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Application of Veneziano Model to Charm Meson Systems

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Abstract

We apply the Veneziano model to study the scattering and decay processes involving charm mesons. Imposing the Adler's PCAC consistency condition on the dual amplitude we predict the masses of the charm axial-vector mesons m_{F_A} and m_{D_A} to be 2.30 and 2.25 GeV, respectively. A comparison between chiral symmetry and duality, as extended to $SU(4)_L \otimes SU(4)_R$, is made. Invoking current-field identity we apply the method to the semi-leptonic decays of charm mesons and calculate the ratios
$r_{K^{**}} = \Gamma(D \to K^{**}(2^+,1420)e\nu_e)/\Gamma(D \to K^*(1^-,892)e\nu_e)$ and
$r_K = \Gamma(D \to K(0^+,1250)e\nu_e)/\Gamma(D \to K^* e\nu_e)$ which turn out to be small, postdicting the data on inclusive lepton spectrum in e^+e^-. Decay widths and lepton spectra are calculated and compared with the current algebra predictions and the experimental data.

I. Introduction

Recently, there have been several attempts [1] to extend the techniques of dual models, the Veneziano model [2] for example, to the realm of charm hadrons. In particular, Igi [1] has studied the mass spectrum of the charm hadrons using the property of factorization of the Regge slopes and the masses of the bound state $c\bar{c}$ systems, J/ψ and $\chi(2^{++}, 3552)$. The predictions are in good agreement with the charm meson masses. This augments the hope of using the duality models to study the detailed dynamical properties of meson systems including the charm hadrons. Our interest in the application of Veneziano model is twofold: Firstly, we want to pursue the connection between duality and chiral symmetry, first pointed out by Lovelace [3] for the case of the $SU(2)_L \otimes SU(2)_R$ systems, and later extended to the $SU(3)_L \otimes SU(3)_R$ case by others [4]. It would be interesting to see if the connection still persists at the $SU(4)_L \otimes SU(4)_R$ level, even though the symmetry is badly broken. An additional spin-off of such investigations are the mass relations, that are usually obtained by imposing the Adler zero on the dual amplitude. In addition to the existing mass relations for the pseudoscalar and the vector mesons, we obtain the mass relations for the axial-vector mesons, namely

$$m^2_{F_A} = \frac{1}{\alpha'_{D^*}} \left[2 - \alpha_{K^*}(m^2_K) - \alpha_{D^*}(0) \right]$$

$$m^2_{D_A} = \frac{1}{\alpha'_{D^*}} \left[2 - \alpha_{\rho}(m^2_\pi) - \alpha_{D^*}(0) \right]$$

which on using the non-degenerate Regge trajectories in Appendix I, yield, respectively,

$$m_{F_A} = 2.30 \quad \text{GeV}$$

$$m_{D_A} = 2.25 \quad \text{GeV}$$

Secondly, we want to apply the method to study the semi-leptonic decays of the charm hadrons. The information on the semi-leptonic decays can be obtained by invoking the field-current identity. The strong interaction vertices so introduced are then extracted from the dual model. In the past, there have been several attempts to calculate the decay modes $D \to K e\nu_e, K^* e\nu_e$ [5],[6] and $F \to \eta(\eta') e\nu_e, \phi e\nu_e$ [7]. However, there has been no attempt to investigate the contribution of the higher mass states, namely, $D \to K^{**}(2^+) e\nu_e$, $\kappa(0^+) e\nu_e$ or the daughters of the K^{**}. (Similarly higher mass states for the F decays). The dual models afford a natural way to study such decays by relating them to the decay $D \to K^* e\nu_e$. We have estimated these decay modes under the assumption of a single term Veneziano model, and find that both

$$r_{K^{**}} = \frac{\Gamma(D \to K^{**}(2^+, 1420) e\nu_e)}{\Gamma(D \to K^* e\nu_e)} \quad \text{and} \quad r_\kappa = \frac{\Gamma(D \to \kappa(0^+, 1250) e\nu_e)}{\Gamma(D \to K^* e\nu_e)}$$

are small, thus postdicting the data on inclusive lepton spectrum in e^+e^- experiments coming from charm decays [8], which do not admit a significant value for $r_{K^{**}}$ and r_κ, as well as for the decay modes $D \to (K n\pi) e\nu_e$. In order to get the absolute rates for $D \to K^* e\nu_e$, $F \to \phi e\nu_e$ etc. we invoke quark model for the strong interaction vertices DK^*F_A, $F\phi F_A$ etc. and compare the results with the ones obtained from current algebra technique.

The paper is organized as follows: In Section II we study the one-term Veneziano model for the four-body processes involving charm axial-vector and vector mesons, and calculated the various strong interaction vertices

by using the Born approximation.
Mass relations for the charm axial-vector mesons are obtained by invoking the Adler's PCAC consistency condition.

Section III contains a comparison between chiral symmetry, exemplified by current algebra, and duality. In particular, we evaluate the on-shell vertices $G^{DK^*F_A}$, $G^{D\rho D_A}$, $G^{F\phi F_A}$ using current algebra and the Veneziano model and show that the two are identical if the Regge slopes are degenerate and if the anomalous magnetic moment of the charm axial-vector mesons has the value $\delta = -1$.

In Section IV the model is applied to study the semi-leptonic decays of the D and F mesons. The various decay widths so obtained are tabulated through Table 1-3, where comparison with the current algebra results (wherever possible) is also made. Assuming the SU(4) symmetric value for the decays $D \to K e \nu_e$, $F \to \eta(\eta') e \nu_e$, we calculate the lepton spectra which are presented in Figs. (2) to (5). (For relevant formulae, see Appendix III).

Section V contains discussion of our results and comparison with the experimental data.

II. Veneziano Model for Meson Systems

We consider in general the processes such as

$$P^i + P^j \to P^k + A^l$$

and

$$P^i + P^j \to P^k + V^l \qquad (i,j,k,l = 1,2,..,16)$$

where P, A and V stand for the pseudoscalar, axial-vector and vector mesons, respectively. With the application of Veneziano model to semileptonic decays of the charm mesons in view, we shall study the following reactions

$$\begin{aligned} \pi D &\to K F_A \\ &\to K F^* \\ \pi D &\to \pi D_A \\ &\to \pi D^* \\ K F &\to K F_A \\ &\to K F^* \end{aligned} \tag{1}$$

where F_A and D_A are the axial-vector partners of the charm vector mesons, F and D , respectively. Aside from their applications to semileptonic decays which are our main concern, these reactions are also important in obtaining many useful relations between various coupling constants related to the vertices PAV, PAT, PVV, PVT , etc. (T stands for the tensor meson).

a) The Reaction $\pi D \to K F_A$

We begin with the study of the reaction

$$D(p_D) + \pi(-p_\pi) \to K(p_K) + F_A(q) \tag{2}$$

The amplitude T (s,t) for the reaction (2) is given by the following invariant decomposition

$$T(s,t) = -A(s,t)(p_K + p_\pi)\cdot\epsilon + B(s,t)(p_K - p_\pi)\cdot\epsilon \tag{3}$$

where ϵ is the polarization vector of F_A, and s,t,u are given by

$$s = (p_D - p_\pi)^2$$
$$t = (p_K + p_\pi)^2$$
$$u = (p_D - p_K)^2$$

The partical wave expansion for the invariant amplitudes A and B is given in ref. (4), from which the following Regge behaviour for the amplitudes A and B follows:

$$\begin{aligned} A(s,t) &\sim s^{\alpha(t)} , \quad s \to \infty , \ t = \text{fixed} \\ A(s,t) &\sim t^{\alpha(s)} , \quad t \to \infty , \ s = \text{fixed} \\ B(s,t) &\sim s^{\alpha(s)-1} , \quad s \to \infty , \ t = \text{fixed} \\ B(s,t) &\sim t^{\alpha(s)} , \quad t \to \infty , \ s = \text{fixed} \end{aligned} \tag{4}$$

Consequently the Veneziano amplitudes suggested by Eq. (3) are

$$\begin{aligned} A(s,t) &= (\gamma_1 s + \gamma_2 t + \gamma_3)\, V(\alpha_{K^*}(t), \alpha_{D^*}(s)) \\ B(s,t) &= (\beta_2 t + \beta_3)\, V(\alpha_{K^*}(t), \alpha_{D^*}(s)) \end{aligned} \tag{5}$$

where

$$V(\alpha_1, \alpha_2) = \frac{\Gamma(1-\alpha_1)\,\Gamma(1-\alpha_2)}{\Gamma(2-\alpha_1-\alpha_2)} \tag{6}$$

In order to determine the parameters introduced in Eq. (5) we shall impose the Adler's consistency conditions, and calculate

the contributions to T(s,t) coming from the pole terms. In imposing the consistency conditions on the amplitude T(s,t) we shall not invoke the limit $p_D \to 0$ because of the uncertainty due to the large extrapolation. For the soft pion and kaon, we have the relations

$$A(s',t') - B(s',t') = 0 \qquad (p_\pi \to 0) \tag{7}$$

$$A(s'',t'') + B(s'',t'') = 0 \qquad (p_K \to 0) \tag{8}$$

with

$$s' = m_D^2 , \quad t' = m_K^2 , \quad u' = m_{F_A}^2$$
$$s'' = m_{F_A}^2 , \quad t'' = m_\pi^2 , \quad u'' = m_D^2$$

Eqs. (7) and (8) demand respectively

$$2 - \alpha_{K^*}(m_K^2) - \alpha_{D^*}(m_D^2) = 0$$

or

$$\gamma_1 m_D^2 + (\gamma_2 - \beta_2) m_K^2 + (\gamma_3 - \beta_3) = 0 \tag{9}$$

and

$$2 - \alpha_{K^*}(m_\pi^2) - \alpha_{D^*}(m_{F_A}^2) = 0$$

or

$$\gamma_1 m_{F_A}^2 + (\gamma_2 + \beta_2) m_\pi^2 + (\gamma_3 + \beta_3) = 0 \tag{10}$$

Assuming either the linear parallel Regge trajectories or the non-parallel trajectories as suggested by the masses of the J/ψ, D and D^* (1), the first of the conditions (9) is not satisfied. Hence we have the second possibility

$$\gamma_1 m_D^2 + (\gamma_2 - \beta_2) m_K^2 + (\gamma_3 - \beta_3) = 0 \tag{11}$$

The first of the conditions (10) is a mass relation for the axial-vector meson, F_A, and leads to the following prediction:

$$m_{F_A}^2 = \frac{1}{\alpha'_{D^*}} \left[2 - \alpha_{K^*}(m_K^2) - \alpha_{D^*}(0) \right] \tag{12}$$

With the assumption of parallel Regge trajectories, this becomes

$$m_{F_A}^2 = m_{D^*}^2 + m_{K^*}^2 - m_\pi^2 \tag{13}$$

which leads numerically to the value $m_{F_A} \approx 2.2$ GeV.

