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The Last Heavy Lepton and the Next One

by

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The Last Heavy Lepton and the Next One

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Talk at the Summer Institute, Karlsruhe, 1-15 Sept. 1978

Abstract

1. Leptonic Weak Interactions.

2. Hadronic Decays of Υ ; the A_1.

3. The Next Heavy Lepton.

4. Unbounded Fermion Mass Spectrum
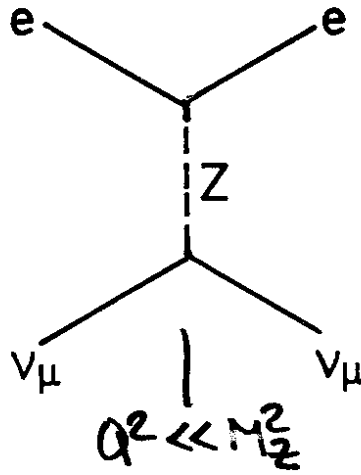
1. Leptonic Weak Interactions

The present leptonic $SU_2 \times U_1$ weak interaction theory was formulated in 1967 [1], ten years after the familiar V-A theory [2]. A decade later still, the low energy limit of this theory is now the successor to the V-A theory [3]. A brief explanation of why this is so (and why I will not discuss models) is in order. Most of us believe that charged current interactions are mediated by exchange of a massive charged vector boson [1],[2]. In the low energy limit the V-A theory with $e\mu$ universality results. ((1)

shows
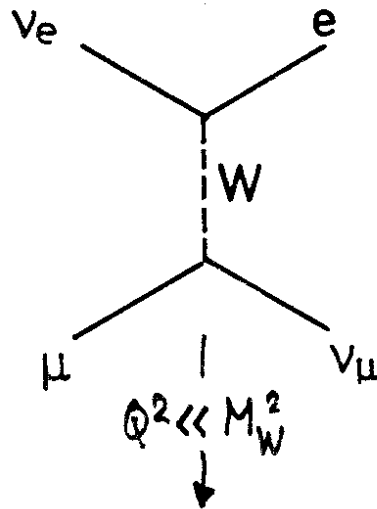
$$\frac{G_F}{\sqrt{2}} \, \bar{\mu} \, \gamma_\alpha (1+\gamma_5) \nu_\mu \, \bar{e} \, \gamma^\alpha (1+\gamma_5) \nu_e$$

μ-decay in this theory; other processes are described similarly.) Neutrinos interact in a chirally pure way; they are left-handed [2]. Only left-handed electrons enter, too. Now we also know the low energy behavior of neutral current processes [3],[4],[5]. ((2) shows $\nu_\mu e \to \nu_\mu e$)

(1)

(2)

$$\frac{G_F}{\sqrt{2}} \, \bar{\nu}_\mu \gamma_\alpha (1+\gamma_5) \nu_\mu \; \bar{e} \, (\gamma^\alpha g_V + \gamma^5 \gamma^\alpha g_A) e$$

$$g_V = -\tfrac{1}{2} + 2\sin^2\theta_W \quad ; \quad g_A = -1/2$$

The neutrino is still left handed, but now the electron is not. The factor $-1/2$ in (2) corresponds to the left-handed piece of the interaction; it is an SU_2 Clebsch-Gordon coefficient, the 3rd component of a "weak isospin" . The extra vector interaction arises because the massive Z^0 and the massless photon are orthogonal mixtures of the 3rd component of an SU_2 triplet and a U_1 boson. Experimentally it happens that [3]

$$\sin^2\theta_W \approx \frac{1}{4}$$

(3)

so that the electron interaction is axial, and left and right handed components enter equally. (This is not so for quarks, because the extra vector interaction is proportional to the fermion charge.)

Recent neutral current experiments and their theoretical analysis convincingly support (2). In fig. 1, I show a plot of g_V versus g_A arrived at from analyses of $\nu_\mu e \rightarrow \nu_\mu e$ scattering, $\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$, $\bar{\nu}_e e \rightarrow \bar{\nu}_e e$ (which has contributions from both charged and neutral current interactions)

and $e^-N \longrightarrow e^-N$ for longitudinally polarized e^- [6]. In this last experiment a cross section different for right and left handed electrons is evidence for a weak interaction contribution. (Its theoretical analysis needs Z^0-quark couplings, which are also now known. The experimentally allowed region is stippled.)

The low energy charged and neutral current leptonic weak interaction is now essentially established. (Of course there is still room for unconventional effects at the 10-30 % level in some experiments.) Besides this, there is now a new heavy lepton τ which appears to behave exactly like e and μ except for its large mass [7]. So $e\mu$ universality is replaced by $e\mu\tau$ universality. Models are not currently of much interest or relevance.
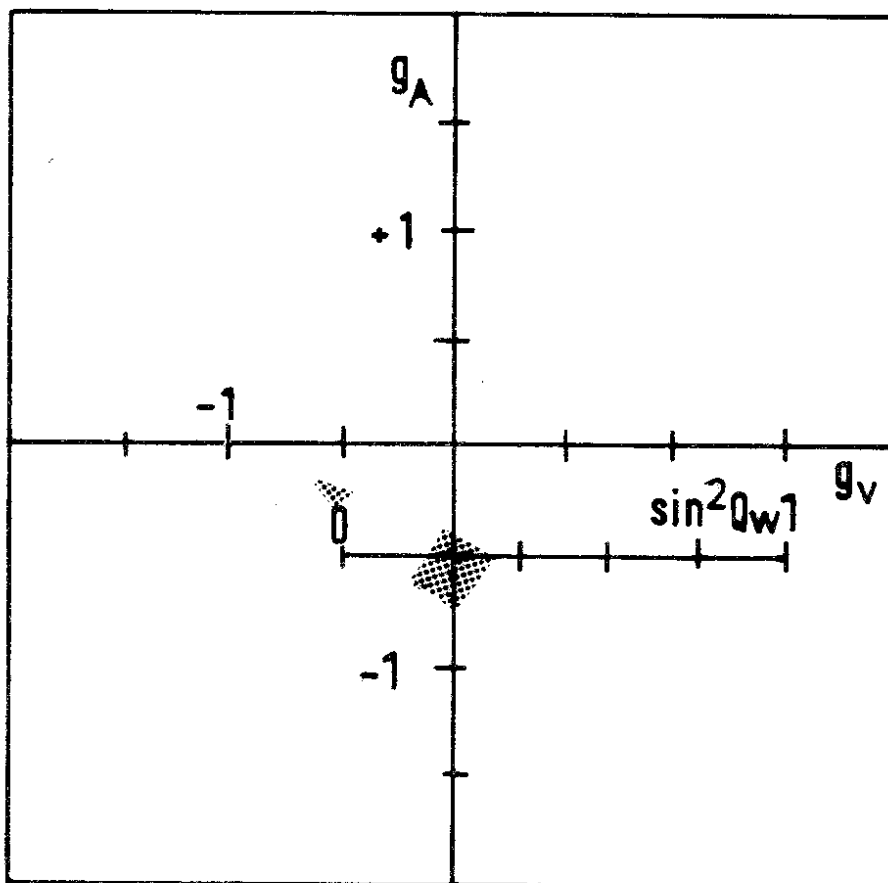

