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JET ANGULAR RADII AND THEIR ENERGY DEPENDENCE

by

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and
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Abstract:

We discuss the opening angle distributions of the hadronic energy radiated by quark and gluon jets in the reactions $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q} \rightarrow 2$ jets and $e^+e^- \longrightarrow Q\bar{Q} \longrightarrow 3g \longrightarrow 3$ jets. Various jet angular radii are defined as a measure of the angular size of jets, and their energy dependence is calculated. We conclude that non-perturbative jets dominate over perturbative QCD jets up to e^+e^- energies of about 3o GeV. A comparison with recent PLUTO data at 9.4 GeV shows nice agreement.

There is now overwhelming evidence for the existence of hadronic jets in high energy e^+e^- annihilation [1) 2)] and in various other collisions at high energies[3)]. The angular distribution of the jet axis relative to the beam axis has been measured in e^+e^- annihilation [1) 2)] and is consistent with that for a pair of spin 1/2 particles. This gives strong support to the idea [4)] that jets are the fragments of quarks, which are the basic constituents of hadronic matter in the framework of quantum chromodynamics (QCD) [5)], the most promising theory of hadrons.

Recently Sterman and Weinberg [6)] initiated a new approach, called jet perturbation theory, which makes it possible to derive certain jet properties from QCD and to prove the dominance of the 2-jet over the n-jet cross section ($n > 2$). However, in order to avoid infrared divergencies and mass singularities due to degenerate states within the resolution of detector devices, physically sensible cross sections had to be used. This lead to the definition of "QCD jets", which are characterized by events, where all but a small fraction of the total energy is emitted within a cone of half angle δ . Thus QCD jets are fixed angle jets rather than fixed transverse momentum jets. Although it is a great success for QCD that the jet structure of hadronic final states could be derived without assuming the parton picture, in particular the transverse momentum cut off, it is nevertheless extremely disturbing that the jets observed experimentally show as a prominent feature a very sharp transverse momentum cut off with a finite $\langle p_\perp \rangle \simeq$ 36o MeV.[1) 2) 3) 7)]

In the following we shall assume that the perturbative calculations are invalidated at present energies by non-perturbative effects associated with $p_\perp$ and the trapping of quarks and gluons. This implies that present fixed transverse momentum jets are of non-perturbative origin and are therefore outside the scope of existing QCD methods. Consequently, any attempt to understand the jets in Nature has to make use of other methods.

It is the purpose of this note to determine the opening angle distribution of the energy flow within quark and gluon jets in e^+e^- annihilation. This allows a direct comparison of non-perturbative jets with QCD jets in terms of a mean jet angular radius $\langle \delta \rangle$ instead of a mean transverse momentum $\langle p_\perp \rangle$.

Let us start with the reaction $e^+e^- \longrightarrow \gamma^* \longrightarrow q\bar{q} \longrightarrow$ 2 jets and assume that the invariant inclusive cross-section for producing a hadron h from one of the two (non-perturbative) quark jets is given by

$$E_h \frac{dn_h}{d^3p} = \int d\Omega_q \frac{1}{\Gamma_{q\bar{q}}} \frac{d\Gamma_{q\bar{q}}}{d\Omega_q} D_h\left(x_\parallel, \frac{p_\perp}{\langle p_\perp \rangle}\right) \quad , \tag{1}$$

where $x_\parallel = 2p_\parallel / E_{CM}$, E_{CM} =total e^+e^- energy, and $p_\parallel$, $p_\perp$ denote longitudinal and transverse momentum, resp., of the detected hadron measured with respect to the quark jet axis, defined by the quark direction, which is integrated over $(d\Omega_q)$.$(1/\Gamma_{q\bar{q}})(d\Gamma_{q\bar{q}}/d\Omega_q)$ $\sim 1 + \cos^2\Theta_q$ describes the angular distribution for the lowest order process $e^+e^- \longrightarrow \gamma^* \longrightarrow q\bar{q}$, Θ_q being the polar angle of the quark with respect to the e^+ beam. The non-perturbative effects ($q \rightarrow h$) are contained in the fragmentation function $D_h\left(x_\parallel, \frac{p_\perp}{\langle p_\perp \rangle}\right)$ +), which is normalized to the quark jet energy $E_{CM}/2$

$$\sum_h \int d^3p \; D_h = \frac{E_{CM}}{2} \quad , \tag{2}$$

the sum extending over all hadrons. The $p_\perp$-dependence appears in the form $p_\perp / \langle p_\perp \rangle$, which shows explicitly that non-perturbative jets have an intrinsic energy scale, i.e. the mean transverse momentum of the jet , $\langle p_\perp \rangle$, averaged over all particles.

Now we ask, what is the energy fraction $d\varepsilon = dE / E_{CM}$ emitted by a quark jet into an angular interval $d\delta$ between δ and $\delta + d\delta$? (Here the jet opening angle δ is measured with respect to the jet axis). From eq. (1) we find

$$\frac{d\varepsilon}{d\delta} = \frac{\sin\delta}{\cos^3\delta} \frac{\pi}{4} E_{CM}^2 \sum_h \int_0^{x_\parallel^{max}} dx_\parallel \, x_\parallel^2 \; D_h\left(x_\parallel, \frac{x_\parallel}{2} \frac{E_{CM}}{\langle p_\perp \rangle} \tan\delta\right) \tag{3}$$

with the normalization

$$\int_0^{\pi/2} d\delta \frac{d\varepsilon}{d\delta} = \frac{1}{2} \; . \tag{4}$$

Eq. (3) allows to deduce the following general behavior of the energy flow near the kinematical boundaries

+) Actually D_h is an averaged fragmentation function, since we have already summed over quark flavors. This is appropriate for a calorimetric measurement.