In the case of the non-parallel trajectories we obtain a mass value for this axial-vector F_A meson as

$$m_{F_A} \approx 2.3 \text{ GeV} \tag{14}$$

where we have used the Regge trajectories parametrized by Igi[1] (see Appendix I) and the known masses.

Next, we evaluate the contributions of the Born terms to the invariant amplitudes A(s,t) and B(s,t). Noting that the s- and the t-channels are dominated by the D^* and K^* poles (and their daughters), the contributions are written explicitly in Appendix II. Comparing the residues of the Born terms and in the Veneziano amplitudes for A(s,t) and B(s,t), we obtain the γ_i and β_i parameters as follows:

$$\begin{aligned}
\gamma_1 &= -2\alpha'_{K^*} g_{K^*K\pi} G_D \\
\gamma_2 = -\beta_2 &= \alpha'_{D^*} g_{D^*D\pi} H_D \\
\gamma_3 &= \alpha'_{K^*} g_{K^*K\pi} \left(G_S + 2m_D^2 G_D\right) \\
&\quad + \alpha'_{D^*} g_{D^*D\pi} \left(m_{K^*}^2 - 2m_K^2\right) H_D \\
\beta_3 &= \alpha'_{K^*} g_{K^*K\pi} G_S + \alpha'_{D^*} g_{D^*D\pi} m_{K^*}^2 H_D
\end{aligned} \tag{15}$$

In addition, we have the relations using Eqs. (11) and (15) (with $m_\pi = 0$):

$$\alpha'_{K^*} g_{K^*K\pi}\left[G_S - G_D\left(m_{D^*}^2 - m_D^2\right)\right] = \alpha'_{D^*} g_{D^*D\pi}\left[H_S - H_D\left(m_{K^*}^2 - m_K^2\right)\right] \tag{16}$$

$$\begin{aligned}
2\alpha'_{K^*} g_{K^*K\pi} G_D\left(m_D^2 - m_{D^*}^2\right) &= \alpha'_{D^*} g_{D^*D\pi}\left\{H_S\left(1 - \frac{m_D^2}{m_{D^*}^2}\right) + \right. \\
&\left. + H_D\left[m_{K^*}^2 - m_{F_A}^2 - \frac{m_D^2}{m_{D^*}^2}\left(m_K^2 - m_{F_A}^2\right)\right]\right\} - 2 g_{\mathcal{D}KF_A} g_{\mathcal{D}D\pi} \alpha'_{D^*}
\end{aligned} \tag{17}$$

$$\begin{aligned}
2\alpha'_{D^*} g_{D^*D\pi} H_D\left(m_K^2 - m_{K^*}^2\right) &= \alpha'_{K^*} g_{K^*K\pi}\left\{G_S\left(1 - \frac{m_K^2}{m_{K^*}^2}\right) + \right. \\
&\left. + G_D\left[m_{D^*}^2 - m_{F_A}^2 - \frac{m_K^2}{m_{K^*}^2}\left(m_D^2 - m_{F_A}^2\right)\right]\right\} - 2 g_{KDF_A} g_{KK\pi} \alpha'_{K^*}
\end{aligned} \tag{18}$$

where G_S and G_D (H_S and H_D) are the coupling constants for the S- and D-wave decays of $F_A \to DK^*$ ($F_A \to D^*K$), as defined in

Appendix II; $\mathcal{D}$ and $\mathcal{K}$ stand for the daughter of the D^* and K^*, respectively.

The relations (16-18) are too involved to be of any practical use. In order to proceed further, one has to resort to some approximation. We shall consider two cases:

Case (i): The daughter contribution is neglected. This amounts to setting

$$g_{\mathcal{D} K F_A} = g_{\mathcal{K} D F_A} = 0 \qquad (19)$$

The relations so obtained take on a particularly simple form, if one assumes the degenerate trajectory mass-relations $m^2_{F_A} \approx m^2_{D^*} + m^2_{K^*}$. Then one obtains

$$\frac{G_S}{G_D} = -\left(m^2_{D^*} + m^2_D\right) \qquad (20)$$

$$\frac{H_S}{H_D} = -\left(m^2_{K^*} + m^2_K\right) \qquad (21)$$

$$\frac{H_D}{G_D} = \frac{\alpha'_{K^*}\, g_{K^* K\pi}}{\alpha'_{D^*}\, g_{D^* D\pi}} \cdot \frac{m^2_{D^*}}{m^2_{K^*}} \qquad (22)$$

A particularly amusing feature of the relations (20) and (21) is that they are obtainable from the Hard-Meson Current Algebra technique (9) for $\delta = -1$ (δ is the anomalous magnetic moment of F_A).

This feature of the Veneziano model was noticed for the ordinary mesons by a number of authors (4). Thus, we have found that the same connection holds also when one studies the APV vertex extended to the chiral SU(4)⊗SU(4).

Case (ii): The D-wave contribution is neglected. This amounts to setting $G_D = H_D = 0$. One then obtains

$$G_S = 2 m_{K^*} \left[\frac{m^2_{K^*} + m^2_K}{m^2_{K^*} - m^2_K}\right]^{1/2} g_{\mathcal{K} D F_A} \qquad (23)$$

$$H_S = 2 m_{D^*} \left[\frac{m^2_{D^*} + m^2_D}{m^2_{D^*} - m^2_D}\right]^{1/2} g_{\mathcal{D} K F_A} \qquad (24)$$

$$\frac{G_S}{H_S} = \frac{\alpha'_{D^*}\, g_{D^* D\pi}}{\alpha'_{K^*}\, g_{K^* K\pi}} \qquad (25)$$

where we have used the following relations obtained by studying the $\pi K \to \pi K$ and $\pi D \to \pi D$ processes in the Veneziano model (i.e. using the Lovelace-Veneziano amplitude)

$$\frac{g_{\mathcal{K} K\pi}}{g_{K^* K\pi}} \approx \left(\frac{m^4_{K^*} - m^4_K}{m^2_{K^*}}\right)^{1/2} \qquad (26)$$

$$\frac{g_{\mathcal{D} D\pi}}{g_{D^* D\pi}} \approx \left(\frac{m^4_{D^*} - m^4_D}{m^2_{D^*}}\right)^{1/2}$$

Similarly, we can go on to study the K^{**} - pole contributions to the amplitudes A(s,t) and B(s,t). Again comparing the residues of K^{**}-pole contributions with those of the Veneziano forms, Eq.(5), and using the parameters in Eq. (15) we obtain relations among the coupling constants involved in the vertices $F_A D K^{**}$, $F_A D \kappa_1$, $F_A D \kappa_2$, $F_A D K^*$ and $F_A D^* K$; where κ_1 and κ_2 are the first and the second daughters of K^{**}, (see: Appendix II). These relations are very involved and again we resort to the approximations made for the K^*- and D^*- contributions.

Case (i) for K^{**}- contribution: With the approximation in Eq.(19) we obtain

$$G_F = \alpha'_{D^*} \frac{g_{K^* K\pi}}{g_{K^{**} K\pi}} \cdot \frac{G_S}{(m^2_{D^*} + m^2_D)}$$

$$G_P = -\alpha'_{D^*} \frac{g_{K^* K\pi}}{g_{K^{**} K\pi}} \left[\frac{m^2_{D^*}(m^2_{K^{**}} - m^2_{K^*})}{m^2_{K^*}(m^2_{D^*} + m^2_D)} + 1 \right] G_S \tag{27}$$

We shall not write down here the expressions for G^1_S, G^1_D and $g_{\kappa_2 D F_A}$; one can easily extract them in terms of G_S from Eqs. (II-16)-(II-18) with the help of the relations (20)-(22). G_P and G_F are the P- and F-wave coupling constants for the vertex $F_A D K^{**}$ respectively, and G^1_S and G^1_D are related to the vertex $F_A D \kappa_1$.

Case (ii) for K^{**}- contribution: Neglecting D-wave contributions in K^* and D^* terms, i.e. with $G_D = H_D = 0$ we get

$$G_F = 0 \quad , \quad G_P = -\alpha'_{D^*} \frac{g_{K^* K\pi}}{g_{K^{**} K\pi}} G_S \tag{28}$$

One can again easily obtain the G^1_S, G^1_D and $g_{\kappa_2 D F_A}$ in terms of G_S from Eqs. (II-16)-(II-18) using the relation (25). We give below those coupling constant ratios, obtained from the Lovelace-Veneziano amplitude for $\pi K \to \pi K$ elastic scattering, which are relevant for both the cases.

$$\frac{g_{K^* K\pi}}{g_{K^{**} K\pi}} = \left(\frac{2}{\alpha'_{K^*}} \right)^{1/2} \tag{29}$$

$$\frac{g_{\kappa_1 K\pi}}{g_{K^{**} K\pi}} \simeq \sqrt{2} \left[m^2_{K^*} - \frac{m^4_K}{m^2_{K^{**}}} \right]^{1/2} \simeq 1.24 \text{ GeV} \tag{30}$$

$$\frac{g_{\kappa_2 K\pi}}{g_{K^{**} K\pi}} \simeq \left[\left(2m^2_{K^*} - m^2_{K^{**}} - \frac{m^4_K}{m^2_{K^{**}}} \right) \left(m^2_{K^{**}} - \frac{m^4_K}{m^2_{K^{**}}} \right) + \frac{1}{3} \frac{(m^2_{K^{**}} - m^2_K)^4}{m^4_{K^{**}}} \right]^{1/2} \tag{31}$$

The last ratio is about 0.1 GeV. Therefore we shall not take into account the contribution of κ_2 in the applications.