Fig. 1

2. Hadronic Decays of τ

Hadronic decays of τ provide us access to the matrix element [8]

$$\langle HADRONS | J_\alpha^{HAD} | 0 \rangle : \qquad (4)$$
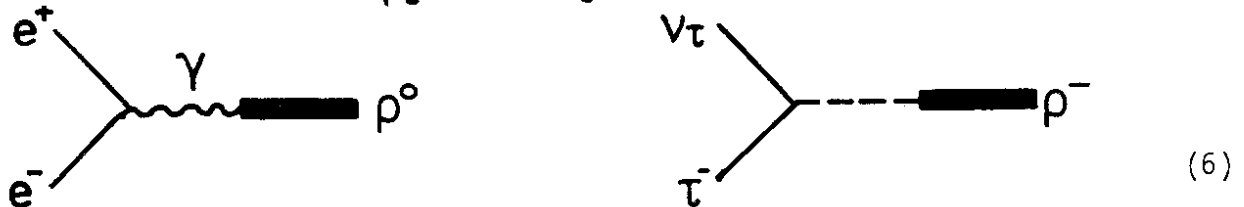

Since the quark couplings are known, this actually gives us access to the strong interaction through

$$\bar{u}d \rightarrow HADRONS \qquad (5)$$

for virtual confined quarks. Interesting issues include

(i) CVC

This is just the statement that the hadronic current behaves just as the vector quark current $\bar{u}\gamma_\alpha d$ (for zero Cabibbo angle). This and $e\mu\tau$ universality give $\tau^- \rightarrow \nu_\tau \rho^-$ from $\rho^0 \rightarrow e^+e^-$
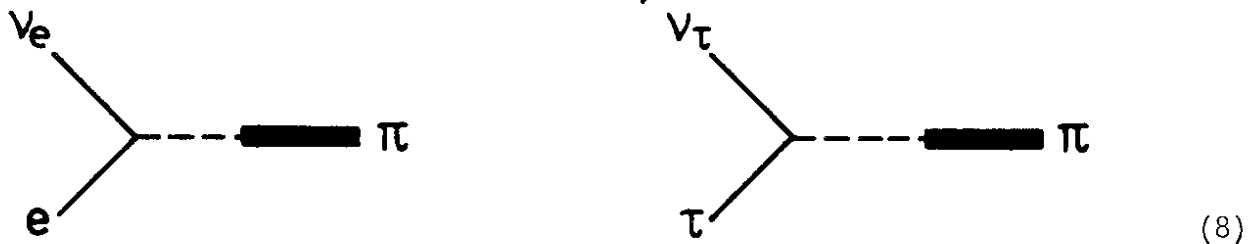

$$(6)$$

$$\frac{\Gamma(\tau \rightarrow \nu_\tau \rho^-)}{\Gamma(\tau \rightarrow \nu_\tau e\nu)} = \begin{cases} 1.2 \quad THEORY^{[8]} \\ 1.3 \pm .5 \quad DASP^{[7]} \end{cases}$$

For the 4π decay [10],

$$\frac{\Gamma(\tau^- \to \nu_\tau (4\pi)^-)}{\Gamma(\tau^- \to \nu_\tau e\nu)} \simeq 0.56 \tag{7}$$

from $e^+e^- \to 4\pi$ data.

For the <u>axial</u> current, $\pi \to e\nu_e$ and $e\mu\tau$ universality give [8]

$$\tag{8}$$

$$\frac{\Gamma(\tau \to \nu_\tau \pi)}{\Gamma(\tau \to \nu_\tau e\nu)} = \begin{cases} .55 \quad \text{THEORY} \\ .5 \ (\pm 30\%) \quad \text{EXP.}^{(7)} \end{cases}$$

This brings us to

(ii) The $J^{PC} = 1^{++}$ A_1 Meson and Current Algebra

The simplest theory for $\tau \to \nu_\tau \rho\pi$ is shown below

$$\tau \to \nu_\tau A_1 \searrow \rho\pi \tag{9}$$

where the coupling of the A_1 to the current is fixed by the Weinberg

sum rules, and A_1 to ρ is fixed by current algebra. This simplest

theory does not work. It gives an acceptable $\Gamma(T \rightarrow \nu_T A_1)$ but
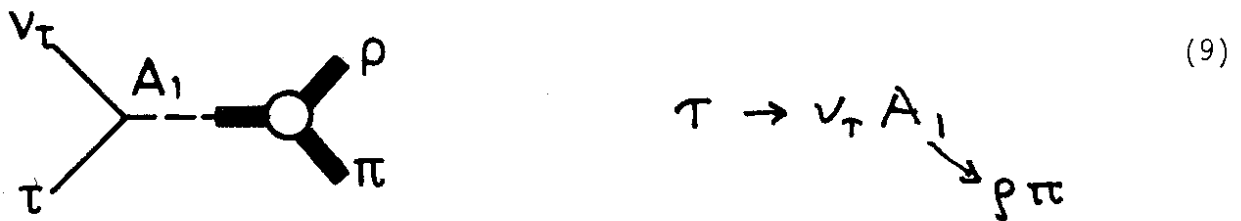
$$\frac{\Gamma(T \rightarrow \nu_T A_1)}{\Gamma(T \rightarrow \nu_T e \nu)} = \begin{cases} .44 & \text{THEORY} \\ .56 \pm .2 & \text{EXPT}^{(7)} \end{cases} \quad ; \quad \Gamma(A_1 \rightarrow \rho \pi) = 25 MeV^{(10)}$$

much too small an A_1 width.$^{(11)}$

A less simple theory (due to Geffen and Wilson $^{(12)}$) deals with the

matrix element,

$$\langle \rho \pi | J_\alpha^{HAD}(0) | 0 \rangle = \epsilon_\alpha F_0 + \epsilon \cdot \pi \left[(\rho - \pi)_\alpha F_+ + (\rho + \pi)_\alpha F_- \right]$$

$$C_0 + \frac{C_0^A}{(\rho + \pi)^2 - M_A^2} \quad ; \quad \frac{C_+^A}{(\rho + \pi)^2 - M_A^2} \quad ; \quad \frac{C_-^\pi}{(\rho + \pi)^2 - M_\pi^2} + \frac{C_-^A}{(\rho + \pi)^2 - M_A^2} \tag{11}$$

