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by

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THE RELATIVISTIC HARMONIC OSCILLATOR RECONSIDERED

by

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Abstract:
The bound states of scalar quarks interacting through a scalar harmonic oscillator are investigated. In the presence of this interaction the dressed quark propagator differs substantially from the free one. This leads to a Bethe Salpeter equation which does not allow for any stable bound states of positive mass.

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I. Introduction

It is a common hope that QCD will be the correct field theory for the hadronic constituents. Calculations are notoriously plagued with the confinement problem which renders the conventional perturbation theory around the free vacuum inapplicable. It has been emphasized [1] that the formal resummation of such perturbation series into the set of Schwinger-Dyson (SD) integral equations might well survive phase transitions and still be valid for a confining theory. Although at present it is not known whether this is indeed so, one may turn around the question and ask, whether conventional field theory as it is expressed through the SD equations is able to accommodate confinement.

There have been a number of attempts in this direction using either QCD itself [2] or some other set of elementary particle fields [3]. The basic procedure is always the same: one starts with an ansatz for either the irreducible interaction kernel, or some vertex function or propagators and proceeds to calculate the Green's functions of the theory and investigates the internal consistency of the ansatz. It should be pointed out that it is in principle straight forward to calculate physical quantities, although the nonlinearity of some equation and numerical calculations may make work a bit tedious sometimes. Thus this framework provides a laboratory for building quark models on the basis of relativistic quantum field theory. Although nobody has as yet seriously attempted to realize this programme, there are some promising steps in this direction [2,3].

Specifying this approach to the two-particle (quark-antiquark) sector, the mesonic bound state problem is formulated in terms of the Bethe-Salpeter (BS) equation. The bound states are determined by the two-particle irreducible interaction kernel which works as an effective potential, allowing for a specific phenomenological ansatz for the infrared content of the interaction. Thus the

BS equation, being the analogue of the Schrödinger equation in relativistic field theory, provides a framework for generalizing nonrelativistic potential models, such as e. g. the charmonium picture.

There is, however, one important difference with regard to the customary treatment of the BS equation in the ladder approximation where one uses free propagators: In order to obtain a consistent approximation scheme, one has to use the fully dressed quark propagator as it is determined by the interaction kernel through another SD equation. In general the dressed quark propagator will differ drastically from the free one for a theory which leads to confinement, becaus of the presence of strong long range forces. Indeed, for a confining interaction the quark propagator should no longer have any singularity in the mass squared variable: Thus the SD equation for the propagator itself represents an important tool for deciding whether a given interaction kernel may lead to confinement [4,5].

One usually assumes that a confining interaction will become increasingly strong as the quark-antiquark pair separates over long distances. This corresponds to an interaction kernel which develops a sufficiently strong singularity as the momentum transfer between the quarks approaches zero. In QCD one often ascribes such a singular behaviour to the running quark-gluon coupling constant g(t) as $t \to 0$. It is then possible to approximate the kernel by an effective one-gluon exchange, which leads to an approximation to leading order 1/N if used within the framework of SD equations described above.

By the example given below it will become clear that not any kernel one might choose this way will lead to confinement. We will investigate the bound state problem for spin zero quarks and a kernel which represents a scalar relativistic harmonic oscillator interaction [6]. Though the interaction diverges quadratically in the quark separation, this interaction does not lead to confinement and does not even allow for any bound states. There are basically three reasons for this specific choice of the kernel: First, one might think of the relativistic scalar harmonic oscillator as being a realistic description of the quark-antiquark system. Secondly, the relativistic harmonic oscillator has been around for quite some time to deserve some careful analysis for its own sake. The third reason for our choice is purely technical, as for this kernel the mesonic bound state equations may be solved explicitly.

In section II we will define the kernel together with the appropriate Bethe Salpeter framework. The solutions for the quark propagator have been derived by Alabiso and Schierholz [4] and are reported in section III. We then investigate in section IV for the various possible propagators the bound state equations which are solved explicitly in two cases. We come to the conclusion that no positive mass bound state exists for the given kernel and comment upon the results in section V.

