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DEEP INELASTIC STRUCTURE FUNCTIONS: DECISIVE TESTS OF QCD

by

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DECISIVE TESTS OF QCD

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1. INTRODUCTION

The purpose of this talk is to discuss the present status of scaling violations in deep inelastic lepton-nucleon scattering processes. Specifically, I will discuss in great detail which experimentally measured structure functions (or moments thereof) have to be used in order to discriminate between various field theories of the strong interactions such as (non-abelian) QCD, abelian vector-gluon ($\bar{\Psi}\gamma_\mu\Psi A^\mu$) , non-abelian scalar-gluon ($\bar{\Psi}\lambda_a\Psi\phi_a$) , and abelian scalar-gluon ($\bar{\Psi}\Psi\phi$) theories. Moreover I will delineate the sensitivity of certain structure functions to the fundamental couplings of the theory, i.e., to the quark-gluon coupling and/or to the gluon self-couplings (triple gluon vertex) - a measurement of the latter would provide us with a direct and unambiguous test of the Yang-Mills structure of QCD which is so very essential for asymptotic freedom. Furthermore, we will consider three-jet events with special emphasis on deep inelastic heavy quark (c,b,...) production via virtual Bethe-Heitler processes. Finally, I shall briefly discuss the relevance of non-leading 2-loop contributions to anomalous dimensions and the effects of "finite terms" in Wilson coefficients and their importance for the analysis of deep inelastic reactions such as the determination of moments of "sea" distributions from measured neutrino cross sections. The relevance of these "finite terms" for Drell-Yan dimuon production cross sections will also briefly be mentioned.

For a detailed discussion of semi-inclusive deep inelastic reactions, such as hadronic final states and QCD predictions for gluon jets in lepton-hadron interactions I refer you to the talks of P. Landshoff and P. Binétruy.

In Section 2 I will briefly recapitulate the structure of QCD and other asymptotically non-free (finite fixed point) field theories and those parts of their predictions for scaling violations in structure functions which will be essential for our present analysis. Here we shall use the classical language of the light cone expansion and the renormalization group. Section 3 will be devoted to a detailed comparison of these predictions with experiment using (of increasing complexity) (A) moments of non-singlet structure functions ($F_2^{ep}-F_2^{en}$, $F_3^{\nu,\bar{\nu}N}$), (B) moments of flavor singlet structure functions ($F_2^{\nu N}$, etc.) and (C) the explicit x- and Q^2-dependence of structure functions. Here, QCD will prove to be the only renormalizable strong interaction field theory compatible with experiment! However, as we shall see, present measurements of scaling violations in $F_2(x,Q^2)$ are rather insensitive to the gluon content of the hadron. In Section 4 we proceed to discuss (very fundamental but equally difficult) measurements which are particularly sensitive to the gluon self-couplings of a (locally gauge invariant) Yang-Mills theory, and which would provide us with a direct and sensitive test of asymptotic freedom. In Section 5 we give predictions for deep inelastic heavy quark (c,b,...) production via virtual Bethe-Heitler processes and, finally, in Section 6 we briefly discuss non-leading 2-loop corrections to anomalous dimensions and "finite terms" in Wilson coefficients and their relevance for comparing QCD and the parton model with experiment; we conclude with a brief discussion of the effects of "finite terms" in Drell-Yan processes.

2. FIELD THEORIES AND SCALING VIOLATIONS

The quark-gluon interactions in QCD are described by the Lagrangian

$$\mathcal{L}_F = i\bar{\Psi}\gamma_\mu(\partial^\mu - ig\frac{\lambda_a}{2}A^\mu_a)\Psi \qquad (1)$$

where λ_a are the Gell-Mann matrices of color $SU(3)_c$ acting on the color triplets $\Psi = (q_2, q_1, q_3)$ with q = u,d,s,c,... . Renormalizability requires the theory to be locally gauge invariant which, because of the non-abelian interaction in Eq. (1), implies self-couplings of the gluon fields A^μ_a via the Yang-Mills Lagrangian $\mathcal{L}_{YM} = -\frac{1}{4}F^{\mu\nu}_a F_{a\mu\nu}$ with

$$F^{\mu\nu}_a = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a + g f_{abc} A^\mu_b A^\nu_c \qquad (2)$$

where $[\frac{1}{2}\lambda_a, \frac{1}{2}\lambda_b] = i f_{abc}\frac{1}{2}\lambda_c$. It is well known[1] that the self-interactions of the vector fields A^μ_a implied by Eq. (2) are responsible for asymptotic freedom, i.e. the effective coupling constant $\alpha_s \equiv g^2/4\pi$ decreases for increasing momentum transfer squared Q^2 (or decreasing distances)

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2N_f)\ln Q^2/\Lambda^2} \tag{3}$$

with N_f being the number of flavors and Λ is the only free parameter of QCD which has to be fixed by experiment ($\Lambda \simeq 0.5$ GeV). Equation (3) is the solution of the renormalization group equation $Q\, d\bar{g}/dQ = \beta(\bar{g})$ for the effective coupling $\bar{g}(g,Q^2)$, where the Callan-Symanzik function $\beta[g(\mu)] \equiv \mu\, \partial g(\mu)/\partial\mu$ describes the variation of the renormalized coupling $g(\mu)$ if one changes the arbitrary renormalization mass parameter μ. The reason for asymptotic freedom is that the gluon self-couplings in Eq. (2) allow for a color charge transfer from the quark fields ψ to the gluon fields A_a^μ and thus create an anti-shielding around the bare charge g_0 which dominates the color charge shielding (opposite sign charges) around g_0 stemming from the usual vacuum polarization effects, due to the coupling in Eq. (2), which do not allow for a color charge exchange with the field. The net effect is that $\beta(g) = -b g^3 < 0$ as shown in Fig. 1 which implies that perturbation theory becomes better (α_s small) the larger Q^2 ($\gtrsim 2$ GeV^2).

The situation is very different for "conventional" field theories such as abelian vector gluon theories where, as in QED, there is no group structure in the quark-gluon coupling in Eq. (1), i.e. the vertex is described by $g\bar{\psi}\gamma_\mu\psi A^\mu$, and consequently there is no gluon self-coupling term in Eq. (2) and the field strength tensor is simply $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$. Similarly, the interaction in Eq. (1) takes the form $g\bar{\psi}\lambda_a\psi\phi_a$ for non-abelian scalar gluon theories, and $g\bar{\psi}\psi\phi$ for abelian scalar gluon (Yukawa) theories. In all these cases we have $\beta(g) = +b g^3 > 0$ and thus these theories are not UV stable near the origin $g = 0$, i.e. the coupling will become larger the larger Q^2 since now the only

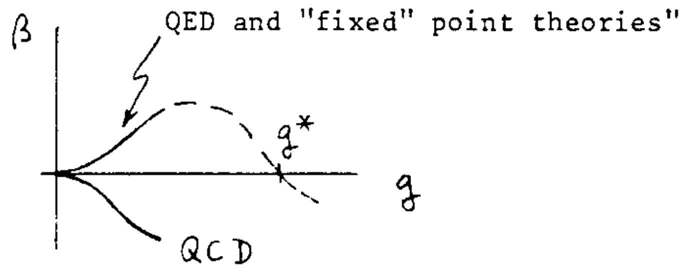


Fig. 1. The β function for QCD and conventional field theories which are assumed to develop a finite UV fixed point g^*.

quantum effects which contribute to charge renormalization are due to vacuum polarization (shielding, but no anti-shielding exists) because no self-couplings of the gluon fields (A^μ or ϕ_a, ϕ) exist. In order to proceed perturbatively at large energies (small distances) we have to make the (so far unproven) assumption that there exists a finite fixed point coupling g^* as $Q^2 \to \infty$, i.e. $\beta(g^*) = 0$, such that the effective coupling $\alpha^*/4\pi \ll 1$ - a necessary requirement in order not to reject a priori conventional field theories as possible candidates for explaining (perturbatively) the experimentally observed small scaling violations. This situation is shown in Fig. 1 and we shall call hereafter these conventional field theories simply "fixed point theories". The fixed point coupling g^* is then the only free parameter of the theory to be determined experimentally (as is the case for Λ in QCD).

The importance of deep inelastic lepton-nucleon scattering processes is that there exists an operator product expansion (OPE) on the light cone,[3] a direct generalization of Wilson's short distance expansion of the product of two currents, which can be used, together with renormalization group techniques, to calculate scaling violations in field theory. Symbolically we may write

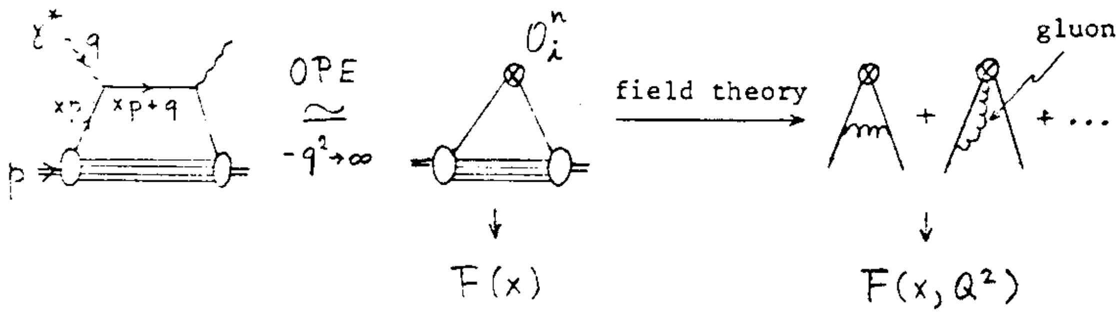


Fig. 2

where O_i^n are the various light-cone (or Wilson) operators of different type i and spin n, and $x = Q^2/2p\cdot q$, $Q^2 \equiv -q^2 \geq 0$. Note that in any interacting field theory the structure function F will depend on the two variables x and Q^2, say, in contrast to the naive parton model where in the Bjorken limit $F = F(x)$. This additional Q^2 dependence is usually referred to as scaling violation. The OPE together with field theory makes predictions[4] only for (Mellin) moments of deep inelastic structure functions defined by

$$\langle F(Q^2)\rangle_n \equiv \int_0^1 dx\, x^{n-2}\, F(x, Q^2) \tag{4}$$

where $F = xF_1$, F_2 or xF_3. In terms of these moments the above OPE can be formally written as

$$\langle F(Q^2)\rangle_n = \sum_i C_i^n(Q^2/\mu^2, \alpha_s(\mu^2))\, \langle p|\, O_i^n \,|p\rangle \tag{5}$$