Note that all the couplings in Eqs. (20)-(28) depend only on one undetermined coupling constant, which we choose to be G_S . The method described above for the process $\pi D \to K F_A$ can be extended in a straightforward way to the other members of the SU(4) multiplet. Hence, we only quote the resulting relations.

b) The Reaction $\pi D \to \pi D_A$

We now consider the process

$$D(p_D) + \pi_i(-p_1) \longrightarrow \pi_j(p_2) + D_A(q) \tag{32}$$

The invariant decomposition is similar to Eq. (3):

$$T(s,u,t) = -A(s,u,t)(p_1+p_2)\cdot\epsilon + B(s,u,t)(p_2-p_1)\cdot\epsilon \tag{33}$$

The isospin decompositions are as follows:

$$A = \delta_{ij} A^{(+)} + \tfrac{1}{2}[\tau_j, \tau_i] A^{(-)}$$
$$B = \delta_{ij} B^{(+)} + \tfrac{1}{2}[\tau_j, \tau_i] B^{(-)} \tag{34}$$

Crossing symmetry ($s \leftrightarrow u$) demands

$$A^{\pm}(s,u,t) = \pm A^{\pm}(u,s,t)$$
$$B^{\pm}(s,u,t) = \mp B^{\pm}(u,s,t) \tag{35}$$

The asymptotic behaviours (in Eq.4) and the crossing symmetry properties of $A^{\pm}$ and $B^{\pm}$ require the following Veneziano representations

$$A^{\pm} = \left[(\gamma_1^{\pm} s + \gamma_2^{\pm} t + \gamma_3^{\pm}) V(\alpha_{D^*}(s), \alpha_\rho(t)) \pm (s \to u)\right]$$
$$B^{\pm} = \left[\quad (\beta_2^{\pm} t + \beta_3^{\pm}) V(\alpha_{D^*}(s), \alpha_\rho(t)) \mp (s \to u)\right] \tag{36}$$

Imposing the Adler's PCAC consistency conditions one obtains $A + B = 0$ for $p_2 \to 0$ ($i.e.$, $s = m^2_{D_A}$, $t = m^2_\pi$, $u = m^2_D$) and $A - B = 0$ for $p_1 \to 0$ ($i.e.$, $s = m^2_D$, $t = m^2_\pi$, $u = m^2_{D_A}$). These constrains can be converted into the following relations

$$2 - \alpha_{D^*}(m^2_{D_A}) - \alpha_\rho(m^2_\pi) = 0 \tag{37}$$

$$\gamma_1 m^2_D + (\gamma_2 - \beta_2) m^2_\pi + (\gamma_3 - \beta_3) = 0 \tag{38}$$

Eq. (37) is a mass relation for the charm axial-vector meson D_A:

$$m^2_{D_A} = \frac{1}{\alpha'_{D^*}}\left[2 - \alpha_\rho(m^2_\pi) - \alpha_{D^*}(0)\right] \tag{39}$$

which for the parallel trajectories leads to the relation

$$m^2_{D_A} = 2 m^2_{D^*} - m^2_D \tag{40}$$

The numerical evaluation of Eq. (39) for the non-parallel trajectories (see Appendix I) gives the value $m_{D_A} \approx 2.25$ GeV.

Proceeding in exactly the same way as for the $\pi D \to K F_A$ case, one obtains the following relations between the coupling constants:

Case (i): Neglecting the ρ and D^* daughters, we get

$$\frac{G_S'}{G_D'} = -\left(m_{D^*}^2 + m_D^2\right)$$

$$\frac{H_S'}{H_D'} = -m_\rho^2 \tag{41}$$

and $$2\, m_\rho^2\, \alpha_\rho'\, g_{D^* D\pi}\, H_D' = m_{D^*}^2\, \alpha_{D^*}'\, G_D'$$

Case (ii): Neglecting the D-wave contributions, i.e., setting $G_D' = H_D' = 0$, we get

$$G_S' = 2\, m_\rho\, g_{\sigma D D_A}$$

$$H_S' = 2 m_{D^*}\left[\frac{m_{D^*}^4 - m_D^4}{m_\rho^4}\right]^{1/2} g_{\not{\rho}\pi D_A} \tag{42}$$

$$\frac{G_S'}{H_S'} = 2\, \frac{g_{D^* D\pi}}{g_{\rho\pi\pi}} \cdot \frac{\alpha_{D^*}'}{\alpha_\rho'}$$

where we have used the relation $g_{\sigma\pi\pi}/g_{\rho\pi\pi} \approx m_\rho$ which can be easily obtained by studying the $\pi\pi \to \pi\pi$ process in the Veneziano model and the second relation in Eq. (26).[4] σ is the daughter of ρ.

c) The Reaction KF $\longrightarrow$ KF$_A$

The process

$$F(p_F) + K(-p_1) \longrightarrow K(p_2) + F_A(q) \tag{43}$$

is very similar to that in Eq. (2). Only replace the π, D and K^* by the K, F and ϕ respectively. Following exactly the

same procedure in Sec. II-a, we obtain the mass relation

$$m_{F_A}^2 = \frac{1}{\alpha_{D^*}'}\left[2 - \alpha_\phi(m_K^2) - \alpha_{D^*}(0)\right] \tag{44}$$

For parallel trajectories this becomes

$$m_{F_A}^2 = m_{D^*}^2 + m_\phi^2 - m_K^2 \tag{45}$$

which is equivalent to Eq. (13) since $m_\phi^2 - m_K^2 = m_{K^*}^2 - m_\pi^2$. For non-parallel trajectories Eq. (44) gives a value very close to the one obtained from Eq. (14).

The relations between the coupling constants are as follows:

Case (i): Ignoring the ϕ and D^* daughters, we get

$$G_S''/G_D'' = -\left(m_{D^*}^2 + m_F^2\right)$$

$$H_S''/H_D'' = -m_\phi^2 \tag{46}$$

and $$\alpha_{D^*}'\, g_{D^* KF}\, m_\phi^2\, H_D'' = \alpha_\phi'\, g_{\phi K\bar{K}}\, m_{D^*}^2\, G_D''$$

Case (ii): Assuming no D-wave contributions, we obtain

$$G_S'' = 2\, m_\phi\, g_{\varphi F F_A}$$

$$H_S'' = 2 m_{D^*} \left[\frac{m_{D^*}^2 + m_F^2}{m_{D^*}^2 - m_F^2} \right]^{1/2} g_{\mathcal{L} K F_A}$$

$$\frac{G_S''}{H_S''} = \frac{g_{D^* F K}}{g_{\phi K \bar{K}}} \cdot \frac{\alpha'_{D^*}}{\alpha'_{\phi}} \qquad (47)$$

where φ stands for the daughter of ϕ.

d) The Reaction $\pi D \to K F^*$

This process has only one invariant amplitude

$$T(s,t) = \epsilon_{\mu\nu\rho\sigma}\, \epsilon^{\mu} p_D^{\nu} p_K^{\rho} p_\pi^{\sigma}\, C(s,t) \qquad (48)$$

where the Veneziano form of $C(s,t)$ is directly proportional to the representation given in Eq. (6). Studying the K^*- and K^{**}-pole contributions to T(s,t) and comparing their residues with those from the Veneziano form, we immediately find the relation

$$g_{DF^*K^{**}} = \alpha'_{D^*} \left(\frac{2}{\alpha'_{K^*}} \right)^{1/2} g_{DF^*K^*} \qquad (49)$$

where we have used Eq. (29).

Similarly, we obtain

$$g_{FF^*\varphi'} = \alpha'_{D^*} \left(\frac{2}{\alpha'_{\phi}} \right)^{1/2} g_{FF^*\phi} \qquad (50)$$

from the reaction $KF \to KF^*$.

III. Comparison with Current Alegbra

The hard meson technique of Schnitzer and Weinberg [9] has been applied to study the Axial-vector-Pseudoscalar meson system ⟨AVP⟩. In ref. (5) and (7) the technique was used to study the semileptonic decays of the charm D and F mesons, respectively. Without going into the details, we quote the result for the coupling constants.

a) The DK^*F_A vertex: For the on-mass-shell vertex, one could derive the following relation:

$$\frac{G_S}{G_D} = \frac{(2+\delta)(m_{K^*}^2 + m_D^2 - m_{F_A}^2) - 2 m_D^2}{(-\delta)} \qquad (51)$$

where δ is the anomalous magnetic moment of the F_A meson. Using $\delta = -1$ and the Veneziano model prediction for the F_A mass, $m_{F_A}^2 \simeq m_{D^*}^2 + m_{K^*}^2$ this reduces to

$$\frac{G_S}{G_D} = -(m_{D^*}^2 + m_D^2) \qquad (52)$$

which is exactly the duality result, Eq. (20).

b) The $D\rho D_A$ vertex: The hard meson technique gives

$$\frac{G_S'}{G_D'} = \frac{(2+\delta)(m_\rho^2 + m_D^2 - m_{D_A}^2) - 2 m_D^2}{(-\delta)} \qquad (53)$$

which on using the mass relation

$$m^2_{D_A} = 2m^2_{D^*} - m^2_D = m^2_{D^*} + m^2_\rho - m^2_\pi \simeq m^2_{D^*} + m^2_\rho$$

again leads to the duality prediction in Eq. (41) for $\delta = -1$.

c) <u>The $F\phi F_A$ vertex:</u> Here one gets

$$\frac{G''_S}{G''_D} = \frac{(2+\delta)(m^2_\phi + m^2_F - m^2_{F_A}) - 2m^2_F}{(-\delta)} \tag{54}$$

which on using $\delta = -1$ and the mass relation $m^2_{F_A} = m^2_{D^*} + m^2_\phi$ leads to the duality prediction in Eq. (46).

