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Quark and Colour Confinement through Dynamical Higgs Mechanism

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Abstract: We suggest that quarks and colour can be confined through a dynamical Higgs mechanism. Two possibilities can be thought of. 1. Quarks may combine to scalar diquark Cooper pairs which then condense into the vacuum. Quarks and gluons may then screen their colour by combining with the Cooper pairs to form physical particles. 2. Instead of quarks, the Higgs scalars may form out of gluons, and screen the colour charge of gluons (and anything with zero triality) by a "local Higgs mechanism". In this case, long range forces between quarks are expected to persist.

Sect. 2-4 are based on a lecture presented at the topical conference on Quantum Chromodynamics at Chania (Crete), June 1977.

1. Introduction

It seems attractive to believe that strong interaction dynamics is described by a nonabelian gauge theory of quarks and gluons with colour group SU(3) [1]. The colour group extends to a gauge group of the 2^{nd} kind due to minimal coupling of an octet of flavorless gluons, while the quarks are colour triplets with an unspecified number of flavors. The Lagrangean density

$$\mathcal{L} = -\tfrac{1}{2}\,\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} - i\bar{q}^i \gamma_\mu \mathcal{D}^\mu q^i - M^{ij}\bar{q}^i q^j \,, \tag{1.1}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - if[A_\mu, A_\nu]_- \;;\quad \mathcal{D}_\mu = \partial_\mu - ifA_\mu \;;\quad A_\mu = \tfrac{1}{2}\lambda^c A^c_\mu \,.$$

It is hoped that the Lagrangean (1.1) leads to quark and colour confinement, so that all physical states are colour singlets. If so, there should exist an effective Lagrangean which describes the interaction of the low mass physical states at not too high energies, and which involves only colour singlet fields. (It would have an UV-cutoff and could be quite complicated but local to the extent that the cutoff permits). The problem is to derive it from (1.1). More generally, the question arises how one can produce a Lagrangean involving only colour singlet fields from a gauge field theory such as e.g. (1.1)

Notation: We use letters a, b (= 1,2,3), c (= 1...8) for colour indices, i.j, ... for flavor indices, $\alpha, \beta \ldots$ for spinor indices, and μ,ν for Lorentz-vector indices. Minkowski space metric is -+++. Quark fields $q^i = (q^{ai}_\alpha)$ while A^c_μ are the gluon potentials. Summation convention is understood; often we use vector notation in 3-dimensional colour space. The bracket [a,b] will stand for antisymmetrization in indices a,b. λ^c are Gell Mann's 3x3 matrices, f the coupling constant. For references to the Higgs-mechanism see ref. [2].

In this paper we call attention to the fact that a "complete Higgs mechanism" with no surviving massless vector mesons achieves just that, at least if a further condition is met which guarantees that the charges of the Higgs scalars in the theory match with the charges of the other fields. We demonstrate this in Sec. 2 at the example of an SU(2)-gauge theory with a quark doublet and a complex doublet of Higgs scalars on a lattice. This model possesses a global SU(2) flavor symmetry in addition to the colour SU(2) symmetry, a fact which appears to have been overlooked in previous discussions [2].
We present heuristic arguments that there is quark and colour confinement in this model if the Higgs mechanism takes place. All states are colourless, and none of them has the quantum numbers of a quark, even apart from colour. In particular, the "quark field in the unitary gauge" creates states which do not have the quantum numbers of a quark. Intuitively it is easy to understand what happens: The quarks screen their colour by combining with Higgs scalars. But since the Higgs scalars carry flavor, also the falvor will be changed in the process.

The lesson from the model cannot be carried over in a straightforward fashion to the QCD Lagrangean (1.1) because that Lagrangean has no Higgs scalars in it. The next simple possiblity is then to imagine that Higgs scalars are formed dynamically.

As a first possibility we imagine the formation of scalar diquark Cooper pairs [3] in a colour 3^*- state. They could be bound by Coulomb forces. these are attractive because the diquark consists of quarks of different colours. The Higgs mechanism would then provide for condensation of such diquarks and antidiquarks into the vacuum. Quarks could screen their colour for instance by combining with a diquark Cooper pair to a physical baryon. Similarly the gluons (or string bits) could screen their charges to form a flavor SU(3) octet of gluonoids, presumably they are all multiparticle states since at least four quarks are involved. Assuming that baryon number conservation is not spontaneously broken in the process one will again have quark and colour confinement in the same sense as for the earlier model.

Note that it is essential that the colourgroup is SU(3). The mechanism would not work with SU(2) or U(1) colour group because the charges of the Cooper pairs would not match then with the charges of the quarks in such a way that a local screening of colours is possible. In other words, the Kronecker product of $3 \otimes 3 \otimes 3$ of three fundamental representations of SU(3) contains the singlet, but the analogous statement is not true for SU(2) or U(1). It is also essential that there are at least three flavors, otherwise there are not enough Higgs scalars to prevent surviving massless gluons.

The condition of no spontaneous baryon number nonconservation turns out not to be trivial though.

There are further problems with this scheme in that it does apparently not explain some outstanding features of the real world in a natural and obvious way:

We do not know whether there are strings hidden somewhere, and there is

thus so far no explanation of linearly rising Regge trajectories.*) A mildly encouraging sign is that the SU(2) model appears to show signs of Reggeization in the $I = 1$ channel [4].

Also we have no natural explanation of the absence of exotics. Why are exotic objects which involve more than one Cooper pair multiparticle configurations and not resonances? Besides one might fear strong violation of SU(6) for baryons: The octet can be made from a quark and a Cooper pair, but to make a decuplet requires flip of spin and flavor (UVW)-isospin inside the Cooper pair. We can only hope that this will not cost too much energy given that the gluon forces are flavor independent.

In conclusion we have not arrived at a fully satisfactory picture in this way. But the considerations show that the Higgs mechanism has potentialities that have not been recognized before.

One may hope that the Higgs mechanism is capable of generalizations that have not been found and explored yet. In particular, colour confinement may always be thought of as a Higgs phenomenon in the very wide sense alluded to at the beginning of this section.

In the last section we present some speculations what the hoped for generalization of the Higgs mechanism could look like. It is suggested to make coloured Higgs scalars out of gluons instead of quarks and to have the gluons screen their colour and acquire a mass through a

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*) The matter is complicated by the fact that the response to static and time dependent external electric fields may be very different as in superconductors.

"local Higgs mechanism", which bears some resemblance to the local phase transitions proposed by Nambu [5]. Such Higgs scalars cannot screen the charges of quarks locally, therefore one may speculate that long range forces between quarks persist, as is known to happen in the abelian 2-dimensional Higgs model for nonintegral external charges [6].