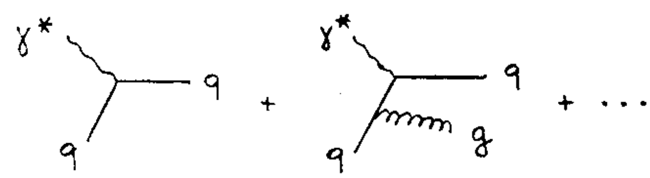
where the Q^2-independent matrix elements $\langle p | \mathcal{O}_i^n | p \rangle$ which describe the bound state (wave function) of the nucleon cannot, at the present state of art, be calculated perturbatively and will be related to the parton distributions in the nucleon at a fixed value of $Q^2 = Q_o^2$, which in turn will be determined from experiment. On the other hand, the Q^2-behavior of the Wilson coefficients C_i^n - the expansion coefficients of the OPE - can be calculated perturbatively which will uniquely determine the Q^2 evolution of quark and gluon distributions, provided Q^2 is large enough. To this end one writes a renormalization group (RG) equation for the Wilson coefficients (which expresses the invariance of any measurable physical quantity with respect to changes of the arbitrary renormalization point μ) in order to sum the leading log contributions to all orders α_s, i.e. one improves simple order by order perturbation theory. For non-singlet (NS) structure functions there is only <u>one</u> leading fermionic Wilson operator[4] O_{NS}^n which contributes to the sum in Eq. (5), for which case the RG equation reads

$$\left(\mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} - \gamma_{NS}^n\right) C_{NS}^n(Q^2/\mu^2, \alpha_s(\mu^2)) = 0 \tag{6}$$

where the "anomalous dimension" γ_{NS}^n of O_{NS}^n, which governs the Q^2 dependence of structure functions, is the coefficient of the logarithmic contribution[4] of the one-gluon-loops in Fig. 2. The solution of Eq. (6) is well known to be

$$C_{NS}^n(Q^2/\mu^2, \alpha_s(\mu^2)) = C_{NS}^n(1, \alpha_s(Q^2)) \exp\left[-\int_\mu^Q \frac{dQ'}{Q'} \gamma_{NS}^n(\alpha_s(Q'^2))\right] \tag{7}$$

where $C_{NS}^n(1, \alpha_s(Q^2)) \simeq 1 + \mathcal{O}(\alpha_s)$ results from



and the non-leading $\mathcal{O}(\alpha_s)$ contributions are the so called "finite terms", i.e., all non-logarithmic contributions from $\gamma^* q \to g q$. As long as we work in the leading 1-loop approximation of γ_{NS}^n, we have to keep only the leading naive parton model contribution ($\simeq 1$) and disregard the $\mathcal{O}(\alpha_s)$ terms which we will do for the time being. Thus the final result for the Q^2 evolution of the moments of a non-singlet structure function can be written as[4-6] (I follow closely the notation of Refs. 5 and 6)

$$\langle F_{NS}(Q^2) \rangle_n = \langle F_{NS}(Q_o^2) \rangle_n \, e^{-s\, a_{NS}(n)} \tag{8}$$

where Q_o^2 ($\simeq$ 2-4 GeV^2) is the input reference momentum at which the structure function has to be determined experimentally, and for QCD the renormalization group exponents are given by

$$a_i = \frac{\gamma_i}{8\pi\alpha_s b} \quad , \qquad s = \ln \frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \tag{9}$$

with $b = \frac{1}{16\pi^2}(11 - \frac{2}{3}N_f)$. For fixed point theories these exponents read

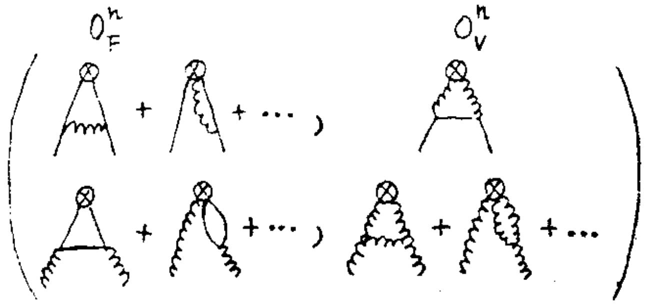
$$a_i = \frac{\gamma_i}{2} \quad , \qquad s = \ln \frac{Q^2}{Q_0^2} \tag{10}$$

where now the value of the UV finite fixed point $\alpha_s \equiv \alpha^*$, appearing in γ_i, has to be determined by experiment[4]. The NS anomalous dimension $\gamma_{NS}^n \equiv \gamma_{FF}^F(n)$ is given by[4]

$$\gamma_{FF}^F = \frac{\alpha_s}{2\pi} C_2(R) \left[1 - \frac{2}{n(n+1)} + 4 \sum_{j=2}^{n} \frac{1}{j} \right] \ . \tag{11}$$

In the general case, structure functions (e.g. $F_2^{\mu p}$) receive contributions also from flavor singlet operators. There are two different types of singlet Wilson operators[4] contributing to the sum in Eq. (5), one fermionic operator O_F^n and one gluonic operator O_V^n, the latter being constructed[4] from the fundamental vector fields A_a^μ. In this case the RG equation becomes a matrix equation according to the 2x2 singlet anomalous dimension matrix $\hat{\gamma}(n)$, because of the four possible matrix elements of $O_{F,V}^n$ between the external fermionic (F) and gluonic (V) states: the operators mix under renormalization which is usually referred to as "singlet operator mixing". This singlet mixing will play a crucial role for discriminative tests of QCD! This singlet matrix reads (in a straightforward notation)

$\hat{\gamma}(n) =$



$$\equiv \begin{pmatrix} \gamma_{FF}^F \ , & \gamma_{FF}^V \\ \gamma_{VV}^F \ , & \gamma_{VV}^V \end{pmatrix}$$

with γ^F_{FF} given by Eq. (11) and[4]

$$\gamma^V_{VV} = \frac{\alpha_s}{2\pi}\left\{C_2(G)\left[\frac{1}{3} - \frac{4}{n(n-1)} - \frac{4}{(n+1)(n+2)} + 4\sum_{j=2}^{n}\frac{1}{j}\right] + \frac{4}{3}T(R)\right\}$$

$$\gamma^F_{VV} = -\frac{\alpha_s}{2\pi}\frac{4(n^2+n+2)}{n(n+1)(n+2)}T(R) \tag{12}$$

$$\gamma^V_{FF} = -\frac{\alpha_s}{2\pi}\frac{2(n^2+n+2)}{n(n^2-1)}C_2(R)$$

where the group invariants are as follows: for non-abelian vector theories (QCD) we have

$$C_2(G) = 3, \quad C_2(R) = \frac{4}{3}, \quad T(R) = \frac{1}{2}N_f \tag{13}$$

and for an abelian gluon field theory these quantities simply read

$$C_2(G) = 0, \quad C_2(R) = 1, \quad T(R) = 3N_f. \tag{14}$$

For all subsequent considerations we shall take $N_f = 4$. Note that only $C_2(G)\delta_{ab} \equiv f_{acd}f_{bcd}$ in γ^V_{VV} is a direct measure of the gluon self-interactions (triple gluon vertex) in Eq. (2). Similar expressions for anomalous dimensions can be derived for non-abelian and abelian scalar gluon theories.[6,7] The Q^2 evolution of singlet structure functions is now obtained by diagonalizing the RG matrix equation via $\hat{\gamma} = \gamma_-\hat{P}^- + \gamma_+\hat{P}^+$ with

$$\gamma_\pm = \frac{1}{2}\left[\gamma^F_{FF} + \gamma^V_{VV} \pm \sqrt{(\gamma^V_{VV} - \gamma^F_{FF})^2 + 4\gamma^F_{VV}\gamma^V_{FF}}\right] \tag{15}$$

and where the projection operators are given by

$$\hat{P}^- = \begin{pmatrix} p^-_{11}, & p^-_{12} \\ p^-_{21}, & 1-p^-_{11} \end{pmatrix}$$

and $\hat{P}^+ = 1 - \hat{P}^-$ with

$$p^-_{11} = \frac{\gamma^F_{FF} - \gamma_+}{\gamma_- - \gamma_+}, \qquad p^-_{21} = \frac{\gamma^F_{VV}}{\gamma_- - \gamma_+}, \qquad p^-_{12} = \frac{\gamma^V_{FF}}{\gamma_- - \gamma_+}. \tag{16}$$