(ϵ is the ρ polarization and particle momenta and labels are the same).
The constants are fixed by current algebra and C_0 allows for a possible low

energy non resonant piece in F_0. (It seems unwise to add such constants to

$F_\pm$, as then the "cross section" $\sigma(\nu_e e \rightarrow \rho \pi)$ would be badly

behaved above the A_1 peak.) The authors have a "no A_1" solution with C_0,

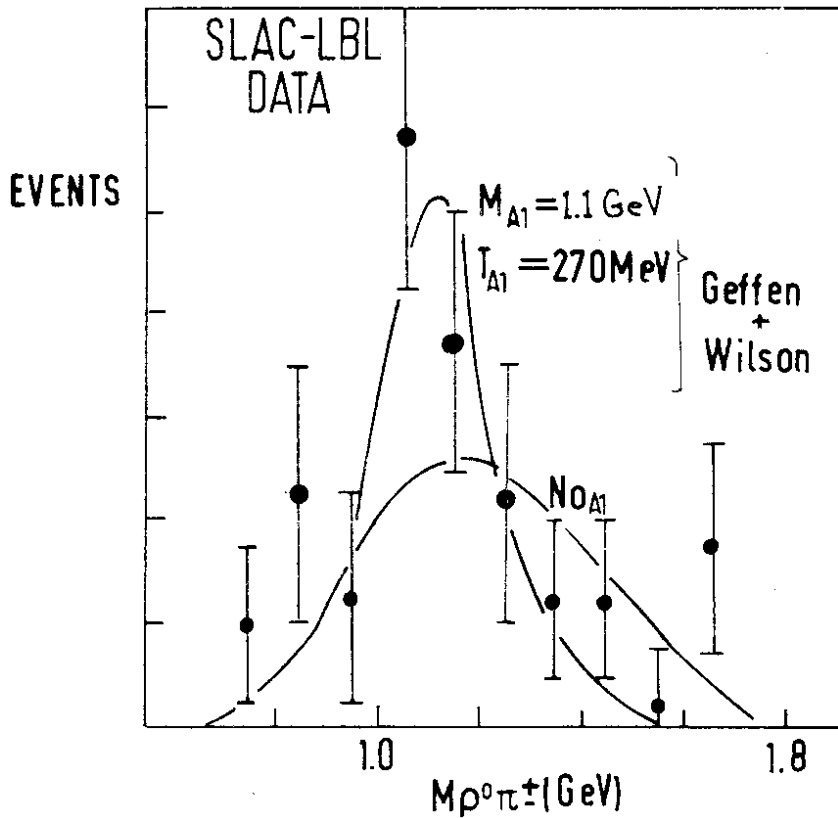a pion pole but no A_1 pole. They also have an A_1 solution (fig. 2).

$$M_{A1} = 1.1\,GeV$$
$$T_{A1} = 270\,MeV$$ Geffen + Wilson

No$_{A1}$

SLAC-LBL DATA

EVENTS

$M\rho^0\pi^\pm(GeV)$

1.0   1.8

Fig. 2

Evidently, the A_1 is not yet seen in $T \rightarrow V_T A_1 \rightarrow V_T \rho \pi$. Once it is established, we will have a nice opportunity to test the decade old ideas of current algebra [13].

(iii) Second Class Currents

Besides the standard weak current which transforms like the ρ , and A_1 (or π) mesons,

current	C_n	P	$G = C_n (-)^I$	meson	
$\bar{u}\gamma_\alpha d$	$-$	$-$	$+$	ρ	(12)
				$A_1 (\pi)$	
$\bar{u}\gamma_\alpha\gamma_5 d$	$+$	$+$	$-$		

One can imagine currents which have "wrong" G parity properties (e.g. a G = + axial piece), and which don't appear in the usual quark current,

"current"	C_n	P	G	meson
$\sim \bar{u}(p_u - p_d)_\alpha d$	$+$	$+$	$-$	$\delta(960)$

$$(13)$$

$$\sim i\, \bar{u}\, \sigma_{\alpha\beta} \frac{(p_u - p_d)^\beta}{m_u + m_d}\, d \qquad - \qquad + \qquad - \qquad B(1250)$$

The former are "first class" currents and the latter "second class" [9]. It has been suggested that "second class" currents do not even exist. However, it may well be that small _effective_ second class currents can appear in nature due to $m_u \neq m_d$ and the effects of quark confinement, or virtual gluon interactions of u and d not envisaged in (12) [14].

Because of the interesting possibilities it would open up, one ought to look for processes forbidden for first class currents, [15]

$$\tau^- \to \nu_\tau\, \delta^-(960)$$
$$\searrow \eta \pi^-,\ K^- K_s$$

$$(14)$$

$$\tau^- \to \nu_\tau\, B^-(1250)$$
$$\searrow \omega \pi^-$$

and also their SU_3 rotated compatriots,

$$\tau^- \longrightarrow \nu_\tau \, \varkappa \, (0^{++})$$
$$\searrow K\pi \tag{15}$$

$$\tau^- \longrightarrow \nu_\tau \, K_B \, (1^{+-})$$
$$\searrow \overset{*}{K}\pi, \; K\rho$$

(These are, of course, suppressed also by a Cabibbo factor $\sin^2\theta_c$.)

