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by

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1. THE EDUCATED GUESSES ON HADRON DYNAMICS

1.1. Quantum Chromodynamics

The opinions of a large group of theoreticians on the dynamics of hadrons seem to converge towards the following picture [(1)]: Similar to QED, hadron dynamics can be described by a renormalizable quantum field theory, in which coloured and flavoured quark fields are coupled to gluon fields according to the principles of local non-abelian gauge invariance. The Lagrangean of this field theory, called quantum chromodynamics (QCD) is

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\psi} \not{D} \psi - \bar{\psi} M \psi$$

with the notation

$\psi_{\alpha f C}(x)$: the coloured and flavoured quark field

α: the Dirac index of spin-1/2 quarks;

f = u,d,s,c: the flavour index describing the phenomenological structure of isospin, hypercharge and charm of hadrons in the quark model;

C = r,w,b: the colour index introduced first to solve the puzzle of quark statistics in the baryon states;

$A_\mu^C(x)$: the coloured gluon field;

C = 1, ..., 8 or $\bar{r}w$, $\bar{r}b$, ...; gluons did not yet show up very strongly in phenomenological discussions;

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(++) Lectures delivered by Hans Joos.

$$F_{\mu\nu}^C(x) = \partial_\mu A_\nu^C(x) - \partial_\nu A_\mu^C(x) - e f^{CC'C''} A_\mu^{C'}(x) A_\nu^{C''}(x),$$

$f^{CC'C''}$ structure constants of $SU(3)_{\text{colour}}$:

$$\left[\frac{\lambda^C}{2}, \frac{\lambda^{C'}}{2}\right] = i f^{CC'C''} \frac{\lambda^{C''}}{2};$$

$\alpha_s = e^2/4\pi$: strong fine structure constant;

$D_\mu = \partial_\mu + ie\frac{\lambda^C}{2} A_\mu^C$: $SU(3)_c$-covariant derivative;

M : Quark mass matrix.

Because of its renormalizability, such a theory is in principle reasonably well defined.

1.2. Asymptotic Freedom

There is one reason why people believe in QCD: it is asymptotically free [(2)]. This means the following:

Let $\tau(p_1,\ldots,p_n; e; m_i)$ be a generalized S-matrix element (Green's function) depending on the momenta p_i, the coupling constant e and the masses m_i. Then a change in the momentum scale can be expressed by a change in the coupling constant:

$$\tau(\lambda p_1,\ldots,\lambda p_n; e; m_i) = C(\lambda, e)\cdot\tau(p_1,\ldots,p_n; e(\lambda); m_i) + \ldots .$$

Asymptotic freedom is defined as $e(\lambda) \to 0$, for $\lambda \to \infty$. (In QCD we have $\alpha_s(\lambda) \approx \alpha_s\left(1 + \frac{25\alpha_s}{6\pi}\log\lambda\right)^{-1}$, $\alpha_s \ll 1$ [(2)].) Under these circumstances the theory at large momenta can be described by a theory with small coupling constant, i.e. by a theory close to the free theory. Therefore one considers asymptotic freedom as the correct formulation of the "successful" quark parton model. It implies relations like the well known formula for the asymptotic value of $\sigma_T(e^+e^- \to \text{hadrons})$, approximate scaling in deep inelastic lepton scattering, etc. [(3)].

1.3. "Infrared Slavery"

The most striking puzzle of the quark model, namely the absence of free quarks, is supposedly solved by the unproven hypothesis of infrared instability or infrared slavery [4]. This means, in terms of the renormalization group technique, $e(\lambda) \to \infty$ for $\lambda \to 0$. At large distances we have the situation of strong coupling. Since our knowledge of strong coupling theory is not very profound, there are only speculations on the mechanism by which the regime of infrared slavery achieves quark confinement. There are two types of ideas:

(i) The "potential" between quarks mediated by the gluons is changed by infrared slavery, simply from a Coulomb type potential e^2/r for $r \to \infty$ to a "confining" potential $e^2(r)/r$, rising at infinite distances.

(ii) Strong coupling leads to a strongly polarized vacuum state describing another "phase" of matter. Phenomenologically this might be simulated by an essential modification of the Lagrangean $\mathcal{L}$. In particular the introduction of Higgs fields leads to a simple description of spontaneous symmetry breaking. The related field equations have classical soliton solutions, i.e. they describe field theoretical bound states like the vortex strings in a superconductor of the second kind. It has become evident that the ideas on infrared slavery are not yet very precise. Hence the corresponding properties of QED_2 in the infrared limit are particularly interesting.

1.4. Quark Confinement

$SU(3)_c$-symmetry and quark confinement are considered to be intrinsically connected: The $SU(3)_c$-gauge invariant QCD contains eight conserved currents of colour charges Q^c, which generate $SU(3)_c$-symmetry, $[Q^c, Q^{c'}] = i f^{cc'c''} Q^{c''}$.

In the coloured quark model, mesons and baryons are "colourless":

$$|\mathrm{Mes}\rangle \sim |r\bar{r}\rangle + |w\bar{w}\rangle + |b\bar{b}\rangle$$