The RG prediction for the Q^2 dependence of a general structure function in Eq. (5) is then obtained to be[5]

$$\langle F(Q^2)\rangle_n = \sum_{i=NS,\pm} \langle F_i(Q_0^2)\rangle_n \; e^{-s\,a_i(n)} \tag{17}$$

with the RG exponents given by Eqs. (9) and (10), and

$$\langle F_\pm(Q_0^2)\rangle_n = \begin{pmatrix} 1-\alpha_n \\ \alpha_n \end{pmatrix} \langle x\Sigma(Q_0^2)\rangle \mp \beta_n \langle xG(Q_0^2)\rangle_n \tag{18}$$

where for brevity we have defined $\alpha_n \equiv p_{11}^-(n)$ and $\beta_n \equiv p_{21}^-(n)$; as we shall see α_n will play a crucial role in discriminating between different field theories. The gluon distribution $G(x,Q^2)$ in the nucleon is defined by $\langle xG(Q_0^2)\rangle_n \equiv \langle p|O_V^n|p\rangle$ and the fermionic singlet, defined by $\langle x\Sigma(Q_0^2)\rangle_n \equiv \langle p|O_F^n|p\rangle$, is just

$$x\Sigma(x,Q^2) \equiv x\sum_q \left[q(x,Q^2)+\bar{q}(x,Q^2)\right] \tag{19}$$

which is directly measured by $F_2^{\nu N}$ (above charm threshold and always assuming $s = \bar{s}$, $c = \bar{c}$). Using Eq. (18) the singlet contributions to Eq. (17) can be rewritten in the following convenient form

$$\begin{aligned}\langle x\Sigma(Q^2)\rangle_n = {} & \left[\alpha_n\langle x\Sigma(Q_0^2)\rangle_n + \beta_n\langle xG(Q_0^2)\rangle_n\right] e^{-s\,a_-(n)} \\ & + \left[(1-\alpha_n)\langle x\Sigma(Q_0^2)\rangle_n - \beta_n\langle xG(Q_0^2)\rangle_n\right] e^{-s\,a_+(n)}\end{aligned} \tag{20a}$$

$$\begin{aligned}\langle xG(Q^2)\rangle_n = {} & \left[(1-\alpha_n)\langle xG(Q_0^2)\rangle_n + \frac{\alpha_n(1-\alpha_n)}{\beta_n}\langle x\Sigma(Q_0^2)\rangle_n\right] e^{-s\,a_-(n)} \\ & + \left[\alpha_n\langle xG(Q_0^2)\rangle_n - \frac{\alpha_n(1-\alpha_n)}{\beta_n}\langle x\Sigma(Q_0^2)\rangle_n\right] e^{-s\,a_+(n)} \, .\end{aligned} \tag{20b}$$

It should be emphasized that these equations are not independent: once $\langle x\Sigma(Q_0^2)\rangle_n$ and $\langle xG(Q_0^2)\rangle_n$ are fixed from experiment by fitting, say, Eq. (20a) to the measured Q^2 dependence of $\langle x\Sigma(Q^2)\rangle_n$, then Eq. (20b) is trivially satisfied and does not constitute an independent test of QCD. We shall come back to this point later.

3. COMPARISON WITH EXPERIMENT

The most straightforward and simple, although in many cases not very stringent and instructive, tests of field theories are to compare just moments of structure functions with experiment which are directly predicted by any field theory. Let us start with the theoretically most simple case of non-singlet structure functions.

3.A. Non-Singlet Moments

As we have seen in Eq. (8) the Q^2 evolution of the moments of a non-singlet structure function $F_{NS} = F_2^{ep} - F_2^{en}$, $F_3^{\nu N}$, etc. is governed by only one anomalous dimension γ_{FF}^F. Considering first $F_3^{\nu N}$, Eq. (8) tells us that for QCD

$$\langle x F_3^{\nu N}(Q^2) \rangle_n^{-1/a_{NS}(n)} \sim \ln \frac{Q^2}{\Lambda^2} \tag{21}$$

i.e. the $(-1/a_{NS})$ -th power of the n-th moments are expected to lie along straight lines when plotted against $\ln Q^2$ with a common intercept $\ln Q^2 = \ln \Lambda^2$. These predictions have been found to be in very good agreement[8] with the data for $\Lambda \simeq 0.5$ GeV as shown in Fig. 3(a). Similar conclusions have been reached from analyzing[9] the BEBC data but it should be emphasized, however, that these latter results rely heavily on measurements between $Q^2 = 0.6$ and 2 GeV^2 – a region neither appropriate for the parton model nor for the legitimacy of perturbative calculations. Even in the CDHS experiment,[3] where $Q^2 \gtrsim 6.5$ GeV^2, ill understood kinematical target mass effects $(\sim x^2 m_N^2/Q^2)$ play a non-negligible role: Assuming that these effects can be in part accounted for by Nachtmann moments[10] (which results from the trace-terms in the NS Wilson operator of definite spin), the fitted slopes decrease by more than 10 % as shown in Fig. 3(b). The importance of this statement will become clear in a moment.

However, these measured NS moments can be equally well explained, in the presently measured region of Q^2, by conventional fixed point field theories. For an abelian vector gluon theory, Eqs. (9)-(11), (13) and (14) tell us that

$$a_{NS}^{vector} = \frac{25\alpha^*}{16\pi} a_{NS}$$

where a_{NS} is the RG exponent of QCD, and thus Eq. (8) predicts

$$\langle x F_3^{\nu N}(Q^2) \rangle_n^{-1/a_{NS}} = C_n(Q_0^2) \left(\frac{Q^2}{Q_0^2} \right)^{\frac{25\alpha^*}{16\pi}} \tag{22}$$

where the unknown normalization constants $C_n(Q_0^2) \equiv \langle x F_3^{\nu N}(Q_0^2) \rangle_n^{-1/a_{NS}}$ have to be fitted to the data at an arbitrary value of $Q^2 = Q_0^2$.[11]

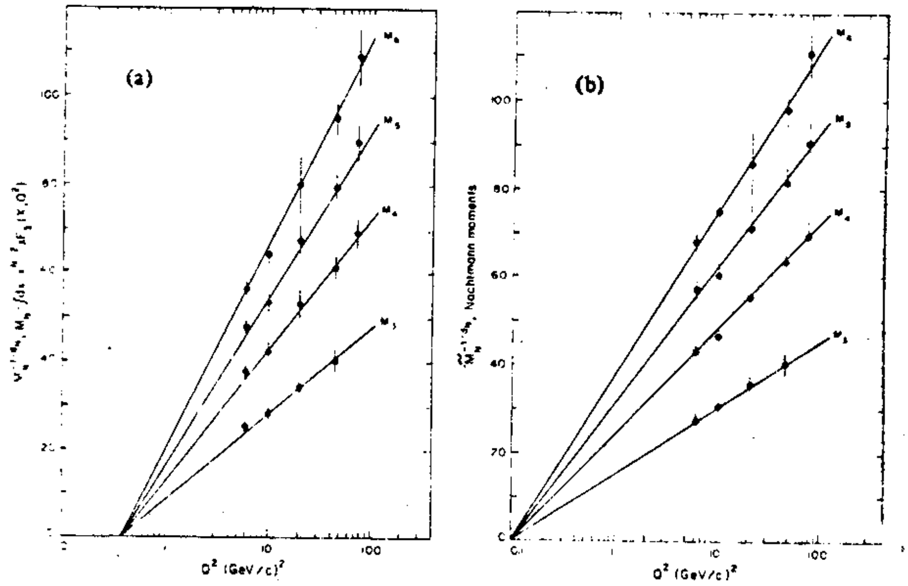


Fig. 3. Fit to the Q^2 dependence of (a) ordinary Cornwall-Norton moments $M_n \equiv \langle x F_3^{\nu N}(Q^2)\rangle_n$ and (b) Nachtmann moments according to QCD. The figures are taken from Ref. 8.

A similar power-like behavior in Q^2 is predicted by, for example, non-abelian scalar-gluon theories[5-7,11] where

$$a_{NS}^{scalar} = \frac{\alpha^*}{8\pi}\,\tilde{a}_n \tag{23}$$

with $\tilde{a}_n = \frac{4}{3}\left[1 - \frac{2}{n(n+1)}\right]$ which gives for Eq. (8)

$$\langle x F_3^{\nu N}(Q^2)\rangle_n^{-1/\tilde{a}_n} = \tilde{C}_n(Q_0^2)\left(\frac{Q^2}{Q_0^2}\right)^{\frac{\alpha^*}{8\pi}} \tag{24}$$

The predictions of abelian scalar-gluon theories are as in Eq. (24) with α^* multiplied by a factor of 3/4. From Fig. 4 it can be seen that the predictions according to Eqs. (22) and (24) are in equally good agreement[11] with experiment as are the straight line fits in Fig. 3. Thus non-singlet quantities can only provide us with a consistency check of a given theory but cannot discriminate between QCD and other finite fixed point theories of strong interactions (unless precision measurements can be extended to $Q^2 = 200$ or 300 GeV^2, as it is evident from Fig. 4). This, however, is not too surprising since the Q^2 dependence of NS moments is uniquely determined by just one anomalous dimension and, therefore, quantities such as $\langle F_{NS}(Q^2)\rangle_n^{-1/a_{NS}}$ are mainly sensitive to