However, we would like to point out that in comparing the current algebra result with the duality prediction for the $F\phi F_A$ vertex, one must set the kaon mass (and for the other two vertices pion mass) to zero, everywhere, otherwise the two approaches do not yield the same S to D ratio.[10]

IV. <u>Application to the Semi-Leptonic Decays of Charm Mesons</u>

The relations between the various coupling constants derived in Sec. II can be used in conjunction with the Field-Current Identity to determine the semi-leptonic decays of the charm mesons. In the GIM model, for the $\Delta C = \Delta S$ current, this amounts to

$$\begin{aligned} A_\mu^{\Delta C = \Delta S} &= g_{F_A} F^a_{A\mu} \\ V_\mu^{\Delta C = \Delta S} &= g_{F^*} F^{*a}_\mu \end{aligned} \tag{55}$$

For the $\Delta C = \mp 1$, $\Delta S = 0$ currents, one has

$$\begin{aligned} A_\mu^{\Delta C = \mp 1, \Delta S = 0} &= g_{D_A} D^a_{A\mu} \\ V_\mu^{\Delta C = \mp 1, \Delta S = 0} &= g_{D^*} D^{*a}_\mu \end{aligned} \tag{56}$$

Here $F^a_{A\mu}$ ($D^a_{A\mu}$) and F^{*a}_μ (D^{*a}_μ) represent respectively the F_A (D_A) and F^*(D^*) mesons.

Let us now consider the decay of the pseudoscalar mesons D and F. The matrix element for the process $\alpha \to \beta \ell \nu_\ell$ is given by

$$\mathcal{M} = \frac{G}{\sqrt{2}} \bar{\nu}_\ell \gamma_\mu (1-\gamma_5) \ell \langle \beta | (V_\mu + A_\mu) | \alpha \rangle \tag{57}$$

with the hadronic matrix element determined by

$$\begin{aligned} \langle \beta | V_\mu(q) | \alpha \rangle &= g_V \Delta_V(q)_{\mu\mu'} \Gamma^{\alpha\beta V}_{\mu'} \\ \langle \beta | A_\mu(q) | \alpha \rangle &= g_A \Delta_A(q)_{\mu\mu'} \Gamma^{\alpha\beta A}_{\mu'} \end{aligned} \tag{58}$$

Here $\Delta_V(q)_{\mu\mu'}$ and $\Delta_A(q)_{\mu\mu'}$ are the vector and axial-vector meson propagators

$$\Delta_{V,A}(q)_{\mu\mu'} = \left(g_{\mu\mu'} - \frac{q_\mu q_{\mu'}}{m^2_{V,A}} \right) \frac{1}{q^2 - m^2_{V,A}} \tag{59}$$

and $\Gamma^{\alpha\beta V,A}_\mu$ are the various vertices that have to be determined on the mass-shell. In the Appendix III, we give the complete Lorentz covariant decompositions of the hadronic vertices for the cases of interest. Let us first concentrate on the semi-leptonic decays of the D meson.

a) <u>Semi-leptonic decays of D meson:</u> The main semi-leptonic decays of the D meson are expected to be

$$D \to K\ell\nu_\ell \,,\; K^*\ell\nu_\ell \,,\; \kappa\ell\nu_\ell \,,\; K^{**}\ell\nu_\ell \,,\; \kappa_1\ell\nu_\ell$$

There are varying estimates of the axial-vector form factor $F_1^A(q^2)$ for the $K^*\ell\nu_\ell$ mode in literature.[(5),(6)] However, the form factors $f_+(q^2)$ for $D \to K\ell\nu_\ell$, $F \to \eta(\eta')\ell\nu_\ell$ and $F_1^V(q^2)$ for $D \to K^*\ell\nu_\ell$, $F \to \phi\ell\nu_\ell$ are not expected to be significantly renormalized from their SU(4) symmetric values, due to the symmetry breaking corrections. (see Appendix III for the definitions of the form factors). Explicit model dependent calculations support this conjecture[(11)]. In order to calculate the total decay rates, one could set the contribution of the vector form factors to zero, since with the SU(4) symmetric value of $F_1^V(q^2)$, its contribution to the total rate is suppressed due to kinematics. The ratios $r_{K^{**}}$ and r_K then become independent of any parameter, and are given in Table 2. The absolute rates, however, are not determined by the Veneziano model alone, since the normalization is not provided by the model. So, there is one overall form factor, left undetermined, which we choose to be G_S. We adopt the following procedure to estimate G_S. Looking at the non-relativistic limit of the invariant decomposition for the matrix element DK^*F_A, Eq. (II-7), we see that in this limit only G_S contributes, and it has the dimension of mass. It is then most natural to assume that G_S be proportional to the energy available, namely,

$$G_S \propto m_{q_1} + m_{q_2} - 2m_{q_3} \tag{60}$$

where the m_{q_1}, m_{q_2} are the quark masses of the initial particle and $2m_{q_3}$ the mass of the quark pair produced in the decay process, as shown in Fig. 1. In other words, this amounts to breaking the SU(4) symmetry by quark mass terms. We shall then determine G_S from the decays of the SPEAR Q_1, Q_2 mesons [(12)], by using

$$G_S^{DF_AK^*} = g_A^S \frac{m_c + m_\lambda}{m_\lambda + m_q} \,, \quad (q = u, d) \tag{61}$$

with g_A^S the S-wave SU(4)-invariant coupling constant.
A minimum χ^2-fit of the data yields $g_A^S \approx 3$ GeV. So assuming $m_c/m_\lambda \simeq 5$, we get

$$G_S \simeq 6\, g_A^S \simeq 18 \text{ GeV} \tag{62}$$

With the value of G_S, so obtained, and using the necessary relations derived in Sec. II-a, we have calculated all the decays $D \to K^*\ell\nu_\ell$, $K^{**}\ell\nu_\ell$, $\kappa\ell\nu_\ell$, $\kappa_1\ell\nu_\ell$. The second daughter of K^{**}, κ_2 (an $J^P = 0^+$ object) has been left out of consideration because of the known troubles with the second and higher daughters [(13)]. The rate for $D \to K\ell\nu_\ell$ (and $\pi\ell\nu_\ell$) has been calculated by assuming the SU(4) symmetric value. With these assumptions the results are given in Table 1. We also give the current algebra results for comparison wherever possible. Since the decay $D \to K\ell\nu_\ell$ is not very model dependent, we can assume the SU(4) symmetry result for its width. The ratio $\Gamma(D \to Ke\nu_e)/\Gamma(D \to K^*e\nu_e)$ so emerged in the Veneziano model is presented in Table 2, together with the other ratios. The lepton spectra for the individual decay modes $D \to K^{**}\ell\nu_\ell$, $K^*\ell\nu_\ell$ $(V \mp A)$,

$K\ell\nu_\ell$ and $\pi\ell\nu_\ell$ are drawn in Fig. (2), with the area normalized to unity in each case. In Fig. (3) we compare the Veneziano model result for the inclusive lepton spectra from D meson decays (calculated for case (i)) and compare it with the current algebra prediction (5).

b) Semi-leptonic decays of F meson: The main semi-leptonic decays of the F meson are expected to be

$$F \rightarrow \eta\ell\nu_\ell\ ,\ \eta'\ell\nu_\ell\ ,\ \phi\ell\nu_\ell\ ,\ \omega\ell\nu_\ell\ ,\ f'\ell\nu_\ell$$

There is no contribution from any hadronic state other than the ones having $I = Y = 0$, since both the $\Delta C = \Delta S$ part of the GIM current and the F mesons are $I = Y = 0$ objects. If one assumes ideal mixing for the vector mesons ω, ϕ the $\omega\ell\nu_\ell$ mode can be neglected. Assuming an Octet-Singlet mixing angle of 11^o, the rates for $F \rightarrow \eta\ell\nu_\ell$ and $F \rightarrow \eta'\ell\nu_\ell$ are simply related (7)

$$\frac{\Gamma(F^\pm \rightarrow \eta\ell^\pm\nu_\ell)}{\Gamma(F^\pm \rightarrow \eta'\ell^\pm\nu_\ell)} = 0.92 \times \text{phase space}$$

The rates $F^\pm \rightarrow \eta\ell^\pm\nu_\ell$ and $D \rightarrow K\ell^\pm\nu_\ell$ are related through SU(3). One could infer the $F^\pm \rightarrow \eta\ell^\pm\nu_\ell$ rate either from $D \rightarrow K\ell^\pm\nu_\ell$ or else evaluate the form factor $f_+^\eta(q^2)$ from SU(4) symmetry as discussed in the preceding case. The ratio $\frac{F\rightarrow f'\ell\nu_\ell}{F\rightarrow\phi\ell\nu_\ell}$ turn out to be very small and we neglect it in all subsequent discussions. The absolute rate for $F \rightarrow \phi\ell\nu_\ell$ is calculated using Eq. (60) and (61). The results are given in table 3. For comparison we also give the current algebra result. (7) The individual lepton spectra from $F \rightarrow \eta\ell\nu_\ell, \eta'\ell\nu_\ell$ and $\phi\ell\nu_\ell$ are given in Fig. (4), with the area normalized to unity in each case. The lepton spectrum from the F-meson decay after summing over the decay modes is shown in Fig. (5) for both the Veneziano model and current algebra.

V. Discussion

In this paper we have applied the method of Veneziano model to the various scattering processes involving charm mesons. The information so obtained on the strong interaction vertices $\Gamma_{DK^*F^*}$, $\Gamma_{DK^*F_A}$ etc. is then used in conjunction with the current-field identity to study the semi-leptonic decays of the charm mesons D and F. The exercise was undertaken to compare the results of current algebra wherever possible and to supplement it where current algebra is unable to provide an answer. There are several comments in order. The connection between the current algebra results and the Veneziano model is conventionally studied for the APV system through the comparison of the ratio G_S/G_D in the two approaches. (4) This is so, since the Veneziano model, by itself, does not fix the normalization of an amplitude. We have found that the connection between the two approaches valid for the chiral SU(3)$\otimes$SU(3) case, holds for the chiral SU(4)$\otimes$SU(4) case as well under identical assumptions, namely degenerate Regge trajectories and the anomalous magnetic moment of the axial vector mesons fixed at $\delta = -1$. However, if one uses the Regge slopes for the $J/\psi - \chi$ trajectory obtained by using the experimentally determined masses, then there are significant departures in the two approaches. Next, the standard method of imposing Adler's zeros on the amplitude is used to derive the masses of the F_A and D_A mesons which turn out to be 2.30 GeV and 2.25 GeV, respectively. It is tempting to identify the fourth peak in the recoil mass spectrum seen at 2.44 GeV at SPFAR (14) as due to the charm axial vector meson $D_A(2.25)$ production through the production $D_A D^*$.