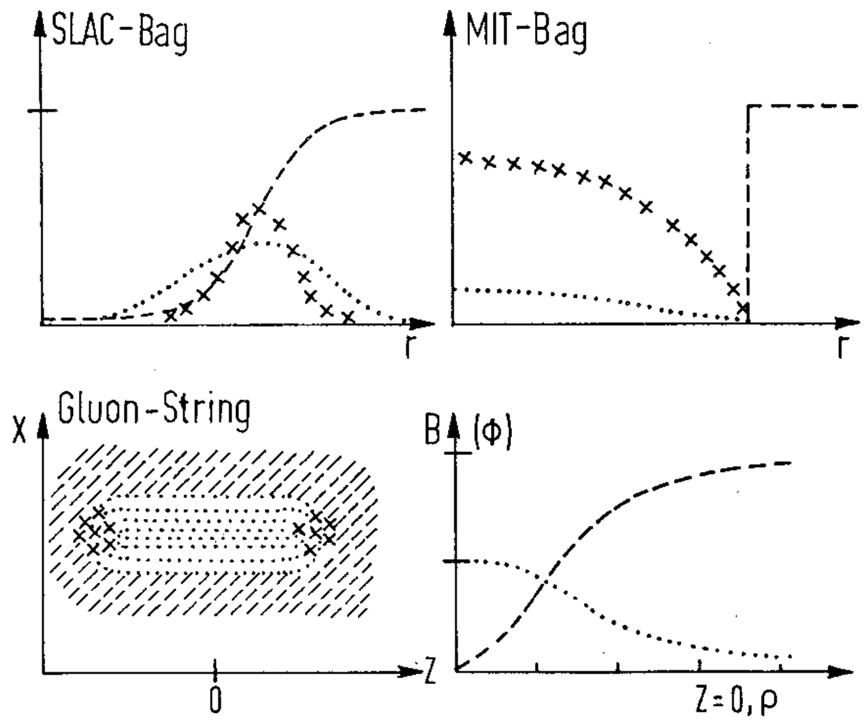
$$|\mathrm{Bar}\rangle \sim |rwb\rangle + |wbr\rangle + |brw\rangle - |wrb\rangle - |rbw\rangle - |bwr\rangle.$$

This means in terms of colour charges: $Q^c\,|\mathrm{hadrons}\rangle = 0$.

Hence the problem of quark confinement is related to the question: Is it possible that in a theory with coloured fields $\psi(x)$: $[\psi_c(x), Q^a] = i\left(\frac{\lambda^a}{2}\right)_{cc'} \psi_{c'}(x)$ all physical states have colour charge zero. K. Wilson [5] argued - with the help of a lattice approximation - that precisely this should happen in non-abelian gauge theories of the QCD type, and it was emphasized by A. Casher, J. Kogut and L. Susskind [6] that QED_2 describes such a situation.

1.5. Bags and Strings

We already mentioned above (1.3ii) the conjecture, that infrared slavery leads to the formation of field theoretical bound states imbedded in a strongly polarized vacuum. There are several phenomenological models which pursue this idea. We mention the MIT Bag [7], the SLAC bag [8], and several kinds of gluon strings [9]. As a reminder, the following pictures give some impression of the different field configurations.



$E(\Phi)$: Energy density, Φ: Higg's field, B: Gluon field, $|\psi|^2$: Quark density

In all these models the strongly polarized vacuum shows up as a constant Higgs field different from zero nearly everywhere. Quarks and gluons dig a hole into it. The pressure of the kinetic energy of the quarks balances the static pressure of the Higgs vacuum field. In those cases where the field theoretical bound states are stabilized by topological conservation laws, they are often called solitons.

1.6. Meson Dynamics

The excitation of bags or strings are supposed to describe the physics of the meson and baryon spectrum. One of the fascinating suggestions is, that the gluon strings might represent a physical realization of the strings introduced for the description of dual dynamics based on the Veneziano formula [10]. Of course, in the real world - and QCD is supposed to describe the real world - such strings can break, resonances can decay. Hence what we expect in QCD are "metastable solitons". Since we see in this notion a "contradictio in adjecto" we call them "insolitons". QED_2 is an excellent testing ground for insolitons (Sect. 2.5-2.6).

2. THE QED_2 AS A MODEL OF QCD

2.1. Quantum Electrodynamics in 1 + 1 Dimensions

It is the aim of the main part of these lectures to show how most of the expected dynamical properties of QCD are realized in 1 + 1 - dimensional quantum electrodynamics (QED_2). As mentioned at several occasions above, we hope that such a discussion helps to clarify some of the new notions of hadron dynamics and makes the hypothetical structure of QCD more plausible. In particular, the renormalization group technique [11] suggests the solution of QCD in three steps: 1. treat the small distance limit, 2. treat the long distance limit, 3. connect the two limits. We find it interesting that in QED_2 such a program can be performed rather explicitly. For all these reasons we consider QED_2 as quantum chromodynamics with one colour (and one flavour) degree of freedom in 1 + 1 dimensions.

The QED_2 is defined by the well-known quantized field equations

for "quark fields" $\psi(x)$ and gluon fields $A_\mu(x)$:

Maxwell Equation

$$\partial^\nu F_{\nu\mu}(x) = -e\,\bar{\psi}(x)\gamma_\mu\psi(x) \equiv -e\,j_\mu(x) \qquad (1)$$
$$F_{\nu\mu}(x) = \partial_\nu A_\mu(x) - \partial_\mu A_\nu(x) \equiv \varepsilon_{\nu\mu}F(x)$$

Dirac Equation

$$(i\gamma^\mu(\partial_\mu - ieA_\mu(x)) - m)\,\psi(x) = 0$$

They are derived from the gauge invariant Lagrangean density

$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi - m\bar{\psi}\psi$$

with the U(1)-covariant derivative $D^\mu = \partial^\mu - ieA^\mu$. These fields are considered in one space and one time dimension with the notation $x = (x^0, x^1) \equiv (t, y)$ for the coordinates, $(g^{\mu\nu}) = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ for the metric tensor and $(\varepsilon^{\mu\nu}) = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}$ for the antisymmetric (Levi Civita) tensor. The algebra of the γ-matrices $\gamma^0, \gamma^1, \gamma^5 = \gamma^0\gamma^1$ with $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$ forms the well-known algebra of Pauli matrices; we fix the representation as

$$\gamma^0 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},\ \gamma^1 = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix},\ \gamma^5 = \begin{pmatrix}-1 & 0\\ 0 & 1\end{pmatrix}.$$

Because of this simple structure of the Dirac algebra we can discuss many things explicitly in QED_2.

The definition of QED_2 has to be completed by the addition of quantization conditions. Since QED_2 is not a simple canonical system, quantization by canonical commutation relations is not straightforward. We shall not specify here one of the different quantization procedures, which in general depend on the choice of a special gauge. In our treatment the quantization will be developed together with the solution of the field equations.