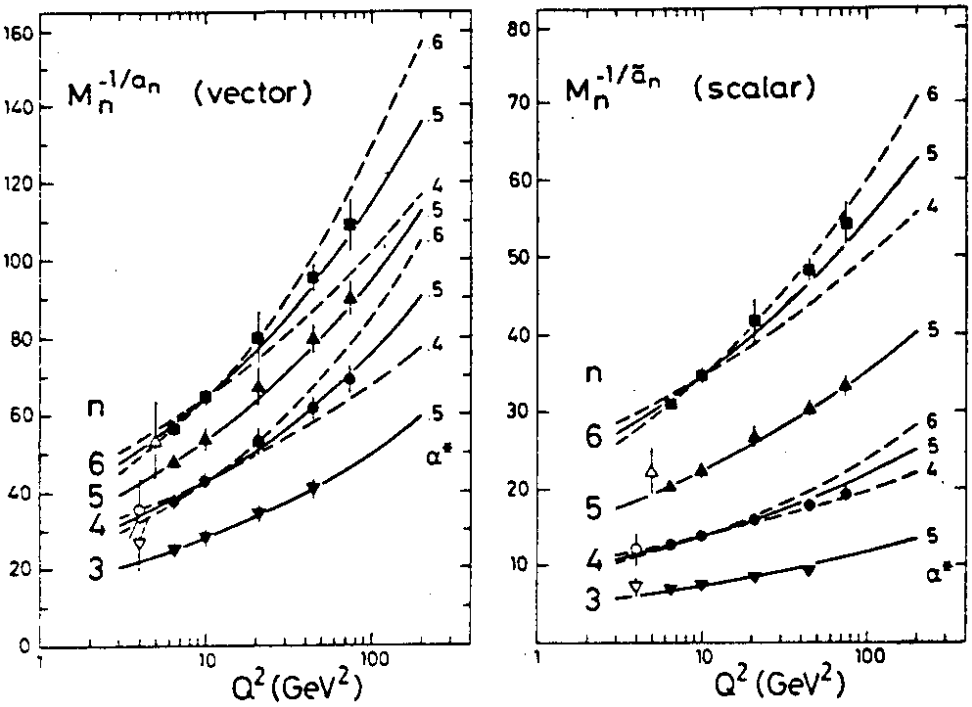


Fig. 4. Comparison of measured[8] moments $M_n \equiv \langle x F_3^{\nu N}(Q^2) \rangle_n$ with the predictions[11] of abelian vector-gluon theories, Eq. (22), and non-abelian scalar-gluon theories, Eq. (24), for various choices of the fixed point α^*. The low-statistics data (open circles and triangles) are from Ref. 9.

differences in a logarithmic and a power-like behavior in Q^2. This is in contrast to structure functions which receive also contributions from flavor-singlet Wilson operators, such as F_2, the Q^2 dependence of which is determined by three different anomalous dimensions in Eq. (17): These subtleties of singlet-mixing will play a crucial role in discriminating between different field theories which will be discussed in the next Section.

Another theoretically very attractive test of field theories, which measures ratios of anomalous dimensions directly, is obtained[9] by comparing the logarithms of two moments $\langle F_{NS} \rangle_n$ and $\langle F_{NS} \rangle_{n'}$ which, according to Eq. (8), should result in straight lines (in the 1-loop order) with slopes $a_{NS}(n)/a_{NS}(n')$:

$$\frac{d \ln \langle F_{NS} \rangle_n}{d \ln \langle F_{NS} \rangle_{n'}} = \frac{a_{NS}(n)}{a_{NS}(n')} . \tag{25}$$

These slopes are obviously independent of Λ and α^*, as well as of the number of flavors. Moreoever it should be emphasized that Eq. (25) can discriminate only between vector and scalar gluons, but not between subtleties such as their abelian or non-abelian group structure: This is because the only difference between an abelian and non-abelian structure of the qqg-coupling in vector-gluon theories is due to the "color charge of a quark." $C_2(R)$ in Eq. (11) which cancels in the ratio in Eq. (25); and similarly for scalar-gluon theories. For illustration we compare in Table I a typical prediction for the ratio of anomalous dimensions, according to Eq. (11), with the measured $F_3^{\nu N}$-moments.[8,9] As we can see, the measured slope[12] of ordinary x-moments $\langle x F_3^{\nu N}(Q^2)\rangle_n$ is in good agreement with the predictions of vector-gluon theories. However, as in the previous case, target mass effects (Nachtmann moments) play a non-negligible role, although $Q^2 \geqslant 6.5\ \mathrm{GeV}^2$ for the CDHS experiment,[8] which decrease the slopes by more than 10 % as compared to our ordinary moments. On the other hand the slope predictions of scalar theories are typically 20 % smaller than those of vector theories. Thus, at present, not even scalar-gluon theories can be ruled out within 1σ on the basis of this non-singlet moment-slope test in Eq. (25).

A similar conclusion has been reached by Harari[13] as far as the slopes of NS moments are concerned. He assumes that $xF_3(x,Q^2)$ is a slowly varying function in the whole x-region for increasing values of Q^2 which, by the usual bremsstrahlung effects of field quanta in any field theory, led him to the ansatz

$$xF_3(x,Q^2) \simeq x^{\frac{1}{2f(Q^2)}}\,(1-x)^{3g(Q^2)} \tag{26}$$

with f and g arbitrary but slowly varying functions of Q^2, obeying $f(Q_0^2) = g(Q_0^2) = 1$ and $f', g' \geqslant 0$. From $g'/f' = 0$ and $f'/g' = 0$ one easily finds lower and upper bounds, respectively, for the slopes of the logarithmic moment ratios in Eq. (25) which should be valid for a general class of field theories. Since, as we have already discussed, the measurements[2,9] for ordinary and Nachtmann-moment ratios scatter throughout the whole region allowed by these upper and lower bounds, Harari[13] concludes that this specific test provides no evidence for QCD. It should be pointed out, however, that in general Harari's "general" bounds are not general. This comes about because Harari assumes more than can be tested by $n > 2$ moments: whereas $n > 2$ moments are sensitive to the large-x region only ($x \gtrsim 0.3$), the ansatz in Eq. (26) assumes the structure function to be a slowly varying function of x and Q^2 in the whole x-region ($0 \leqslant x \leqslant 1$). It is well known[5,14] that scalar-gluon theories do not satisfy this latter assumption over the whole x-region, and thus it is not surprising that the ratios of anomalous dimensions in Eq. (25) predicted by scalar-gluon theories, using Eq. (23), lie outside Harari's "upper" and "lower bounds": specifically, the

	experiment		theory	
	$\langle xF_3^{\nu N}\rangle_n$	Nachtmann	vector	scalar
$a_{NS}(6)/a_{NS}(4)$	1.34 ± 0.07	1.18 ± 0.09	1.29	1.06

Table 1. Comparison of the theoretical predictions for the $n/n' = 6/4$ moment ratio with CDHS measurements.[8,12]

ratios $a_{NS}^{\mathrm{scalar}}(n)\ /\ a_{NS}^{\mathrm{scalar}}(n')$ lie always below Harari's "lower bounds".

Recently, a similar moment analysis has been performed[15,16] using the Fermilab data for $F_2^{\mu p}$ and $F_2^{\mu p}-F_2^{\mu n}$ in addition to the SLAC-MIT ep,n data for large values of x. Besides the leading α_s contributions also the subleading α_s^2 corrections[17] (2-loops in anomalous dimensions) have been taken into account. If one naively uses 2-loop corrections to $F_{NS} = F_2^{\mu p} - F_2^{\mu n}$ one expects $d\ln\langle F_{NS}\rangle_6 / d\ln\langle F_{NS}\rangle_4$ to change from 1.29 in the 1-loop order to 1.33 in the 2-loop order[17] (for $Q_o^2 = 4$ GeV^2, $Q^2 = 50$ GeV^2 and $\Lambda = 0.5$ GeV). The observed value turns out to be somewhat larger, 1.6 ± 0.2, but a very good agreement with the measurements of the various structure functions is obtained[15,16] by taking into account n-dependent Λ's, i.e. Λ_n increases for increasing n which is "naturally" explained theoretically[18] by going beyond the leading 1-loop order: typically[15] $\Lambda_4 \simeq 0.4$ GeV and $\Lambda_6 \simeq 0.5$ GeV.

To summarize, we have seen that present measurements of non-singlet moments cannot discriminate between different field theories of the strong interactions and can only provide us with a (necessary) consistency check of QCD. In order to discriminatively test QCD we therefore must turn to structure functions such as F_2 which contain dominant singlet components. This should prove more awarding since fixed point theories differ from QCD mainly in their singlet mixing properties, because of their very different gluonic anomalous dimensions[5,6] as for example implied by Eq. (12).

3.B. Singlet Moments

The most important and instructive moment to study is the lowest $n = 2$ moment of F_2, i.e. the area under $F_2(x,Q^2)$. Recall that the fermionic singlet in Eq. (19) is directly measured in neutrino scattering

$$F_2^{\nu N}(x,Q^2) = x\,\Sigma(x,Q^2) \quad , \qquad (27)$$

whereas deep inelastic e (or μ) scattering off nucleons measures in addition also the NS part, as for example

$$F_2^{\mu p}(x,Q^2) = \frac{5}{18}\, x\Sigma(x,Q^2) + \frac{1}{6}\, x[u+\bar{u}-d-\bar{d}-s-\bar{s}+c+\bar{c}] \qquad (28)$$

with $u = u(x,Q^2)$ etc., and where the Q^2 dependence of the NS expression in square-brackets is determined solely by a_{NS} as in Eq. (8). For brevity we will discuss the area under the pure singlet structure function $F_2^{\nu N}$ although the discussion of $\langle F_2^{\mu p}(Q^2)\rangle_2$ is very similar.[6] According to Eq. (20a) the Q^2-dependence of the lowest (n=2) moment of the singlet component is given by

$$\langle x\Sigma(Q^2)\rangle_2 = \alpha_2 + [\langle x\Sigma(Q_0^2)\rangle_2 - \alpha_2]\, e^{-s\,a_+(2)} \qquad (29)$$

where we have used $\alpha_2 = \beta_2$, and $\langle xG\rangle_2 = 1 - \langle x\Sigma\rangle_2$ and $a_-(2)=0$, by energy-momentum conservation. This quantity, being the total fractional momentum carried by the fermionic constituents in the nucleon, is then directly measured by

$$\int_0^1 F_2^{\nu N}(x,Q^2)\,dx = \langle x\Sigma(Q^2)\rangle_2 \quad . \qquad (30)$$

