S_1(x)$ by using the method of least squares. The value of ρ_{eff} so determined is very sensitive to the lower limit of x. The values of ρ_{eff} for the various ranges of x are presented in table 2. Since there will always be an experimental cut on the electron (muon) momentum, and there is always a finite energy resolution we have not evaluated ρ_{eff} for the entire theoretical range $0 \leqslant x \leqslant 1$. #

There are several equivalent ways of incorporating the effect of the radiative corrections for the processes (1), in order to enable a comparison with the experimental results. Supported by the argument of the preceding paragraph, one can adopt the attitude that the experimental curves have to be compared with the theoretical distributions obtained by using the one-parameter Michel formula, however, with an effective ρ, which takes into account the effect of the radiative correction adequately. It is known that the electron (muon) energy distribution for a heavy lepton in motion can be evaluated analytically. One can then calculate in a straightforward way the corrected energy spectrum replacing ρ by ρ_{eff} in these formulae to be compared directly with the experimental result. We reproduce here the analytic formula to calculate the electron (muon) energy spectrum for a heavy lepton in motion, valid for both the V $\pm$ A interactions.

$$\frac{d\Gamma}{dx_\ell} = \frac{2}{\beta}\left[1 - \frac{4x_\ell}{(1+\beta)^3}\left(3+3\beta-4x_\ell\right) - \frac{2}{9}\rho\left\{1-\frac{4x_\ell^2}{(1+\beta)^3}\left(9+9\beta-16x_\ell\right)\right\}\right] \tag{9}$$

$$\text{for} \quad \frac{1-\beta}{2} \leqslant x_\ell \leqslant \frac{1+\beta}{2} ,$$

$$= \frac{32x_\ell^2}{(1-\beta^2)^3}\left[(3-2\rho)(1-\beta^2) + 2x_\ell\left(1+\frac{1}{3}\beta^2\right)\left(\frac{8}{3}\rho-3\right)\right] \tag{10}$$

$$\text{for} \quad 0 \leqslant x_\ell \leqslant \frac{1-\beta}{2} ;$$

where

$$x_\ell = \frac{E_\ell}{E_{beam}} \qquad (\ell = e, \mu)$$

and

$$\beta = \sqrt{1-4m_L^2/W^2} , \qquad W = 2E_{beam} .$$

The electron - energy spectrum for the decay $L^{\mp} \rightarrow e^{\mp}\nu_e\nu_L$ is plotted in Fig. (4) for β = 0.8 both with and without the radiative correction. We have used ρ_{eff} = 0.52, - 0.34 for the V - A, V + A interactions respectively, to indicate the extreme effect of the radiative corrections. The effect for both the V $\mp$ A is significant. However, the two interactions are still distinguishable.

Alternatively, one can adopt the attitude of Kinoshita and Sirlin [(6)], by incorporating the radiative corrections through modifications of the coupling constants G_L, G_R introduced in (2). One then obtains a formula, in which the Michel Parameter remains unchanged, and the decay distribution is modified multiplicatively by a function depending on α and x . For the details we refer to ref. (6) and merely quote the relevant formula here (using ρ_{V-A} = 3/4 and ρ_{V+A} = 0):

$$\frac{1}{\Gamma}\frac{d\Gamma}{dx} = \frac{4x^2(1+h_L(x))}{(2+\Lambda_1+\Lambda_2)}(3-2x) \qquad (\text{for } V-A)$$

$$= 12x^2(1+h_R(x)-\Lambda_3)(1-x) \qquad (\text{for } V+A) \tag{11}$$

where

$$h_L(x) = \frac{\alpha}{2\pi}\frac{f_L(x)}{(3-2x)}$$
$$h_R(x) = \frac{\alpha}{12\pi}\frac{f_R(x)}{(1-x)}$$
$$\Lambda_1 = \frac{2\alpha}{\pi}\int_0^1 \frac{3x^2(1-x)}{(3-2x)} f_L(x)\,dx$$
$$\Lambda_2 = \frac{2\alpha}{\pi}\int_0^1 \frac{x^3}{(3-2x)} f_L(x)\,dx \tag{12}$$
$$\Lambda_3 = \frac{\alpha}{\pi}\int_0^1 x^2 f_R(x)\,dx$$

The values of $h_L(x)$, $h_R(x)$ can be read from Table 1, those of Λ_i can be obtained by analytic integration of the expression appearing above and are presented in table 3.