2. The SU(2) - gauge theory with Higgs doublet on a Euclidean lattice.

The theory lives on a Euclidean cubic lattice with lattice spacing a, vertices x and bonds $b = (x,\mu)$ joining nearest neighbor vertices x and $x+\hat{\mu}$. It involves the Euclidean fields (= random variables) for the

$$\text{gluons:} \quad U(b) = U(x,\mu) \in SU(2) \tag{2.1}$$

$$\text{quarks:} \quad q(x) = \begin{pmatrix} q_1(x) \\ q_2(x) \end{pmatrix}$$

$$\text{Higgs scalars} \quad \phi(x) = (\phi_1(x), \phi_2(x)), \quad \phi^+(x) = \begin{pmatrix} \phi_1^*(x) \\ \phi_2^*(x) \end{pmatrix}$$

Quarks and Higgs scalars are both colour doublets. The string bit variables are used in place of vector potentials in the continuum theory

$$U(x,\mu) \sim \exp i\int db^\mu A_\mu(b) \approx 1 + iaA_\mu(x) + \ldots, \quad A_\mu = f\Sigma\frac{\tau^a}{2}A_\mu^a \tag{2.2}$$

with Pauli matrices τ^a.

Under space time dependent gauge transformations $V(x) \in SU(2)$

$$\begin{aligned} U(x,\mu) &\mapsto V(x)U(x,\mu)V(x+\hat{\mu})^{-1} \\ q(x) &\longrightarrow V(x)q(x) \\ \phi(x) &\longrightarrow \phi(x)V(x)^{-1} \end{aligned} \tag{2.3}$$

The gauge invariant Euclidean Lagrangean density is

$$\begin{aligned} \mathcal{L}_E(x) = &- M\bar{q}(x)q(x) + K\sum_\mu \bar{q}(x)[1+\gamma_\mu]U(x,\mu)q(x+\hat{\mu}) \\ &- \tfrac{1}{2}\sum_\mu \{\phi(x)U(x,\mu)\phi^+(x+\hat{\mu}) + h.c.\} + \phi\phi^+(x) - \mathcal{V}(\phi\phi^+(x)) \\ &- \text{plaquette term} \qquad \equiv \mathcal{L}_1(x) - \mathcal{V}(\phi\phi^+(x)) \end{aligned} \tag{2.4}$$

The plaquette term is the lattice version of $\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}$, it has the usual $\mathrm{tr}[UUUU]$ - form [14].

Observables are functions

$$F = F\left(\{\phi(x), q(x), U(b)\}\right) \tag{2.5}$$

of the random variables. Their expectation values

$$\begin{aligned} \langle F \rangle = Z^{-1}\int \prod_x d\phi(x)d\phi^+(x)\,dq(x)d\bar{q}(x) \prod_b dU(b) \\ \cdot F(\{\ldots\}) \exp\int_x \mathcal{L}_E(x) \qquad ; \quad \int_x \equiv a^4\sum_x \end{aligned} \tag{2.6}$$

dU is Haar measure on SU(2) and Z is determined by $\langle 1 \rangle = 1$.

Transformation to gauge invariant field variables.

We may write (cp. the early papers on Higgs mechanism cited in [2])

$$\begin{aligned} \phi(x) &= \hat{\phi}(x)L[\phi(x)]^{-1} \\ \text{with } L[\ldots] &\in SU(2), \quad \hat{\phi}(x) = (\rho(x), 0), \quad \rho(x) \geq 0. \end{aligned} \tag{2.7}$$

This defines $L[\phi(x)]$ uniquely for generic $\phi(x)$ and determines $\rho(x) = [\phi\phi^+(x)]^{1/2}$. Uniqueness holds because the little group of $\hat{\phi}$ in SU(2) is trivial,

$$\hat{\phi}V = \hat{\phi}, \quad V \in SU(2) \text{ implies } V = 1, \quad \text{if } \hat{\phi} \neq 0 \tag{2.8}$$

We may therefore define

$$W(x,\mu) = L[\phi(x)]^{-1} U(x,\mu) L[\phi(x+\hat{\mu})]$$
$$\Psi(x) = L[\phi(x)]^{-1} q(x) \qquad (2.9)$$

Under gauge transformations, $L[\phi(x)] \rightarrow V(x) L[\phi(x)]$ by definition (2.7), therefore $W(x,\mu)$, $\Psi(x)$, $\rho(x)$ are gauge invariant.

Remark: W, Ψ are often referred to as gluon and quark fields in a particular (unitary gauge). We prefer to consider them as new, gauge invariant objects. There is no contradiction between both views: Also the mass is equal to the energy in a particular Lorentz frame and is at the same time a Lorentz invariant. Consistency comes from trivial transformation law under the little group in both cases. ∎

Explicitly, one finds

$$L[\phi] = \rho^{-1}\begin{pmatrix} \phi_1^* & -\phi_2 \\ \phi_2^* & \phi_1 \end{pmatrix} ; \qquad (2.10)$$

$$\Psi(x) = \rho^{-1}\begin{pmatrix} \phi q \\ \phi^{+}\varepsilon q \end{pmatrix} ; \quad W(x,\mu) = [\rho(x)\rho(x+\hat{\mu})]^{-1}\begin{pmatrix} \phi U \phi^{+} & \phi U \varepsilon^{-1}\phi^{+} \\ \phi^{+}\varepsilon U \phi^{+} & \phi^{+}\varepsilon U \varepsilon^{-1}\phi^{+} \end{pmatrix}$$

We have omitted arguments x and $x+\hat{\mu}$, for instance

$$\phi U \phi^{+} = \phi(x) U(x,\mu) \phi(x+\hat{\mu})^{+} \quad \text{etc.}$$

ε is the antisymmetric tensor in two dimensions, it transforms column vectors into row vectors. Nobody can object to making the (singular) transformation of variables $U, q, \phi \rightarrow W, \Psi, \rho, L$ under the integral in (2.5), this is just like going from Cartesian to polar coordiantes under an integral sign.