At moderate $Q^2 \simeq 2\text{-}4$ GeV^2, corresponding to our input Q_0^2, experiment tells us that[9,19] $\langle x\Sigma(Q_0^2)\rangle_2 \simeq 0.52$ and hence, according to Eq. (29) and since $a_+(2) = 56/75 > 0$, $\langle x\Sigma(Q^2)\rangle_2$ is an increasing or decreasing function of Q^2 depending on whether α_2 is larger or smaller than 1/2, respectively. Substituting the different possible values of the group invariants of Eqs. (13) and (14) into Eqs. (11) and (12) and also into the appropriate expressions for anomalous dimensions of scalar-gluon theories,[5] it turns out that $\alpha_2 < 1/2$ <u>only</u> for QCD where $\alpha_2 = 3/7$. It is a unique feature[5] of <u>all</u> other presently known field theories that $\alpha_2 > 1/2$ (specifically[6] $\alpha_2 = 6/7$, 9/10 and 72/73 for abelian vector-gluon, non-abelian scalar-gluon and abelian scalar-gluon theories, respectively) which forces $\int_0^1 F_2(x,Q^2)\,dx$ to <u>increase</u> with Q^2. Since $\int_0^1 F_2(x,Q^2)\,dx$ is experimentally observed[9,19,20] to <u>decrease</u> with Q^2 (or at most to be constant), all theories except QCD are already excluded on the basis of this single <u>qualitative</u> observation! In Fig. 5 we compare the data for $\int_0^1 F_2(x,Q^2)\,dx$ with the predictions of QCD (solid curves) and of the abelian vector field theory (dotted curves), for which we have taken the fixed point α^* to be 0.5, in agreement with our analysis[11] of NS moments in Fig. 4. The predictions of scalar-gluon theories are in even worse agreement with the data since their values for α_2 are always larger than 6/7. Similar conclusions have been already reached some time ago by a more detailed quantitative analysis[5,14] of $F_2^{e,\mu N}(x,Q^2)$.

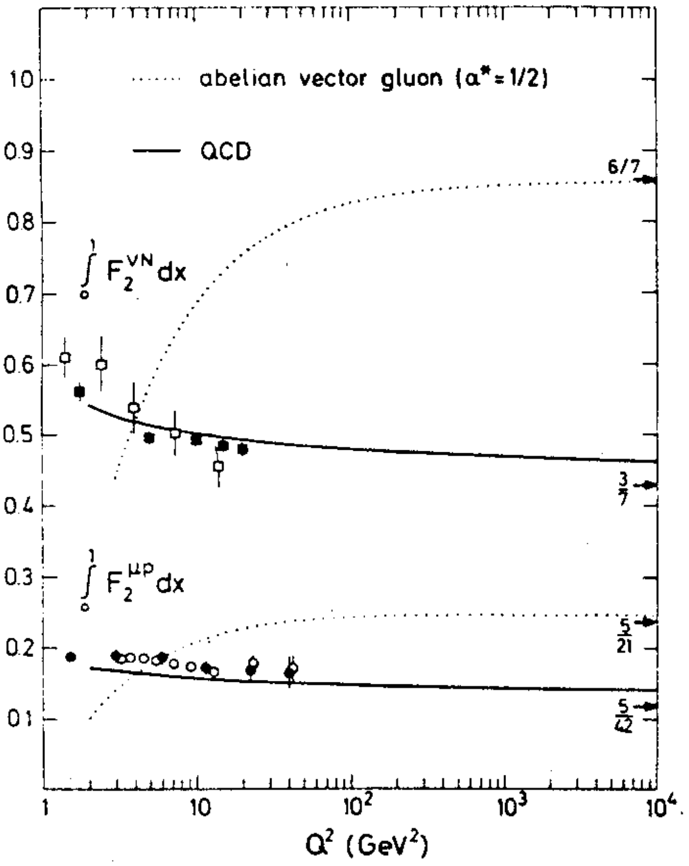
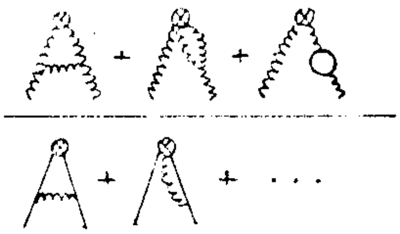


Fig. 5. Comparison[6] of the Q^2 evolution of the area under F_2, predicted by vector gluon theories, with the νN data of Refs. 9 and 19, and with the μp data of Ref. 20.

It should be noted that, although for n = 2 we have

$$\alpha_2 = \frac{\gamma^V_{VV}(2)}{\gamma^F_{FF}(2) + \gamma^V_{VV}(2)} = \qquad , \qquad (31)$$



this discriminative test of QCD is <u>not</u> sensitive to the triple gluon coupling since the coefficient of $C_2(G)$ in $\gamma^V_{VV}(2)$ vanishes, and therefore the whole contribution to $\gamma^V_{VV}(2)$ is due to the term proportional to T(R) in Eq. (12), i.e. to the external wave function renormalization, the vacuum polarization in Eq. (31). Thus, α_2 measures mainly the color charge of quarks, i.e. the quark-gluon coupling in Eq. (1) but not the color charge of gluons, i.e. the Yang-Mills structure of Eq. (2).

Since higher n ($\gtrsim 3$) moments weigh mainly the large x region ($x \gtrsim 0.3$), the study of $n > 2$ moments of any structure function cannot provide us with additional information on the gluon structure of the theory. This is so because $\gamma_+(n) > \gamma_-(n) = \gamma^T_{FF}(n) - O(1/n^2 \ln n)$ for $n > 2$ and thus always just one anomalous dimension $\gamma_{NS} \equiv \gamma^T_{FF}$ dominates. Therefore, the subtle and very important singlet-mixing properties of the theory, which allows us to study the detailed gluon structure, is only effective for small n, i.e. in the small x-region, not accessible to any moment analysis.

To summarize, recent measurements on $\int_0^1 F_2(x,Q^2)\,dx$ enable us already to eliminate all possible finite fixed point theories by purely qualitative arguments, leaving us with QCD as the only viable theory of the fundamental strong interactions.[21] Since this very discriminative test, as well as any higher $(n > 2)$ moment analysis of F_2, is sensitive only to the quark-gluon coupling of QCD in Eq. (1), we have to resort to the full x- and Q^2-dependence of structure functions (especially in the small x-region, $x \lesssim 0.2$) in order to test and learn about the specific gluon structure of QCD.

3.C. Scaling Violations in $F_2(x,Q^2)$

The most efficient and direct way to obtain the explicit x-dependence of structure functions is to do a numerical Mellin-inversion[22] of the moments predicted by QCD in Eq. (17):

$$F(x,Q^2) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} dn\; x^{-n+1} \langle F(Q^2) \rangle_n \qquad (32)$$

As it is apparent from Eq. (20a) we now need, in addition to the input quark distributions (which we have fitted in a standard way[6] to experiment at $Q_0^2 \simeq 4$ GeV^2), the gluon distribution $G(x,Q_0^2)$. To check the sensitivity of the predicted scaling violations to the choice of $G(x,Q_0^2)$, we have performed the calculations once with the "standard" gluon distribution $xG(x,Q_0^2 \simeq 4\ \mathrm{GeV}^2) = 2.6(1-x)^5$ and once with $G(x,Q_0^2) = 0$. This latter choice obviously violates the energy momentum sum rule and is intended only as a check on the above mentioned sensitivity to $G(x,Q_0^2)$. To further make sure that the results do not sensitively depend on our "standard" input gluon distribution chosen, we have repeated the calculations using a broad gluon $xG(x,Q_0^2) = 0.88(1+9x)(1-x)^4$ as suggested by the Caltech group[23] and which appears to be in better agreement with recent experiments[3,15,19]: within a few percent our predictions in Figs. 6 and 7 (solid curves) remain unchanged. As one can see from Figs. 6 and 7 the scaling violations with the "standard" gluon distribution (full lines) do not differ significantly (i.e., by less than a standard deviation) from the ones with a zero input gluon distribution (dashed lines). A distinction can be made only in the small

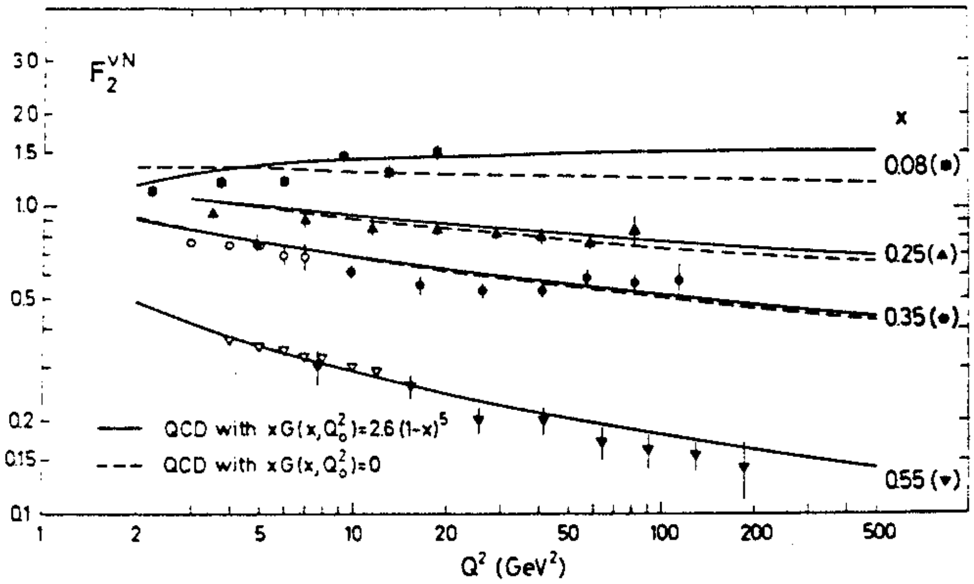


Fig. 6. Predictions[6] of scaling violations according to QCD as compared with neutrino data[19] (solid points) and ed data[24] (open points) multiplied by 9/5.