3. The Next Heavy Lepton

Perhaps τ is the last heavy lepton. (Many people think so.) But it may be that there are more leptons (and quarks),

$$
\begin{array}{cccc}
u & d & \nu_e & e \\
c & s & \nu_\mu & \mu \\
t & b & \nu_\tau & \tau \\
g & h & \nu_\sigma & \sigma \\
\vdots & \vdots & \vdots & \vdots
\end{array}
\tag{16}
$$

How would we recognize a next heavy lepton like σ ? The classic signatures are [7]

$$e^+ e^- \rightarrow \sigma^+ \quad \sigma^- \searrow \mu^- (e^-) \nu \nu \tag{17}$$
$$\swarrow e^+ (\mu^+) \nu \nu$$

and

$$e^+ e^- \rightarrow \sigma^+ \sigma^- \searrow \text{anything} \tag{18}$$
$$\searrow \text{hand } e^+ \alpha \mu^+ + \nu \nu$$

There is no evidence for such a signal for $\sqrt{s} \lesssim 7.4$ GeV [7].

(i) $\sigma^\downarrow$ Decays

We have

$$\sigma^- \rightarrow \nu_\sigma W^- \qquad\qquad \sigma^- \rightarrow \nu_\sigma W^- \tag{19}$$
$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$
$$\bar{\nu}_e e^- \qquad\qquad\qquad d\bar{u} + O(\theta_c^2) s\bar{u}$$
$$\bar{\nu}_\mu \mu^- \qquad\qquad\qquad s\bar{c} + O(\theta_c^2) d\bar{c}$$
$$\bar{\nu}_\tau \tau^- \qquad\qquad\qquad b\bar{t} + O((\theta')^2) b\bar{c}$$
$$+ O((\theta'')^2) b\bar{u}$$

Estimates of m_b, m_t are $m_b \sim 4.5$ GeV, $m_t \gtrsim 8$ GeV (from the absence of a pN $\rightarrow \mu^+ \mu^- + \cdots$ signal [16]). Since θ', $\theta'' \lesssim \theta_c$ [17], no significant b production is expected, and no $b\bar{t}$ for $M_\sigma \lesssim 12.5$ GeV. So σ decays will be dominated by $\bar{\nu}_e e, \bar{\nu}_\mu \mu, \bar{\nu}_\tau \tau, d\bar{u}, s\bar{c}$. To estimate the

branching ratio $\sigma \to \nu_\sigma e \nu$ or $\sigma \to \nu_\sigma \mu \nu$, it's necessary to calculate the V-A rate in (20)

$$x = \frac{(p_1 + p_2)^2}{M_\sigma^2} \tag{20}$$

The decay rate differential in x is [18]

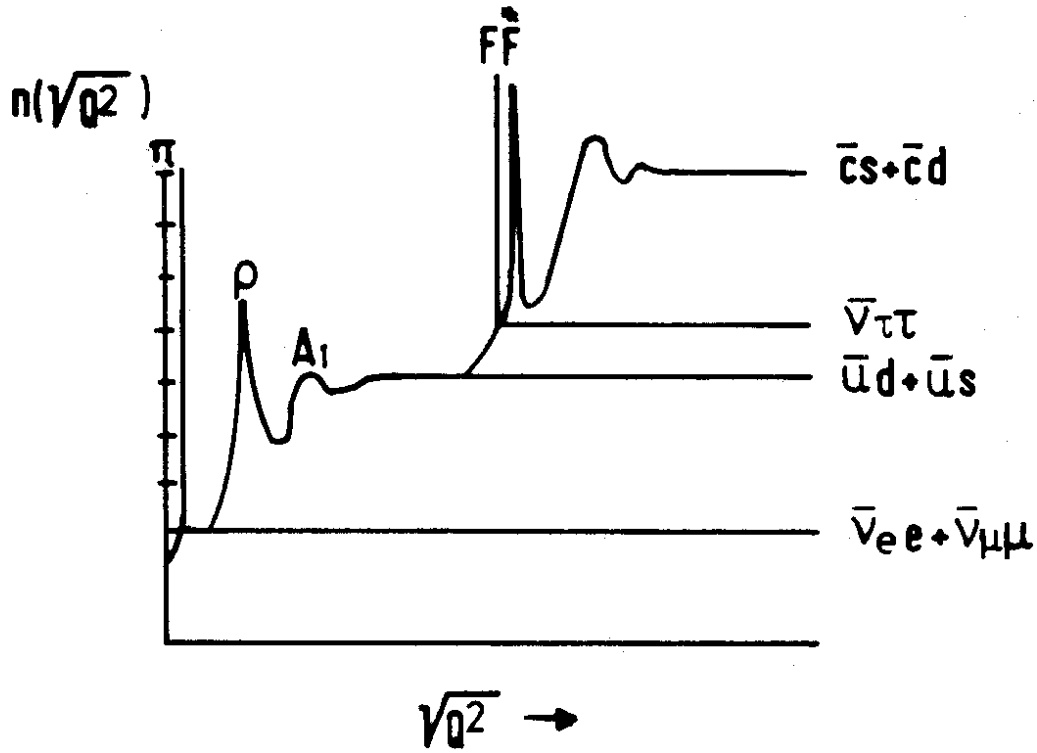$$\frac{1}{\Gamma(\sigma \to \nu_\sigma e \nu)} \frac{d\Gamma}{dx} = 2\,n(x)\,(1-x)^2\,(1+2x) \tag{21}$$

where

$$n(\sqrt{Q^2}) = \frac{\sigma(\bar{\nu}_e e \to \Sigma f_1 \bar{f}_2)}{\sigma(\bar{\nu}_e e \to \bar{\nu}_\mu \mu)}$$

is the number of active fermion flavors in the charged weak current (3 per quark pair, 1 for a lepton pair). It is the analog of

$$R = \frac{\sigma(e^+ e^- \to \Sigma f \bar{f})}{\sigma(e^+ e^- \to \mu^+ \mu^-)} \tag{22}$$

$n(\sqrt{Q^2})$ looks as follows:

In order to estimate branching ratios as a function of σ mass, approximate these thresholds by θ -functions,

$$n(\sqrt{Q^2}) \approx 5 + 4\,\theta(\sqrt{Q^2} - 1.8\ GeV) \tag{23}$$
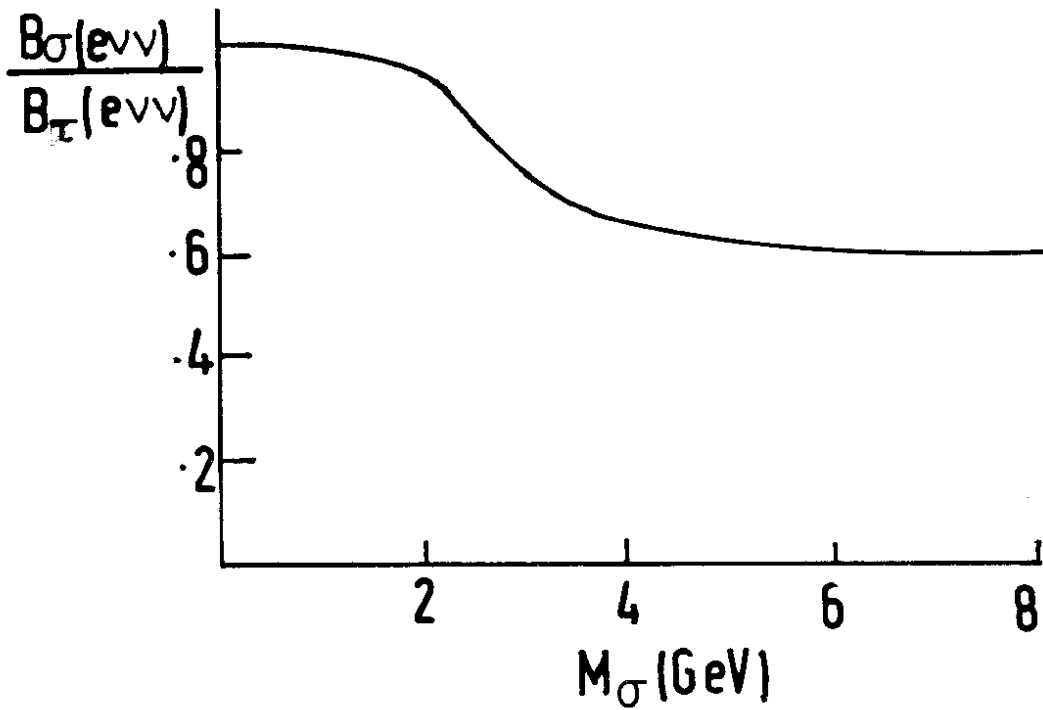
The result is fig. (3)

$$\frac{B_\sigma(e\nu\nu)}{B_\tau(e\nu\nu)}$$

Fig. 3

Since we expect that $M_\sigma > 3.5$ GeV,

$$\frac{\sigma(e^+e^- \to \sigma^+\sigma^- \to e\mu + \cdots)}{\sigma(e^+e^- \to \tau^+\tau^- \to e\mu + \cdots)} \approx \frac{1}{3} \qquad (24)$$

provided $M_\sigma < M_b + M_t$

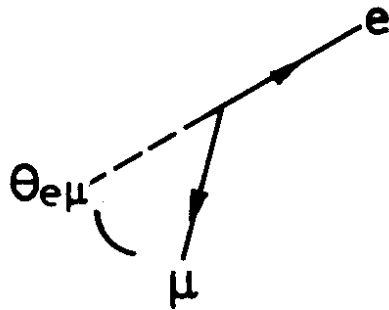(ii) Telling $\sigma^+\sigma^-$ and $\tau^+\tau^-$ Apart

Both $e^+e^- \rightarrow \tau^+\tau^-$ and $e^+e^- \rightarrow \sigma^+\sigma^-$ give an $e\mu$ signal. In order to check for a new component in $e^+e^- \rightarrow e\mu$ + missing energy, it is simplest to plot data in a way such that it would exactly scale if there were no new $e\mu$ source. The following distributions scale exactly for τ[18]:

$$E_e(E_\mu) \text{ spectrum}: \quad \frac{1}{\Gamma}\frac{d\Gamma}{dz}\frac{2\beta}{1+\beta} \quad ; \quad z = \frac{2\beta}{1+\beta}\frac{E_e}{E_\tau} \quad , \quad z > \frac{1-\beta}{1+\beta}$$

$$(25)$$

$$\theta_{e\mu} \text{ distribution}: \quad \frac{1}{\Gamma}\beta^2\frac{d\Gamma}{dx} \quad ; \quad x = \frac{\beta^2\gamma^2}{2}(1+\cos\theta_{e\mu})$$

Fig. (4) shows these functions for a V$\pm$A τ
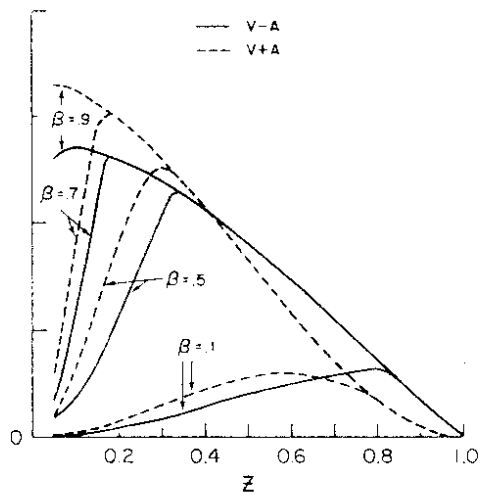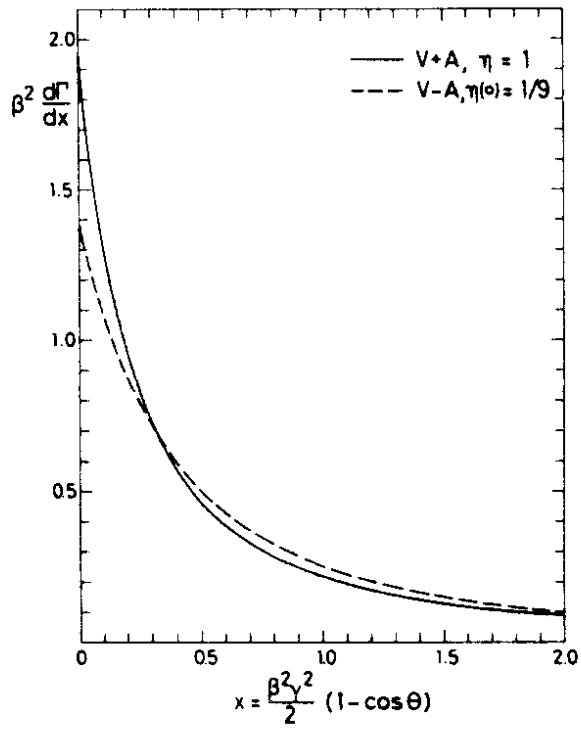
Fig. 4

On passing a new $\sigma^+\sigma^-$ threshold, dramatic violations of the scaling laws (24) will appear. Qualitatively,

$$P_e \; , \; P_\mu \; \sim \; O\left(\frac{E_\sigma}{3}\right) \quad , \quad \theta_{e\mu} \; \sim \; O\left(\frac{m_\sigma}{E_\sigma}\right) \tag{26}$$

both much larger on average than P_e , P_μ or $\theta_{e\mu}$ for the τ .