We have tried to extract information on the semi-leptonic decay modes of the D and F mesons, involving scalar and tensor particles. This was done with the view of providing a rationale for the assumption commonly used in the analysis of the data, namely that the D and F semi-leptonic decay modes are dominated by $D \to K \ell \nu_\ell$, $K^* \ell \nu_\ell$, $F \to \eta(\eta') \ell \nu_\ell$ and $F \to \phi \ell \nu_\ell$ respectively.

We find that the ratios $\Gamma(D \to K^{**}(2^+, 1420) \ell \nu_\ell)/\Gamma(D \to K^* \ell \nu_\ell)$, $\Gamma(D \to K(0^+, 1250) \ell \nu_\ell)/\Gamma(D \to K^* \ell \nu_\ell)$ are almost independent of any parameter within the Veneziano model and are all negligible (see table (2) for exact values). The same is true of the ratio $\Gamma(F \to f'(2^+, 1514) \ell \nu_\ell)/\Gamma(F \to \phi \ell \nu_\ell)$. Although, intuitively expected to be small due to the phase space arguments, we believe ours is the just attempt to prove, within a model, that the decay modes $D \to K^{**} \ell \nu_\ell$, $D \to \kappa \ell \nu_\ell$, $D \to K_A \ell \nu_\ell$ and $F \to f' \ell \nu_\ell$ etc. are indeed very much suppressed. Coupled with the low energy argument [15], based on the Isoscalar nature of the $\Delta C = \Delta S$ GIM current, which suppresses the direct $D \to K(n\pi) \ell \nu_\ell$ modes, our calculation then provides a complete picture of the semi-leptonic decays of the D and F meson. This is in agreement with the inclusive lepton spectra in e^+e^- experiments which is well described by $D \to K^* \ell \nu_\ell$ and $D \to K \ell \nu_\ell$ modes.

Finally, we would like to comment on the absolute values of the various decay modes. The decay rates $D \to K \ell \nu_\ell$, $F \to \eta \ell \nu_\ell$, $\eta' \ell \nu_\ell$, using current algebra and SU(4) symmetry come out to be rather close [5],[7]. This presumably is due to the Ademollo-Gatto Theorem for $f_+(q^2)$ [16],

the relevant form factor for these decays, which states that the SU(4)-symmetry breaking effects come in the second order. This has the intuitive support from the fact that $f_+(q^2)$ is dimensionless. In general departures from symmetric results are expected to be rather soft. The decay $D \to K^* \ell \nu_\ell$ and $F \to \phi \ell \nu_e$ are rather model dependent. This is mainly due to the fact that the dominant form factor in these decays, namely $F_1^A(q^2)$ has the dimensions of mass. Clearly, it depends on how the mass-scale sets in. This is the reason why current algebra where $F_1^A(0) \simeq \sqrt{2}\, m_{K^*}$ [5] gives $\Gamma(D \to K^* \ell \nu_\ell)/\Gamma(D \to K \ell \nu_\ell) \approx 0.33$ as opposed to the Quark model result [7] $F_1^A(0) \simeq m_D + m_{K^*}$, which leads to $\Gamma(D \to K^* \ell \nu_\ell)/\Gamma(D \to K \ell \nu_\ell) \approx 1.2$. The Veneziano model, by itself, cannot be used to support one or the other, as we have pointed out earlier. However, one could invoke the non-relativistic Quark model and the SLAC data on the $Q_1(1^+, 1290)$, $Q_2(1^+, 1400)$ decays to provide an alternative result. The results so obtained interpolate between the current algebra value and the Quark model results.

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Appendix I

We quote here the parametrization of Regge trajectories obtained from the property of factorization of the Regge slopes and the J/ψ , $\chi(3552)$, D , D^* masses by Igi (ref. 1):

$$\alpha_\rho(s) = 0.47 \; + 0.88 \; s$$
$$\alpha_{K^*}(s) = 0.34 \; + 0.83 \; s$$
$$\alpha_\phi(s) = 0.18 \; + 0.79 \; s$$
$$\alpha_{D^*}(s) = -1.18 + 0.54 \; s$$
$$\alpha_{J/\psi}(s) = -2.17 + 0.33 \; s$$

Appendix II

The pole contributions to the invariant amplitudes A(s,t) and B(s,t) are

$$A_{K^*} = \frac{g_{K^*K\pi}}{t-m_{K^*}^2}\left\{ G_S\left(2s+m_{K^*}^2-2m_K^2-m_D^2-m_{F_A}^2\right) - \frac{(m_K^2-m_\pi^2)}{m_{K^*}^2}\left[G_S+G_D\left(-m_{K^*}^2+m_D^2-m_{F_A}^2\right)\right]\right\} \qquad (\text{II-1})$$

$$B_{K^*} = -\frac{g_{K^*K\pi}}{t-m_{K^*}^2}\, G_S \qquad (\text{II-2})$$

$$A_{D^*} = -\frac{1}{2}\frac{g_{D^*D\pi}}{s-m_{D^*}^2}\left\{3H_S+H_D\left(2t+m_{D^*}^2-2m_D^2-m_K^2-m_{F_A}^2\right) - \frac{(m_D^2-m_\pi^2)}{m_{D^*}^2}\left[H_S+H_D\left(-m_{D^*}^2+m_K^2-m_{F_A}^2\right)\right]\right\} \qquad (\text{II-3})$$

$$B_{D^*} = -\frac{1}{2}\frac{g_{D^*D\pi}}{s-m_{D^*}^2}\left\{H_S-H_D\left(2t+m_{D^*}^2-2m_D^2-m_K^2-m_{F_A}^2\right) + \frac{(m_D^2-m_\pi^2)}{m_{D^*}^2}\left[H_S+H_D\left(-m_{D^*}^2+m_K^2-m_{F_A}^2\right)\right]\right\} \qquad (\text{II-4})$$

$$A_\kappa = -2\,\frac{g_{\kappa DF_A}\, g_{\kappa K\pi}}{t-m_\kappa^2} \; , \qquad B_\kappa = 0 \qquad (\text{II-5})$$

$$A_\chi = -B_\chi = -\frac{g_{\chi KF_A}\, g_{\chi D\pi}}{s-m_\chi^2} \qquad (\text{II-6})$$

Here $\mathcal{K}$ and $\mathcal{D}$ are the daughters of K^* and D^*, respectively. The coupling constants G_S, G_D and H_S, H_D for the S- and D-wave decays of $F_A \to DK^*$ and $F_A \to D^*K$ are defined as follows:

$$\langle K^* | j_D | F_A(q) \rangle = i \left[G_S \, \epsilon\cdot\eta + 2 G_D \, \epsilon\cdot p_{K^*} \, \eta\cdot q \right] \quad (\text{II-7})$$

$$\langle D^* | j_K | F_A(q) \rangle = -i \left[H_S \, \epsilon\cdot\eta + 2 H_D \, \epsilon\cdot p_{D^*} \, \eta\cdot q \right] \quad (\text{II-8})$$

where ϵ_μ and η_μ are the polarization vectors of the axial-vector and the vector particles.

The residues of the Veneziano forms of A(s,t) and B(s,t), Eq. (5), are

$$\mathrm{Res}\, A = -(\gamma_1 s + \gamma_2 m^2_{K^*} + \gamma_3)/\alpha'_{K^*}$$

$$\mathrm{Res}\, B = -(\beta_2 m^2_{K^*} + \beta_3)/\alpha'_{K^*}$$

at the pole $t = m^2_{K^*}$, and (II-9)

$$\mathrm{Res}\, A = -(\gamma_1 m^2_{D^*} + \gamma_2 t + \gamma_3)/\alpha'_{D^*}$$

$$\mathrm{Res}\, B = -(\beta_2 t + \beta_3)/\alpha'_{D^*}$$

at the pole $s = m^2_{D^*}$. These residues contain contributions coming from the daughters $\mathcal{K}$ and $\mathcal{D}$ as well. We now compare these residues with those from Eqs. (II-1)-(II-6), namely we can write ResA = Res A_{K^*} + Res $A_{\mathcal{K}}$, Res B = Res B_{K^*} + Res $B_{\mathcal{K}}$ at $t = m^2_{K^*}$ and Res A = Res A_{D^*} + Res $A_{\mathcal{D}}$, Res B = Res B_{D^*} + Res $B_{\mathcal{D}}$ at $s = m^2_{D^*}$. These four equations plus the consistency condition constraint (11) immediately give us the eight relations already written down in the text (Eqs. (15)-(18)).

Now we study the K^{**}. The pole contributions of the K^{**} and its daughters $\mathcal{K}_1$(vector) and $\mathcal{K}_2$(scalar) are as follows (with $m_\pi = 0$):

$$A_{K^{**}} = \frac{g_{K^{**}K\pi}}{t - m^2_{K^{**}}} \left\{ \frac{m_K^2}{2m^2_{K^{**}}} (2s + X) + \frac{1}{6m^2_{K^{**}}} YZ \right\} G_P$$
$$- \frac{2g_{K^{**}K\pi}}{t - m^2_{K^{**}}} \left\{ \frac{1}{4}(2s+X)^2 + \frac{1}{3} Z \left(-m^2_{F_A} + \frac{Y^2}{4m^2_{K^{**}}} \right) \right\} G_F \quad (\text{II-10})$$

$$B_{K^{**}} = \frac{g_{K^{**}K\pi}}{t - m^2_{K^{**}}} \left(s + \frac{1}{2} X \right) G_P \quad (\text{II-11})$$

$A_{\mathcal{K}_1}$ and $B_{\mathcal{K}_1}$ are exactly in the same form as Eqs. (II-1) and (II-2) with the obvious replacements $K^* \to \mathcal{K}_1$, $G_S \to G_S^1$ and $G_D \to G_D^1$. Similarly $A_{\mathcal{K}_2}$ and $B_{\mathcal{K}_2}$ have the same form as Eq. (II -5). Here we have used the following short notations:

$$X = m^2_{K^{**}} - m^2_K - m^2_D - m^2_{F_A} - \frac{m^2_K}{m^2_{K^{**}}}(m^2_D - m^2_{F_A})$$

$$Y = m^2_D - m^2_{K^{**}} - m^2_{F_A}$$

$$Z = m^2_{K^{**}} - 2m^2_K + \frac{m^4_K}{m^2_{K^{**}}}$$

G_P and G_F in Eqs. (II-10) and(II-11) are the P- and F-wave coupling constants for the vertex $F_A D K^{**}$ and defined as follows

$$\langle K^{**} | j_D | F_A(q) \rangle = i \left[G_P \epsilon_\mu \epsilon^{\mu\nu} q_\nu + 2 G_F \, \epsilon\cdot p_{K^{**}} \, \epsilon^{\nu\lambda} q_\nu q_\lambda \right] \quad (\text{II-12})$$

where $\epsilon_{\mu\nu}$ is the polarization tensor of K^{**}. G_S^1 and G_D^1 describe the vertex $\langle K_1 | j_D | F_A \rangle$ and are defined in the same way as Eq. (II-7).