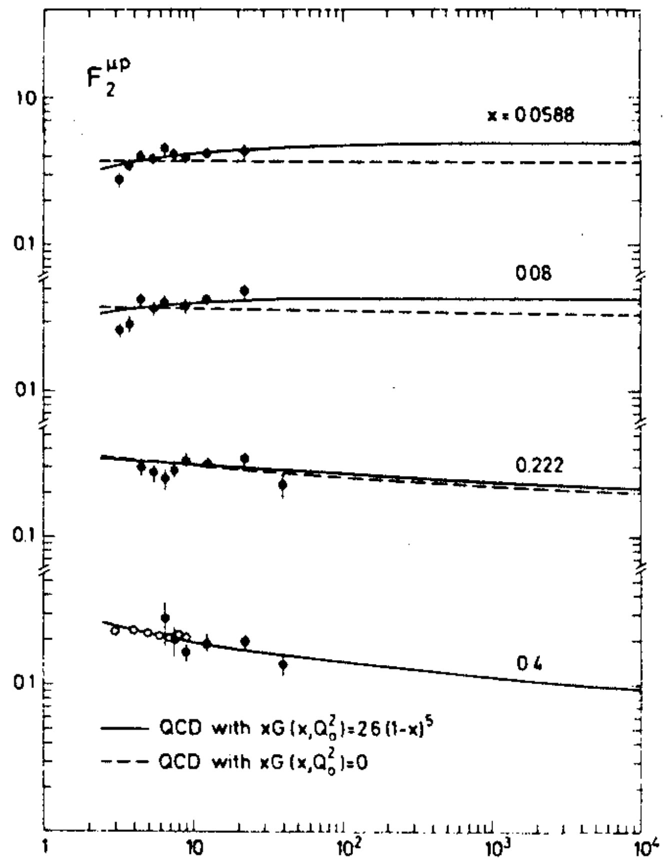


Fig. 7. Comparison of the predictions[6] for scaling violations with μp data[20] (solid points) and with ep data[24] (open points).

x (< 0.2) region and at higher values of Q^2: future precision measurements of F_2 for $0.05 \lesssim x \lesssim 0.2$ (heavy quark production should become important[25] only for smaller values of x) and for Q^2 up to 100-200 GeV^2, say, should prove very useful to pin down the gluon distribution in the nucleon! Thus any moment analysis of F_2 with $n \gtrsim 3$, which is sensitive to the large x-region only, for testing QCD and determining the gluon distribution[9,15,26] is rendered meaningless and statistically insignificant.

The fact that the predictions for scaling violations in F_2 are insensitive to the gluon content of the nucleon for $x > 0.1$ can be easily understood from the explicit values of the projection matrix elements α_n and β_n and how they enter Eq. (20a): From Table 2 we see that $\alpha_n \simeq 1$ and $\beta_n \ll 1$ for $n > 2$, and therefore only the term proportional to $\alpha_n \langle x\Sigma(Q_0^2)\rangle_n$ in Eq. (20a) will survive, except for small values of n (small x) where the gluon-terms gradually begin to contribute. This is, of course, in contrast to the Q^2-evolution of the gluon distribution itself where the terms proportional to $\langle xG(Q_0^2)\rangle_n$ in Eq. (20b) always dominate. We shall turn to this point in the next Section.

Alternatively, instead of using the "heavy artillery" of the renormalization group, the insensitivity of scaling violations to the detailed shape of the gluon distribution in the nucleon can be understood using the physical more transparent and intuitive language of Altarelli and Parisi.[27] Here the Q^2 evolution of $q(x,Q^2)$ and $G(x,Q^2)$ is described by a coupled set of integro-differential equations which are uniquely determined by the parton i → parton j decay probabilities $P_{ji}(x)$. These probabilities can be obtained from the fundamental interaction vertices of QCD in a probe (beam) independent (!) way,[27] which is in contrast to our renormalization group approach using the light cone expansion. Moreover, the (Mellin)

n	$a_{NS}(n)$	$a_-(n)$	$a_+(n)$	α_n	β_n
2	0.427	0	0.747	0.429	0.429
3	0.667	0.609	1.39	0.925	0.288
4	0.837	0.817	1.85	0.98	0.17
5	0.971	0.960	2.19	0.992	0.119
6	1.08	1.07	2.46	0.996	0.091

Table 2. Values for the renormalization group exponents $a_i(n)$ and for the projection matrix elements $\alpha_n \equiv p_{11}^-(n)$ and $\beta_n \equiv p_{21}^-(n)$ for a four flavor QCD ($N_f = 4$).

moments of these coupled set of equations are identical to the RG moment-equations (20a) and (20b) by keeping in mind that

$$\int_0^1 dx\, x^{n-1} \begin{pmatrix} P_{qq}(x) \\ 2N_f P_{qg}(x) \\ P_{gq}(x) \\ P_{gg}(x) \end{pmatrix} = -\frac{\pi}{\alpha_s} \begin{pmatrix} \gamma^F_{FF}(n) \\ \gamma^F_{VV}(n) \\ \gamma^V_{FF}(n) \\ \gamma^V_{VV}(n) \end{pmatrix} \tag{33}$$

Since P_{qg}, which couples $G(x,Q^2)$ to the evolution equation for $q(x,Q^2)$, is small only the term proportional to P_{qq} dominates the effects of scaling violations and therefore the latter are rather insensitive to the gluon content of the nucleon.

For alternative, non-field-theoretic (Regge-like) approaches to scaling violations we refer to Refs. 28 and 29, and to the lecture of G. Preparata. However, these (generalized) vector-meson dominance models can be "trusted" mainly in the small x-region where they might serve to extrapolate[29,30] structure functions down to $Q^2 \simeq 0$ - a region which cannot be reached by perturbative QCD. The power and beauty of explaining scaling violations with field theoretic methods (i.e. radiative corrections in QCD) remains, however, unchallenged in as much as they provide us with a framework for the whole x-region with essentially only one free parameter Λ.

4. MEASURING THE TRIPLE GLUON VERTEX

So far we have seen that all deep inelastic tests strongly favor QCD over any other field theory, and these tests were sensitive only to the quark-gluon coupling and left the non-abelian vertex in Eq. (1) unchallenged. Needless to say that, once the non-abelian character of the quark-gluon coupling is established, renormalizability of the theory requires the gluon self-couplings of Eq. (2), i.e. a Yang-Mills gauge-field Lagrangian, which are so essential for asymptotic freedom. This argument in favor of the full QCD Lagrangian is certainly convincing enough to most theorists, it probably is of little - if any - "proof" to most experimentalists. It is therefore also desirable to look for additional measurements which are directly sensitive to the gluon content of the nucleon and thus to the triple-gluon vertex of QCD in order to "see" experimentally the Yang-Mills structure which plays such a prominent role in QCD.

The most direct way to test the triple-gluon vertex directly would be the Q^2 evolution of $G(x,Q^2)$ itself[31] as predicted by the moments in Eq. (20b). As it has been already discussed, the terms in Eq. (20b) proportional to $\langle x\Sigma(Q_0^2)\rangle_n$ are suppressed for $n \gtrsim 3$ because of the values of α_n and β_n in Table 2: Thus $\langle xG(Q^2)\rangle_n / \langle xG(Q_0^2)\rangle_n \simeq \exp(-s\, a_+(n))$ with $a_+(n)$ being critically dependent on $\gamma^V_{VV}(n)$ in Eq. (12) and thus on the triple-gluon vertex. This sensitivity to the triple-gluon vertex (the term proportional to $C_2(G)$ in γ^V_{VV}) is demonstrated in Fig. 8 by the difference between the solid and dashed curves, the latter being the result with $C_2(G) = 0$ in γ^V_{VV}. For these quantitative predictions we have used[31] the input ratios $\langle x\Sigma(Q_0^2)\rangle_n / \langle xG(Q_0^2)\rangle_n$ determined experimentally[9,19] from the Q^2 variation of F_2-moments predicted by Eq. (20a). (Similar results hold of course also for the explicit x-dependence, but in order to avoid any ambiguities on the theoretically ill understood x-dependence of $G(x,Q^2)$ we discuss only moments of structure functions). It should be emphasized that the predictions of Eq. (20b) for $\langle xG(Q^2)\rangle_n$ cannot serve as an independent test of QCD if one determines[9,19] the gluon distributions from fitting to the scaling violations of F_2 predicted by Eq. (20a) since, once $\langle x\Sigma(Q_0^2)\rangle_n$ and $\langle xG(Q_0^2)\rangle_n$ are fixed by experiment via $\langle x\Sigma(Q^2)\rangle_n$ in Eq. (20a), Eq. (20b) is trivially satisfied. Thus we need an <u>additional</u>, <u>independent</u> determination of $G(x,Q^2)$! This can be achieved[31] by measuring the longitudinal structure function $F_L \equiv F_2 - 2xF_1$ which, to leading order α_s,