Finally, we would like to comment on the connection of our results with the ones obtainable in renormalizable gauge field theory models. Our remarks are based on the conclusion of ref. (7), where radiative corrections to μ - decay in gauge models are calculated and compared with the effective current $\otimes$ current theory predictions. A straight - forward extension of the μ -decay to the L -decay shows that in an $SU(2) \times U(1)$ model of the Weinberg-Salam type (8), with (L, ν_L) being sequential leptons, the Michel parameter ρ is unchanged from its current $\otimes$ current theory prediction upto terms of order $\alpha (m_L^2/m_W^2)$. In fact most gauge theory models will reproduce the effective current $\otimes$ current prediction for ρ upto terms of order $\alpha(\text{lepton mass})^2/m_W^2$. The reason is that the Born terms, the Bremsstrahlung diagrams and the relevant virtual photon diagrams, in such models can only differ by terms of order $(\text{lepton mass})/m_W$. Since typically $m_L/m_W \sim \frac{1}{20}$, all such terms are undetectable.

To conclude. we remark that our calculations suggest that the radiative corrections to the decay $L^- \to e^- \nu_e \nu_L$ will modify the Michel parameter and the electron energy spectrum appreciably from its bare $V \pm A$ values. The radiative corrections leave the bare $V \pm A$ values practically unchanged for the decay .

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Footnotes

† Here $F_L(0)$ is defined as

$$F_L(0) = 2\int_0^1 x^2 f_L(x)\,dx$$

The functions $S(x)$ and $S_1(x)$ are chosen so as to satisfy the normalisation condition

$$\int_0^1 S(x)\,x^2\,dx = \int_0^1 S_1(x)\,x^2\,dx = 1$$

For the V + A case, the corresponding function is

$$S(x) = \frac{12}{1+\frac{\alpha}{2\pi}F_R(0)}(1-x)\left[1+\frac{\alpha}{12\pi}\frac{f_R(x)}{(1-x)}\right]$$

where

$$F_R(0) = 2\int_0^1 x^2 f_R(x)\,dx .$$

\# The value of ρ_{eff} so determined is insensitive to the variation in the heavy lepton mass in the range 1.8 GeV $\leqslant m_L \leqslant$ 2.0 GeV, relevant for the SLAC - LBL experiments.

Table Captions

Table 1. Values of the function $f_L(x)$, $f_R(x)$ appearing in Eq. (6) of the text. Columns (3) and (5) give the relative contribution of the radiative corrections for the V-A case. The same quantity is given in columns (7) and (9) for the V + A case.

Table 2. Values of the effective Michel Parameter obtained by the method of least squares for the various ranges of $x = 2E_\ell/m_L$.

Table 3. Values of the parameters Λ_i introduced in Eq. (11) of the text.

Figure Captions

Figure 1. Feynman diagrams for the first order radiative correction to the decay $L^{\mp} \longrightarrow e^{\mp} \nu_e \nu_L$. The diagrams for $L^{\mp} \to \mu^{\mp} \nu_\mu \nu_L$ are similar. (a) Virtual photon contribution (b) Bremsstrahlung Contribution.

Fig. 2 Electron energy distribution as a function of $x = 2\, E_e/m_L$, from the decay $L^- \longrightarrow e^- \bar{\nu}_e \nu_L$ in the rest frame of L^- for the V $\pm$ A interactions. Dashed Curves correspond to radiatively corrected case, solid lines without radiative corrections. We have assumed $1.8 \leqslant m_L \leqslant 2.0$ GeV, $m_{\nu_L} = 0$.

Fig. 3 The function $\left[3 - 2x + \frac{\alpha}{2\pi} f_L(x) \right]$ corresponding to the radiative correction for the decay $L^{\mp} \longrightarrow e^{\mp} \nu_e \nu_L$ with V – A interaction as a function of x.

Fig. 4 Electron energy spectrum as a function of $x = E_e / E_{beam}$ from the decay of L^- in flight, with $\beta = 0.8$. Solid curves for the uncorrected spectrum, dashed ones for radiatively corrected spectrum. (a) V-A interaction $\rho = 3/4$, (b) V + A interaction $\rho = 0$, (C) V – A interaction $\rho = 0.52$ (d) V + A interaction $\rho = -0.34$. We have assumed $1.8 \leqslant m_L \leqslant 2.0$ GeV and $m_{\nu_L} = 0$.