Another potentially interesting signal for a new heavy lepton occurs when $e^+e^- \rightarrow \sigma^+\sigma^-$ followed by $\sigma^+ \rightarrow e^+\nu\nu$ or $\mu^+\nu\nu$ and $\sigma^- \rightarrow \nu_\sigma \bar{u}d$ or $\nu_\sigma \bar{c}s \rightarrow$ 2 jets,[19]

$$e^+e^- \rightarrow \sigma^+\sigma^- \rightarrow \ell^\pm + 2\,jets + missing\ energy \tag{27}$$

For a V-A σ, the invariant mass distribution of the 2 jets and its mean are shown on fig. (5)

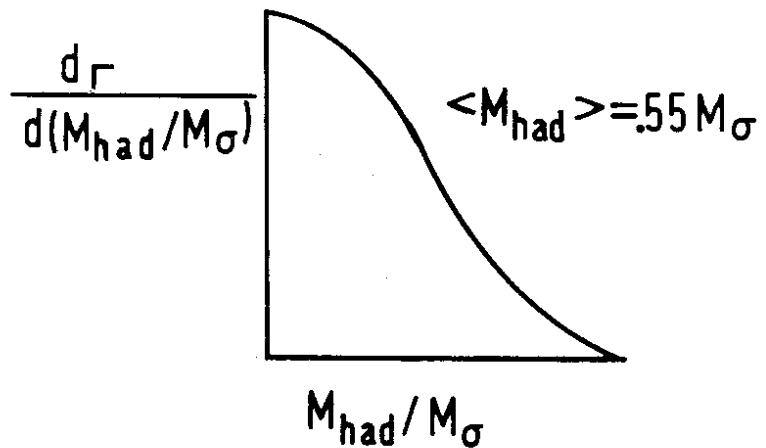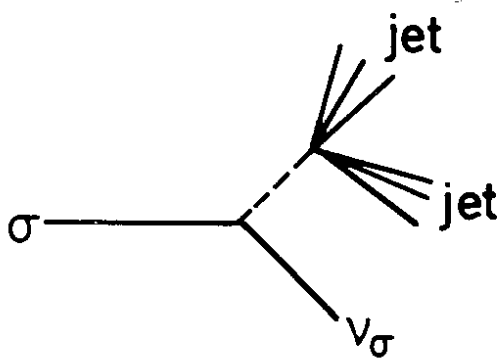
Fig. 5

Two well defined jets will require $\langle M_{had} \rangle > $ 7 GeV or $M_\sigma > $ 15 GeV.

(iii) Other backgrounds

Separating $e\mu$ events from $\tau^+\tau^-$ and $\sigma^+\sigma^-$ is clearly not a problem. It may be worth remark that other (quantum electrodynamic) backgrounds generally depend only logarithmically on energy for experimental cuts which scale with the e^+e^- beam energy. Thus if

$$ P_e, P_\mu \gtrsim \epsilon E_B \quad , \quad \theta_{e\mu} > \theta_0 \tag{28} $$

Such backgrounds will be not materially worse for $\sigma^+\sigma^-$ and $\tau^+\tau^-$ provided ϵ and θ_0 are the same. (e.g. at $E_B \sim$ 2-3.5 GeV θ_0 is 20^0 and ϵ typically 0.5 [7]). Other backgrounds with a threshold behavior, e.g. the 2γ process

$$ e^+e^- \to e^+e^-\tau^+\tau^- \to e\mu + \dots $$

have to be dealt with separately.

4. Unbounded Fermion Mass Spectrum

It would be very exciting to find a new heavy lepton at PETRA or PEP. Perhaps the spectrum of "elementary" fermions continues up to very high mass indeed. But there are limitations. We do not expect that calculable radiative corrections to known processes which are finite for a few leptons become infinite if we allow an unbounded lepton (and quark) mass spectrum [20]. Take the fermion

loop corrections to the muon anomalous moment as an example,
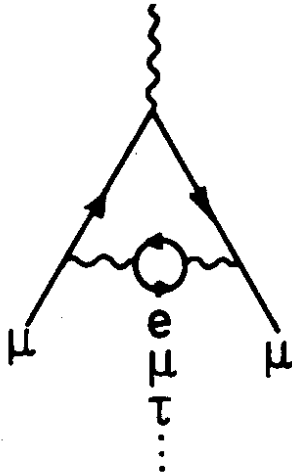
$$a_\mu^{loop} = m_\mu^2 \left(\frac{\alpha}{3\pi}\right)^2 \int \frac{ds}{s^2} R(s) \qquad (29)$$

Then we require that for an unbounded spectrum [21]

$$a_\mu^{loop} \sim \sum_{n=1}^{\infty} \frac{1}{M_n^2} < \infty \qquad (30)$$

where M_n is the mass of the nth pointlike fermion. The simplest (but not the only) way to assure $a^{loop} < \infty$ is for [21]

$$M_n^2 = const \; n^\alpha \qquad \alpha > 1 \qquad (31)$$

and it is easy to show that

$$\frac{M_{n+1}}{M_n} \xrightarrow[n\to\infty]{} 1 \quad ; \quad M_{n+1} - M_n \xrightarrow[n\to\infty]{} \infty \quad if \; n > 2 \qquad (32)$$

This suggests that if we want to look for fermion mass regularities, we try plotting $\ln M_n$ versus $\ln n$. This is done in fig. (6). I plotted $\ln M_n/M_e$ (n = 1 for e, 2 for   , ...) versus $\ln n$, $\ln M_n/M_u$ for charge + 2/3 quarks, and $\ln M_n/M_d$ for charge -1/3 quarks. In the latter case n = 1 for u or d, 2 for c or s, ...

Lepton masses are well known; for quark masses I took

$$m_u = 4 \text{ MeV} \qquad m_c = 1.2 \text{ GeV}$$

$$m_d = 6 \text{ MeV} \qquad m_s = 135 \text{ MeV} \qquad m_b = 4.5 \text{ GeV}^{(33)}$$

These are, of course, more ambiguous than lepton masses. So any regularity will be less obvious.