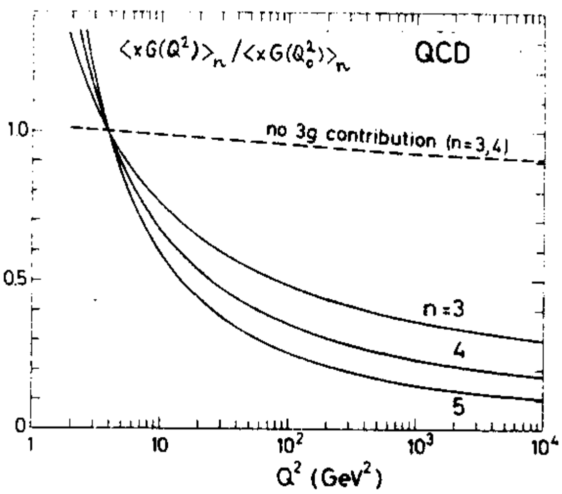
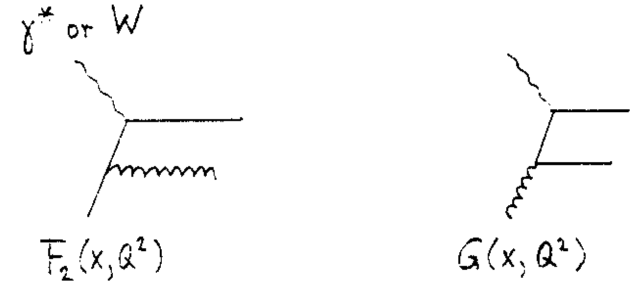


Fig. 8. Predictions[31] for the Q^2 evolution of gluon moments according to Eq. (20b). The dashed curve for n = 3,4 demonstrates the sensitivity of these predictions to the gluon self-couplings and corresponds to the contribution with the triple-gluon coupling turned off ($C_2(G) = 0$ in γ^V_{VV}), and is similar but slightly larger for n = 5.

receives its contribution from



i.e.

$$\langle F_L(Q^2)\rangle_n = C_n^q \langle F_2(Q^2)\rangle_n + a\, C_n^g \langle xG(Q^2)\rangle_n \tag{34}$$

with $a = \sum_q e_q^2 = 10/9$ for electroproduction and $a = 4$ for ν and $\bar{\nu}$ scattering on matter, and where the moments of the longitudinal projections of the fundamental parton processes are given by[32]

$$C_n^q = \frac{4\alpha_s(Q^2)}{3\pi(n+1)} \quad , \qquad C_n^g = \frac{2\alpha_s(Q^2)}{\pi(n+1)(n+2)} \quad . \tag{35}$$

From Eq. (34) we see that good data on $\langle F_L(Q^2)\rangle_n$, together with the experimental knowledge of[9,19,20] $\langle F_2(Q^2)\rangle_n$, n = 2,3..., can be translated into a reasonable knowledge of the gluon density

$$\langle xG(Q^2)\rangle_n = \frac{\pi(n+1)(n+2)}{2a\,\alpha_s(Q^2)}\langle F_L(Q^2)\rangle_n - \frac{2(n+2)}{3a}\langle F_2(Q^2)\rangle_n \quad . \tag{36}$$

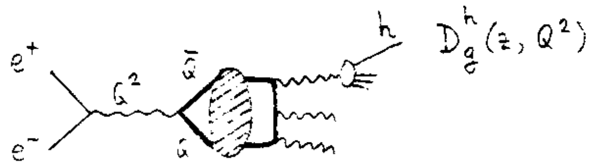
For $n > 2$ moments, measurements for $x \gtrsim 0.3$ should suffice. Furthermore, as can be seen from Fig. 8, measurements in the region $10\ \mathrm{GeV}^2 \lesssim Q^2 \lesssim 100\ \mathrm{GeV}^2$ will be required in order to clearly pin down the triple-gluon coupling; at these large values of Q^2, non-perturbative contributions to F_L can be safely neglected[33] since they are of the order k_T^2/Q^2 or m^2/Q^2, with k_T being the intrinsic transverse momentum of partons and m some typical hadronic mass scale. We are aware of the fact that measurements of F_L, or $R = F_L/F_2$, are exceedingly difficult,[34] but feasible[35] in the not too distant future. However, a precision measurement of F_L would be equally fundamental in providing us with a direct and sensitive test of the Yang-Mills structure (gluon self-couplings) of QCD.

For the present status of the measurements of $R \equiv \sigma_L/\sigma_T = F_L/F_2$ we refer to the talk of R. Taylor.[36] Here we only would like to mention that at least the qualitative trend expected by the parton model and the QCD prediction in Eq. (34) (using a "standard" gluon distribution $xG(x,Q_0^2) \simeq 2.6(1-x)^5$ as <u>input</u>) is in agreement with the scarce data available up to now. Let us compare these data with the predictions for[33,37] $R(x,Q^2) = R^{intrinsic} + R^{QCD}$ where the intrinsic part is due to taking into account kinematical target mass effects (intrinsic transverse momenta) and R^{QCD} results from Eq. (34): The SLAC ep measurements ($0.3 \lesssim x \lesssim 0.8$, 3 $GeV^2 \lesssim Q^2 \lesssim 18$ GeV^2) give on the average[36] $R = 0.21 \pm 0.1$ to be compared with $R(0.5, 8\ GeV^2) = 0.025 + 0.035 = 0.06$, whereas the Fermilab μp experiment ($0.003 < x < 0.1$, 1 $GeV^2 \lesssim Q^2 \lesssim 30$ GeV^2) gives an average value of[15,16] $R = 0.52 \pm 0.35$, which can be compared with the prediction $R(0.02, 15\ GeV^2) = 0.0 + 0.2 = 0.2$. The expected increasing trend of R for decreasing x seems to be reproduced by the data. To illustrate the expected[33,37] x- and Q^2-dependence of $R(x,Q^2)$ we give a few predictions in Table 3. These values can be increased if one chooses a harder (flatter) gluon distribution than our "standard" choice.

Similar direct tests of the triple-gluon vertex can be obtained by looking[38] for the Q^2 evolution of gluon jets in heavy quarkonium decay, i.e. measuring the gluon decay function $D_g^h(z,Q^2)$ in $e^+e^- \rightarrow Q\bar{Q} \rightarrow 3g \rightarrow h$ + anything of successive 1^3S_1 quarkonium states (e.g. at $Q^2_\Upsilon \simeq 100$ GeV^2 and, provided toponium exists, at $Q^2_{t\bar{t}} \simeq 1000$ GeV^2):

$Q^2 (GeV^2)$	x	R	$R^{intrinsic}$	R^{QCD}
2	0.8	0.1	0.08	0.02
	0.5	0.16	0.1	0.06
	0.2	0.19	0.04	0.15
	0.02	0.53	0.0	0.53
10	0.8	0.025	0.015	0.01
	0.5	0.05	0.02	0.03
	0.2	0.08	0.01	0.07
	0.02	0.23	0.0	0.23
20	0.8	0.014	0.006	0.008
	0.5	0.035	0.01	0.025
	0.2	0.063	0.003	0.06
	0.02	0.18	0.0	0.18
100	0.8	0.007	0.002	0.005
	0.5	0.02	0.002	0.018
	0.2	0.04	0.0	0.04
	0.02	0.12	0.0	0.12

Table 3. Predicted values for[33] $R(x,Q^2)=R^{intrinsic}+R^{QCD}$ for electroproduction. The predictions at $Q^2 = 2$ GeV^2 correspond to naive (Q^2-independent) parton distributions.

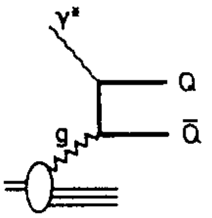


Since D_g^h satisfies[39] a similar renormalization group equation as in Eq. (20b) (or, equivalently, similar Altarelli-Parisi equations as do the distribution functions), the predicted[38] Q^2 evolution is again critically sensitive to the gluon self-couplings and is similar to the predictions in Fig. 8.

Finally it has been suggested[40] to look for T-odd asymmetries in the hadronic decays of heavy quarkonia produced in e^+e^- collisions, where electrons and positrons are longitudinally polarized with opposite helicities. For Υ production this asymmetry is expected to be as small as[40] 0.3 %, and therefore non-perturbative effects due to final state interactions may totally mask these predictions.