The physical observables are invariant under the gauge transformations

$$\psi'(x) = e^{-i\lambda(x)}\psi(x), \quad A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu \lambda(x).$$

These gauge invariant quantities are the field strength F(x) and the "string operator"

$$T_{\alpha\beta}(x,x';\mathcal{L}) = \psi_\alpha(x)\, \exp\Big(ie\int_{x\ \mathcal{L}}^{x'} d\ell^\mu A_\mu(\ell)\Big)\, \bar\psi_\beta(x).$$

It is important for understanding quark confinement to describe confined quarks by gauge invariant quantities. In this context one associates a physical picture with the gauge invariant string operators $T(x,x';\mathcal{L})$. They are creation and annihilation operators of quark-antiquark states connected by gluon lines. String operators are particularly important in gauge invariant lattice approximations to QCD.

2.2. Asymptotic Freedom in QED_2

The QED_2 is superrenormalizable and hence asymptotically free. This depends on a simple dimensional argument: From the consideration of the Lagrangean one concludes that the fields etc. have the natural dimensions (in length ℓ, $\hbar = c = 1$):

$$\dim A_\mu = \ell^0, \quad \dim F_{\mu\nu} = \ell^{-1}, \quad \dim\psi = \ell^{-1/2},$$
$$\dim e = \ell^{-1}, \quad \dim m = \ell^{-1}.$$

Considering the Fourier transform of the quark propagator in a perturbation theoretic expansion with respect to the unrenormalized coupling constant $\alpha_s = e^2/4\pi$:

$$\mathcal{FT}\langle 0|T\bar\psi(x)\psi(x')|0\rangle = \tau(p,p';e;m) =$$

$$= \Big\{\frac{(2\pi)^{-1}}{\gamma p - m} + \alpha F^{(1)}(p,m) + \alpha^2 F^{(2)}(p,m) + \ldots\Big\}\,\delta(p-p'),$$

we find from simple dimensional counting $\dim F^{(n)}(p,m) = \ell^{1-2n}$, and hence Euler's differential equation:

$\left(m\frac{\partial}{\partial m} + \lambda\frac{\partial}{\partial\lambda} + (2n-1)\right)F^{(n)}(\lambda p, m) = 0$. The differential $m\frac{\partial}{\partial m}$ is related to a change in mass, corresponding to a substitution of a quark line ——— by a mass vertex ——x——, i.e. $(\gamma p - m)^{-1}$ by $(\gamma p - m)^{-1}\, m\, (\gamma p - m)^{-1}$. These terms drop more rapidly for large λ, therefore we get for the leading term in λ:

$$F^{(n)}(\lambda p, m) = \frac{1}{\lambda^{2n-1}} F^{(n)}(p,m) + \ldots .$$

We conclude that the leading term at small distances is the free one.

This short remark indicates that asymptotic freedom is a trivial property of theories with coupling constants of positive mass dimension, i.e. of super-renormalizable theories. However, in renormalizable theories with dimensionless coupling constants, terms of each order in the perturbation expansion contribute to the asymptotic limit. Therefore asymptotic freedom is a non-trivial property of renormalizable theories restricted mainly to non-abelian gauge theories [12]. It must be treated by similar, but more refined methods. For a discussion of these methods in QED_2 we refer to papers by R.J. Crewther et al. and M. Gomes et al. [13]. Here we only want to emphasize that QED_2, although in a trivial manner, may serve as an illustration of asymptotic freedom, too. We shall pursue the physical discussion of this property with the explicit solution below (Sect. 2.4, Eq. (17) f.)

2.3. The Infrared Limit of QED_2. Vacuum Structure

The infrared limit with strong interaction between the quarks cannot be treated so easily. But we learn from our dimensional consideration that m/e is the dimensionless parameter, which determines the structure of the theory. Since $m/e \to 0$ describes the strong interaction limit, we guess that the limiting case m = o shows all the characteristic features we expect in the infrared region such as strong vacuum polarization, spontaneous symmetry breaking, quark confinement etc. Fortunately we can solve this special case ("Schwinger model" [14]) explicitly. Since this solution enables us to study all the phenomena just mentioned, we have to go through some technical effort in order to get at it (§§ (i) - (vii)).

(i) The simple structure of the Dirac algebra in two dimensions allows the solution of the massless Dirac equation

$$i\gamma^\mu(\partial_\mu - ie A_\mu(x))\psi(x) = 0 \qquad (2)$$

for an arbitrary gluon field. For this we assume the Lorentz condition $\partial^\mu A_\mu(x) = 0$. It is a peculiarity of two dimensional space-time, that a divergenceless vector field $B^\mu(x)$, $\partial^\mu B_\mu(x) = 0$, can be expressed as the curl of a scalar $B(x)$

$$B^\mu(x) = \varepsilon^{\mu\nu}\partial_\nu B(x), \qquad B(x) = \int^x dx'^\mu \varepsilon_{\mu\nu} B^\nu(x').$$

Therefore, writing

$$A^\mu(x) = -\frac{\sqrt{\pi}}{e}\varepsilon^{\mu\nu}\partial_\nu A(x) \qquad (3)$$

and using the formula for the Dirac matrices in two dimensions

$$\gamma^\mu\gamma^5 = \varepsilon^{\mu\nu}\gamma_\nu, \qquad (4)$$

one easily verifies that

$$\psi(x) = e^{i\sqrt{\pi}\gamma^5 A(x)}\psi_0(x) \qquad (5)$$

is a solution of Equ. (2), iff

$$i\gamma^\mu\partial_\mu \psi_0(x) = 0. \qquad (6)$$

We add the following remark on the vector and the axial vector current of the free Dirac field

$$j_f^\mu(x) = \bar\psi_0(x)\gamma^\mu\psi_0(x), \qquad j_f^{\mu 5}(x) = \bar\psi_0(x)\gamma^\mu\gamma^5\psi_0(x).$$