Doing so we find that $\mathcal{L}_E$ is a function of the local gauge invariants W, Ψ, ρ only.

$$\mathcal{L}_E(x) = \text{as (2.4), with } W, \Psi, (\rho, 0) \text{ substituted for } U, q, \phi . \qquad (2.11)$$

The L-integration may be performed. Taking account of the Jacobian yields

$$< F > = Z^{-1}\int \prod_x d\rho\, d\Psi\, d\bar{\Psi} \prod_b dW \; F_{ave}(\{\hat{\phi}, \Psi, W\}) \exp\int_x (\mathcal{L}_1 - \vartheta_{eff}) \qquad (2.12a)$$

where $\vartheta_{eff}(\rho^2) = \vartheta(\rho^2) - \frac{3}{2} a^{-4} \ln \rho^2$; $\hat{\phi} = (\rho, 0)$.

with

$$F_{ave}(\{\phi(x), q(x), U(x,\mu)\}) = \text{average of F over gauge group} \qquad (2.12b)$$
$$= \int \prod_x dV(x) F(\{\phi(x)V(x)^{-1}, V(x)q(x), V(x)U(x,\mu)V(x+\hat{\mu})^{-1}\}) .$$

More precisely, suppose that we are dealing with a finite lattice to begin with.

We admit arbitrary boundary conditions (except that they should preserve the Markov property to the extent that it is needed to have Osterwalder Schrader positivity [7]) in order not to rule out propagating colour charged states a priori. The boundary conditions are allowed to break gauge invariance, in particular there may be charges at the boundary. Eqs. (2.12) are correct as they stand if F does not depend on fields with arguments x, resp. b = (x,μ) in resp. touching the boundary of the lattice. Otherwise, they are true with the following modifications: Averaging in (2.12b) is only over V(x) for x not on the boundary (put V(x)=1 on the boundary), F_{ave} will continue to depend on $L[\phi(x)] \equiv L(x)$ for x on the boundary, and there will be extra integrations over $L(x)$ for x on the boundary in (2.12a).

Higgs mechanism:

For purposes of illustration it will suffice to consider a semiclassical approximation. Suppose $\mathcal{V}(\phi\phi^+) = \mathcal{V}(\rho^2)$ has its absolute minimum at $\rho^2 = \lambda^2 \neq 0$. Then the action $\int_x \mathcal{L}_E(x)$ has an isolated absolute maximum at

$$\rho(x)^2 \equiv \lambda^2 \quad , \quad W(x,\mu) \equiv 1 \quad , \quad \Psi(x) = 0 \qquad (2.13)$$

Expanding around the maximum up to quadratic terms (semiclassical approximation) gives a free field Lagrangean which would in the continuum limit on Minkowski space describe

particles	with interpolating fields
3 massive vector mesons	$[W(x,\mu) - \mathbb{1}]_{11} \sim \frac{a}{\lambda^2}\, \phi(x)[\partial_\mu + iA_\mu(x)]\phi^+(x)$ etc.
2 fermions	$\Psi(x) \sim \lambda^{-1}(\phi q(x)\, ,\, \phi^+ \varepsilon q(x))'$ (2.14)
1 massive scalar	$\rho(x) - \lambda \sim (2\lambda)^{-1} : \phi\phi^+(x) :$

There is freedom in the choice of interpolating fields, we have used this to approximate $\rho(x)$ by λ in the denominators. $'$ means transpose.

All these particles are colourless since they are created by local gauge invariant fields. It is also interesting to note that they all admit composite interpolating fields. This matches with the results of refs. [4] that the model seems to reggeize in the I = 1 channel (about I see below).

To avoid misunderstanding, let us make it clear what the issue is. We have worked with a Lagrangean without a gauge fixing

term (except possibly on the boundary). In such circumstances, the Hilbert space of physical states is positive definite and colour charged particles, if they exist, cannot be created by local operators. Therefore it does not suffice to inspect the fields in the theory, one must determine the particle content. If all the particles that there are can be created by gauge invariant fields, then none of them is coloured.*) In the semiclassical approximation, the particle content is clear. Note, however, that without Higgs mechanism there would be no gluonoid mass term in (2.11) and hence no isolated absolute maximum of $\int_x \mathcal{L}_E(x)$, semiclassical approximation would then be unjustified.

Remark: In view of (2.12a) it would seem equally natural to expand around the absolute minimum of $\mathcal{V}_{eff}(\rho^2)$. $\mathcal{V}_{eff}$ always has its minimum at $\rho \neq 0$. Hence in this approximation, the Higgs mechanism always takes place (for lattice spacing a > 0) independent of how V looks. It is however costumary to use integral representations [15]

$$\rho(x) \propto \int d\xi(x) d\xi^*(x) \exp -\tfrac{1}{\hbar}\, \xi^* \rho\, \xi(x) \qquad , \xi \text{ scalar Fermi field},$$

and to treat the effect of the functional determinant $\prod \rho(x)^3$ as radiative corrections which are neglected in the semiclassical approximation.

Flavor:

The model has an SU(2) flavor symmetry group besides the colour group, i.e.

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*) This is our definition of a colourless particle, and it should be true in any reasonable definition of a colourless particle. It implies that its physical states are invariant under global, space time independent, gauge transformations.

the global symmetry group of the Lagrangean (2.4) is SU(2) x SU(2) . Only the Higgs scalars are flavored, while quarks and gluons carry no flavor. This symmetry is present already if we omit the gauge fields: Consider ϕ as a real 4-component field, then global SO(4) - symmetry is obvious since only bilinears $\phi(x)\phi(y)^{+}$ appear in the Lagrangean. Since SU(2) x SU(2) is a covering of SO(4) we may consider it as the symmetry group just as well. Adding the flavorless gluons makes one of the SU(2)-ideals into a gauge symmetry of the 2nd kind (colour) while the second SU(2)-ideal is preserved as a global symmetry group.

To exhibit the transformation law, let us collect the Higgs fields in a 2 x 2 matrix

$$\Phi = \begin{pmatrix} \phi_1^* & -\phi_2 \\ \phi_2^* & \phi_1 \end{pmatrix} \qquad (2.15)$$

With this notation, $L[\phi(x)] = \rho(x)^{-1}\Phi(x)$. $\rho = [\frac{1}{2} \operatorname{tr} \Phi\Phi^{+}]^{1/2}$

The transformation laws under $(V_1, V_2) \in SU(2) \times SU(2)$ read then

$$\begin{array}{lll} \text{colour} & SU(2) & \Phi(x) \to V_1 \Phi(x),\ U(b) \to V_1 U(b) V_1^{-1},\ q(x) \to V_1 q(x) \\ \text{flavor} & SU(2) & \Phi(x) \to \Phi(x) V_2^{-1},\ U(b) \to U(b),\ q(x) \to q(x)\ . \end{array} \qquad (2.16)$$

The colour transformation law of Φ agrees with (2.3) since $\bar{V}_{11} = V_{22}$, $\bar{V}_{12} = -V_{21}$ for any $V = (V_{ab}) \in SU(2)$.