	$L \to e \nu_e \nu_L$ (V−A)		$L \to \mu \nu_\mu \nu_L$ (V−A)		$L \to e \nu_e \nu_L$ (V+A)		$L \to \mu \nu_\mu \nu_L$ (V+A)	
x	$\frac{\alpha}{2\pi} f_L(x)$	$\frac{(\alpha/2\pi) f_L(x)}{3-2x}$ %	$\frac{\alpha}{2\pi} f_L(x)$	$\frac{(\alpha/2\pi) f_L(x)}{3-2x}$ %	$\frac{\alpha}{4\pi} f_R(x)$	$\frac{\alpha}{12\pi} \frac{f_R(x)}{(1-x)}$ %	$\frac{\alpha}{4\pi} f_R(x)$	$\frac{\alpha}{12\pi} \frac{f_R(x)}{(1-x)}$ %
0.1	1.457	52.0	0.070	2.5	0.980	36.3	0.097	3.6
0.2	0.484	18.6	0.060	2.3	0.358	14.9	0.086	3.8
0.3	0.248	10.3	0.040	1.6	0.185	8.8	0.069	3.3
0.4	0.138	6.3	0.025	1.2	0.101	5.6	0.052	2.9
0.5	0.072	3.6	0.014	0.7	0.050	3.3	0.038	2.5
0.6	0.027	1.5	0.006	0.3	0.016	1.3	0.024	2.0
0.7	−0.006	−0.3	−0.002	−0.1	−0.002	−0.6	0.014	1.5
0.8	−0.033	−2.4	−0.009	−0.6	−0.017	−2.8	0.006	1.0
0.9	−0.062	−5.2	−0.017	−1.4	−0.017	−5.8	0.000	0.1
0.95	−0.083	−7.6	−0.023	−2.1	−0.013	−8.5	−0.001	−0.6
0.99	−0.132	−13.2	−0.037	−3.7	−0.004	−14.3	−0.001	−2.2

TABLE I

Range of x	$L \to e \nu_e \nu_L$		$L \to \mu \nu_\mu \nu_L$	
	ρ (V-A)	ρ (V+A)	ρ (V-A)	ρ (V+A)
$0.1 \leqslant x \leqslant 0.95$	0.52	-0.34	0.73	-0.07
$0.2 \leqslant x \leqslant 0.95$	0.63	-0.18	0.73	-0.07
$0.3 \leqslant x \leqslant 0.95$	0.66	-0.13	0.73	-0.07
bare values	0.75	0	0.75	0

TABLE II

	$L \to e \nu_e \nu_L$	$L \to \mu \nu_\mu \nu_L$
Λ_1	0.027	0.002
Λ_2	-0.035	-0.010
Λ_3	-0.022	-0.022

