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Depolarisation Effects in PETRA

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1. Introduction

The natural radiative polarisation occuring in electron storage rings is in competition with various depolarising effects. In PETRA these are associated mainly with resonances of the form:

$$\gamma a = k + k_z Q_z + k_x Q_x \tag{1}$$

where γ is the Lorentz factor,

$a = \frac{g-2}{2} \simeq 1.1617 \times 10^{-3}$ is the anomalous part of the gyromagnetic ratio,

k, k_z, k_x are integers, possibly negative, and

Q_z, Q_x are the vertical and horizontal betatron wave numbers.

In contrast to low-energy machines, such as ACO or VEPP 2, the quantum fluctuations at PETRA energies are strong enough to produce depolarisation effects even if the precession frequency γa is appreciably away from the resonance condition (1).

Depolarisation of a beam is almost entirely due to the abrupt changes in electron-trajectory parameters resulting from photon recoil in perturbing magnetic fields. Subsequent "mixing" of amplitudes and phases leads to so-called spin diffusion. At PETRA energies the main perturbing fields are radial and arise from vertical betatron oscillations and closed-orbit distortions in the vertically-focussing fields. Longitudinal fields have a much weaker effect at these energies and are only likely to be significant in the neighbourhood of systematic resonances. Variations of vertical field along a particle trajectory contribute only indirectly through coupling from x to y motion.

The polarisation time constant τ_p is given by:

$$\tau_p = \frac{8}{5\sqrt{3}} \cdot \frac{m_o R \rho^2}{\hbar r_o \gamma^5} \tag{2}$$

where: R is the average radius and ρ the bending radius,
r_o is the electron classical radius and m_o the rest mass,
and $\hbar$ is the reduced Planck's constant. The depolarisation time constant τ_d is given by different expressions depending on the nature of the mechanism. The evolution of the polarisation P follows the equation

$$\frac{dP}{dt} = -\left(\frac{1}{\tau_p} + \frac{1}{\tau_d}\right) P + \frac{8}{5\sqrt{3}} \cdot \frac{1}{\tau_p} \tag{3}$$

whose solution is:

$$P = \frac{8}{5\sqrt{3}} \cdot \frac{\tau_d}{\tau_d + \tau_p} \left\{ 1 - e^{-\left(\frac{1}{\tau_p} + \frac{1}{\tau_d}\right) \cdot t} \right\} \tag{4}$$

In the absence of depolarising effects the limit $P(t \to \infty) \simeq 92.4$ %; otherwise it is reduced by the factor $\tau_d/(\tau_d+\tau_p)$.

2. Depolarisation from vertical betatron oscillations

It is important in the present context to make a clear distinction between two components of vertical betatron amplitude, viz.

(a) the vertical amplitude induced directly by quantum jumps due to a vertical component of momentum recoil of the emitted photon, and
(b) the vertical amplitude resulting from smooth adiabatic coupling from the horizontal betatron motion.

In an idealised machine without coupling only (a) is present; in practical machines both (a) and (b) components occur and have to be considered separately.

2 (a). Direct vertical quantum excitation

This has been calculated independently by Derbenev and Kondratenko[1] and by Robinson[2]. Both papers give essentially the same formula for the depolarisation rate in the vicinity of a resonance, though ref. 1) is more general:

$$\frac{1}{\tau_d} \approx \frac{(\gamma a)^2 Q_z^2}{(\gamma a + k \pm Q_z)^2} \cdot \frac{\lambda\!\!\!^-}{\rho} \cdot \frac{1}{2\tau_E} \qquad (5)$$

where $\lambda\!\!\!^- = \frac{\hbar}{m_o c} = 3.8616 \times 10^{-13}$ m is the Compton wavelength, τ_E the energy damping time and k a positive or negative integer. In a perfect machine of superperiodicity S only those values of k which are multiples of S are significant and this is also likely to be true of an imperfect machine unless one is very close to a resonance.

For PETRA at 15 GeV the polarisation time from (2) is τ_p = 1754 s. If we require that $\tau_d \geq 10\ \tau_p$ we find, with $Q_z \sim 22$, ρ = 192 m and $\tau_E = 4.94 \times 10^{-3}$ s, that

$$|\gamma a + k \pm Q_z| \geq 0.0448,$$

corresponding to a distance from the resonance of 19.7 MeV in energy units, almost exactly 1 σ_E. Since one unit in k corresponds to $m_o c^2/a \approx 440$ MeV in energy, there should be no difficulty in choosing the operating energy far enough from these resonances to make their contribution to depolarisation negligible.

From the form of the resonance denominator in (5) one sees that the resonances occur in pairs, corresponding to $\pm Q_z$, members of a pair being separated by twice the non-integral part of Q_z. Thus, to have available the maximum energy range between pairs it is advantagous to operate with a small non-integral part of Q_z.