From this we deduce the transformation law of the gauge invariant fields under flavor SU(2).

$$W(b) \to V_2 W(b) V_2^{-1}\ ,\quad \Psi(x) \to V_2 \Psi(x)\ ,\quad \rho(x) \to \rho(x)\ . \qquad (2.17)$$

This gives us the flavor isospin I of the physical particles

$$\text{vector mesons: } I = 1,\ \text{fermions: } I = \frac{1}{2},\ \text{scalar } I = 0\ . \qquad (2.18)$$

We emphasize that despite the formal similarity of Eq. (2.17) with (2.3) the remaining global SU(2)-symmetry has nothing to do with colour but is a flavor symmetry which was there from the beginning in addition to colour symmetry.

We see from Eq. (2.18) that the physical particles have acquired flavor from the Higgs scalars, since they screened their colour by combining with Higgses, compare Eq. (2.14) for the interpolating fields.

Let us supply the demonstration that the Lagrangean (2.4) is indeed flavor symmetric. Since $W(b) \in SU(2)$ we may write $W(x,\mu) = \exp i a f \vec{\tau}\vec{B}_{\mu}(x)$ with $\vec{B}$ having isospin I = 1 . Then

$$\tfrac{1}{2}\, \phi(x) U(x,\mu) \phi^{+}(x+\hat{\mu}) + h.c. = \tfrac{1}{2}\rho(x)\rho(x+\hat{\mu})[W(x,\mu) + W(x,\mu)^{+}]_{11}$$
$$= \rho(x)\rho(x+\hat{\mu}) \cos af [\vec{B}_{\mu}(x)^2]^{1/2}\ .$$

since $(\vec{\tau}\vec{B})^2 = \vec{B}^2 \cdot \mathbb{1}$. The r.h.s. only depends on $\vec{B}^2$ and is therefore invariant. Alternatively, $[W + W^{+}]_{11} = \operatorname{tr} W$, cp. after Eq. (2.16)

Conclusion:

Lesson No. 1. The physical particles are colourless: Colour confinement

Lesson No. 2. The physical particles carry flavor acquired from the Higgses. There are no states with the quantum numbers of the quark (spin $\frac{1}{2}$, I = 0), since according to (2.18) half integral spin goes with half integral isospin: Quark confinement. The physical particles are created by composite interpolating fields (2.14).

Generalization of the model.

With an SU(2) gauge theory one cannot mimick the real world completely because it is not possible to make a colour singlet out of three quarks in the fundamental representation of colour SU(2). It is nevertheless instructive to consider the generalization of the model which is obtained by giving flavor to the quarks; the model then resembles the real world a little more. Thus, instead of one quark colour doublet we introduce two of them, q^1 and q^2. We group them into a 2 x 2 matrix

$$q = (q_a{}^i) \quad , \quad i = 1,2 \;\; (\text{flavor}) \qquad a = 1,2 \;\; (\text{colour})$$

and we continue to describe the scalars by the 2 x 2 matrix introduced in (2.15),

$$\Phi = \begin{pmatrix} \phi_1^* & -\phi_2 \\ \phi_2^* & \phi_1 \end{pmatrix} \equiv (\Phi_a{}^i)$$

In addition we have the string bit variables $U(b) \in SU(2)$ as before. The symmetry transformation laws are now

colour-SU(2) $\quad \Phi(x) \to V_1 \Phi(x) \; ; \; U(b) \to V_1 U(b) V_1^{-1} \; ; \; q(x) \to V_1 q(x)$.

flavor-SU(2) $\quad \Phi(x) \to \Phi(x) V_2^{-1} \; ; \; U(b) \to U(b) \; ; \; q(x) \to q(x) V_2^{-1}$

The Euclidean action may be taken to be

$$\mathcal{L}_E(x) = -M \operatorname{tr} \bar{q}(x) q(x) + \kappa \sum_\mu \operatorname{tr} \bar{q}(x)[1+\gamma_\mu] U(x,\mu) q(x+\hat{\mu})$$
$$-\tfrac{1}{2} \operatorname{tr} \Big\{ \sum_\mu \Phi^+(x) U(x,\mu) \Phi(x+\hat{\mu}) - \Phi^+(x)\Phi(x) \Big\} - \vartheta \left(\tfrac{1}{2} \operatorname{tr} \Phi^+ \Phi(x) \right)$$
$$- \lambda \operatorname{tr}(q_\alpha \Phi^+) \operatorname{tr}(\Phi \bar{q}_\alpha)$$

it is cleary invariant. The last term is included to break still higher symmetry. The analysis of this model proceeds in the same fashion as before and is left to the reader. One finds that the physical fermions will have integral flavor isospin while the quarks had half-integral isospin. Otherwise the conclusions are the same as before.

3. Higgs models with special features.

The model of Sec. 2 had properties

(A) No massless vector mesons survive the Higgs mechanism.

(B) The Lagrangean could be rewritten in terms of local gauge invariants.

We note that it is always possible to rewrite gauge theories in terms of gauge invariants, as has been shown by Mandelstam, but usually this introduces nonlocalities through the appearance of variables like $[\exp i \int_\infty^x A_\mu(y) dy^\mu] \Psi(x)$; [18].

The criterium for validity of (A) is well known: Combine all Higgs scalars into one row vector ϕ and let H the connected component of the stability group of ϕ in the gauge group G . Then the surviving massless vector mesons transform according to the adjoint representation of H . There are none if H is trivial, i.e. if $\phi V = \phi$ implies $V = 1$ for infintesimal gauge transformations V and generic ϕ.

To have (B) , one must be able to define $L[\Phi(x)]$ uniquely through Eq. (2.7), with $\hat{\phi}$ a suitable gauge invariant depending on ϕ . This requires that $\phi V = \phi$ implies $V = 1$ for arbitrary finite gauge transformations V, and generic ϕ .

Condition (B) is stronger than (A) , it requires that the charges of the Higgses match so that they can screen all colour charges locally.

Example: Lattice QED (gauge group U(1)) with spinor fields of electric charge Q = 1 plus Higgs scalars of charge 2 satisfies (A) but violates (B) since $\phi V = \phi$ for $V = \exp i\pi Q$.

We shall append some comments on the physical meaning of the Higgs mechanism. IT IS SAID THAT Higgs mechanism means spontaneous breakdown of colour symmetry; if (A) holds one has complete breaking. We only consider this case here.

THIS IS TRUE in the sense that no nontrivial symmetry is left to act on the physical state space.