TABLE III

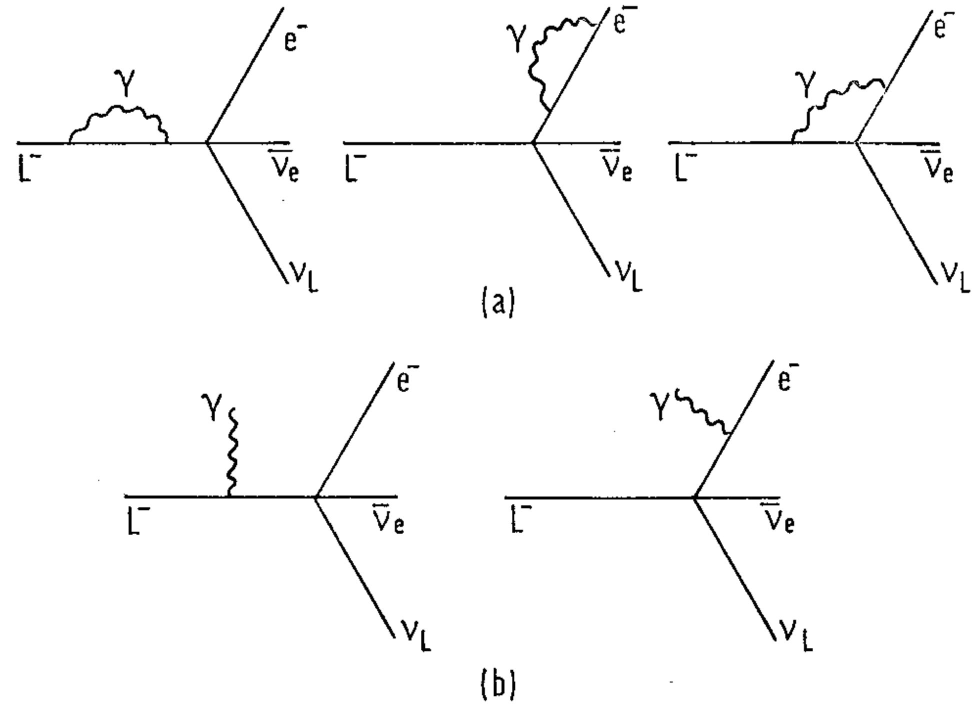


Fig.1

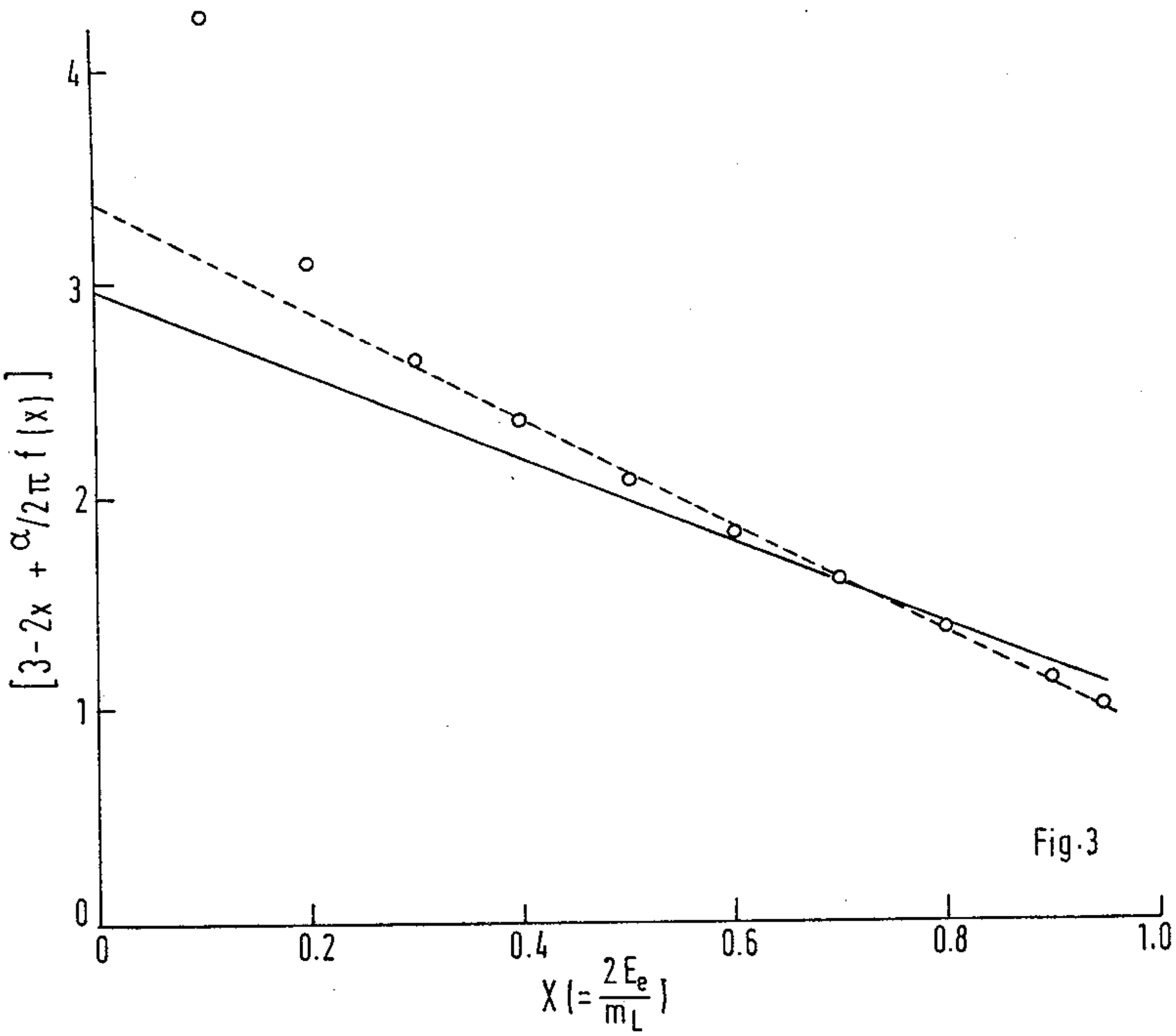

[3-2x + α/2π f(x)]
4
3
2
1
0
0
0.2
0.4
0.6
0.8
1.0
X (= 2Eₑ/m_L)