They are related by $j_f^{\mu 5}(x) = \varepsilon^{\mu\nu} j_{f\nu}(x)$, (Eq. (4)). Since they are conserved, they satisfy the wave equation

$$\partial^\mu\partial_\mu j_f^\nu(x) = 0 \qquad (7)$$

$$(\text{i.e. } \partial^0\partial_0 j^1 = \partial^0\partial_0 j^{05} = -\partial^0\partial_1 j^{15} = -\partial_1\partial^0 j^0 = -\partial_1\partial^1 j^1)$$

(ii) The discussion of the quantized Maxwell equation

$$\partial^\nu F_{\nu\mu}(x) = -e j_\mu(x) \qquad (8)$$

requires a careful definition of $j_\mu(x)$ in terms of the interacting Dirac field $\psi(x)$. The gauge invariant definition of $j_\mu(x)$ is, according to Schwinger [14]:

$$j_\mu(x) = :\bar\psi(x)\gamma_\mu\psi(x): \equiv$$

$$\equiv -\lim_{\varepsilon\to 0} \mathrm{Trace}\,\gamma_\mu\left(\psi(x+\varepsilon)e^{ie\int_{x+\varepsilon}^{x} dt^\nu A_\nu(t)}\bar\psi(x) - \text{V.E.V.}\right)$$

$$\equiv -\lim_{\varepsilon\to 0} \mathrm{Trace}\,\gamma_\mu\left(T(x+\varepsilon,x) - \text{V.E.V.}\right).$$

Using the structure of the solution Eq. (5), he shows that the short distance behaviour of $\psi(x)\bar\psi(x')$ implies

$$j_\mu(x) = \frac{e}{\pi}A_\mu(x) + j_{f\mu}(x), \qquad (9)$$

where $j_f^\mu(x)$ is the free current of the quantized $\psi_0(x)$, which also satisfies Eq. (7). Inserting this $A_\mu(x)$ into the Maxwell equation (8) we get, as a necessary condition, the Klein-Gordon equation for the gauge invariant current:

$$(\partial^\nu \partial_\nu + \frac{e^2}{\pi}) j_\mu(x) = 0. \tag{10}$$

(iii) These considerations, i.e. Eqs. (5), (6) and (3), (7), (9), (10), show that the quark and gluon fields of the Schwinger model are related to free fields. Consequently a relativistic covariant solution in operator form [15] can be expressed by the free quantum fields

$$\phi(x): \quad (\partial^\mu \partial_\mu + \frac{e^2}{\pi})\phi(x) = 0, \; [\phi(x), \phi(x')]_- = i\Delta(x-x', \tfrac{e}{\sqrt{\pi}}), \tag{11a}$$

$$\psi_0(x): \quad i\gamma^\mu \partial_\mu \psi_0(x) = 0, \; [\psi_0(x), \bar{\psi}_0(x')]_+ = -\gamma^\mu \partial_\mu D(x-x'), \tag{11b}$$

$$\eta(x): \quad \partial^\mu \partial_\mu \eta(x) = 0, \; [\eta(x), \eta(x')]_- = -iD(x-x'), \tag{11c}$$

in the following form

$$A^\mu(x) = -\frac{\sqrt{\pi}}{e}\varepsilon^{\mu\nu}\partial_\nu(\phi(x)+\eta(x)) = -\frac{\sqrt{\pi}}{e}(\varepsilon^{\mu\nu}\partial_\nu\phi(x) + \partial^\mu\tilde{\eta}(x)), \tag{12a}$$

$$\psi(x) = :e^{i\sqrt{\pi}\gamma^5(\phi(x)+\eta(x))}: \psi_0(x), \tag{12b}$$

$$F(x) = -\frac{e}{\sqrt{\pi}}\phi(x), \tag{12c}$$

$$j^\mu(x) = -\frac{1}{\sqrt{\pi}}\varepsilon^{\mu\nu}\partial_\nu\phi(x) + j^\mu_f(x) - \frac{1}{\sqrt{\pi}}\partial^\mu\tilde{\eta}(x), \tag{12d}$$

(compare Eqs. (3), (5), (1), (9) respectively).

The canonical quantization of the free fields (11) defines the quantization of the Schwinger model. In particular quantum mechanical ordering of operator products will be expressed with the help of Wick ordering with respect to these free fields (: :). In the following we have to discuss some peculiarities of this procedure.

(iv) In the definition of the commutator function $\Delta(x,\mu) = \Delta^+(x,\mu) - \Delta^+(-x,\mu)$, $\Delta^+(x,\mu) = \frac{i}{2\pi}\int d^2p\, \delta(p^2-\mu^2)\theta(p^0)e^{ip\cdot x}$, the integral does not exist for $\mu = 0$. In this case it must be regularized by an infrared cutoff κ: $D^+(x) = \frac{1}{4\pi i}\log\kappa^2(-x^2 - i\varepsilon x^0)$. This characterizes the infrared structure of our model. It has also implications for the quantization of the scalar field in two dimensions [16].

(v) The field $\eta(x)$ is a pure gauge field, because we may put $\varepsilon^{\mu\nu}\partial_\nu\eta(x) = \partial^\mu\tilde{\eta}(x)$ in Eq. (12a).
(Notice $\eta(t,y) = \eta^+(t+y) + \eta^-(t-y)$, $\tilde{\eta} = \eta^+ - \eta^-$).
It is quantized with indefinite metric and infrared cutoff κ. This leads to local commutation relations

$$[\psi_\alpha(t,y), \psi_\beta(t,y')^+]_+ = Z_2\, \delta_{\alpha\beta}\, \delta(y-y'),$$

$$[A^\mu(x), A^\nu(x')]_- = -i(g^{\mu\nu}\Delta(x-x', \tfrac{e}{\sqrt{\pi}}) + \frac{\pi}{e^2}\partial^\mu\partial^\nu(\Delta(x-x', \tfrac{e}{\sqrt{\pi}}) - D(x-x'))),$$

$$[A^\mu(x), \psi(x')]_- = \frac{\pi}{e}\varepsilon^{\mu\nu}\partial_\nu(\Delta(x-x', \tfrac{e}{\sqrt{\pi}}) - D(x-x'))\gamma^5\psi(x'),$$

with a finite renormalization constant $Z_2 = \left(\frac{2\kappa}{e}\right)^{1/2}$ [20] and "smoothed" light cone behaviour.