BUT breaking is very different from e.g. ferromagnets at $T < T_c$:

There are no Goldstone particles - no long range correlations (mass gap!) - no degeneracy of the vacuum.*)

In a FERROMAGNET at $T < T_c$ one may compute a state ω through $\omega(A) \equiv \langle A \rangle = Z^{-1} \mathrm{Tr}\, A e^{-\beta H}$ with symmetric Hamiltonian H (no ext. magnetic field, free boundary conditions), this gives $\omega(\vec{s}) = 0$ for the spin variables $\vec{s}$. But the state ω is impure (" degeneracy of the vacuum"): $\omega = \int d\Omega_{\vec{e}}\, \omega_{\vec{e}}$ ($\vec{e}$ labels possible directions of the magnetization) and

$$\omega_{\vec{e}}(\vec{s}) = \mathcal{M}\vec{e} \quad , \quad \mathcal{M} = \text{magnetization} \neq 0 \quad . \qquad (3.1)$$

The pure states $\omega_{\vec{e}}$ can be constructed by adding an external magnetic field $\mathcal{H}$ and letting $\mathcal{H} \to 0$.

In a HIGGS model with free boundary conditions one also obtains

$\langle \phi \rangle = \langle \phi_{ave} \rangle = 0$ from (2.12). But adding $\int_x \mathcal{H}\phi(x)$ to $\mathcal{L}_E$

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*) NB: We are talking about the statistical mechanical system defined by $\mathcal{L}_E$ without a gauge fixing term in a lattice gauge theory.

and letting $\mathcal{H} \to 0$ still gives $\langle \phi \rangle = 0$ in contrast with (3.1). This has recently been shown by Lüscher [8] and independently the authors of ref. [9].

In view of all this one would better speak of colour confinement instead of colour symmetry breaking. This view is also supported by a result of Swieca [10]. He showed that absence of massless photons implies absence of charged particles in an abelian gauge theory.

It would be extremely important to extend this result to the non-abelian case, even from a phenomenological point of view. While one is not so sure about quarks, it is certain that there are no massless strongly interacting gluons as real particles in nature. If the non-abelian version of Swiecas theorem were true, this would leave a clear-cut alternative: Either coloured quarks do not exist as free particles, or QCD is wrong.

Now it may not be clear how to observe colour, since there is no gauge invariant local colour charge density. However, absence of colour also rules out the existence of quarks with non integral baryon number: Under a baryon number transformation a field or state of baryon number B transforms as $\Psi \to \Psi e^{2\pi i \alpha B}$. In particular, for quarks $q \to q e^{2\pi i \alpha/3}$. If α is integer, then this is <u>identical</u> to the action of a transformation in the center $\mathbb{Z}_3$ of colour-SU(3), i.e. it is a <u>gauge</u> transformation. [In other words, the global symmetry group is <u>not</u> the direct product of colour and flavor symmetry groups, even though the same is true for the Lie algebras!]

Thus $\mathcal{B}$ must be integer if Ψ is gauge invariant. Similarly the flavor group $SU(3) \times U(1)_{charm}$ contains a subgroup $\mathbb{Z}_3$ in common with the gauge group. This leads to flavor-SU(3) triality zero for uncharmed particles in the same way, and so on.

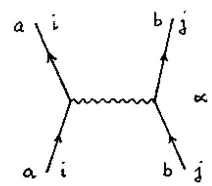
These considerations also show that one only need consider the action of the center of the gauge group to see whether there is quark confinement.

We have seen that condition Ⓑ is somewhat stronger than Ⓐ.
What happens if Ⓐ is true but B is not? Local screening of charges is then impossible. On the other hand, if the non abelian generalization of Swiecas theorem were true, one would expect that colour confinement still takes place. We will come back to that in Sect.5.

4. Cooper pairs in quantum chromodynamics.

We now turn to the consideration of quantum chromodynamics (QCD) with Euclidean Lagrangean (1.1) in continuous space time.
We note that the Coulomb force (1 gluon exchange) is attractive between two quarks q^i, q^j in a relative 3^* colour state. This is so because 3^* is the antisymmetric 2-spinor representation of SU(3), the pair consists therefore of two quarks with different colour $a \neq b$, and the sign of the Born term is given by Fig. 1

Fig.1



$$\propto \sum_{c=1}^{8} \lambda^c_{aa} \lambda^c_{bb} = -\frac{2}{3}\,, \quad (a \neq b)\,. \tag{4.1}$$

We assume that this leads to the formation of diquark and anti-diquark Cooper pairs, and that the physical vacuum (= groundstate of the Hamiltonian) contains a finite and equal density of them. They are colour triplets transforming according to 3^* resp. 3 representation of colour SU(3).

Effective Lagrangean

According to the renormalization group philosophy [11] one may(in principle)

compute from (1.1) UV-cutoff effective Lagrangeans $\mathcal{L}_{eff}$ which describe the same physics at distances $\xi > a$. The UV-cutoff may be introduced through a lattice, with lattice constant a.

As a consequence of Cooper pair formation, we may expect that corresponding scalar fields ϕ appear in $\mathcal{L}_{eff}$.

$$\phi^{ij} = C'_{\alpha\beta}\, q^i_\alpha \times q^j_\beta \quad ; \quad C' = iC\gamma_0 = \begin{pmatrix} \varepsilon & 0 \\ 0 & \varepsilon^{-1} \end{pmatrix} \tag{4.2}$$

in a basis where γ_5 is diagonal

ε = antisymmetric tensor in two dimensions, $\times$ is the vector product in 3-dimensional colour space, α, β are spinor indices. Because of Fermi statistics

$$\phi^{ij} = -\phi^{ji} \tag{4.3}$$

Because of gauge invariance of the 2nd kind, ϕ must appear coupled to the gluons. The effective Lagrangean might look something like

$$\begin{aligned} \mathcal{L}_{eff}(x) = & -M'_{ij}\, \bar{q}^i q^j + \kappa \sum_\mu \bar{q}^i(x)[1+\gamma_\mu]\, U(x,\mu)\, q^i(x+\hat{\mu}) \\ & - \kappa \sum_\mu \phi^i(x)\, U(x,\mu)\, \phi^j(x+\hat{\mu})^+ - \vartheta\,(\phi^i \phi^{j*}) \\ & - \frac{1}{2f^2}\, \text{plaquette term} - f' \left| C'_{\alpha\beta}\, q^i_\alpha \times q^j_\beta - \phi^{ij} \right|^2 . \end{aligned} \tag{4.4}$$

Herein $U(x,\mu) \in SU(3)_{colour}$, ϕ^{ij}, $\bar{q}^i$ are row-3-vectors in colour space, κ, ϑ, f, f' depend on a with $\kappa \to 0$, $f' \to \infty$ as $a \to 0$.
The last term in the Lagrangean incorporates the information that the

Cooper pairs are made of two quarks. The effective Lagrangean could be much more complicated, but its precise form is not crucial for what follows.