$$\frac{d\varepsilon}{d\delta} \longrightarrow \begin{cases} k \left(\frac{E_{CM}}{\langle p_\perp \rangle}\right)^2 \sin\delta & \text{for} \quad \delta \to 0 \\ a \left(\frac{E_{CM}}{\langle p_\perp \rangle}\right)^{-1} & \text{for} \quad \delta \to \frac{\pi}{2} \end{cases} \tag{5}$$

Here k is a dimensionless constant +), which depends on the details of the fragmentation functions, while a is the "multiplicity strength" determined by the asymptotic behavior of the total e^+e^- multiplicity

$$\langle n \rangle = 2a \ln E_{CM} + \cdots \tag{6}$$

The energy distribution $d\varepsilon/d\delta$ depends on two independent variables, δ and E_{CM}. Inspection of eq. (3) suggests, however, that a particular combination of these two variables should minimize an eventual residual energy dependence. To see this, let us define an "angular scaling variable" τ, $0 \leq \tau < \infty$ by

$$\tau = \frac{\tan\delta_0}{\tan\delta} = \frac{\cot\delta}{\cot\delta_0} \quad , \quad \tan\delta_0 = \frac{a\,\langle p_\perp \rangle}{E_{CM}} \tag{7}$$

which scales the angle δ by an energy dependent reference angle δ_0. Rewriting $d\varepsilon/d\delta$ as $d\varepsilon/d\tau$ we obtain

$$\tau^3 \frac{d\varepsilon}{d\tau} = \frac{\pi}{4} a^2 \langle p_\perp \rangle^2 \sum_h \int_0^{x_\parallel^{max}} dx_\parallel\, x_\parallel^2\, D_h\left(x_\parallel, \frac{a}{2}\,\frac{x_\parallel}{\tau}\right) . \tag{8}$$

It is obvious, that an explicit E_{CM}-dependence of eq. (8) can only come from an explicit energy dependence of the fragmentation functions (apart from trivial phase space effects related to $x_\parallel^{max}$). Since in our calorimetric measurement we are not interested in individual particle properties but rather sum over all hadrons, both charged and neutral ones, it is not inconceivable that the dependence of $d\varepsilon/d\tau$ on E_{CM} is negligible over a wide energy range. This then leads us to the conjecture, that quark jets in e^+e^- annihilation may show "angular scaling"

+) Streaktly speaking, k is constant in the case of perfect scaling.

$$\frac{d\varepsilon}{d\tau} = F(\tau) \quad , \quad \int_0^\infty d\tau\, F(\tau) = \frac{1}{2} \quad , \tag{9}$$

where the "angular scaling function" F (τ) is a function of τ only.+)
At $\tau = 0$ ($\delta = \pi/2$) and for $\tau \to \infty$ $(\delta \to 0)$ we obtain from eq. (5) the general result

$$F(0) = 1 \quad , \quad F(\tau) \xrightarrow[\tau \to \infty]{} \frac{k a^2}{\tau^3} \quad . \tag{10}$$

The following discussion will be based on the scaling hypothesis (9), which simplifies considerably our problem to calculate jet angular radii. Instead of many fragmentation functions which are functions of two variables, we have to find only one scaling function F (τ).

Recently Fesefeldt et al [9] made a detailed analysis of 12 and 24 GeV pp data, and demonstrated that perfect scaling of the type (9) holds, where the scaling function can be represented by

$$F(\tau) = \frac{1}{(1+4\tau^2)^{3/2}} \quad . \tag{11}$$

This scaling function, which is shown as curve (I) in Fig.1 satisfies all the conditions (9), (10), and will be used from now on.++)

There remains to fix the reference angle δ_0, eq. (7). We choose $\langle p_\perp \rangle$ = 360 MeV, which is consistent with the PLUTO results [2] at 9.4 GeV and with the results of SPEAR [1) 7)] at lower energies.+++)
As to the multiplicity strength a, we remark that a = n + 1, if the averaged longitudinal scaling function (summed over all particles) behaves like $(1-x_\parallel)^n$. With n=3 we obtain a = 4, which is consistent with the data on charged multiplicity, see also eqs.(19,20). δ_0 is then given by

$$\tan \delta_0 = \frac{4\langle p_\perp \rangle}{E_{CM}} = \frac{1.44}{E_{CM}} \quad , \tag{12}$$

+) Scaling of this type has already been discussed by Ochs and Stodolsky[8]. See also ref.9) and the discussion below.

++) From a study of special fragmentation functions we obtained the scaling function $F(\tau) = (1+\tau)^{-3}$. The following calculations have also been carried out for this scaling function. It turns out that most of the results, in particular the values for the jet angular radii, remain essentially unchanged.

+++) We do not think that the weak energy variation of $\langle p_\perp \rangle$ found by the PLUTO group[2] contradicts our assumption of a constant $\langle p_\perp \rangle$. The measurement of $\langle p_\perp \rangle$ for all particles depends crucially on the correct determination of the jet axis and is very sensitive to detector acceptance and cuts.

E_{CM} in GeV, which yields $\delta_0 \simeq 9^\circ$ at 9.4 GeV.

As a first check of our assumptions we calculate the fraction $f(\delta)$ of the energy outside a cone of half angle δ

$$f(\delta) = \frac{2\tau}{\sqrt{1+4\tau^2}} = \frac{2\tan\delta_0 \cos\delta}{\sqrt{\sin^2\delta + 4\tan^2\delta_0 \cos^2\delta}} \quad . \qquad (13)$$

Recently the PLUTO group[2] has measured f (δ) at 9.4 GeV, just below the Υ(9.46) resonance, the results being indicated by the dotted area in Fig.2. The experimental jet axis is defined by either the thrust or sphericity axis, the width of the dotted area reflects the difference between these two definitions. The prediction (13) is shown as the lower curve in Fig.2. We find excellent agreement with the PLUTO data.