(vi) The quantum fields Eq. (12) satisfy the Dirac equation (2) and

$$\partial^\nu F_{\nu\mu}(x) + e j_\mu(x) = e(j_{f\mu}(x) - \frac{1}{\sqrt{\pi}}\partial_\mu\tilde{\eta}(x)) \equiv e j_{L\mu}(x).$$

Therefore the Maxwell equation (8) is only satisfied for physical states. These are defined with the help of the positive frequency part $j^{(-)}_{L\mu}(x)$ of the "longitudinal" current by $j^{(-)}_{L\mu}(x)|phys\rangle = 0$.

(vii) From QED_4 we are familiar with the fact, that the field equations are only satisfied on physical states which form a sub-space of an indefinite Hilbert space defined by a subsidiary condition [17]. In view of this involved structure, the physical content of the solution (12) might be discussed best using the gauge invariant observables F(x) and $T(x,x';\mathcal{L})$, which leave the physical sub-space invariant. We calculate the string operator from the solution (12)

$$T_{\alpha\beta}(x,x';\mathcal{L}) = :e^{i\sqrt{\pi}\left(-\int_x^{x'} d\ell^\mu \epsilon_{\mu\nu}\partial^\nu \phi(\ell) - \alpha\phi(x) - \beta\phi(x')\right)}: N_{\alpha\beta}, \tag{13}$$

with

$$N_{\alpha\beta} = e^{i\sqrt{\pi}\,\chi^{(+)}_{\alpha\beta}(x,x')}\,\psi_{o\alpha}(x)\,\bar{\psi}_{o\beta}(x')\,e^{i\sqrt{\pi}\,\chi^{(-)}_{\alpha\beta}(x,x')},$$

$$\chi^{(\pm)}_{\alpha\beta}(x,x') = \tilde{\eta}^{(\pm)}(x) - \alpha\eta^{(\pm)}(x) - \tilde{\eta}^{(\pm)}(x') - \beta\eta^{(\pm)}(x'), \quad \alpha,\beta = \pm 1.$$

On physical states the use of the condition $\partial_\mu \tilde{\eta}^{(-)}|phys\rangle = \sqrt{\pi}\, j^{(-)}_{f\mu}|phys\rangle$ leads to

$$(N_{\alpha\beta}) = \begin{pmatrix} -\frac{\kappa}{2\pi}\sigma_+\sigma_-^+ & \frac{1}{2\pi i}\frac{1}{t+y-(t'+y')-i0} \\ \frac{1}{2\pi i}\frac{1}{t-y-(t'-y')-i0} & -\frac{\kappa}{2\pi}(\sigma_+\sigma_-^+)^+ \end{pmatrix} \tag{14}$$

with

$$\sigma_\alpha(x) = \left(\frac{2\pi}{\kappa}\right)^{1/2} e^{i\frac{\pi}{2}(Q+Q^5)}\, e^{i\sqrt{\pi}\left(j_f(x)-\alpha j_f^5(x)\right)^{(+)}}\,\psi_{o\alpha}(x)\, e^{i\sqrt{\pi}\left(j_f(x)-\alpha j_f^5(x)\right)^{(-)}}, \tag{15}$$

where Q, Q^5 denote the free charges and $j_f(x)$, $j_f^5(x)$ the current potentials of the free current $j_{f\mu}(x)$ and free axial current $j^5_{f\mu}(x)$ respectively: $j_{f\mu}(x) = \frac{1}{\sqrt{\pi}}\partial_\mu j_f(x)$ (see 2.3 (i)). In a weak sense, the appropriately smeared integral in (15) converges towards

an operator σ with the properties [18]:

$$\sigma_\alpha \sigma_\alpha^+ = 1, \quad [\sigma_+, \sigma_-] = 0, \tag{16a}$$

$$[\sigma_\alpha, P_\mu] = 0, \tag{16b}$$

$$[\sigma_\alpha, \phi(x)] = 0, \tag{16c}$$

$$[\sigma_\alpha, Q] = \sigma_\alpha, \quad [\sigma_\alpha, Q^5] = \gamma^5_{\alpha\beta}\sigma_\beta. \tag{16d}$$

This concludes the technical part on the solution of the Schwinger model. For the discussion of the physics it is important that the observables depend only on the massive free field ϕ, indicating quark confinement, and on the operators σ_α, describing vacuum polarization and spontaneous symmetry breaking.

Let us first discuss the physics related to the σ-fields [19]. The operators σ_α are independent of x (Eq. (16b)). They generate an infinite dimensional vacuum space $\mathcal{H}_o = \{\sigma_+^{n_+}\sigma_-^{n_-}|o\rangle\}$: It is: $P_\mu \sigma_+^{n_+}\sigma_-^{n_-}|o\rangle = 0$ because of Eq. (16b). The eigenstates of σ_α in $\mathcal{H}_o$

$$\sigma_\alpha |\theta_+\theta_-\rangle = e^{i\theta_\alpha}|\theta_+\theta_-\rangle$$

form an orthonormal base. Since $[\sigma_\alpha, \phi(x)] = 0$, all states $T(x,x';\mathcal{L})|\theta_+\theta_-\rangle$, $F(x)|\theta_+\theta_-\rangle$ belong to the same eigenvalues of σ_α. As a consequence of Eq. (16d), these base transforms under the U(1) x U(1) group generated by Q and Q^5 like

$$e^{iQ\alpha}|\theta_+\theta_-\rangle = |\theta_+ + \alpha, \theta_- + \alpha\rangle,$$

$$e^{iQ^5\beta}|\theta_+\theta_-\rangle = |\theta_+ - \beta, \theta_- + \beta\rangle.$$

The degeneracy of the vacuum indicates strong vacuum polarization. Since the ground state (vacuum), belonging to a pure (irreducible, θ_+,θ_--fixed) representation of the observables, is not invariant under the

U(1) x U(1)-group, this symmetry is spontaneously broken. According to Eq. (14), the observables depend only on $\theta = \theta_+ - \theta_-$. Therefore they are invariant under phase transformations $e^{iQ\alpha}$. However, the parameter θ, appearing in the chiral non-invariant part of the string operator, is changed by $e^{iQ^5\beta}$. Hence in a perturbed dynamics depending on such an observable, e.g. QED_2 with a mass term, the vacuum polarization measured by θ influences strongly the dynamical structure (sec. 2.5).