2 (b). Vertical excitation from coupling

In practical electron storage rings the vertical betatron amplitude is usually dominated by betatron coupling from the horizontal motion or by effects of vertical dispersion. However, the excitation of spin resonances by such coupling mechanisms is intrinsically much weaker than for the case of direct vertical quantum excitation. Physically this is because the quantum jumps in energy, which determine the horizontal betatron amplitude and energy spread, manifest themselves mainly by way of the relatively slow process of adiabatic coupling. The slowness of this mechanism can be likened to a narrow-band filter which strongly attenuates the quantum excitation spectrum except in very close proximity to a spin resonance.

The error in the calculation of Chao and Schwitters[3] is to attribute the full vertical betatron amplitude to direct vertical quantum excitation. Their approximate formula in fact reduces to Eqn.(5) with k = 0, provided the "natural" vertical beam size is used.

The contribution to the depolarisation rate from the coupled betatron motion is given, in smooth approximation, by Derbenev (private communication) as:

$$\frac{1}{\tau_d} \approx \frac{\langle z^2 \rangle}{R^2} \cdot \frac{16\ (\gamma a)^2 \sin^2 \{\pi(Q_z - Q_x)\}}{Q_x(\tau_z + \tau_x)} \qquad (6)$$

where $\langle z^2 \rangle$ is the square of the vertical beam size resulting from coupling and τ_z, τ_x are the vertical and horizontal damping times respectively. The expression is valid if one is not too close to a resonance of type $\gamma a + k \pm Q_x = 0$. One notes that the depolarisation from this source vanishes if Q_z and Q_x differ by an integer; the coupling mechanism then becomes infinitely slow and the bandwidth of the equivalent filter vanishes. This is in fact the situation for the nominal PETRA optics, but to be safe we will assume a fractional integer Q-split of 0.1 .

If we evaluate (6) for $\langle z^2 \rangle = (10\ \text{mm})^2$, corresponding to $10\sigma_z$ for $\sigma_z \sim 1$ mm, $Q_x \sim 26$ and $\tau_z + \tau_x \approx 4\tau_E \approx 20 \times 10^{-3}$s, we find at 15 GeV:

$$\tau_d \sim 4 \times 10^5\ \text{s} \qquad \sim 110\ \text{hrs.}$$

This is large compared with the polarisation time so very little depolarisation is to be expected from this effect.

A more precise expression for the depolarising rate due to vertical betatron motion is given in ref. 1). The contribution from the coupled betatron motion is:

$$\frac{1}{\tau_d} = \frac{1}{2} \cdot \frac{1}{\gamma^2} \frac{d}{dt} \langle \delta\gamma^2 \rangle \cdot \frac{(\gamma a)^6}{\{(\gamma a)^2 - Q_z^2\}^2} \sum_{k=-\infty}^{\infty} \frac{|g_k|^2}{(\gamma a - k)^2 \{(\gamma a - k)^2 - Q_x^2\}^2} \qquad (7)$$

where $\frac{1}{\gamma^2} \frac{d}{dt} \langle \delta\gamma^2 \rangle = \frac{55}{24\sqrt{3}} \cdot \frac{r_o c \lambda\!\!\!^- \gamma^5}{\rho^3}$ is the quantum excitation rate and g_k is the amplitude of the k^{th} harmonic of the coupling function from horizontal to vertical motion. The form of the denominators indicates which resonances are to be avoided, but a numerical evaluation of (7) requires a knowledge of the harmonic amplitudes g_k.

3. Depolarisation from vertical closed-orbit distortions

In ref. 1) the following expression is given for the depolarisation rate arising from the harmonic number k:

$$\frac{1}{\tau_d} = \frac{1}{2} \cdot \frac{1}{\gamma^2}\frac{d}{dt}\langle\delta\gamma^2\rangle \cdot \frac{(\gamma a)^4 k^4}{(\gamma a-k)^4} \cdot \frac{Q_x^4}{\{(\gamma a-k)^2-Q_x^2\}^2} \cdot \frac{z_k^2}{R^2} \qquad (8)$$

where z_k is the k^{th} Fourier harmonic of the orbit distortion and the other quantities are as previously defined. The factor containing Q_x takes account of quantum jumps in horizontal betatron motion which were not included in the earlier paper of Baier[4]. Eq. (8) has been derived for a regular lattice.

The behaviour of (8) is dominated by the resonance denominator $(\gamma a-k)^4$. If one assumes that the Fourier spectrum of the closed orbit is smooth, i.e. not modulated with the machine superperiodicity, then adjacent values of k (for k >> Q) contribute almost equal harmonic amplitudes z_k. The best we can then do is to choose the energy so that γa is half integral, making $(\gamma a-k)^{-4} = 16$. If we evaluate the expression for PETRA at ∿ 15 GeV, assuming $\tau_d = 10\ \tau_p = 1.754 \times 10^4$s, $Q_x \approx 26$, $\gamma a = 33.5$, $k = 34$, $R = 367$ m and $\frac{1}{\gamma^2}\frac{d}{dt}\langle\delta\gamma^2\rangle = 1.329 \times 10^{-3} s^{-1}$, we find:

$$z_k = 0.021 \text{ mm}.$$

The harmonic amplitude may be related to the maximum closed-orbit amplitude following Guignard[5].