On the basis of eqs. (9,11,12) we are now in a position where we can calculate all angular properties of quark jets. In Fig.3 we show our prediction for the opening angle distribution of the emitted energy from quark jets at 9.4 GeV

$$\frac{d\varepsilon}{d\delta} = \frac{\tan\delta_0 \sin\delta}{\left(\sin^2\delta + 4\tan^2\delta_0 \cos^2\delta\right)^{3/2}} \quad . \qquad (14)$$

The energy flow vanishes on the jet axis, has a pronounced maximum at $\delta \simeq 13^\circ$ and then gradually decreases with increasing opening angle leading to a kind of plateau around 9o$^\circ$, the height of which is determined by the multiplicity strength a, see eq. (5). In order to compare the prediction (14) with experimental data we used the PLUTO data[2] at 9.4 GeV on the angular distribution of charged and neutral energy measured with respect to the thrust axis for thrust bins o.85-1.o, o.75-o.85 and o.5-o.75. From these data we reconstructed an angular distribution for the total energy flow, i.e. the visible energy distribution, not yet corrected for acceptance losses.[+)] For a proper comparison we renormalized the visible energy distribution by multiplication with an overall factor 1.66 ensuring the correct normalization (4). The energy distribution reconstructed in this way is indicated by the dotted area in Fig.3. It is seen that the position of the peak and the qualitative shape are correctly predicted from eq.(14). Since our reconstruction procedure does not take into account any detector acceptance, it is likely that the corrected data agree even better with the theoretical curve, especially near 9o$^\circ$.

+) We are grateful to G.Alexander for a discussion on the PLUTO data.

As a quantitative measure of the angular radius of a quark jet, we define an average opening angle $\langle\delta\rangle$ by[+)]

$$\langle\delta\rangle = \int_0^{\pi/2} d\delta\, \delta\, \frac{d\varepsilon}{d\delta} \Big/ \int_0^{\pi/2} d\delta\, \frac{d\varepsilon}{d\delta} \quad . \tag{15}$$

From eq. (14) we obtain

$$\langle\delta(E_{CM})\rangle = \frac{\tan\delta_0}{\sqrt{1-4\tan^2\delta_0}}\, \ln\left(\frac{1+\sqrt{1-4\tan^2\delta_0}}{1-\sqrt{1-4\tan^2\delta_0}}\right) . \tag{16}$$

The energy dependence of the average jet angular radius $\langle\delta\rangle$ is shown as the full curve in Fig.4. The large values obtained, 47° and 34° at the energies 5 and 9.4 GeV, resp., demonstrate in a quantitative fashion that present quark jets are broad. Even at a total e^+e^- energy of 4o GeV $\langle\delta\rangle$ is as broad as 14°. For large energies $\tan\delta_0 \ll 1$ and eq.(16) simplifies

$$\langle\delta(E_{CM})\rangle \simeq \frac{\langle p_\perp\rangle\langle n\rangle}{E_{CM}} \quad , \quad E_{CM}\rightarrow\infty \; . \tag{17}$$

This is a very satisfactory result, because it corresponds exactly to what one would have expected naively. Defining a "naive jet angular radius" δ naive by

$$\tan\delta_{naive} = \frac{\langle p_\perp\rangle}{\langle p_\parallel\rangle} \tag{18}$$

and replacing $\langle p_\parallel\rangle$ by [++)]

$$\langle p_\parallel\rangle = \frac{E_{CM}}{\langle n\rangle}\left(1 - \varkappa\, \frac{\langle p_\perp\rangle}{E_{CM}}\right) \tag{19}$$

+) Notice that $\langle\delta\rangle$ is an energy-weighted average.

++) Eq.(19) follows from eq.(1), $\varkappa$ being a dimensionless constant, which depends on the details of the fragmentation functions. With $\varkappa$ =3.5 eq.(19) reproduces nicely the experimentally found 2) energy dependence of $\langle p_\parallel\rangle$.

we obtain immediately

$$\tan \delta_{naive} \simeq \frac{\langle p_\perp \rangle \langle n \rangle}{E_{CM}} \tag{2o}$$

which is identical to eq. (17), i.e. $\langle \delta \rangle \longrightarrow \delta$ naive. The energy dependence of δnaive, eq. (2o), is shown as the upper dashed curve in Fig. 4. Here we used for the multiplicity $\langle n \rangle = 8 \ \ln E_{CM} - 5$. One should notice, however, that δ naive underestimates $\langle \delta \rangle$ appreciably in the energy range between 5 and 1o GeV, and should therefore be used only at much higher energies.

Another useful measure of the angular size of a jet is given by the angle $\delta_{1/2}$, which is defined as the half angle of a cone that contains half of the available jet energy, $f(\delta_{1/2}) = 1/2$. Eq. (13) gives

$$\tan \delta_{1/2} = \sqrt{12} \tan \delta_o \simeq \frac{5}{E_{CM}} \ . \tag{21}$$

$\delta_{1/2}(E_{CM})$ is shown as the dot-dashed line in Fig. 4. At 9.4 GeV we obtain $\delta_{1/2} = 28^o$ in excellent agreement with the result from PLUTO. With increasing energy $\delta_{1/2}$ decreases faster than $\langle \delta \rangle$ or δnaive; at 4o GeV one obtains $\delta_{1/2} = 7^o$.