2.4. Quark Confinement in QED_2

Where are the quarks in the solution of the Schwinger model?

All physical states are generated by the application of the gauge invariant observables (2.3 (vii)) on the vacuum $|\theta_+\theta_->$; therefore they consist of the free particles with mass $\mu_0 = e/\sqrt{\pi}$ described by the field $\phi(x)$. Since we have $[Q^c, \phi(x')] = [\int j^0(t,y)\,dy, \phi(x')] = 0$, these particles are colourless. Furthermore, their field is derived from a quark-antiquark ($q\bar{q}$)-expression (see Eq. (12d))

$$\partial^\nu \phi(x) = -\sqrt{\pi}\,\varepsilon^{\nu\mu} :\bar{\psi}(x)\gamma_\mu \psi(x):$$

hence they represent mesons in our model, which we will call "ϕ-mesons" in the following. Since all particles are colourless, we have complete quark confinement. In order to get a dynamical explanation of this phenomenon, we consider a special gauge invariant state of a quark-antiquark linked by a gluon line: $|\psi_t(y,y')> = T(t,y;t,y')|0>$ and calculate the energy expectation value $<\psi_t(y,y')|H\,\psi_t(y,y')>/<\psi_t(y,y')|\psi_t(y,y')> \sim |y-y'|$. According to the Coulomb law in one space dimension, one needs infinite energy to separate q and $\bar{q}$ to infinite distances. A single quark with its gluon field has infinite energy! On the other hand, a "naked" quark without its gluon field is not a physical state. It was shown by Coleman et al.[20] that in the massive QED_2 confinement occurs also in this way.

Because of this simple explanation, one might believe that this type of quark confinement is a typical 1+1-dimensional effect. Even if

this should be true, it is important to have a relativistic, field theoretical model, in which infinitely strong, "confining" forces between quarks are compatible with Einstein causality, and short range interactions between hadrons[21] - like the ϕ-mesons in massive QED_2. In addition there are indications from lattice approximations of 3 + 1 - dimensional QCD, that confinement happens there in a similar way.

Where are the quarks? Our theory is asymptotically free, hence we can observe the quarks as partons. A. Casher et al.[22] recommend to consider as an example the correlation function of the scalar density $s(x) = :\bar{\psi}(x)\psi(x):$ which follows from Eq. (13):

$$i <0|T\,s(x)\,s(0)|0> = \frac{i}{2\pi^2}\left(\frac{\mu_0}{2e^{\Gamma'(1)}}\right)^2 e^{4\pi i \Delta_F(x,\mu_0)} + \ldots$$
$$= \frac{-i}{2\pi^2}\,\frac{1}{x^2 - i0} + \ldots \quad \text{for } x^2 \to 0 \tag{17}$$

It shows for $x^2 \to 0$ the parton behaviour.

From expression (17) they gain an intuitive picture for the process $e^+e^- \to$ hadrons. In the first moment the pair, created by the annihilation of the pointlike (scalar!) photon, propagates freely. Then the field between them strongly polarizes the vacuum. Finally the propagating dipole field catches the original pair, and the polarization charge annihilates it. Thus the strong polarizability of the vacuum prevents the appearance of free charges.

2.5. Bag Approximation in QED_2

Now we pursue further the essential dynamical pattern of our model with $m \neq 0$. Does the vacuum polarization induce the formation of field theoretical bound states?

For such an investigation, we use the string operator and the gluon field of the exact solution of the Schwinger model (Eqs. (12c),(13)) as an Ansatz to calculate

$$\mathcal{H} = :\tfrac{1}{2} F^2(x): + i :\bar\psi(x)\gamma^\nu D^\nu \psi(x) \quad + m\ \bar\psi(x)\psi(x): \ =$$

$$= :\tfrac{1}{2}\mu_0^2 \phi^2(x): - \lim_{\varepsilon\to 0} \mathrm{Trace}\,(i\gamma^\nu \partial_\varepsilon^\nu \{T(x+\varepsilon,x) - V.E.V.\} + m\ T(x+\varepsilon,x))$$

in the massive case. This operation transforms QED_2 into a canonical system defined by the Hamilton operator [20]

$$H = :\int dy \left[\tfrac{1}{2}\left(\mu_0^2\phi^2 + \left(\tfrac{\partial\phi}{\partial t}\right)^2 + \left(\tfrac{\partial\phi}{\partial y}\right)^2\right) + \tfrac{em}{2\pi}\cos(2\sqrt{\pi}\phi - \theta)\right]:$$

$$= \qquad H_0 \qquad + \qquad H_I \ , \tag{18}$$

$$\left[\tfrac{\partial\phi}{\partial t}(t,y),\ \phi(t,y')\right] = -i\,\delta(y-y'),$$

which is particularly suited to treat the limit of strong coupling $e^2 \gg m^2$. In the following we restrict our discussion to this limit, where H_I is small compared to H_0. Physically this Hamiltonian describes the interaction of the ϕ -mesons via H_I. The important influence of the vacuum polarization on the ϕ -meson dynamics is expressed by the phase θ. What happens in a state with a fixed energy in the limit $e^2 \to \infty$? There are very heavy particles with a mass $\sim e/\sqrt{\pi}$, moving slowly. Therefore we may neglect in a first approximation many-particle collisions and we may use non-relativistic approximations. This leads to the following approximations [23]:

(i) restriction to 2-particle collisions: this simplifies H to

$$H' = :\int dy \left[\tfrac{1}{2}\left(\mu^2\phi^2 + \left(\tfrac{\partial\phi}{\partial t}\right)^2 + \left(\tfrac{\partial\phi}{\partial y}\right)^2\right) - \tfrac{\lambda}{4}\phi^4\right]: ,$$

$$\mu^2 = \mu_0^2\left(1 \pm \tfrac{2\pi m}{e}\right), \qquad \lambda = \pm\tfrac{4\pi}{3}\, em\ ; \tag{19}$$

We restrict ourselves to the two characteristic phases $\theta = \pi, 0$, for which vacuum polarization leads to attractive or repulsive forces between the ϕ -mesons, respectively.

(ii) non-relativistic approximation, i.e. particle number conservation and non-relativistic kinematics:
We use the decomposition of $\phi(x) = \phi^{(+)}(x) + \phi^{(-)}(x)$ with non-relativistic kinematics

$$\phi^{(+)}(x) = (\phi^{(-)}(x))^+,$$

$$\phi^{(-)}(x) = \frac{1}{\sqrt{2\mu}}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dk\, b(k)\, e^{-i\left(\left(\mu + \frac{k^2}{2\mu}\right)t - ky\right)} \equiv \frac{\varphi(x)}{\sqrt{2\mu}},$$

$$[b(k),\ b(k')^+] = \delta(k-k'),$$

in order to perform the n.r. limit in H':

$$H_{n.r.} = \int_{-\infty}^{\infty} dk\, b(k)^+\left(\mu + \frac{k^2}{2\mu}\right) b(k) - \tag{20}$$

$$- \frac{3\lambda}{16\pi\mu^2}\int dk_1\, dk_2\, dk_1'\, dk_2'\, b(k_1)^+ b(k_2)^+ \delta(k_1 + k_2 - k_1' - k_2')\, b(k_1')\, b(k_2').$$

In this approximation, we get from Heisenberg's equation of motion

$$i\frac{\partial}{\partial t}\varphi(t,y) = [\varphi(t,y),\ H_{n.r.}] \equiv \tag{21}$$

$$\equiv \left(\mu - \frac{1}{2\mu}\frac{\partial^2}{\partial y^2}\right)\varphi(t,y) - \frac{3\lambda}{4\mu^2}(\varphi^+\varphi\varphi)(t,y),$$

i.e. the fields φ and φ^+ are solutions of the so-called non-linear Schrödinger equation. It is known, that for $\lambda > 0$ the classical non-linear Schrödinger equation has a soliton solution [24].

In our model we have soliton formation in the strongly polarized attractive vacuum phase. Therefore it also shows this important aspect of the possible dynamical pattern of QCD.

Let us give some details of this field theoretical bound state[25]. Direct calculation shows, that

$$\varphi(t,y) = \sqrt{\frac{8\mu^2}{3\lambda}}\,|\eta|\,\frac{e^{-i(\eta^2-\mu)t}}{\cosh\sqrt{2\mu}\,\eta y}\,, \quad \eta \in \mathbb{R},$$

is a solution of Eq. (21), i.e. $|\varphi(t,y)|$ has a bag like shape. Hence $\varphi(t,y)$ is a non-dissipative, non singular solution of finite energy: a soliton. Of course, this solution can be boosted according to Galilei invariance of the non-linear Schrödinger equation. Since it is a periodic solution in time, one may quantize it according to the WKB or Bohr-Sommerfeld method. The result for the energy of the stationary states is

$$\epsilon_n = -\frac{3}{128}\frac{\lambda^2}{\mu^3}n^3 + \mu n\,, \tag{22}$$

$$\eta_n = \frac{3}{\sqrt{32}}\frac{\lambda}{\mu^{3/2}}n\,, \quad n = 1,2,3,\dots\,.$$

We shall elaborate on the physical content of this soliton solution in the following section.

2.6. Remarks on Meson Dynamics

In the discussion of QCD one usually pays most attention on the consideration of the fundamental questions like quark confinement etc. However, if one considers QCD as a complete theory of hadrons one should also be able to derive the actual dynamics of mesons and baryons from the fundamental Lagrangean. In the final section we want to give an example which illustrates that QED_2 is also an inspiring guide to this type of problems.

We have already seen in Sect. 2.5 that the non-relativistic approximation of the "bosonized" Hamiltonian (Eq. (18)) is a good starting point to find a "trajectory" of bag excitations. In order to consider this description of the meson resonances as zero-order approximation of a consistent relativistic perturbation theory, we regard these bound states in a more conventional quantum mechanical frame work. We remember that $H_{n.r.}$ (Eq. (20)) conserves the particle number n. Therefore we consider the Schrödinger equation $H\psi = E\psi$ in a sub-space of fixed n:

$$\left(-\frac{1}{2\mu}\sum_{i=1}^{n}\frac{\partial^2}{\partial y_i^2} + n\mu - \frac{3\lambda}{4\mu^2}\sum_{k<l}\delta(y_k - y_l)\right)\psi(y_1,\dots,y_n) = \epsilon_n\psi(y_1,\dots,y_n).$$

It describes the interaction of n non-relativistic mesons by a pointlike potential. We know[26] that there is precisely one bound state for each n, to which belongs the energy

$$\epsilon_n = \frac{-3\lambda^2}{128\mu^3}n(n^2-1) + n\mu + \frac{p^2}{2n\mu} \tag{23}$$

and the wave function

$$\psi(y_1,\dots,y_n) = N_n\, e^{ip\frac{y_1+\dots+y_n}{n}}\, e^{-\frac{3\lambda}{8\mu}\sum_{k<l}|y_k - y_l|}\,. \tag{24}$$