II. Bethe-Salpeter Equation and Kernel

In a world of spin zero quarks the mesonic bound states are described by the BS amplitude

$$\phi(p,q) = (2\pi)^{3/2} \int d^4x \, e^{iqx} \langle 0 | T(q(\tfrac{x}{2}) q^+(-\tfrac{x}{2})) | \phi_p \rangle \tag{2.1}$$

which for the case of stable bound states obeys the homogeneous BS equation

$$\phi(p,q) = \Delta_F(q_+)\Delta_F(-q_-)\, i \int d^4k \, K(q,k;p)\phi(p,k) \tag{2.2}$$

and is normalized according to

$$(2\pi)^4 i \int d^4q \, \phi(p,q) \left\{ \left[\frac{\partial}{\partial p^2} \Delta_F^{-1}(q_+)\Delta_F^{-1}(-q_-) \right] \phi(p,q) - \int d^4k \left[\frac{\partial}{\partial p^2} K(q,k;p) \right] \phi(p,k) \right\} = 1 \tag{2.3}$$

where we have introduced the notation

$$q_{\pm} = \tfrac{1}{2} p \pm q$$

Here $\Delta_F(q)$ is the dressed quark Green's function

$$\Delta_F(q) = \int d^4x \, e^{iqx} \langle 0 | T(q(x) q^+(0)) | 0 \rangle \tag{2.4}$$

which will be calculated in the next section and $K(q,k;p)$ is the two particle irreducible interaction kernel which we will assume to have scalar coupling to the quarks and behave like t^{-3} for $t = (q-k)^2$, regularized in a canonical way to give

$$K(q,k;p) = \underset{\lambda=3}{Res} \frac{\beta}{t^\lambda} = -i\beta \, \Box_{q-k} \, \delta^4(q-k) \tag{2.5}$$

In configuration space this kernel behaves like

$$K(x,y;p) \sim (x-y)^2 \tag{2.6}$$

which is the reason why we will refer to is as a relativistic harmonic ascillator interaction.

III. The Quark Green's Function

The kernel (2.5) which we will use as an input represents long range forces which may drastically change the propagation of a quark as compared to the free situation. Thus the first object to be calculated is the quark Green's function (2.4). This can be done via the SD equation for the current vertex

$$\Gamma_\mu(p,q) = 2gZ q_\mu + i\int d^4k \, K(q,k;p) \Delta_F(q_+) \Delta_F(-q_-) \Gamma_\mu(p,k) \tag{3.1}$$

Letting $p_\mu \to 0$ and using the Ward identity one obtains

$$\frac{\partial \Delta_F^{-1}(q)}{\partial q^\mu} = 2Zq_\mu - i\int d^4k \, K(q,k;p=0) \frac{\partial \Delta_F(k)}{\partial k^\mu}$$

For a convolution type kernel like (2.5) this can be converted into

$$\frac{\partial \Delta_F^{-1}(q)}{\partial q^\mu} = 2q_\mu Z - i\frac{\partial}{\partial q^\mu}\int d^4k \, K(q,k;p=0) \Delta_F(k)$$

from which by integration we get the nonlinear SD equation for the propagator

$$\Delta_F^{-1}(q) = Z \Delta_0^{-1}(q) - i\int d^4k \, K(q,k;p=0) \Delta_F(k) \tag{3.2}$$

where

$$\Delta_0^{-1}(q) = q^2 - m_0^2 \tag{3.3}$$

with m_0 being an arbitrary integration constant playing the role of the mass of the free, undressed propagator.

Inserting the kernel (2.5) into the equation for the propagator one obtains

$$\Delta_F(q)\left\{q^2 \frac{d^2}{(dq^2)^2} + 2\frac{d}{dq^2}\right\}\Delta_F(q) - \frac{Z}{4\beta}(q^2 - m_0^2)\Delta_F(q) + \frac{1}{4\beta} = 0 \tag{3.4}$$

Alabiso and Schierholz [4] gave an extensive discussion of this equation for which we refer to the literature. We will however, briefly state their results.