Yet another measure of the jet size is provided by the angle δmax, which is the position of the maximum of $d\varepsilon/d\delta$, eq. (14). The result is

$$\sin \delta_{max} = \frac{\sqrt{2} \tan \delta_o}{\sqrt{1 - 4 \tan^2 \delta_o}} \ . \tag{22}$$

$\delta_{max}(E_{CM})$ is shown as the lower dashed curve in Fig. 4. It starts at $3o^o$ at 5 GeV and falls down to 3^o at 4o GeV. The maximum energy flow is $\sim E_{CM}$.

The various angular radii discussed in this note illustrate clearly, that non-perturbative jets are rather broad at present energies and show an angular shrinkage propotional to $1/E_{CM}$ (for $\delta 1/2$ and δ max) or $\ln E_{CM}/E_{CM}$ (for $\langle \delta \rangle$ or δ_{naive}).

We now want to compare our non-perturbative estimates with a corresponding QCD prediction. Because of the already mentioned singularities in QCD, not every definition of a jet angular radius is meaningful. It turns out, however, that the quantity $\langle \sin^2 \delta \rangle$ ($\langle \ \rangle$ refers again to an energy-weighted average, as in eq.(15)) can be calculated perturbatively in QCD with the result [10]

$$\langle \sin^2\delta \rangle_{QCD} = 2\left(\frac{\alpha_s}{\pi}\right) \tag{23}$$

which is correct to order α_s, where α_s is the running coupling constant in QCD, asymptotically given by [11]

$$\alpha_s(E_{CM}) = \frac{6\pi}{(33-2N_f)\,\ln(E_{CM}/\Lambda)} \quad . \tag{24}$$

N_f is the number of flavors, and $\Lambda \simeq$ 5oo MeV. In Fig. 5 $\langle \sin^2\delta \rangle_{QCD}$ is plotted as a function of E_{CM} [+)] and compared with the non-perturbative (NP) result derived from $d\varepsilon/d\delta$ (eq.(14)

$$\langle \sin^2\delta \rangle_{NP} = \frac{2\tan\delta_0}{\sqrt{1-4\tan^2\delta_0}}\left\{\frac{\pi/2 - \arctan\left(\frac{2\tan\delta_0}{\sqrt{1-4\tan^2\delta_0}}\right)}{\sqrt{1-4\tan^2\delta_0}} - 2\tan\delta_0\right\} \tag{25}$$

which asymptotically approaches

$$\langle \sin^2\delta \rangle_{NP} = \pi\tan\delta_0 = \frac{4\pi\langle p_\perp \rangle}{E_{CM}} \quad , E_{CM} \to \infty \; . \tag{26}$$

It is seen from Fig. 5 that the non-perturbative contribution is clearly dominating at present energies, but since it decreases like $1/E_{CM}$ it vanishes much faster than the QCD prediction, which vanishes only like $1/\ln E_{CM}$. At 3o GeV the two contributions are about equal, and we conclude that it will be extremely difficult to detect genuine QCD jet properties below 3o GeV, since the non-perturbative smearing effects due to fragmentation are so large.

+) In Fig. 5 we assumed $N_f = 4$ for $E_{CM} \lesssim 11$ GeV, $N_f = 5$ for $11 \lesssim E_{CM} \lesssim 3o$ GeV and $N_f = 6$ for $E_{CM} \gtrsim 3o$ GeV.

Finally we would like to discuss the angular properties of gluon jets, where we take the reaction $e^+e^- \longrightarrow Q\bar{Q} \longrightarrow 3g \longrightarrow 3$ jets as an example.[12] (Here $Q\bar{Q}$ is a heavy quark-antiquark $^3S_1$ bound state of mass $M_{Q\bar{Q}}$). Asking again for the energy fraction emitted by a gluon jet into an angular interval $d\delta$, we obtain the same eq.(3), but with D_h now replaced by the smeared gluon fragmentation function

$$D_h \longrightarrow \int_{x_\parallel}^{1} d\xi\, \mathcal{F}(\xi)\, \mathcal{D}_h\left(\frac{x_\parallel}{\xi}, \frac{x_\parallel}{2} \frac{E_{CM}}{\langle p_\perp \rangle_g} \tan\delta\right) . \qquad (27)$$

Here $\xi = 2\, E_{gluon}/E_{CM}$, $\mathcal{D}_h$ denotes the gluon fragmentation function and $\mathcal{F}(\xi)$ is well-known from the analysis of positronium[13]. $\langle p_\perp \rangle_g$ denotes the transverse momentum of the gluon jet. It is straightforward to show that the relations (5) are true also for the gluon case (with the appropriate replacements), the "multiplicity strength" a_g being defined by

$$\langle n \rangle_{3g} = 3\, a_g \ln E_{CM} + \cdots \qquad (28)$$

in analogy to eq.(6). One then repeats the considerations following eq. (6), and ends up with a similar scaling hypothesis for the gluon jet as expressed in eqs.(9), (1o), but with the modified normalization condition

$$\int_0^\infty d\tau\; F_g(\tau) = \frac{1}{3} \qquad , \qquad (29)$$

since one gluon carries on the average 1/3 of the bound state energy. τ is defined as in eq.(7) with $\tan \delta_0^g = a_g \langle p_\perp \rangle_g / M_{Q\bar{Q}}$.

It is obvious how to generalize the quark scaling function (11). For the following discussion we shall use

$$F_g(\tau) = \frac{1}{(1+9\tau^2)^{3/2}} \qquad , \qquad (3o)$$

which leads to the symmetry relation

$$F_g(\tau) = F(\tfrac{3}{2}\tau) \,. \tag{31}$$

The gluon function $F_g(\tau)$ is shown as the curve (II) in Fig. 1.