Transformation to gauge invariant field variables

The model (4.4) has properties (A) and (B) of Sect. 3 provided

i) there are at least three flavors

ii) the colour group is SU(3).

If there were less than three flavors, there would be at most one Higgs triplett because of Fermi statistics. But then $\phi = \phi V$ would be true for at least an SU(2) subgroup of colour -SU(3), and so the criterium for (A) would not be satisfied.

If on the other hand, the colour group were SU(2) or U(1), then (B) would be violated because the Higgs scalars cannot locally screen the colour of quarks. The trivial representation of SU(3) is contained in the Kronecker product $3 \otimes 3 \otimes 3$, but the trivial representation of SU(2) is not contained in the Kronecker product of any odd number of fundamental representations of SU(2). This follows by considering the center of the gauge group. Similar statements hold for U(1).

For simplicity we consider first the model with three flavors i =1,2,3. There are then three colour tripletts of Higgs scalars, and three colour tripletts of quarks. We write $\phi^i_a = \epsilon^{ijk}\phi_a^{jk}$ and introduce 3 x 3 matrices

$$\Phi = (\phi^{+\,i}_a) \;,\; \Phi^* = (\phi^i{}_a) \;,\; q = (q_a{}^i) \;,\; \bar{q} = (\bar{q}^i{}_a) \qquad (4.6)$$

The transformation laws under colour and flavour transformations are then

$$\begin{aligned}
&\text{colour SU(3)} && \Phi(x) \to V_1(x)\Phi(x)\,,\; q(x) \to V_1(x)q(x)\,,\; U(x,\mu) \to V_1(x)U(x,\mu)V_1(x+\hat{\mu})^{-1} && (a)\\
&\text{flavor SU(3)} && \Phi(x) \to \Phi(x)V_2'\,,\; q(x) \to q(x)V_2'\,,\; U(x,\mu) \to U(x,\mu) && (4.5)\;(b)\\
&\text{baryon U(1)} && \Phi(x) \to e^{-2i\alpha}\Phi(x)\,,\; q(x) \to e^{i\alpha}q(x)\,.\; U(x,\mu) \to U(x,\mu) && (c)
\end{aligned}$$

V_2' stands for the transpose of V_2 ; $\alpha = -\pi \ldots \pi$.

For generic ϕ, the 3 x 3 matrix Φ will be non degenerate, $\det \Phi \neq 0$.

Any such matrix can be uniquely factorized in the form

$$\Phi(x) = S(x)\,\sigma(x) \qquad S \text{ unitary},\; \sigma = \sigma^* > 0 \qquad (4.6a)$$

The unitary matrix S may be used to define $L[\Phi(x)]$ uniquely by

$$S(x) = L[\Phi(x)]\,e^{i\varphi/3} \qquad (-\pi < \varphi \leq \pi \;,\; L[\Phi(x)] \in SU(3)) \qquad (4.6b)$$

Under a gauge transformation

$$L[\Phi(x)] \to V_1(x)\,L[\Phi(x)]$$

Therefore we may define new gauge invariant variables by

$$\begin{aligned}
\rho(x) &= L[\Phi(x)]^{-1}\Phi(x) = e^{-i\varphi/3}\sigma(x) \qquad (-\pi < \varphi \leq \pi\,,\, \sigma(x) > 0) \qquad (4.7)\\
\xi(x) &= L[\Phi(x)]^{-1} q(x)\\
W(x,\mu) &= L[\Phi(x)]^{-1} U(x,\mu)\, L[\Phi(x+\hat{\mu})]
\end{aligned}$$

All of them are 3 x 3 matrices. Since the change from old to new variables may be viewed as a gauge transformation, the Lagrangean retains its form when rewritten in terms of the gauge invariant variables

$$\mathcal{L}_{eff}(x) = \text{as (4.4) with } \rho, \xi, W \text{ substituted for } \Phi, q, \mathcal{U} \quad (4.9)$$

Let us now determine the transformation law of the new gauge invariant variables under flavor-SU(3). From (4.6) and (4.5b) we see that

$$S(x) \to S(x)V_2' \quad , \quad \sigma(x) \to V_2'^{-1}\sigma(x)V_2' \tag{4.10}$$

hence

$$L[\Phi(x)] \to L[\Phi(x)]V_2'$$

From this one finds the transformation law of ρ, ξ, W

$$\rho(x) \to V_2'^{-1}\rho(x)V_2' \quad ; \quad \xi(x) \to V_2'^{-1}\xi(x)V_2' \quad ; \quad W(x,\mu) \to V_2'^{-1}W(x,\mu)V_2'$$

Thus ρ and ξ contain singlets and octets, while W contain only an octet because $\det W = \exp \operatorname{tr} \ln W = 1$. This is as expected intuitively, considering that the Higgs scalars carry flavor, and the screening will therefore alter the flavor of quarks and gluons.

Up to this point, the discussion of the model (4.4) runs exactly as in the SU(2) model of Sect. 2. Assuming $\mathcal{V}$ is such that the Higgs mechanism takes place with no surviving massless vector particles, viz. $\langle(\rho\rho^*)_{ij}\rangle = \delta_{ij}\lambda$ with $\lambda > 0$, the same conclusions are reached as in Sect. 2, viz. quark and colour confinement.

A new problem appears however when we consider baryon number. It might appear from (4.6) that the transformation law under baryon group (4.5c) is

$$L[\Phi(x)] \to L[\Phi(x)] \,,\; \varphi \to \varphi + 3\alpha \quad \textit{whence} \quad \xi(x) \to e^{i\alpha}\xi(x) \tag{4.11}$$

i.e. the ξ-field carries the same baryon number as the quark field. This contradicts the statement made at the end of Sect. 3 that gauge invariant fields would have to transform trivially if $\alpha/2\pi = \frac{1}{3}, \frac{2}{3}$, assuming they are covariant under baryon U(1). In fact , (4.11) is incorrect because it violates the constraint (4.6b) on φ , $-\pi < \varphi \leq \pi$ if $\alpha = -\pi \ldots \pi$. This constraint is necessary in order that $L[\Phi(x)]$ is uniquely defined. We conclude that variables ξ (and also ρ, W) do not transform covariantly under baryon U(1).