They find the following solutions:*

$$\Delta_F(q) = \sqrt{\frac{-q^2}{3\beta}} \tag{3.5}$$

for $Z=0$ and m_0^2 finite;

$$\Delta_F(q) = \frac{1}{Zq^2} \tag{3.6}$$

for $m_0 = 0$ and

$$\Delta_F(q) = -\delta^{-1} \tag{3.7}$$

for $Z=0$ and $m_0^2 \to \infty$ such that

* There is a misprint in equation (3.15) of ref. 4 which is corrected here.

$$Z m_o^2 \longrightarrow \delta$$

where δ is an arbitrary but finite constant. For $m_o^2 > 0$ or $Z = 0$ it can be proven that these are the only solutions which do not have unphysical singularities. For $m_o = 0$ and $Z > 0$ Alabiso and Schierholz searched for other solutions than (3.6) using Padé techniques, but all of them appear to have again unphysical singularities which fall out of the context of conventional field theory.

The solutions (3.5) and (3.6) reflect the vanishing of the interaction on the light cone implicit in the kernel (2.6), as both solutions represent zero mass quanta. Eq. (3.5) shows a quark with finite bare mass which has dressed up so as to become a zero mass coherent quark gluon state. Accordingly $Z = 0$ as no single quark can be observed. For vanishing bare mass, solution (3.6) gives the propagator of a free massless quark which escapes the interaction on the light cone. For consistency one should put $Z = 1$ which will, however, be of no importance for our further calculations. The Jost-Schroer theorem [7] states that this solution belongs to a free theory. As it is not clear if the theorem is applicable to our case we will investigate the mesonic bound state problem also for this solution.

Finally, (3.7) is the only solution where the quark submits to the infrared slavery mechanism and ceases to propagate entirely, its Fourier transform into configuration space being a δ-function.

IV. Bound States

Using the kernel (2.5) the BS equation becomes

$$\left\{ \Box_q - \left[\beta \Delta_F(q_+) \Delta_F(-q_-) \right]^{-1} \right\} \phi(p,q) = 0 \tag{4.1}$$

We want to solve (4.1) for the bound state amplitudes of the propagator solutions (3.5) - (3.7).

The constant propagator leads to a Klein Gordon equation with e. g. free plane wave solutions in the variable q_μ which are of no physical interest*.

As for the solutions (3.5) and (3.6) we restrict ourselves to the case where $p^2 = M^2 > 0$. The bound state wavefunctions have to be representations of the little group which leaves p^μ invariant [8] and we may therefore write

$$\phi(p,q) = Y_{\ell m}(\theta, \phi)\, |\vec{q}|^{\ell}\, u(q_0, |\vec{q}|) \tag{4.2}$$

where the parametrization of q^μ in terms of $q_0, |\vec{q}|, \theta$ and ϕ is understood to take place in the restframe of the total momentum p^μ. Thus

$$q_0 = pq/M \quad ; \quad \vec{q}^{\,2} = (pq)^2/M^2 - q^2 \tag{4.3}$$

We then have

$$\left\{ \frac{\partial^2}{\partial q_0^2} - \frac{\partial^2}{\partial \vec{q}^{\,2}} - \frac{2(\ell+1)}{|\vec{q}|} \frac{\partial}{\partial |\vec{q}|} - \left[\beta \Delta_F(q_+) \Delta_F(-q_-) \right]^{-1} \right\} u(q_0, |\vec{q}|) = 0 \tag{4.4}$$

In order to solve this equation we transform from $q_0, |\vec{q}|$ to a pair of new variables x and y which are chosen such that the equation separates in the new variables. The details of these transformations are given in the appendix where we construct the most general transformation which may separate an equation like (4.4) with an arbitrary function of q_0 and $|\vec{q}|$ in the square brackets.