The following gluon estimates are made for what we call a "minimal gluon jet", defined by having the same global jet parameters as a quark jet, i.e. $a_g = a = 4$, $\langle p_\perp \rangle_g = \langle p_\perp \rangle = 360$ MeV. In this case the τ variables are scaled with the same reference angle δ_0, and the symmetry relation (31) tells us, that a gluon jet in the 3 gluon decay of a bound state of mass $M_{Q\bar{Q}}$ has the same angular shape as a quark jet produced in e^+e^- annihilation at an energy $E_{CM} = \frac{2}{3} M_{Q\bar{Q}}$. To be definite, this implies that gluon jets in the decay $\Upsilon(9.46) \longrightarrow 3g$ have the same angular size as quark jets at $E_{CM} =$ = 6.3 GeV. In Figs. 2 and 3 we show our predictions for the Υ decay. We emphasize, that a measurement of these curves will tell a lot about the properties of gluon jets. In Fig.4 we have indicated by arrows the positions of Υ and of a hypothetical $t\bar{t}$ bound state at 28 GeV; the open circles show our predictions for the average opening angle $\langle \delta \rangle_g$. On the Υ resonance we predict $\langle \delta \rangle_g = 42^\circ$, to be compared with a quark opening angle of 34° at the same energy. Thus gluon jets have larger angular radii, even if their average transverse momentum is the same as for quark jets.

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Figure Captions

Fig.1: Angular scaling functions for

$$(\mathrm{I})\quad e^+e^- \to \gamma^* \to q\bar{q} \to 2\ \mathrm{Jets}\quad,\ \mathrm{eq.}\ (11)$$

$$(\mathrm{II})\quad e^+e^- \to \Upsilon \to 3g \to 3\ \mathrm{Jets}\quad,\ \mathrm{eq.}\ (30)\ .$$

Fig.2: Fraction $f(\delta)$ of total energy outside a cone of half angle δ as a function δ .Data at 9.4 GeV from PLUTO [2)].

Fig.3: Hadronic energy distribution as a function of jet opening angle δ . The reconstruction procedure (from PLUTO data [2)]) is explained in the text.

Fig.4: Energy dependence of jet angular radii. The experimental point at 9.4 GeV is from PLUTO [2)]. The open circles are predictions for the average opening angle $\langle \delta \rangle$ for $\Upsilon(9.46) \to 3\,g$ and for a hypothetical bound state $t\bar{t}$ at 28 GeV.

Fig.5: Energy dependence of $\langle \sin^2 \delta \rangle$. Comparison between non-perturbative contribution (NP) and perturbative QCD prediction (QCD).