$$z(\phi) = \sum_{k=1}^{\infty} \frac{Q^2}{Q^2-k^2}\left(f \cos k\phi + g \sin k\phi \right) \qquad (9)$$

The summation over the coskϕ terms can be expressed exactly:

$$Q^2 f \sum_{k=1}^{\infty} \frac{\cos k\phi}{Q^2-k^2} = Q^2 f \left(\frac{\pi}{2} \cdot \frac{\cos\{Q(\pi-\phi)\}}{Q \sin\pi Q} - \frac{1}{2Q^2} \right)$$

$$= \frac{f}{2}\left(\frac{\pi Q \cos\{Q(\pi-\phi)\}}{\sin\pi Q} - 1 \right) \qquad (10)$$

in which the first term clearly dominates.

Since there appears to be no corresponding expression for the sinkϕ series we invoke

the statistical nature of the errors by putting g = f and assuming the same contribution from both sine and cosine terms, adding these in quadrature. Putting $\cos\{Q(\pi-\phi)\} = 1$ then gives the peak closed-orbit amplitude:

$$\hat{z} \approx \frac{f \cdot \pi Q}{\sqrt{2} \sin\pi Q}$$

The amplitude of the k^{th} harmonic in (9) is:

$$z_k = \frac{\sqrt{2} \cdot f Q^2}{Q^2-k^2}$$

and hence the ratio becomes:

$$\frac{z_k}{\hat{z}} = \frac{2Q \sin\pi Q}{\pi(Q^2-k^2)} \qquad (11)$$

Evaluated for $Q_v = 22.15$ and k = 34 (∿ 15 GeV)

$$\frac{z_k}{\hat{z}} = 9.62 \times 10^{-3}$$

which, for a harmonic amplitude $z_k = 0.021$ mm would require a peak closed-orbit distortion of 2.2 mm or less. This estimate is, however, probably pessimistic for the following reasons.

The Fourier expansion of the closed orbit has been made in terms of the betatron phase variable ϕ in the usual way. In fact, what really matters in the present context is the spin precession phase angle, which advances only along the bending arcs and is stationary in the straight sections. Thus the precession resonance corresponds to a higher betatron harmonic number than the 34 assumed. A rough estimate of this correction can be made by taking the ratio of the Q of the whole machine to that of the arcs only. This leads to an effective k of about 45 and a peak c.o amplitude of ∿ 5 mm instead of 2.2 mm. A more detailed harmonic analysis would be necessary to improve the reliability of these estimates.

Even with substantially larger closed-orbit deviations a measurable degree of polarisation should be present. This leaves the possibility of correcting the offending harmonics using measurements of the polarisation as criteria. Such a procedure will require a rather sensitive polarimeter, with a response time preferably in the range of a few tens of seconds. It is evident from Eq.(8) that the 4^{th} power in the resonant denominator causes the sensitivity to adjacent harmonics to fall off rapidly. Consequently, the correction of two harmonics should be enough to bring about a considerable improvement.

4. Beam-beam depolarising effects

These arise for essentially the same reasons as those which limit the beam lifetime, namely, the broadening of resonances due to the non-linear beam-beam forces. A paper by Kondratenko[6] discusses the problem in general terms but gives a formula useful only for weak-focusing machines. The detailed analysis for a specific strong-focusing storage ring is clearly non-trivial, but the author states a general principle that the beam-beam depolarisation scales similarly to the reduction in lifetime due to beam-beam effects. The same point was made by Derbenev in a private discussion. A recent evaluation[7] of beam-beam depolarisation effects for SPEAR and PEP seems to support this conclusion.

We may therefore reasonably suppose that operation of PETRA below the beam-beam limit at a somewhat reduced luminosity would maintain the polarisation.

5. Conclusions

It appears that there are good hopes of operating PETRA with polarised beams, at least up to 15 GeV. With a proper choice of energy, in steps of 440 MeV, it should be possible to avoid betatron resonances. Values of Q_z and Q_x differing by an integer, together with small fractional parts of Q, facilitate this choice.

For vertical closed-orbit distortions the situation may be more marginal, based on the traditional error statistics for regular machine lattices. It seems desirable to make harmonic analyses of a statistical ensemble of PETRA lattices, using the information available from the PETROS program. This would indicate whether or not any special harmonic correction schemes need to be invented. The upper energy limit for polarised beams is likely to be determined by the amplitude of the appropriate vertical closed-orbit harmonics. Normal c.o. correction schemes tend to reduce mainly the harmonic amplitudes nearest to Q and could even enhance the higher harmonics if one were unlucky. There is thus a strong incentive to achieve the best possible uncorrected vertical closed orbit and to minimize the amount of correction required.

Beam-beam depolarisation in PETRA is not yet well quantified, but can most likely be kept sufficiently small by operating below the beam-beam limit at somewhat reduced luminosity.

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