There does seem to be a pattern to the masses, which is clearer for leptons than for quarks. This apparent regularity may well be accidental. If it is not, then there may be a new charged lepton around 16-18 GeV mass, and perhaps a new e_Q = -1/3 quark too. The indication from the figure is that the next e_Q = + 2/3 quark is rather more massive than the next e_Q = -1/3 quark. (Perhaps it lies around $\sim$ 40 GeV mass; this differs from one popular guess that

$$2m_t : M_\Upsilon : M_{J/\psi} : M_\phi \simeq 27 : 9 : 3 : 1 \text{ GeV} \;)$$

It would be surprising if any such naive guess of future fermion masses actually turned out to be correct. However it is already clear that the masses of the "elementary" fermions are one of the basic puzzles of elementary particle physics. The more fermions there are, the more fascinating the puzzle.
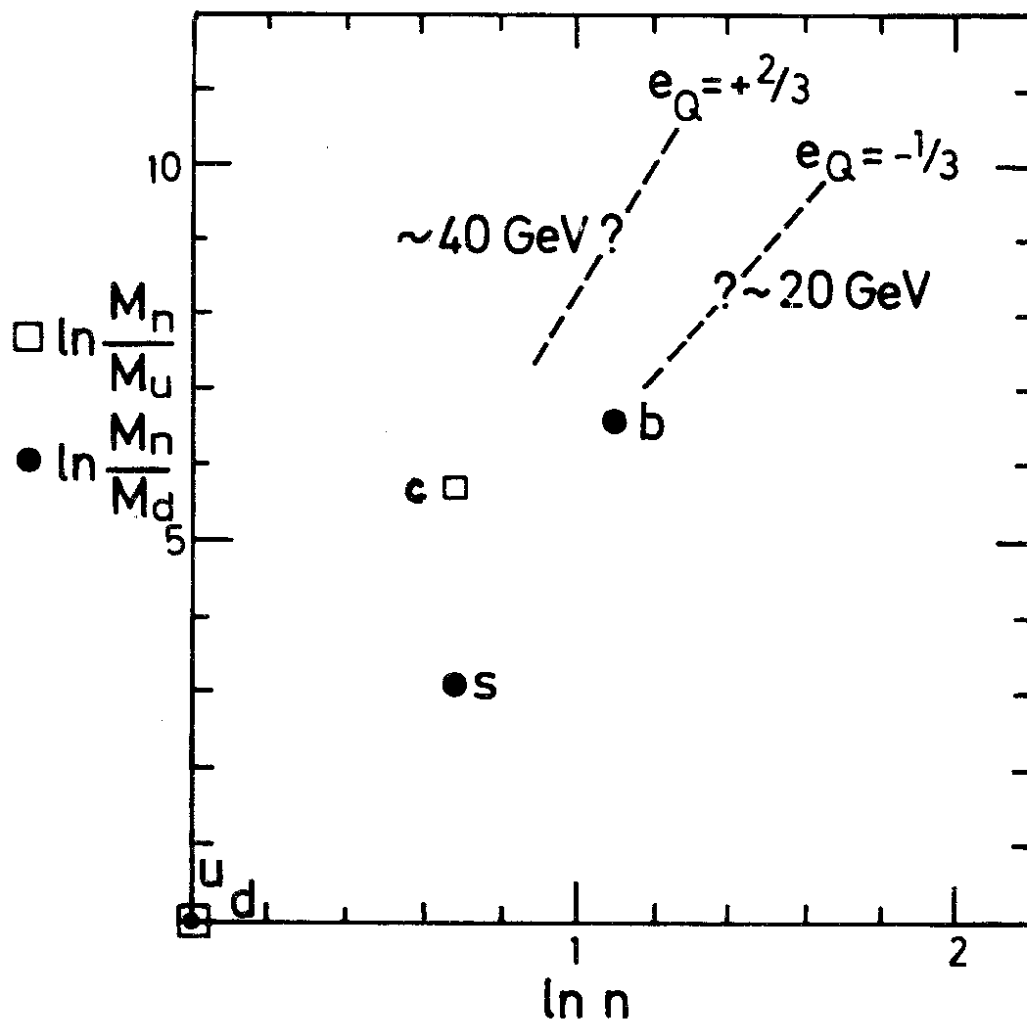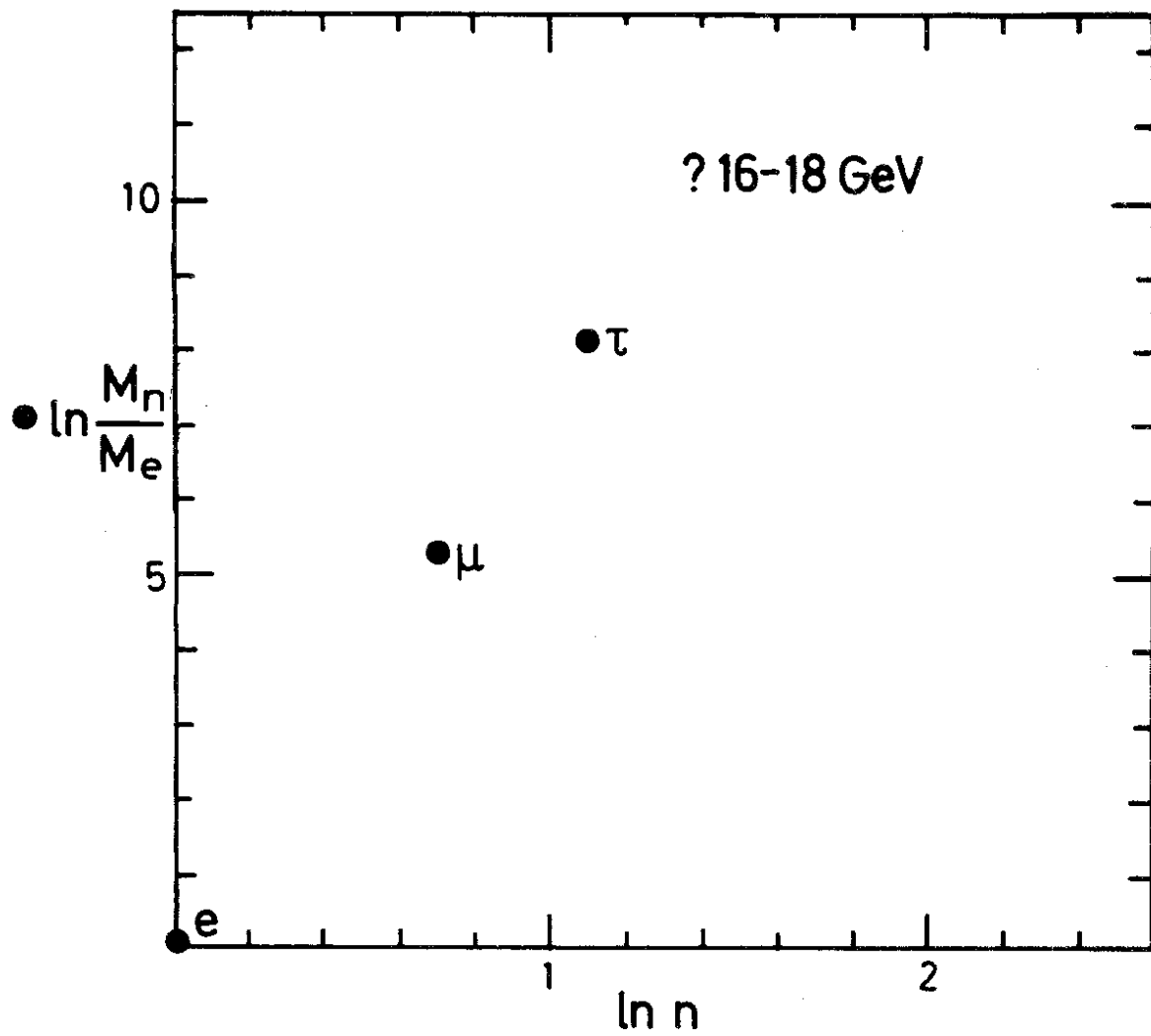
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   A. Salam, Elementary Particle Physics (ed. N. Svartholm, Stockholm 1968).