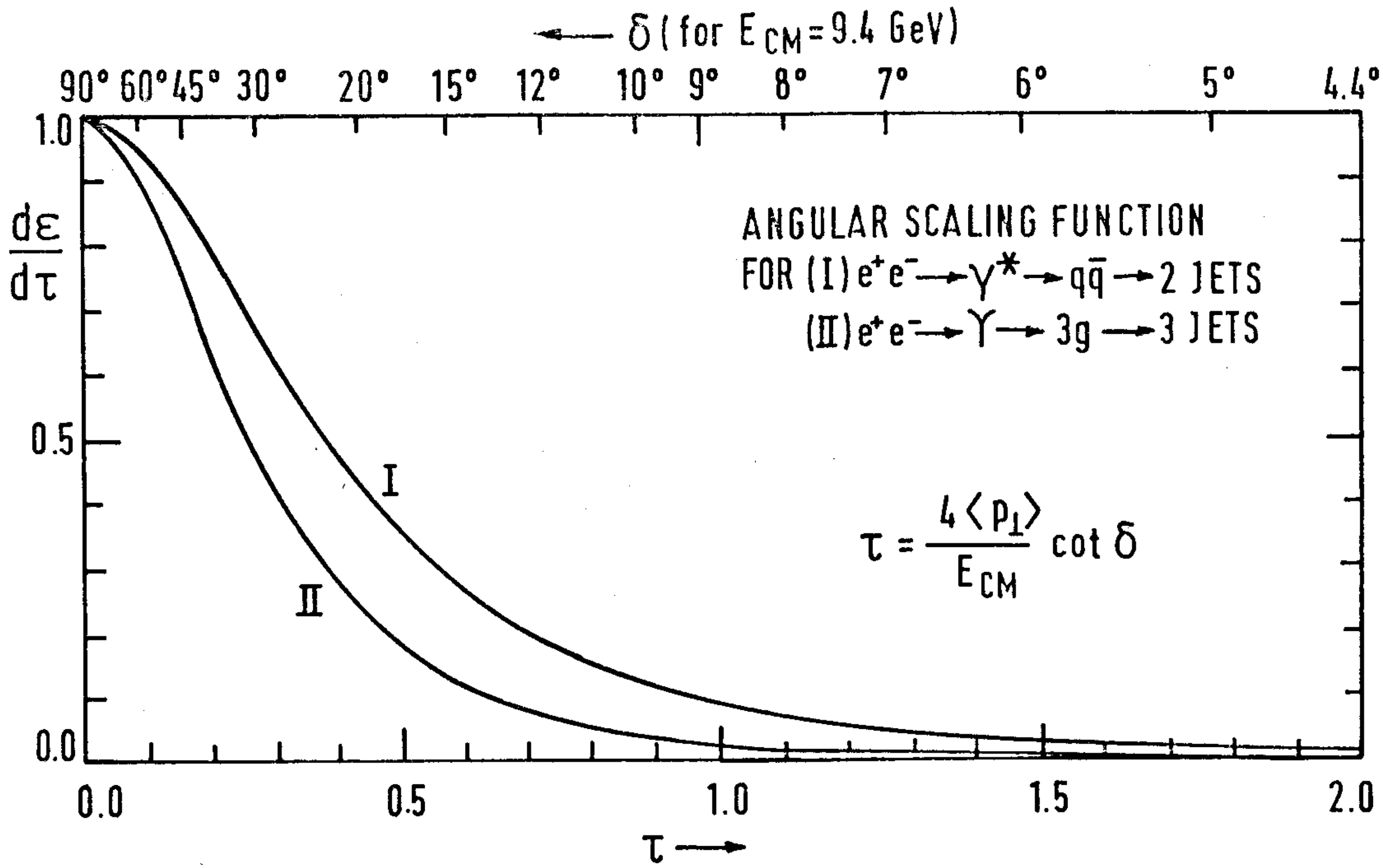

δ (for E CM = 9.4 GeV)
90° 60° 45° 30° 20° 15° 12° 10° 9° 8° 7° 6° 5° 4.4°
dε/dτ
1.0
0.5
0.0
ANGULAR SCALING FUNCTION
FOR (I) e+e- → γ* → qq̄ → 2 JETS
(II) e+e- → Υ → 3g → 3 JETS
τ = 4⟨p⊥⟩/E CM cot δ
I
II
0.0
0.5
1.0
1.5
2.0
τ

Fig. 1

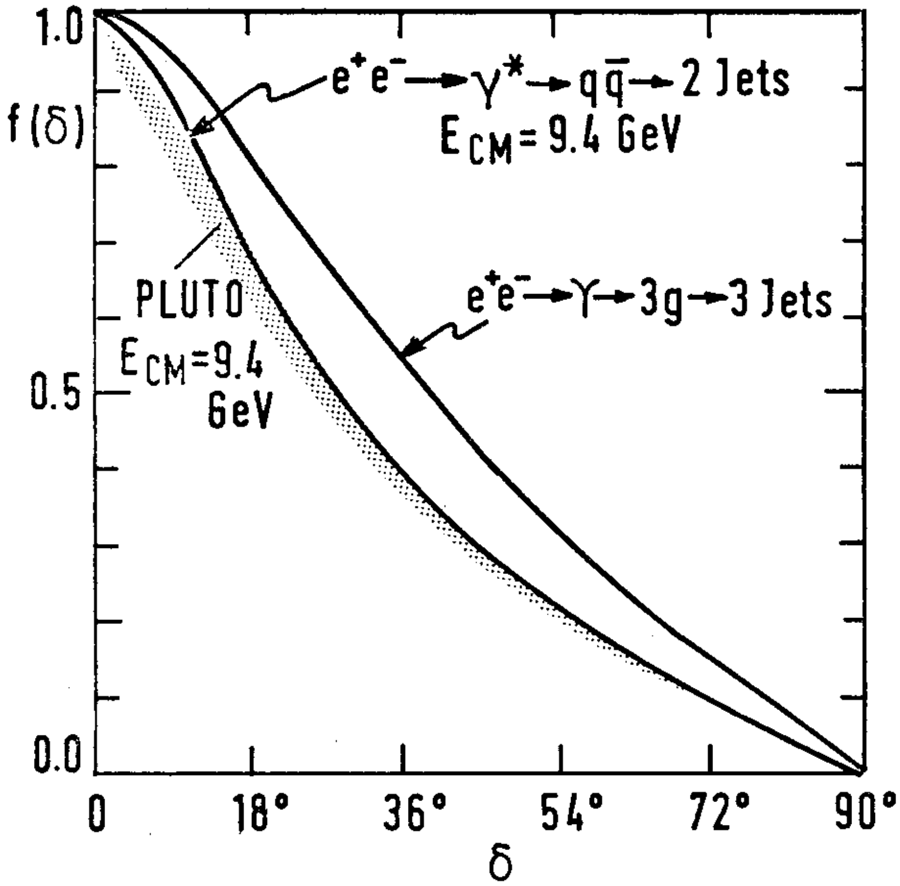

1.0
f(δ)
0.5
0.0
e⁺e⁻ → γ* → qq̄ → 2 Jets
E_CM = 9.4 GeV
e⁺e⁻ → Υ → 3g → 3 Jets
PLUTO
E_CM = 9.4
GeV
0
18°
36°
54°
72°
90°
δ