The exact binding energy of Eq. (23) agrees with that gained from semiclassical quantization of the soliton (Eq. (22)) in the leading order of n, as is to be expected for a WKB-approximation. Now what is the significance of this non-relativistic Schrödinger amplitude in comparison with a relativistic Bethe-Salpeter-amplitude:

$$\chi_n^{(i)}(p_1,\dots,p_i;P)\,\delta\Big(\sum_{k=1}^{i}p_k - P\Big) = \int dx_1\dots dx_i\, e^{i(p_1x_1+\dots+p_ix_i)}\cdot\langle 0|T\phi(x_1)\dots\phi(x_i)|{}^{n}_{P}\rangle \tag{25}$$

Because of particle number conservation, we consider first the leading amplitude i = n. For this case, we have the following answer: The Fourier transform of the Schrödinger amplitude Eq. (24) is the non-relativistic approximation of the B.S.-amplitudes

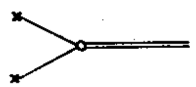
$$\chi_2^{(2)}(p_1,p_2;P) = \frac{1}{(p_1^2-\mu^2)(p_2^2-\mu^2)}$$

$$\chi_3^{(3)}(p_1,p_2,p_3;P) = \frac{1}{\prod_{j=1}^{3}(p_j^2-\mu^2)}\left\{\frac{1}{(p_1+p_2)^2-\mu_2^2} + \text{cyclic}\right\} \qquad (26)$$

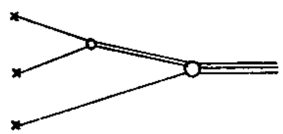
$$\chi_4^{(4)}(p_1,\dots,p_4;P) = \frac{1}{\prod_{j=1}^{4}(p_j^2-\mu^2)}\left\{\sum \frac{4}{((p_i+p_j)^2-\mu_2^2)((p_k+p_l)^2-\mu_2^2)} + \right.$$
$$\left. + \sum \frac{9}{((p_i+p_j)^2-\mu_2^2)((p_i+p_j+p_k)^2-\mu_3^2)}\right\}.$$

For these amplitudes the approximation was done explicitly by setting equal relative times and using non-relativistic kinematics. The result might be expressed by Feynman graphs:

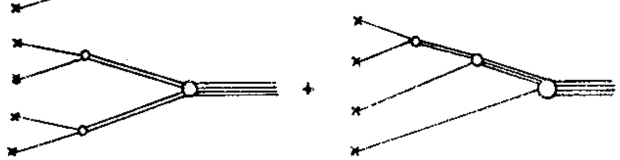
n = 2 :



n = 3 :

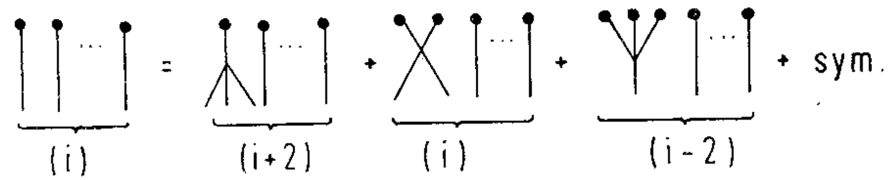


n = 4 :

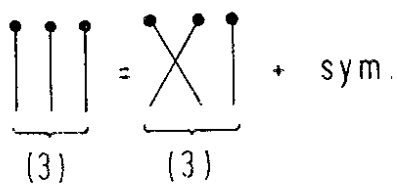


where $\Big\}\, n$ denotes the propagator of the n-particle bound state with mass μ_n corresponding to Eq. (23). The graphical representation suggests the generalization of our result to arbitrary n.

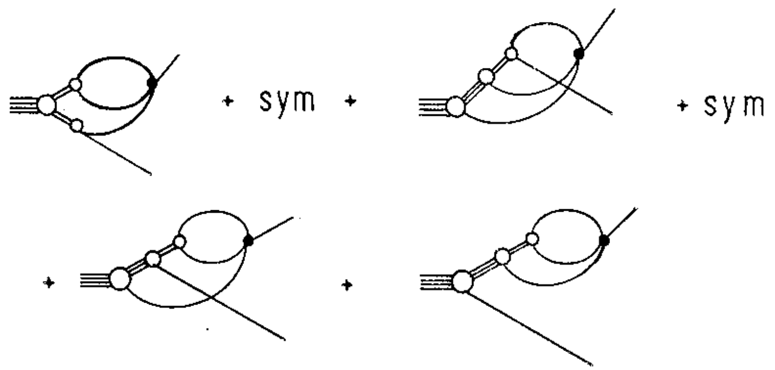
Guided by non-relativistic considerations we found the existence of a series of bound states in the attractive vacuum phase of QED_2. These might be either described semiclassically as excitations of a bag-like structure, or by B.S. amplitudes in zero-width zero-range approximation like in Eq. (26). At this point we should remember, that we considered up to now only an approximation for small λ and small binding energies, where relativistic effects like particle decays are neglected. Energetically the states for n = 4,5,... may decay in an appropriate number of ϕ -mesons and stable bound states (n = 2,3). A more complete treatment should include these effects. In the following we want to add a short hint as to how one may go on. The general B.S. amplitudes (Eq. (25)) are solutions of a coupled system of integral equations [27], which in ϕ^4-approximation (Eq. (19)) has the graphical form



The amplitudes Eq. (26) might be recovered as approximate solutions of (example n = 3)



Iteration leads to first-order decay amplitudes of the type (example $\phi^{****} \to 2\phi$):



Let us summarize our remarks on the series of meson resonances in QED_2: They may be considered as the excitations of an instable soliton state. We suggest to call such a configuration an "insoliton".

With this excursion to "phenomenological" applications we finish our review of QED_2 as a model of chromodynamics. We hopefully were able to demonstrate that QED_2 supplies a valuable guidance to the structure of quark dynamics and that it is a useful testing ground for the new concepts going with it.

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