The residues of A(s,t) and B(s,t) in Eq. (5) at the pole $t = m^2_{K^{**}}$ are

$$\mathrm{Res}\,A = -\frac{1}{\alpha'_{K^*}}\left(\gamma_1 s + \gamma_2 m^2_{K^{**}} + \gamma_3\right)\left(\alpha'_{D^*} s + \alpha_{D^*}(0)\right) \tag{II-13}$$

and

$$\mathrm{Res}\,B = -\frac{1}{\alpha'_{K^*}}\left(\beta_2 m^2_{K^{**}} + \beta_3\right)\left(\alpha'_{D^*} s + \alpha_{D^*}(0)\right)$$

Note that these residues contain the K^{**}, its first daughter (K_1) and its second daughter (K_2) contributions. Therefore from

$$\mathrm{Res}\,A = \mathrm{Res}\,A_{K^{**}} + \mathrm{Res}\,A_{K_1} + \mathrm{Res}\,A_{K_2} \text{ and } \mathrm{Res}\,B = \mathrm{Res}\,B_{K^{**}} + \mathrm{Res}\,B_{K_1} + \mathrm{Res}\,B_{K_2}$$

we get the relations:

$$\gamma_1 = 2\,\frac{\alpha'_{K^*}}{\alpha'_{D^*}}\, g_{K^{**}K\pi}\, G_F \tag{II-14}$$

$$\left(\beta_2 m^2_{K^{**}} + \beta_3\right) = \frac{\alpha'_{K^*}}{\alpha'_{D^*}}\, g_{K^{**}K\pi}\, G_P \tag{II-15}$$

$$\frac{\alpha_{D^*}(0)}{\alpha'_{K^*}}\left(\beta_2 m^2_{K^{**}} + \beta_3\right) = -\frac{1}{2}\, g_{K^{**}K\pi}\, G_P X + g_{K_1K\pi}\, G_S^1 \tag{II-16}$$

$$\frac{1}{\alpha'_{K^*}}\left[\alpha_{D^*}(0)\gamma_1 + \alpha'_{D^*}\left(\gamma_2 m^2_{K^{**}} + \gamma_3\right)\right] = g_{K^{**}K\pi}\left(2G_F X - \frac{m_K^2}{m^2_{K^{**}}} G_P\right) - 2 g_{K_1K\pi}\, G_D^1 \tag{II-17}$$

$$\begin{aligned} -\frac{\alpha_{D^*}(0)}{\alpha'_{K^*}}\left(\gamma_2 m^2_{K^{**}} + \gamma_3\right) = {} & g_{K^{**}K\pi}\left[\frac{m_K^2}{2m^2_{K^{**}}} X + \frac{1}{6m^2_{K^{**}}} YZ\right] G_P \\ & - 2 g_{K^{**}K\pi}\left[\frac{1}{4}X^2 + \frac{1}{3} Z\left(-m^2_{F_A} + \frac{Y^2}{4m^2_{K^{**}}}\right)\right] G_F \\ & + g_{K_1K\pi}\left\{G_D^1\left(m^2_{K_1} - m_K^2 - m_D^2 - m^2_{F_A}\right) - \frac{m_K^2}{m^2_{K_1}}\left[G_S^1 + G_D^1\left(m_D^2 - m^2_{F_A}\right)\right]\right\} \\ & - 2\, g_{K_2DF_A}\, g_{K_2K\pi} \end{aligned} \tag{II-18}$$

where γ_i and β_i are given in Eq. (15).

Appendix III

We give here the relevant formulae for the semi-leptonic decays of the charm mesons.

(a) $D(p) \rightarrow K(k) + e(q_1) + \nu_e(q_2)$:

$$\mathcal{M} = -i \frac{G}{\sqrt{2}} \bar{u}_e(q_1) \gamma^\mu (1-\gamma_5) v_{\nu_e}(q_2) \langle K | V_\mu | D \rangle \cos\theta_c \qquad (\text{III-1})$$

where θ_c is the Cabibbo angle $(\cos^2\theta_c \simeq 0.95)$ and Fermi constant G has a value of $1.02 \times 10^{-5}\, m_p^{-2}$. The hadronic matrix element can be decomposed as follows

$$\langle K | V_\mu | D \rangle = f_+(q^2)(p+k)_\mu + f_-(q^2)\, q_\mu \qquad (\text{III-2})$$

where q = p-k. However, since q_μ times leptonic matrix element is proportional to the lepton mass, m_e, we can neglect $f_-(q^2)$ term. The square of the invariant matrix element is

$$|\mathcal{M}|^2 = 4G^2 \left[2q_1\cdot(p+k)\; q_2\cdot(p+k) - (q_1\cdot q_2)(p+k)^2 \right] |f_+(q^2)|^2 \cos^2\theta_c \qquad (\text{III-3})$$

We can parametrize $f_+(q^2)$ with a monopole form:

$$f_+(q^2) = \frac{m_{F^*}^2\, f_+(0)}{m_{F^*}^2 - q^2} \qquad (\text{III-4})$$

(b) $D \rightarrow \pi e \nu_e$: In order to study this process, one has only to replace $\cos^2\theta_c$ by $\sin^2\theta_c$ and F^* by D^* in the

above formulae. Note that the difference in parametrization comes from the fact that the $\cos\theta_c$ part of the GIM vector current has the quantum number of F^*, whereas the $\sin\theta_c$ part has the quantum number of D^*.

(c) $D(p) \rightarrow K^*(k) + e(q_1) + \nu_e(q_2)$:

$$\mathcal{M} = -i \frac{G}{\sqrt{2}} \bar{u}_e(q_1) \gamma^\mu (1-\gamma_5) v_{\nu_e}(q_2) \langle K^* | V_\mu + \lambda A_\mu | D \rangle \cos\theta_c \qquad (\text{III-5})$$

where $\lambda = \mp 1$ for $(V \mp A)$ current. For the GIM current $\lambda = -1$. The invariant decompositions of the hadronic axial-vector and vector currents are

$$\langle K^* | A_\mu | D \rangle = F_1^A(q^2)\, \epsilon_\mu + F_2^A(q^2)(p\cdot\epsilon)\, k_\mu + F_3^A(q^2)(q\cdot\epsilon)\, q_\mu \qquad (\text{III-6})$$

$$\langle K^* | V_\mu | D \rangle = i F_1^V(q^2)\, \epsilon_{\mu\nu\lambda\sigma}\, \epsilon^\nu k^\lambda q^\sigma \qquad (\text{III-7})$$

where ϵ_μ is the polarization vector of K^*. The square of the invariant matrix element is

$$\begin{aligned} |\mathcal{M}|^2 = 4G^2\cos^2\theta_c \Big\{ & 2|F_1^V|^2 \left[-(q_1\cdot k)(q_2\cdot k) - \tfrac{1}{4} q^2 k^2 + \tfrac{1}{2}(q\cdot k)^2 \right] q^2 \\ & + |F_1^A|^2 \left[q_1\cdot q_2 + 2(q_1\cdot k)(q_2\cdot k)/m_{K^*}^2 \right] \\ & - 2F_1^A F_2^A \Big[(q_1\cdot p)(q_2\cdot k) - 2(p\cdot k)(q_1\cdot k)(q_2\cdot k)/m_{K^*}^2 \\ & + (q_1\cdot k)(q_2\cdot k) \Big] + |F_2^A|^2 \left(-m_D^2 + (p\cdot k)^2/m_{K^*}^2 \right) \Big[2(q_1\cdot k)(q_2\cdot k) \\ & - m_{K^*}^2 (q_1\cdot q_2) \Big] + 4\lambda F_1^A F_1^V (q_1\cdot q_2)(q_1\cdot k - q_2\cdot k) \Big\} \end{aligned} \qquad (\text{III-8})$$

Note that the form factor $F_3^A(q^2)$ has been neglected, since we put $m_e \approx 0$ everywhere. One can again parametrize the form factors using monopole forms

$$F_{1,2}^A(q^2) = \frac{m_{F_A}^2 \, F_{1,2}^A(0)}{m_{F_A}^2 - q^2} \qquad (\text{III-9})$$

$$F_1^V(q^2) = \frac{m_{F^*}^2 \, F_1^V(0)}{m_{F^*}^2 - q^2} \qquad (\text{III-10})$$

where F_A is the axial-vector partner of F^*. The coupling constants F_1^A and F_1^V which determine the VA interference term, are assumed to have the same relative sign. Note that Eq. (II-7) is a special form of Eq. (III-6) such that the A_μ current is dominated by the on-mass-shell particle. In this connection, G_S and $2G_D$ correspond to the on-mass-shell values of F_1^A and F_2^A, respectively.