Fig.2

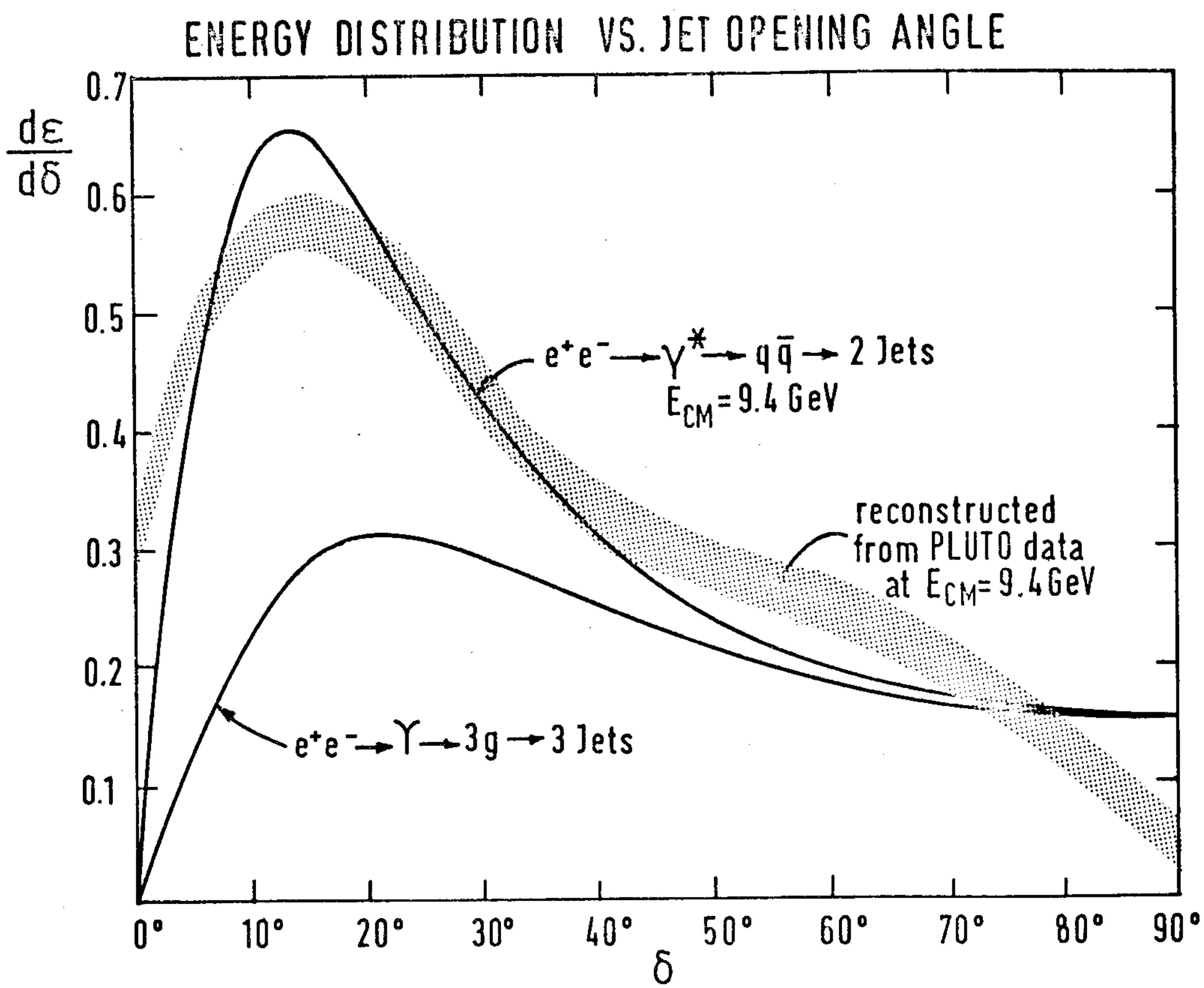

ENERGY DISTRIBUTION VS. JET OPENING ANGLE
dε/dδ
0.7
0.6
0.5
0.4
0.3
0.2
0.1
e+e− → γ* → q q̄ → 2 Jets
ECM = 9.4 GeV
reconstructed
from PLUTO data
at ECM = 9.4 GeV
e+e− → Υ → 3g → 3 Jets
0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
δ