Assume that a pair $x(q_0, |\vec{q}|), y(q_0, |\vec{q}|)$ has been found. Then (4.4) becomes

$$\left\{ \mathcal{D}_1(x) - \mathcal{D}_2(y) \right\} u(q_0, |\vec{q}|) = 0 \tag{4.5}$$

with $\mathcal{D}_i$ being some differential operator. The general solution can be obtained by using a generalized product ansatz for u [9]

* See, however, the discussion in Section V.

$$\mathcal{U}(q_0,|\vec{q}|) = \int d\sigma(K)\, v_1(x,K)\, v_2(y,K) \tag{4.6}$$

with arbitrary measure σ, where the v_i satisfy

$$\begin{aligned} \{D_1(x)+K\}\, v_1(x,K) &= 0 \\ \{D_2(y)+K\}\, v_2(y,K) &= 0 \end{aligned} \tag{4.7}$$

We proceed by investigating the equations for the two remaining quark propagator solutions. (For the details of the equations presented, see Appendix).

A) $\Delta_F(q) = \sqrt{-q^2/3\beta}$

In this case we define the new variables to be

$$\binom{x}{y} = \frac{1}{M^2}\left(\sqrt{q_+^2} \pm \sqrt{q_-^2}\right)^2 \tag{4.8}$$

The equations (4.7) are given by the hypergeometric equations

$$\left\{x(1-x)\frac{d^2}{dx^2} + \left[\tfrac{1}{2} - (\ell+2)x\right]\frac{d}{dx} - K\right\} v_1(x,K) = 0 \tag{4.9}$$

and the same equation for $V_2(y,K)$ with x replaced by y and K by $K' = K - \frac{3}{4}$

The normalization condition (2.3) becomes

$$O(1) = \int d^4q\, |\phi(p,q)|^2 \frac{4(q_0^2+\vec{q}^{\,2}) - M^2}{\sqrt{q_+^2 q_-^2}^{\,3}} \tag{4.10}$$

Due to the divergence of the denominator in (4.10) for $q_\pm^2 = 0$ the bound state wave functions cannot be normalized.

B) $\Delta_F(q) = 1/Z q^2$

Here we choose

$$\binom{x}{y} = \frac{1}{4M^2}\left\{\sqrt{q^2 - M^2 + 2M|\vec{q}|} \pm \sqrt{q^2 - M^2 - 2M|\vec{q}|}\right\}^2 \tag{4.11}$$

and obtain for (4.7)

$$\left\{x(x+1)\frac{d^2}{dx^2} + \left[(\ell+2)x + \ell + \tfrac{3}{2}\right]\frac{d}{dx} - \frac{2^2M^6}{4\beta}x\left(x+\tfrac{3}{4}\right)^2 + K\right\} v_1(x,K) = 0 \tag{4.12}$$

and the same equation holds for $v_2(y,K)$ with x replaced by y. In this case the normalization condition is

$$O(1) = \int d^4q\, |\phi(p,q)|^2 \left\{M^2 - 4(q_0^2 + \vec{q}^{\,2})\right\} \tag{4.13}$$

Though we did not solve equation (4.12) explicitly it is easy to see that there can be no stable bound states. The equation may be solved in the neighbourhood of its singular points. In order that it be normalizable, the cuts in x and y have to extend from -1 to ∞. Reexpressing the variables in terms of q_0 and $|\vec{q}|$ this means that there exists a cut in q_0 covering the complete real axis without any gap which implies that the bound states have to decay.

V. Discussion

We have investigated the mesonic bound states for spin zero quarks interacting through the kernel

$$K(q,k;p) = -i\beta\, \Box_{q-k}\, \delta^4(q-k) \tag{2.5}$$

and have found that no positive mass stable bound states exist for this type of interaction.