Fig. 3

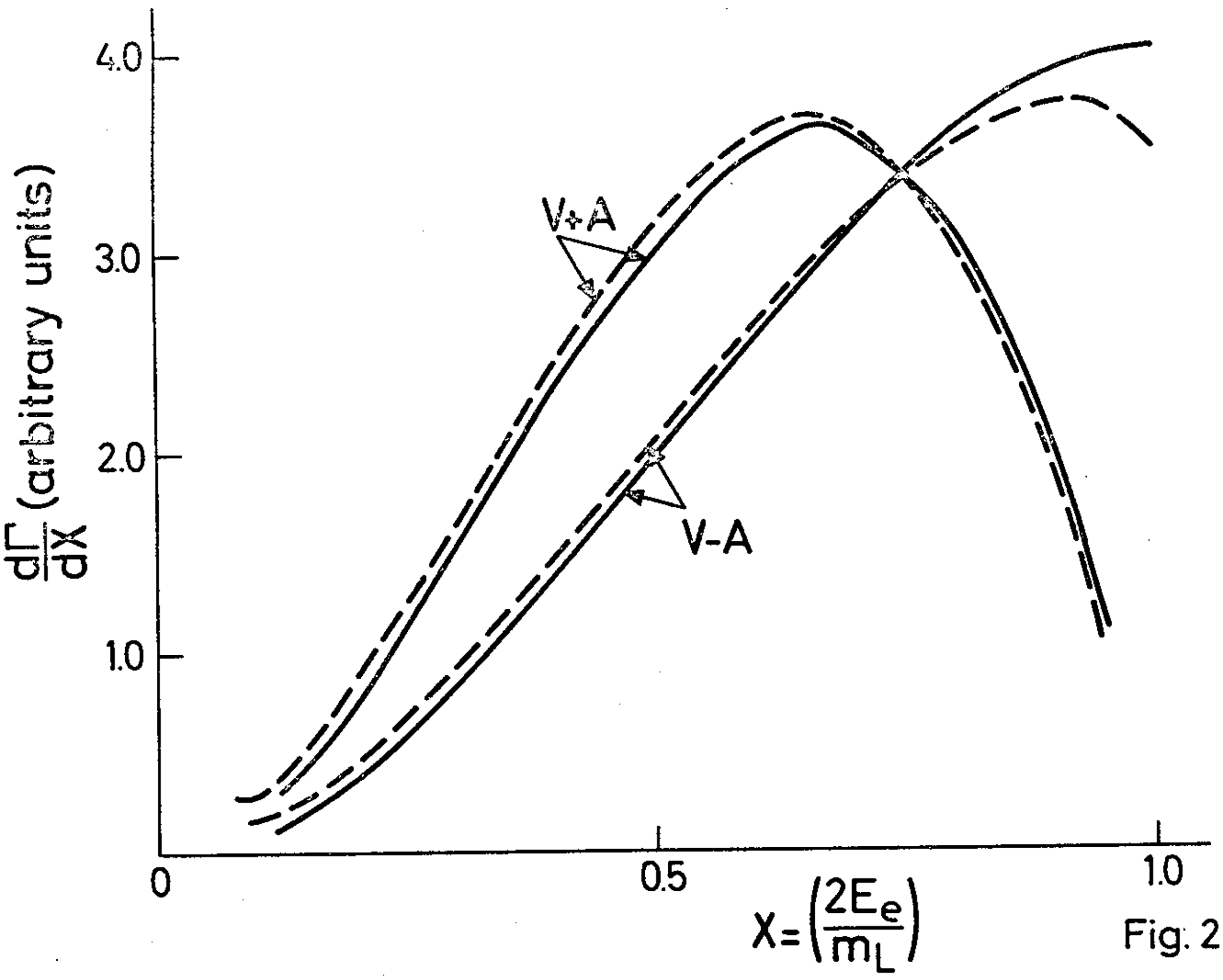
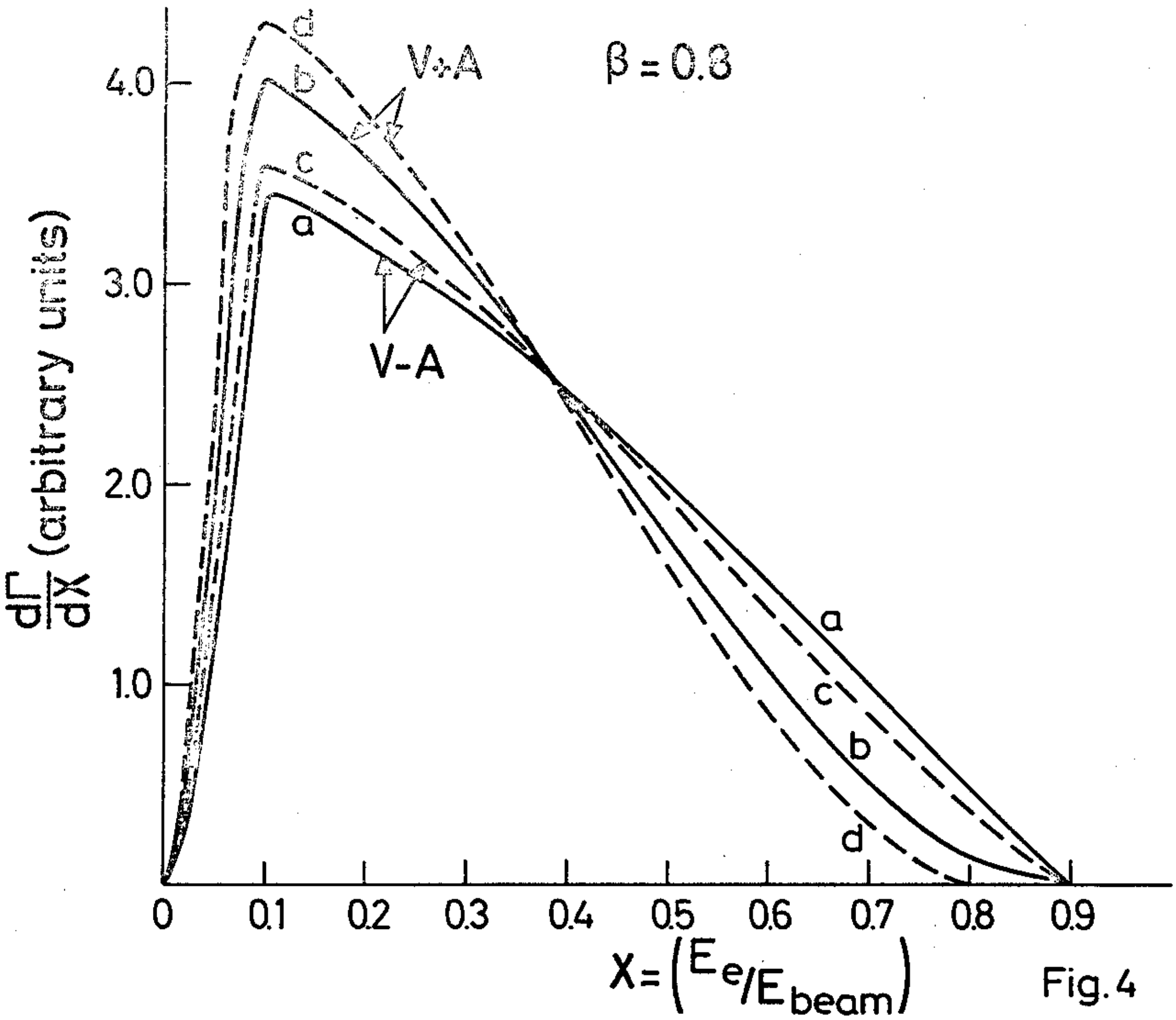

dΓ/dX (arbitrary units)
4.0
3.0
2.0
1.0
V+A
V-A
0
0.5
1.0
X = (2Eₑ/m_L)

Fig. 2



d
b
c
a
V+A
β = 0.8
V-A
a
c
b
d
4.0
3.0
2.0
1.0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
dΓ/dX (arbitrary units)
X = (Ee/Ebeam)

Fig. 4