(d) $D(p) \to K^{**}(k) + e(q_1) + \nu_e(q_2)$

$$\mathcal{M} = -i\frac{G}{\sqrt{2}}\,\bar{u}_e(q_1)\gamma^\mu(1-\gamma_5)v_{\nu_e}(q_2)\langle K^{**}|V_\mu + \lambda A_\mu|D\rangle \cos\theta_c \qquad (\text{III-11})$$

The invariant decompositions of the hadronic axial-vector and vector currents are

$$\langle K^{**}|A_\mu|D\rangle = G_1^A(q^2)\,\epsilon_{\mu\nu}p^\nu + G_2^A(q^2)\,\epsilon_{\nu\lambda}p^\nu p^\lambda k_\mu + G_3^A(q^2)\,\epsilon_{\nu\lambda}p^\nu p^\lambda q_\mu \qquad (\text{III-12})$$

$$\langle K^{**}|V_\mu|D\rangle = i\,G_1^V(q^2)\,\epsilon_{\mu\nu\lambda\rho}\,\epsilon^{\nu\sigma}p_\sigma k^\lambda q^\rho \qquad (\text{III-13})$$

where $\epsilon_{\mu\nu}$ is the polarization tensor of K^{**} normalized as

$$\epsilon^*_{\mu\nu}(k)\,\epsilon_{\mu'\nu'}(k) = \frac{1}{2}P_{\mu\mu'}P_{\nu\nu'} + \frac{1}{2}P_{\mu\nu'}P_{\nu\mu'} - \frac{1}{3}P_{\mu\nu}P_{\mu'\nu'} \qquad (\text{III-14})$$

with

$$P_{\mu\mu'} = -g_{\mu\mu'} + k_\mu k_{\mu'}/k^2$$

The G_3^A term can be neglected, since its contribution is proportional to the lepton mass. The invariant matrix element square is

$$|\mathcal{M}|^2 = 2G^2\cos^2\theta_c\Big[2|G_1^A|^2 m_1 + \frac{4}{3}|G_2^A|^2 m_2 + \frac{8}{3}G_1^A G_2^A m_3 + |G_1^V|^2 m_4 - 4\lambda\, G_1^A G_1^V m_5\Big] \qquad (\text{III-15})$$

where

$$\begin{aligned} m_1 = {} & \frac{1}{2}\left[m_D^2 - (p\cdot k)^2/m_{K^{**}}^2\right]\left[-(q_1\cdot q_2) - 2(q_1\cdot k)(q_2\cdot k)/m_{K^{**}}^2\right] \\ & + \frac{1}{6}\Big\{2(q_1\cdot p)(q_2\cdot p) - \frac{1}{2}q^2 m_D^2 - \frac{2(p\cdot k)}{m_{K^{**}}^2}\Big[(q_1\cdot k)(q_2\cdot p) \\ & + (q_1\cdot p)(q_2\cdot k) - 2(q_1\cdot k)(q_2\cdot k)(p\cdot k)/m_{K^{**}}^2\Big] \\ & + \frac{(p\cdot k)^2}{m_{K^{**}}^4}\Big[-2(q_1\cdot k)(q_2\cdot k) + \frac{1}{2}q^2 m_{K^{**}}^2\Big]\Big\} \end{aligned} \qquad (\text{III-16})$$

$$m_2 = \frac{1}{2}\left[m_D^2 - (p\cdot k)^2/m_{K^{**}}^2\right]\left[4(q_1\cdot k)(q_2\cdot k) - q^2 m_{K^{**}}^2\right] \quad (\text{III-17})$$

$$m_3 = \left[m_D^2 - (p\cdot k)^2/m_{K^{**}}^2\right]\left[(q_1\cdot k)(q_2\cdot p) + (q_1\cdot p)(q_2\cdot k) - 2(q_1\cdot k)(q_2\cdot k)(p\cdot k)/m_{K^{**}}^2\right] \quad (\text{III-18})$$

$$m_4 = \left[m_D^2 - (p\cdot k)^2/m_{K^{**}}^2\right]\left[2(q_1\cdot k)(q_2\cdot k)q^2 + \frac{1}{2}m_{K^{**}}^2 q^4 - (k\cdot q)^2 q^2\right] \quad (\text{III-19})$$

$$m_5 = \frac{1}{2}\left[m_D^2 - (p\cdot k)^2/m_{K^{**}}^2\right] q^2 (q_1 - q_2)\cdot k \quad (\text{III-20})$$

The form factors G_1^A, G_2^A and G_1^V can be parametrized in exactly the same way as for the $D \to K^* e \nu_e$ case. The interference effect from AV term are expected to be very small due to the very small phase space available and the kinematic factors multiplying the G_1^V term. To a very good approximation, the form factor G_1^V could be set to zero here.

Table Caption

Table 1: Decay rates (units: sec^{-1}) for $D \to K e \nu_e$, $K^*(1^-, 892) e \nu_e$, $K^{**}(2^+, 1420) e \nu_e$, $\kappa(0^+, 1250) e \nu_e$ and $\kappa_1(1^-, 1420) e \nu_e$. κ_1 is the first daughter of $K^{**}(2^+, 1420)$ and in dual models is mass degenerate with K^{**}. The current algebra results are taken from ref. (5). For the Veneziano model case (i) corresponds to no daughter of K^* solution and case (ii) to the neglect of the D-wave couplings in the decay $D \to K^* \ell \nu_\ell$.

Table 2: Relative rates for the various semi-leptonic decays of D meson. The SU(4) symmetry value for the ratio $\Gamma(D \to \pi e \nu_e)/\Gamma(D \to K e \nu_e)$ corresponds to $\delta = -1$. Current algebra results are from ref. (5).

Table 3: Decay and relative rates for $F \to \eta \ell \nu_\ell$, $\eta' \ell \nu_\ell$ and $\phi \ell \nu_\ell$ using the Veneziano model and the current algebra. The current algebra results are from ref. (7).

Decay Mode	Veneziano Model		Current Algebra		
	case (i)	case (ii)	$\delta = 0.0$	$\delta = -0.5$	$\delta = -1.0$
$\Gamma(D \to K e \nu_e)$	-	-	1.24×10^{11}	1.39×10^{11}	1.55×10^{11}
$\Gamma(D \to \pi e \nu_e)$	-	-	0.88×10^{10}	1.12×10^{10}	1.41×10^{10}
$\Gamma(D \to K^{*} e \nu_e)$	0.78×10^{11}	1.20×10^{11}	0.37×10^{11}	0.43×10^{11}	0.50×10^{11}
$\Gamma(D \to K^{**} e \nu_e)$	0.11×10^{10}	0.12×10^{9}	-	-	-
$\Gamma(D \to \kappa e \nu_e)$	-	0.55×10^{10}	-	-	-
$\Gamma(D \to K_1 e \nu_e)$	0.62×10^{10}	0.86×10^{9}	-	-	-

Table 1

Figure Caption

Fig. 1. Quark diagram for the two-body decay of a meson.

Fig. 2. Individual lepton spectra from the rest frame semi-leptonic decay of D meson, with the area normalized to unity. Form factor effects are included.

Fig. 3. Inclusive electron spectra from the rest frame D decay. (a) Veneziano model prediction, (b) Current algebra prediction.

Fig. 4. Individual lepton spectra from the rest frame semi-leptonic decay of F meson, with the area normalized to unity. Form Factor effects are included.

Fig. 5. Inclusive electron spectra from the rest frame F decay. (a) Veneziano model prediction, (b) Current algebra prediction.

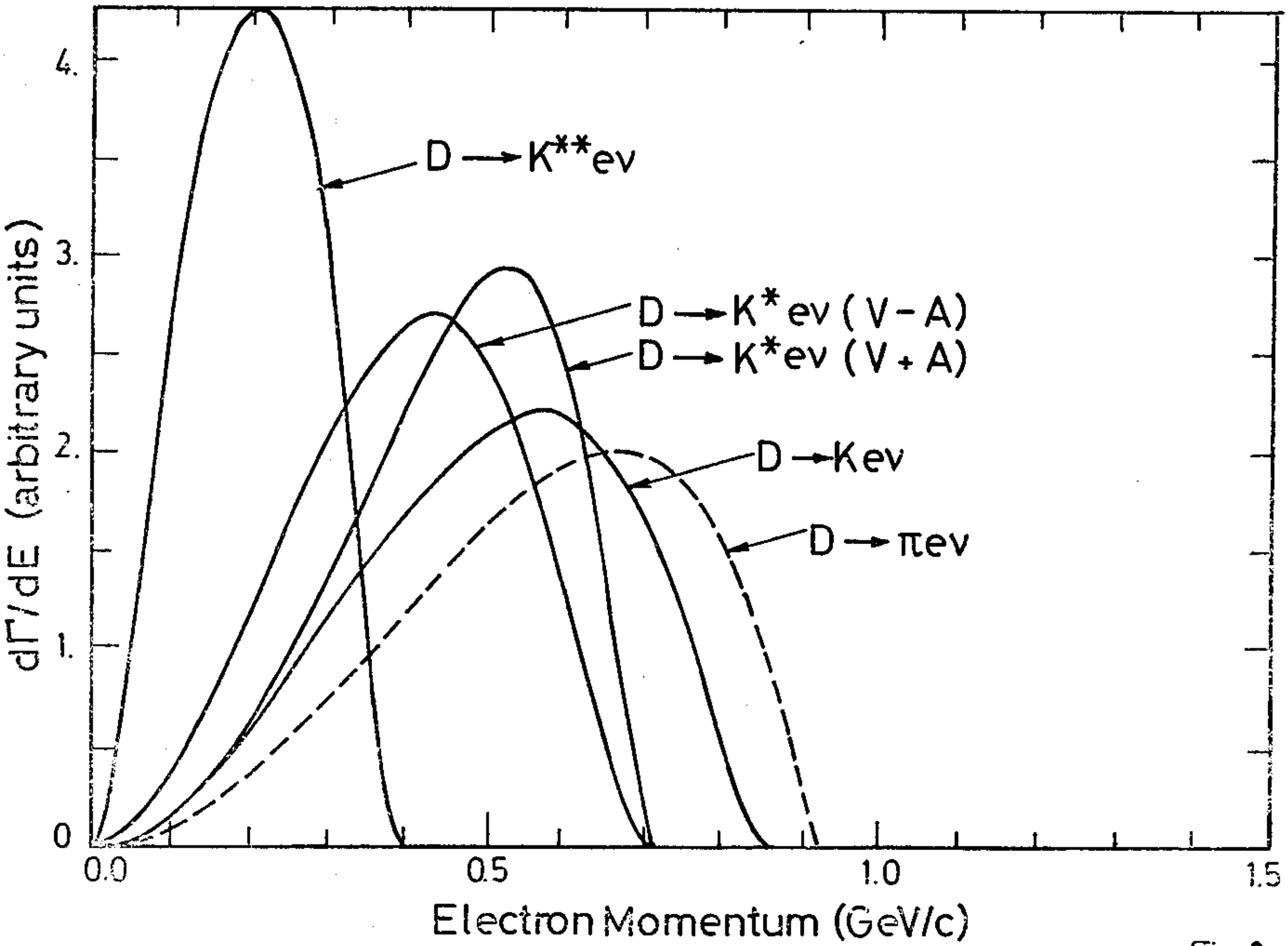
Current Algebra			
	$\delta = 0.0$	$\delta = -0.5$	$\delta = -1.0$
$\Gamma(D \to K^* e \nu_e)/\Gamma(D \to K e \nu_e)$	0.30	0.31	0.32
$\Gamma(D \to \pi e \nu_e)/\Gamma(D \to K e \nu_e)$	0.07	0.08	0.09
Veneziano Model			
$\frac{\Gamma(D \to K e \nu_e)}{\Gamma(D \to K^* e \nu_e)}$ case (i)	1.6	1.8	2.0
$\frac{\Gamma(D \to K e \nu_e)}{\Gamma(D \to K^* e \nu_e)}$ case (ii)	1.0	1.15	1.3

	case (i)	case (ii)
$\Gamma(D \to K^{**} e \nu_e)/\Gamma(D \to K^* e \nu_e)$	0.014	0.001
$\Gamma(D \to \kappa e \nu_e)/\Gamma(D \to K^* e \nu_e)$	-	0.05
$\Gamma(D \to K_1 e \nu_e)/\Gamma(D \to K^* e \nu_e)$	0.08	0.007