This result depends crucially upon the fact that the quark propagator is calculated in the presence of the interaction. It is this treatment which renders the result exact to all orders of perturbation theory and it depends on perturbation theory indeed only through the fact that the set up of SD equations has to take recourse to this expansion.

It is not very surprising that there are no stable bound states for the propagators (3.5) and (3.6), as they correspond to zero mass quanta which implies that for any $p^2 = M^2 \geq 0$ there will be no mass gap to prevent the constituents from going on shell. In ref. 5 it will indeed be proven that for any quasi free constituent propagator obeying a Lehmann representation the bound states will decay above $M^2 = 4m^2$, where m^2 represents the lowest singularity in the mass squared variable of the propagator. There it is also argued that this is likely to hold for any interaction regardless of the strength of its infrared singularity.

However, the fact that the kernel (2.5) leads to propagators with singularities corresponding to finite constituent masses may be regarded as an artifact of the vanishing of the interaction on the light cone. But there is still one solution which is in accord with general ideas about infrared slavery, the propagator (3.7) which is an entire function in q^2. We discarded the formal solution to the BS equation in this case being plane waves in q_μ which have no meaningful physical interpretation. It is an amusing fact, to notice that in configuration space the quark distance is fixed to a hyperboloid $x^2 = const$, which in Euclidean space would lead to perfectly confined quarks one orbiting around the other on a four dimensional sphere. However, we have to get along in Minkowski space and here the proper interpretation for the case of the constant propagator should be that although it is in agreement with confinement ideas* there is too little structure in the scalar interaction kernel, to allow for bound states. Indeed, the BS-equation in this case is not even an eigenvalue equation in M^2, the bound state mass. For a constant propagator such an eigenvalue equation can only come about through the interaction kernel. The simplest possibility to achieve this, is to assume that there is a vector contribtuion to the kernel. However it can be shown that this does not yet lead to a physically sensible spectrum. Another possibility is that the necessary p^2-dependence comes in through higher order corrections involving quark loops which represent intermediate hadronic states.

* And is likely to be the only form of a propagator which can achieve this, as is argued in ref. 5.

Appendix

To find the transformations which separate equation (4.4) we will first determine the class of transformations which separate

$$\left\{ \frac{\partial^2}{\partial\varphi^2} - \frac{\partial^2}{\partial\psi^2} - \frac{N}{\psi}\frac{\partial}{\partial\psi} + F(\varphi,\psi) \right\} u(\varphi,\psi) = 0 \tag{A.1}$$

with an arbitrary function $F(\varphi,\psi)$. Here we have put $q_0 = \varphi$, $|\vec{q}| = \psi$, $2(\ell+1) = N$ for notational convenience. Lateron we will specify F to the two solutions of the quark propagator.

Consider an arbitrary transformation of the set (φ,ψ) to (x,y) and its inverse

$$\begin{aligned} \varphi &= \varphi(x,y) \quad , \quad \psi = \psi(x,y) \\ x &= x(\varphi,\psi) \quad , \quad y = y(\varphi,\psi) \end{aligned} \tag{A.2}$$

which we assume to be independent of N, leading to a unique transformation for all N. Except for this restriction we will construct the most general transformation which may separate (A.1).

Defining

$$\begin{aligned} g &= x_\varphi^2 - x_\psi^2 \ , \ h = y_\varphi^2 - y_\psi^2 \ , \ r = x_{\varphi\varphi} - x_{\psi\psi} \ , \ s = y_{\varphi\varphi} - y_{\psi\psi} \ , \\ t &= x_\varphi y_\varphi - x_\psi y_\psi \ , \ \rho = \varphi_x \psi_y - \varphi_y \psi_x \end{aligned} \tag{A.3}$$

where an index denotes partial differentiation with respect to the indexed variable, equation (A.1) is transformed to