There is a related problem. Suppose we start with a configuration of variables $\Phi(x)$, $q(x)$, $\mathcal{U}(x,\mu)$ which is smooth in the sense that the variables change very little from one lattice site x to a neighboring one. The new variables may then still be very discontinuous because φ may jump by 2π, and $e^{-i\varphi/3}$ appears in (4.7), whereas $\sigma = (\Phi^*\Phi)_+^{1/2}$ is smooth. This suggests that expectation values of products of these variables will blow up when the continuum limit $a \to 0$ is taken, and so they have no correspondence yet with interpolating fields of physical particles. Objects which can be expected to have a continuum limit are functions of the variables (4.7) that can be written as monomials in the original gauge variant variables, this is in accord with the conventional wisdom that normal products are the only functions of fields that can be given meaning in the absence of cutoffs (in > 2 dimensions). Such objects are for instance

$$\rho(x)^* W(x,\mu)\rho(x) = \Phi(x)^* \mathcal{U}(x,\mu)\Phi(x) \qquad \text{(gluonoids)}$$

$$\rho(x)^* \xi(x) = \Phi(x)^* q(x) \qquad \text{(baryons)}$$

$$\rho(x)^* \rho(x) = \Phi(x)^* \Phi(x) \qquad \text{(scalars)}$$

etc.

All of these expressions are matrices with entries indexed by flavor indices i,j, so they are flavor SU(3) singlets and octets; moreover they all have integer baryon numbers. In particular the fields $\Psi^k(x) = \mathrm{tr}\, \Phi^*(x) q(x) \lambda^k$ create the baryon octet.

The real nature of the problem of baryon number becomes apparent when we notice that the Lagrangean (4.9) is expressed in terms of variables (4.7) which satisfy a constraint. The dangerous constraint is the one on φ, it says that $e^{i\varphi}$ takes values in the circle S^1. It is rotated around the circle by baryon U(1). Such a situation is familiar from the nonlinear σ-model, it brings about a danger of spontaneous breaking of baryon number conservation (corresponding to spontaneous magnetization in the XY-model = nonlinear SO(2)-σ- model). Spontaneous symmetry breaking is not inevitable, as is shown by the example of the 2-dimensional XY-model. Whether it takes place could only be answered by computation of the effective potential as a function of the variable conjugate to an appropriate external source carrying baryon number [12]. The present state of the art does not allow to perform such calculation. Quasiclassical approximations in (4.4) or (4.9) are useless because they are inconsistent with the composite nature of the Higgs scalar (they would approximate $q \approx 0$, $\Phi^*\Phi \approx \langle \Phi^*\Phi \rangle = \lambda \mathbb{1} \neq 0$ while $\phi \sim qq$) or, equivalently, with a large value of f' in (4.4).

Of course, in order to decide whether a Higgs mechanism takes place at all one would have to compute appropriate effective potentials also, starting from the Lagrangean (1.1) with source term $J(x)\,\mathrm{tr}\,\phi^*(x)\phi(x)$; Φ given by (4.2). Such computations are unfortunately equally unfeasable.

So far we have considered three flavors. A fourth flavor charm can be added without difficulty, there will then be an extra quark and another triplett of scalars χ. $L[\Phi]$ is computed as before in terms of the uncharmed scalars Φ, there is then no problem with charm conservation. A problem would arise if we would insist on exact SU(4). However, if SU(4) symmetry is broken sufficiently strongly in the QCD Lagrangean (1.1) already, the question of spontaneous breaking of SU(4) will not pose itself.

5. Local Higgs mechanism in non abelian pure Yang Mills theories.

We hope to have convinced the reader through the study of the SU(2)-model in Sec. 2 that a Higgs mechanism is a natural way to confine colour. We have studied the possibility of making Higgs scalars from two quarks. Now we will ask ourselves whether one could instead make the Higgs scalars from gluons rather than quarks.

Let us consider a pure Yang Mills theory with colour group SU(3). The Euclidean Lagrangean is

$$\mathcal{L}_E(x) = -\tfrac{1}{2}\,\mathrm{tr}\, F_{\mu\nu} F_{\mu\nu} \quad , \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i f [A_\mu , A_\nu] \tag{5.1}$$

Of course the Lagrangean is to be considered as a function not of the field strengths $F_{\mu\nu}$ but of the potentials A_μ. We would like to rewrite this Lagrangean in terms of gauge invariant variables without detroying locality. For this one needs scalar fields ϕ to fix a gauge frame.

However, since $\mathcal{L}_E(x) = 0$ when $F_{\mu\nu}(x) = 0$, we do not need

$\phi(x) \neq 0$ everywhere, but only in regions of space time where $F_{\mu\nu}(x) \neq 0$ Therefore one can abandon the requirement that the vacuum expectation value of ϕ (or better $\phi^*\phi$) be non zero. This leads to the concept of a "local Higgs mechanism", which is a variant of the idea of local phase transitions introduced by Nambu.

Colour carrying scalar fields can be made from field strengths, e.g.

$$\phi^c(x) = d^{abc} F^a_{\mu\nu}(x) F^b_{\nu\mu}(x) \text{ , viz } \Phi = \sum \phi^c \frac{\lambda^c}{2} = F_{\mu\nu} F_{\nu\mu} - \text{trace} \quad (5.2a)$$

$$\chi^c(x) = \tfrac{1}{2} \operatorname{tr} \chi(x) \lambda^c \quad \text{with} \quad \chi = F_{\mu\nu} F_{\nu\rho} F_{\rho\mu} - \text{trace} \quad (5.2b)$$

and so on.

For a given x let us call the field strength tensor $F_{\mu\nu}(x)$ <u>generic</u> if

$$U\Phi(x)U^{-1} = \Phi(x) \text{ , } U\chi(x)U^{-1} = \chi(x) \text{ , } U \in SU(3) \quad (5.3)$$

implies $U \in \mathbb{Z}_3$ = center of SU(3) .

So long as we restrict attention to the pure Yang Mills theory without fermions, we may consider $G = SU(3)/\mathbb{Z}_3$ as our gauge group, Eq. (5.3) say then that the little group of $(\Phi(x), \chi(x))$ is trivial.

There is an important special case of a field strength tensor which is not generic, viz

$$F^c_{\mu\nu}(x) = \alpha_{\mu\nu}(x) h^c(x) \quad \text{for all } \mu,\nu \quad (5.4)$$

so that the field is abelian at x. In this case the 3 x 3 matrices $\Phi(x)$ and $\chi(x)$ can be simultaneously diagonalized by the action of a gauge transformation , and the condition (5.3) is fullfilled for the diagonalized matrices ϕ, χ for $U \in S\big(U(1) \times U(1) \times U(1)\big)$ = subgroup of diagonal matrices.