   S. Glashow, Nucl. Phys. $\underline{22}$ (1961) 579.

2) See R.E. Marshak, Riazuddin and C.P. Ryan, Theory of Weak Interactions, Wiley, Interscience, New York, 1969.

3) For a critical approach see J.J. Sakurai, Topical Conference on Neutrino Physics, Oxford, 3-7 July, 1978.

4) L.F. Abbot and R.M. Barnett, SLAC-PUB-2136.

5) E. Paschos, Brookhaven preprint.

   D.P. Sidhu and P. Langacker, Brookhaven preprint.

6) The plot is from ref. (4), modified slightly. In $SU_2 \times U_1$ gauge models g_A has to change in half-units. Only the "standard" model is allowed by the figure, since $g_A \approx - 1/2$.

7) M. Perl, this school.

8) Y.S. Tsai, Phys. Rev. D4 (1971) 2821.

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9) S. Weinberg, Phys. Rev. $\underline{112}$ (1958) 1375.

   M.A.B. Bég and J. Bernstein, Phys. Rev. $\underline{D5}$, 714 (1972)

10) F. Gilman and D. Miller, Phys. Rev. <u>D17</u> (1978) 1846.

    N. Kawamoto and A. Sanda, Phys. Lett. <u>76B</u> (1978) 446.

11) $\Gamma(A_l \rightarrow \rho\pi)$ follows from ref. (12) if $F_A = \frac{1}{2} F_\rho$ $f_{A\rho\pi} = g_{A\rho\pi}$, and $M_{A_l}^2 = 2 M_\rho^2$.

12) D. Geffen and W. Wilson, Minnesota preprint.

13) S. Weinberg, XIV Conference on High Energy Physics, Vienna, 1968.

14) A. Halprin, B.W. Lee and P. Sorba, Phys. Rev. <u>D14</u> (1976) 2343.

15) J. Pestieu, Louvain preprint (unpublished).

16) F. Vanucci, this school.

17) From J. Ellis et al., Nucl. Phys. <u>B131</u> (1977) 285.

    Strictly speaking, $\theta', \theta'' < \theta_c$ hold in a 6-quark, 6-lepton model

    <u>without</u> σ^2 . But the limits are probably more general.

18) K. Fujikawa and N. Kawamoto, Phys. Rev. <u>D13</u> (1976) 2534, S-Y Pi and

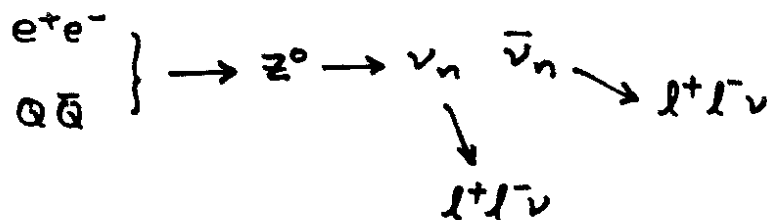   A.I. Sanda, Ann. Phys. (NY) <u>106</u> (1977) 171.

19) See also J. Vermaseren, Purdue preprint.

20) S. Glashow, Topical Conference on Neutrino Physics, Oxford, 3-7 July, 1978.

    H. Nielsen and C. Froggatt, private communication.

21) The argument presented here is a heuristic one. It is, of course, hardly clear that leptons and quarks remain pointlike even if they have very large mass. One can only try to motivate a search for mass regularities.

22) J. Gasser and H. Leutwyler, Nucl. Phys. B94 (1975) 269.

23) Suppose there are very many fermion flavors. Then we also expect many "neutrinos" $\nu_e, \nu_\mu, \nu_\tau \cdots \nu_n \cdots$. Probably most (if not all) are massive. If their masses follow the pattern of e, μ, τ we would expect a lot of them (of order 10' or so) within the mass range up to a few GeV. They will mix with one another and decay (as b, s, do). For ν_n lifetime $\tau \lesssim 10^{-9}$ sec signatures at PETRA are

$$\left.\begin{array}{c} e^+e^- \\ Q\bar{Q} \end{array}\right\} \rightarrow Z^0 \rightarrow \nu_n \ \bar{\nu}_n \searrow \ell^+\ell^-\nu$$
$$\searrow \ell^+\ell^-\nu$$

where $\ell^\pm, \nu$ stands for any charged or neutral lepton of mass $< m_{\nu_n}$. Since most of these decay too, many $e^\pm, \mu^\pm$ can result.