$$\left\{ g\frac{\partial^2}{\partial x^2} + h\frac{\partial^2}{\partial y^2} + 2t\frac{\partial^2}{\partial x\partial y} + \left(r - \frac{N}{\psi}x_\psi\right)\frac{\partial}{\partial x} + \left(s - \frac{N}{\psi}y_\psi\right)\frac{\partial}{\partial y} + F(\varphi,\psi) \right\} u(x,y) = 0 \tag{A.4}$$

where all variables have to be expressed in terms of x and y, of course. If this equation is to separate in x and y it will do so by being multiplied by a properly chosen function $K(x,y)$. For the part of equation (A.4) which involves differential operators this leads to the conditions:

$$(Kg)_y = (Kr)_y = \left(\frac{K}{\psi} x_\psi\right)_y = (Kh)_x = (Ks)_x = \left(\frac{K}{\psi} y_\psi\right)_x = t = 0 \qquad \text{(A.5)}$$

Transforming (A.4) backwards one has to reproduce (A.1) which requires

$$x_\psi = -\frac{\varphi_y}{P}, \; y_\psi = \frac{\varphi_x}{P}, \; g = \frac{\psi_y}{\varphi_x P}, \; h = -\frac{\psi_x}{\varphi_y P}, \; \varphi_x \varphi_y = \psi_x \psi_y$$

$$g\,\varphi_{xx} + h\,\varphi_{yy} + r\,\varphi_x + s\,\varphi_y = \sigma \qquad \text{(A.6)}$$

$$g\,\psi_{xx} + h\,\psi_{yy} + r\,\psi_x + s\,\psi_y = \sigma$$

Using (A.5) and (A.6) one obtains after some algebra

$$(\ln \psi)_{xy} = \left[\ln(\varphi_x / \psi_y)\right]_{xy} = \sigma \qquad \text{(A.7)}$$

which has - except for degenerate transformations like $\varphi = \varphi(x), \psi = \psi(y)$ which are of no interest - the general solution

$$\psi = a(x)\,b(y)$$

$$\varphi = \pm\sqrt{(a^2(x) + C_1)(b^2(y) + C_2)} + C_3 \qquad \text{(A.8)}$$

where the C_i are undetermined constants and $a(x)$ and $b(y)$ are arbitrary functions of x and y, respectively. They may, of course, without loss of generality be put equal to the identity. In our case it is convenient to choose

$$a^2(x) = C_1 x$$

$$b^2(y) = C_2 y \qquad \text{(A.9)}$$

$K(x,y)$ turns out to be

$$K(x,y) = a^2(x)\,C_2 - b^2(y)\,C_1 = C_1 C_2 (x - y) \qquad \text{(A.10)}$$

and the separation of equation (A.1) is possible, if $K(x,y)\cdot F(\varphi(x,y),\psi(x,y))$ separates in x and y. To affect this, one has to choose the proper values for the constants C_i.

In the case of equation (4.4) the separation is achieved by letting

$$C_1 C_2 = \frac{1}{4} M^2 \quad , \quad C_3 = 0 \qquad \text{(A.11a)}$$

for the propagator (3.5) and

$$C_1 C_2 = M^2 \quad , \quad C_3 = 0 \qquad \text{(A.11b)}$$

for the propagator (3.6).

The separated equations defined by (4.5) - (4.7) are then given by

$$\left\{ x(1+x)\frac{d^2}{dx^2} + \left[(\ell+2)x + \ell + \frac{3}{2}\right]\frac{d}{dx} + F_1(x) + K \right\} v_1(x,K) = 0 \qquad \text{(A.12)}$$

and the same equation for $v_2(y,k)$ replacing x by y and $F_1(x)$ by $F_2(y)$.

For the propagator (3.5) we have

$$F_1 = 0 \quad , \quad F_2 = -\frac{3}{4} \qquad \text{(A.13a)}$$

and for the propagator (3.6)

$$F_1(x) = -\frac{z^2 M^6}{4\beta}\, x\left(x + \frac{3}{4}\right)^2 \qquad \text{(A.13b)}$$

and $F_2(y)$ is given by the same formula after replacing x by y.

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