Table 2

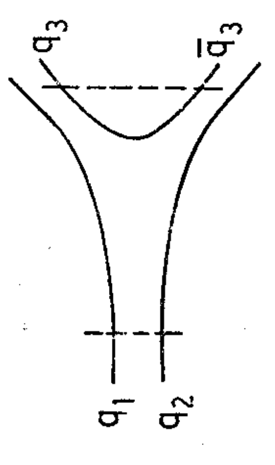
Current Algebra			
$\Gamma(F \to \eta e \nu_e)$	1.2×10^{11}		
$\Gamma(F \to \eta' e \nu_e)$	0.35×10^{11}		
	$\delta = 0.0$	$\delta = -0.5$	$\delta = -1.0$
$\Gamma(F \to \phi e \nu_e)$	0.49×10^{11}	0.57×10^{11}	0.66×10^{11}
$\Gamma(F \to \phi e \nu_e)/\Gamma(F \to \eta e \nu_e)$	0.40	0.48	0.55
Veneziano Model			
	case (i)	case (ii)	
$\Gamma(F \to \phi e \nu_e)$	0.83×10^{11}	1.17×10^{11}	
$\Gamma(F \to \phi e \nu_e)/\Gamma(F \to \eta e \nu_e)$	0.70	0.98	

Table 3



D → K**eν
D → K*eν (V−A)
D → K*eν (V+A)
D → Keν
D → πeν
dΓ/dE (arbitrary units)
0
1.
2.
3.
4.
0.0
0.5
1.0
1.5
Electron Momentum (GeV/c)

Fig.2



q_3
$\bar{q}_3$
q_1
q_2

Fig.1

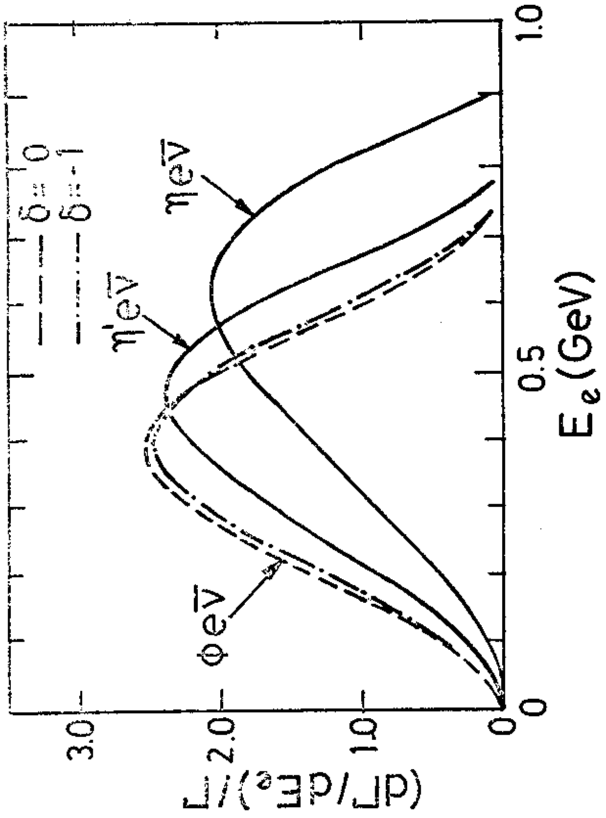


Fig.4

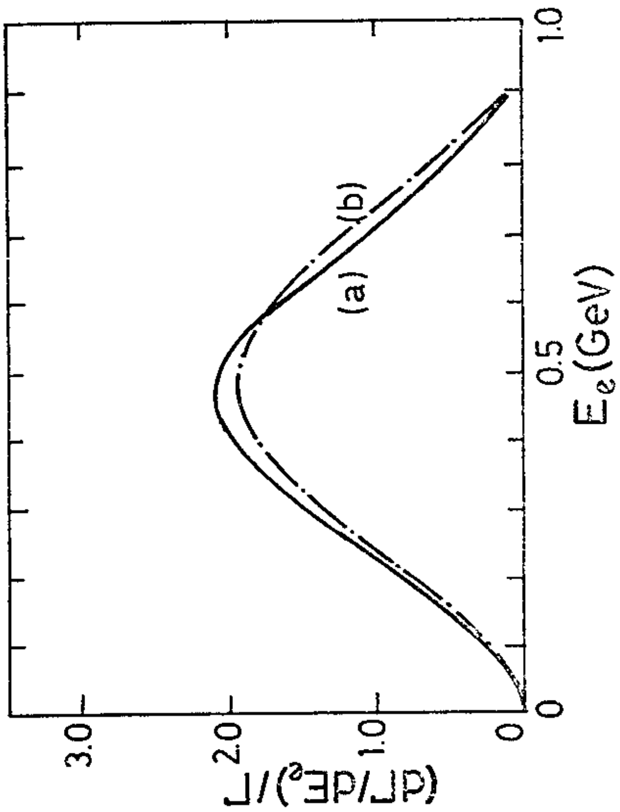


Fig.5

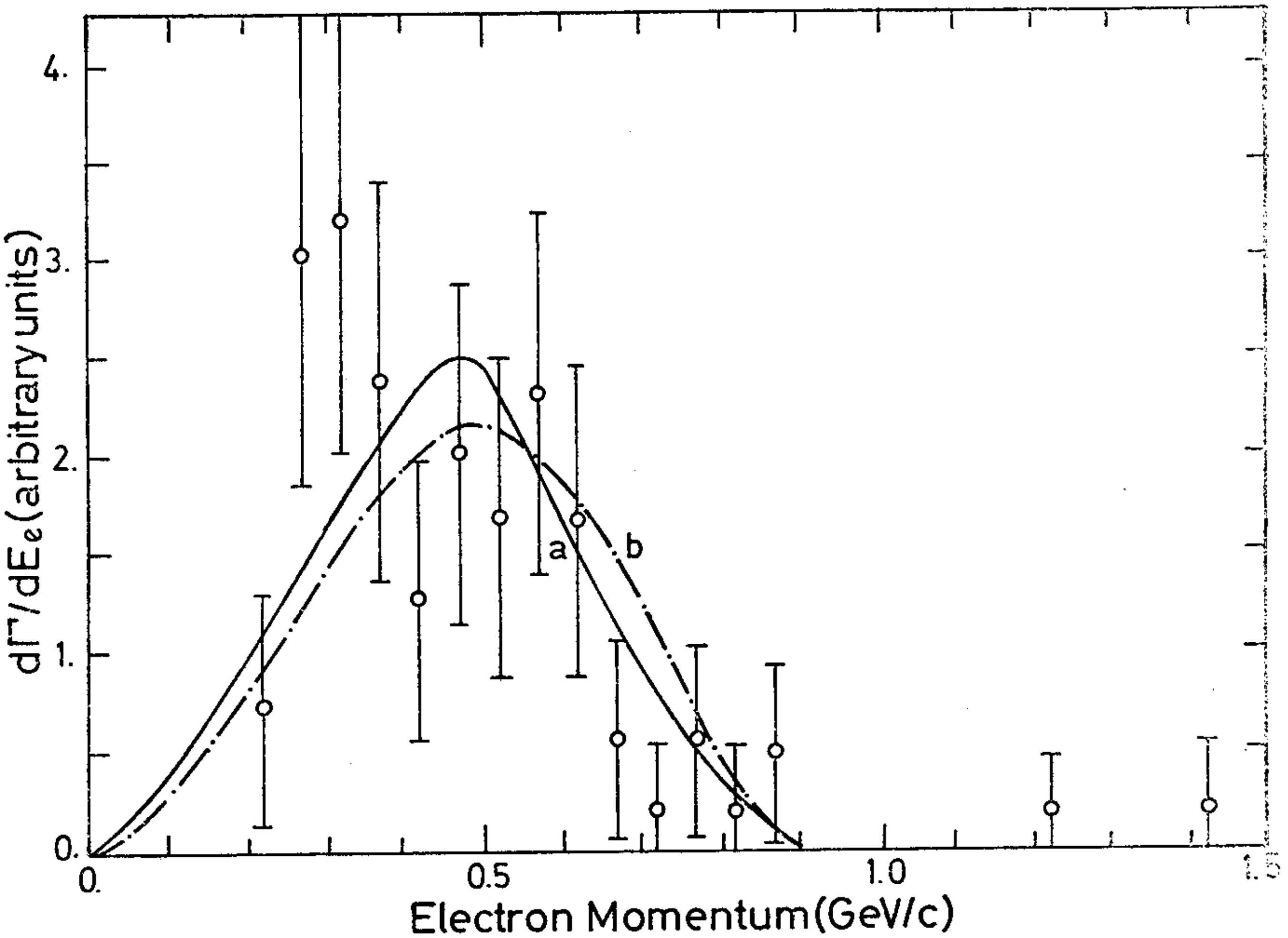


Fig.3