It follows that generic field strengths do not exist in two dimensions, because (5.4) is always true there with $\alpha_{\mu\nu}(x) = \epsilon_{\mu\nu}$.

A sufficient condition for genericity of $F_{\mu\nu}(x)$ is that $\Phi(x)$ has distinct eigenvalues and none of the off diagonal entries of $\chi(x)$ vanish in the basis where Φ is diagonal. We call a field configuration generic if $F_{\mu\nu}(x)$ is generic for all x except on some lower dimensional subset of space-time. It seems reasonable to expect that in more than two space time dimensions the measure in the Euclidean path integral is concentrated on generic field configurations. In other words, nongeneric field configurations have negative infinite entropy.

The further procedure is now the same as for any Higgs model satisfying conditions (A) and (B) of Sec. 3. The space of pairs $(\Phi(x), \chi(x))$ is decomposed into orbits under the gauge group, and one selects once and for all one representative $(\hat{\Phi}, \hat{\chi})$ out of every orbit. It follows that there is a gauge transformation $L(x)$ such that

$$\Phi(x) = L(x)\hat{\Phi}(x)L(x)^{-1} \text{ ; } \chi(x) = L(x)\hat{\chi}(x)L(x)^{-1} \quad (5.5)$$

If $F_{\mu\nu}(x)$ is generic, then $L(x)$ represents a unique element of

$G = SU(3)/\mathbb{Z}_3$ and we may define *)

$$\mathcal{B}_\mu(x) = L(x)^{-1}A_\mu(x)L(x) + \frac{i}{f}L(x)^{-1}\partial_\mu L(x) \tag{5.6}$$
$$G_{\mu\nu}(x) = \partial_\mu \mathcal{B}_\nu - \partial_\nu \mathcal{B}_\mu - if[\mathcal{B}_\mu, \mathcal{B}_\nu]$$

These quantities are gauge invariant. $\mathcal{L}_E$ can now be expressed in terms of these new gauge invariant variables as promised, viz.

$$\mathcal{L}_E(x) = -\frac{1}{2}\,\mathrm{tr}\, G_{\mu\nu}G_{\mu\nu} \tag{5.7}$$

The potentials $\mathcal{B}_\mu$ must satisfy certain constraints, e.g. $G_{\mu\nu}G_{\nu\mu} - \mathrm{tr.} = \hat{\Phi}$ for some Φ. If we are only interested in classical solutions of the field equations in Minkowski space, we may impose almost all of these constraints through introducing Lagrangean multipliers. For instance, to define $\hat{\Phi}, \hat{\chi}$ we may require that $\hat{\Phi}$ is diagonal and the off diagonal entries $\hat{\chi}_{32}, \hat{\chi}_{31}$ are real. (This determines $L(x)$ up to elements of a discrete subgroup of G.) To impose the first constraint, we may add a term

$$\sum_{a\neq b}\sum \mu_{ab}(x)\left(G_{\mu\nu}(x)\,G_{\nu\mu}(x)\right)_{ba} \tag{5.8}$$

to the Lagrangean with multipliers $\mu_{ab}(x)$, and similarly for the second constraint. Validity of this multiplier method in Euclidean QFT would require a special investigation.

Higgs mechanisms can be pictured as screening mechanism in which coloured gluons combine with coloured Higgs scalars to form colourless massive

*) There is a problem here coming from the fact that $L(x)$ need not be continuous (and differentiable) even if Φ, χ are. We will not discuss the topological aspects of gauge theories which are connected with this fact. [19].

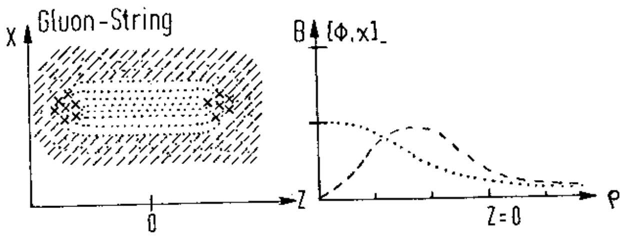
vector particles (possible very unstable ones). For this one does not really need a finite density of Higgs scalars in the vacuum, i.e. everywhere, but one needs them only where the gluons are, i.e. in space time regions where $F_{\mu\nu}(x) \neq 0$. This is consistent with a composite nature of Higgs scalars as described in Eqs. (5.2). It is therefore tempting to speculate that gluons in non-abelian gauge theories can screen their colour and acquire mass through such a "local Higgs mechanism".

Quarks

Suppose this is so, what will then happen when we add quarks to the theory? Standard lore [13] suggests the following speculations. Higgs scalars made of gluons have triality zero, and so they cannot screen the colour charge of the quarks locally. In other words, condition (B) of Sec. 3 is violated while (A) still holds for the complete theory. If the generalization of Swiecas theorem mentioned in Sec. 3 is true we must expect that colour confinement will still occur under these circumstances. On the other hand, since the colour charges of quarks cannot be locally screened, it is suggested that there remain long range forces between these quarks. This view is supported by recent results of Callan, Dashen and Gross [6] for the 2-dimensional abelian Higgs model where it is shown that long range forces between external charges persist in the presence of the Higgs mechanism, for external charges that are not integral multiples of the Higgs charge. Now a force field mediated by gluons cannot enter far into a space region where there is a finite density of (sufficiently many kinds of) Higgs scalars. On the other hand, extended areas with nongeneric $F_{\mu\nu}$ will cost a lot of entropy according

to our previous hypotheses because $F_{\mu\nu}$ is forced into a lower dimensional subset of the field manifold. This leaves us with the possibility of a string joining the two quarks, with a line in the middle along which $F_{\mu\nu}$ is large but non generic - for instance abelian.*) The field configuration might then look something like Fig. 2.
[In Wilsons Cargèse lectures [14] an alternative argument is given why finite coherence length (gluonoid mass) leads to formation of strings] .

Fig. 2



$[\Phi,x]_-$: Higg's field, B: Gluon field, $|\psi|^2$: Quark density

In contrast with individual quarks, the colour of any object with triality zero (e.g. quark - antiquark, three-quark) can be locally screened by the Higgs scalars. This may be responsible for saturation of forces and absence of exotics.

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*) The situation resembles magnetic monopoles in a superconductor and the "dielectric model" of Susskind and t'Hooft [16]. A list of references to string and bag models is found in [17].

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