Fig.3

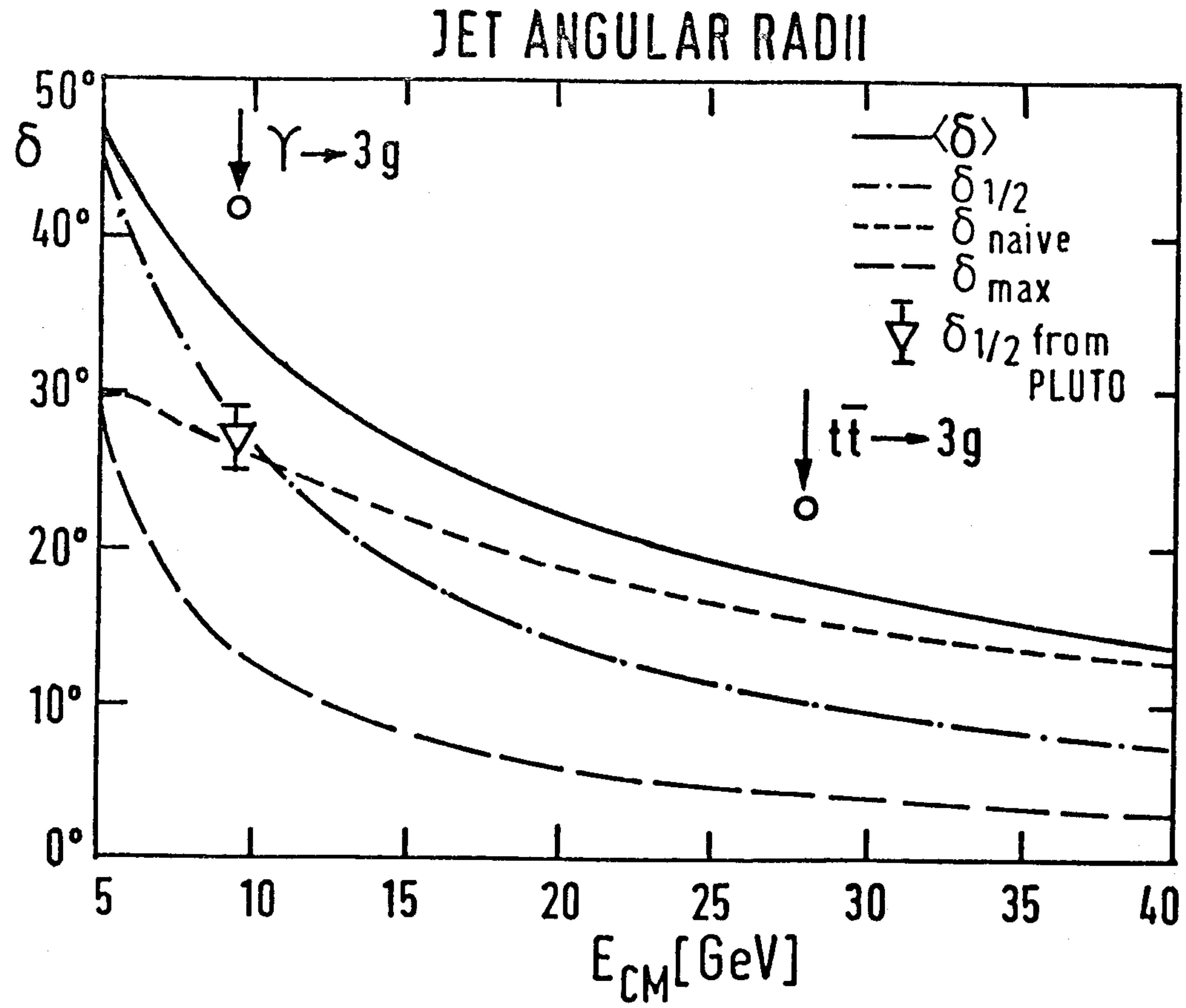
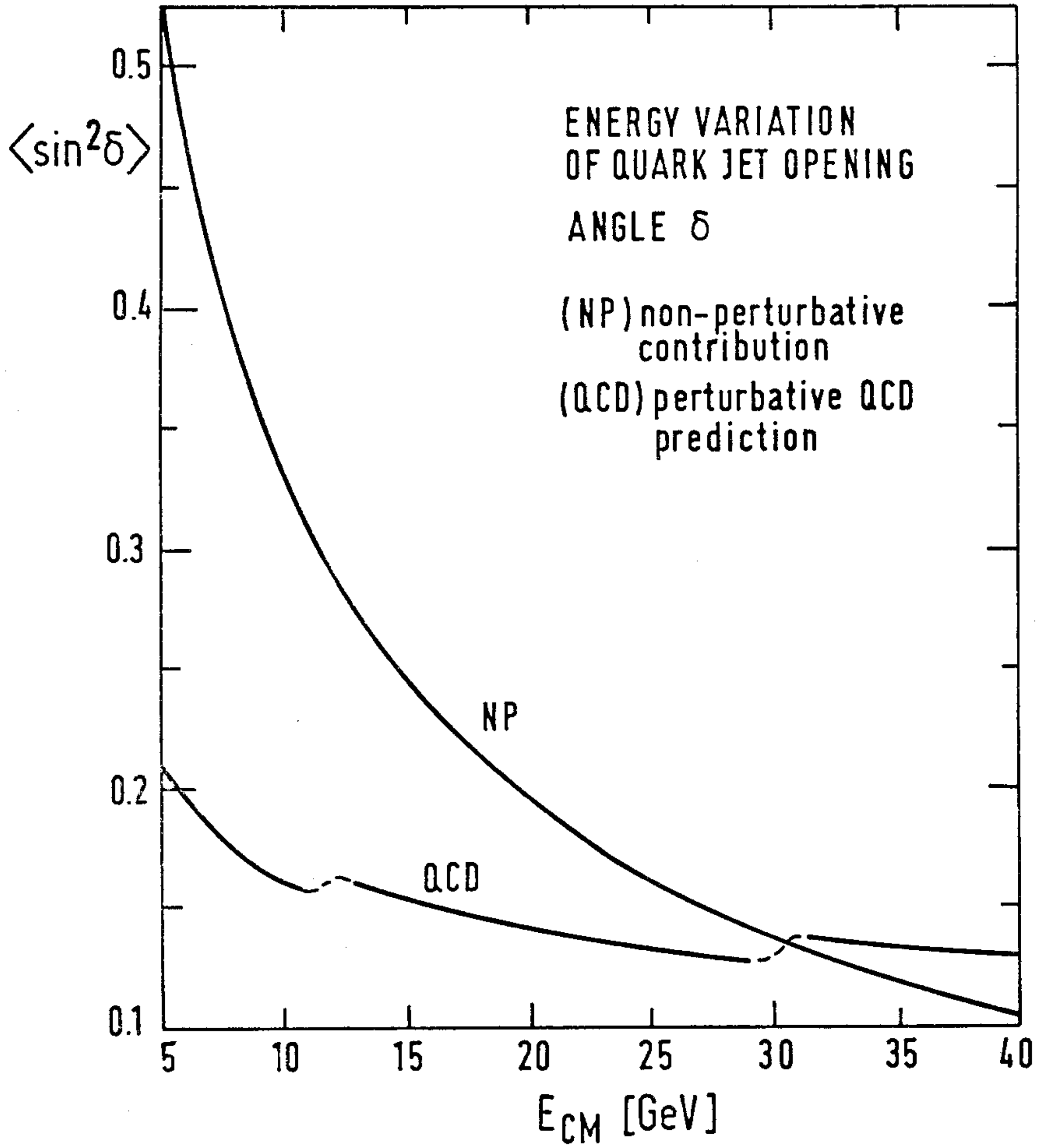

JET ANGULAR RADII
50°
40°
30°
20°
10°
0°
δ
Υ → 3g
$t\bar{t}$ → 3g
⟨δ⟩
$\delta_{1/2}$
δ_{naive}
δ_{max}
$\delta_{1/2}$ from PLUTO
5
10
15
20
25
30
35
40
E_{CM} [GeV]

Fig.4



0.5
0.4
0.3
0.2
0.1
⟨sin²δ⟩
ENERGY VARIATION
OF QUARK JET OPENING
ANGLE δ
(NP) non-perturbative contribution
(QCD) perturbative QCD prediction
NP
QCD
5
10
15
20
25
30
35
40
E_CM [GeV]

Fig.5