5. DEEP INELASTIC HEAVY QUARK PRODUCTION

Due to the large mass, all heavy quark flavors (c,b,...) in the nucleon are expected[41] to exist only as quantum fluctuations at short distances and the corresponding distribution can be consistently calculated within QCD. Therefore at $Q^2 \not\gg 4m_Q^2$, heavy quark Q=c,b,... production in deep inelastic electron (muon) nucleon scattering should, in leading order, be adequately described[42,25] by the renormalization group improved virtual Bethe-Heitler process



which is proportional to the gluon content $G(x,Q^2)$ of the nucleon. Thus the nucleon should be considered to consist only of the three light quarks u,d,s and of gluons G, with all other heavy quark flavors being produced via these light quark and gluon fields. The total charm contribution to F_2 can then be calculated to be[25] ($b\bar{b}$ production is about two orders of magnitudes smaller than $c\bar{c}$)

$$F_2^{c\bar{c}}(x,Q^2) = \frac{1}{N}\int_{ax}^{y_{max}} \frac{dy}{y}\, y\, G(y,Q^2)\, f_2^{\gamma^* g\to c\bar{c}}\left(\frac{x}{y},Q^2\right) \tag{37}$$

with $a = 1 + 4\, m_c^2/\,Q^2$, $Q^2 \equiv -q^2$, and for total charm production $N \equiv 1$, $y_{max} = 1$, whereas for J/ψ production N = (number of charmonium states) $\simeq 8$, $y_{max} = x(1 + 4m_D^2/Q^2)$; furthermore the fundamental subprocess $\gamma^* g \to c\bar{c}$ gives

$$f_2^{\gamma^* g \to c\bar{c}}(z, Q^2) = \frac{4}{9}\,\frac{\alpha_s(Q^2)}{\pi}\Big\{v\Big[4z^2(1-z) - \frac{z}{2} - \frac{2m_c^2}{Q^2}\, z^2(1-z)\Big] + \Big[\frac{z}{2} - z^2(1-z) + \frac{2m_c^2}{Q^2}\, z^2(1-3z) - \frac{4m_c^4}{Q^4}\, z^3\Big]\ln\frac{1+v}{1-v}\Big\} \tag{38}$$

with $v^2 = 1 - 4m_c^2 z/Q^2(1-z)$. From the predictions for "open charm" and J/ψ production according to Eq. (37) as shown in Fig. 9, we observe a very steep and non-negligible contribution for $x \lesssim 0.01$ which increases rapidly not only for decreasing values of x but also for increasing Q^2. For example, fitting the x dependence of the charm predictions at $Q^2 = 4$ GeV^2, the charm sea is expected to behave like (for $m_c = 1.25$ GeV)

$$x\,c(x, Q^2 \simeq 4\ GeV^2) = 0.05\,(1-x)^{30} \quad . \tag{39}$$

Furthermore, Fig. 9 tells us that, at $Q^2 = 10$ GeV^2 and $x \simeq 0.01$, the total charm and J/ψ production in F_2 is about 0.08 and 0.003, respectively, which is about a 20 % contribution to the measured total value of F_2. To illustrate the size of F_2^{charm}, we compare it with actual data in Fig. 10 on top of $F_2^{\mu p}(x \simeq 0.01) = 0.365$. It seems

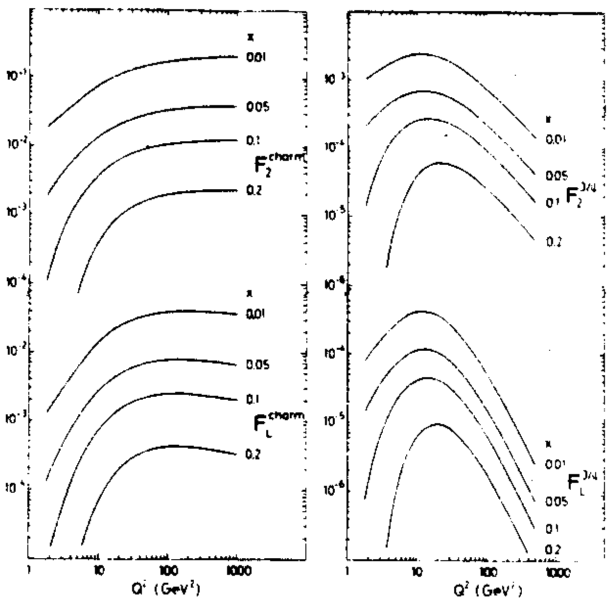


Fig. 9. Predictions[25] for total charm and J/ψ production according to Eq. (37).

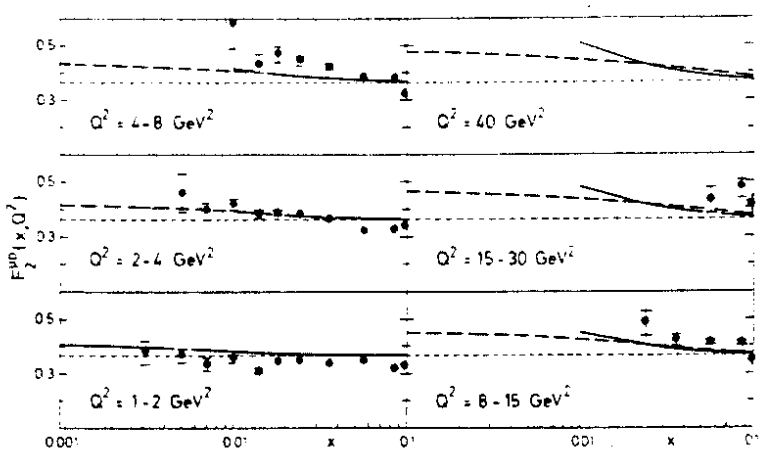


Fig. 10. Comparison of the predictions[25] of total charm production, Eq. (37), with μp data.[20] These predictions are added to the short-dashed curves which correspond to the light quark contributions to $F_2^{\mu p}(x=0.01)=0.365$. The full QCD predictions of Eq. (37) yield the solid curves, while the long-dashed curves correspond to using a naive Q^2-independent gluon distribution in Eq. (37).

appropriate to conclude that the amount of charm produced by the process $\gamma^* g \to cc$ accounts for all the charm component of $F_2(x,Q^2)$ presently observed. The CERN-EMC[43] and the Berkeley-Princeton[44] μp experiment at Fermilab should provide us with stringent tests of this production mechanism of heavy quark flavors.

Furthermore, photoproduction of charm is also expected to proceed via the Bethe-Heitler process $\gamma g \to c\bar{c}$ for which QCD makes firm predictions.[46] Again, μp precision measurements[43,44] should be able to discriminate between alternative production mechanisms.[47]

In neutrino scattering charm is already produced by the weak current in zeroth order, in contrast to electroproduction, via the naive quark-parton process $W^+ s \to c$. Additional (non-leading) corrections originate[41,48] from the virtual weak Bethe-Heitler process $W^+ g \to c\bar{s}$. Such calculations, however, depend now critically on the light s-quark mass chosen because of the appearance of terms like $\ln(\hat{s} - m_c^2)/m_s^2$; these large logarithms should be absorbed as usual into the RG improved strange quark distribution s of the 0-th order $W^+ s \to c$ process.

6. NON-LEADING CORRECTIONS

Finally, I would like to comment briefly on various recent analyses concerning subleading α_s corrections to Wilson coefficients ("finite terms") and 2-loop contributions to anomalous dimensions.

Taking into account these non-leading terms, i.e.

$$\begin{aligned} \beta(\alpha_s) &= -\beta_0 \alpha_s^2 - \beta_1 \alpha_s^3 \\ \gamma^n(\alpha_s) &= \gamma_0^n \alpha_s + \gamma_1^n \alpha_s^2 \\ C_i^n(\alpha_s) &= 1 + c_i^n \alpha_s \quad , \end{aligned} \tag{40}$$

and inserting these expansions into the solution (7) of the RG equation

$$C_i^n(Q^2/\mu^2, \alpha_s(\mu^2)) = C_i^n(1, \alpha_s(Q^2)) \exp\left[-\frac{1}{2}\int_{\alpha_s(\mu^2)}^{\alpha_s(Q^2)} d\alpha \frac{\gamma^n(\alpha)}{\beta(\alpha)}\right] \tag{41}$$

where $C_i^n(1, \alpha_s(Q^2)) \equiv C_i^n(\alpha_s(Q^2))$, the Q^2-dependence of moments of structure functions in Eq. (8) is predicted to be

$$\langle F_i(Q^2)\rangle_n = \langle q_0\rangle_n \left\{1 + \alpha_s(Q^2)\left[c_i^n + \left(\frac{\gamma_1^n}{2\beta_0} - \frac{\gamma_0^n \beta_1}{2\beta_0^2}\right)\right]\right\} e^{-\frac{\gamma_0}{2\beta_0} s} . \tag{42}$$

Here, $\langle q_0\rangle_n$ denotes the moments of the matrix elements of the local Wilson operator in Eq. (5) between target states which are nothing else but appropriate combinations of input quark distributions fitted to experiment, and, because of Eq. (40), the corrected form of $\alpha_s(Q^2)$ is

$$\frac{1}{\alpha_s(Q^2)} = \beta_0 \ln \frac{Q^2}{\Lambda^2} + \frac{\beta_1}{\beta_0} \ln \frac{\ln Q^2/\Lambda^2}{\ln \mu^2/\Lambda^2} \quad , \tag{43}$$

instead of Eq. (3), with $\Lambda^2 \equiv \mu^2 \exp[-1/\beta_0 \alpha_s(\mu^2)]$ and $\beta_0 = 4\pi b$. It is clear from Eq. (42) that $O(\alpha_s)$ corrections to the Wilson coefficients C_i^n have to be taken into account, once the 2-loop contributions β_1^n and γ_1^n to β and γ^n are considered, in order to include consistently all contributions in a given order of perturbation theory. We recall that only the <u>whole</u> Eq. (41) corresponds to a physical measurable quantity, whereas the individual quantities $C_i^n(\alpha_s(Q^2))$ and exp[...] depend upon the precise definition of the Wilson operator (the renormalization prescription): Although the parameters γ_0^n, β_0 and β_1 are gauge and renormalization prescription independent, the quantities c_i^n and γ_1^n depend on the renormalization prescription and on the gauge chosen. Thus c_i^n and γ_1^n must be calculated in the <u>same</u> renormalization scheme in order to obtain a physical, convention independent answer for Eq. (41).

The γ_1^n for non-singlet operators (i.e. for NS structure functions) has been calculated in Ref. 49; comparing these results quantitatively[50] with the data for $F_2^{\mu p}$ for $x \gtrsim 0.4$ showed that the only effect of subleading contributions being a change in Λ by about 20-30 % as compared to the value of Λ obtained by fitting the leading order (1-loop) expressions to experiment.

Furthermore, the rather lengthy calculations of γ_1^n for singlet structure functions have recently been completed[17,51] which, together with the $O(\alpha_s)$ contributions[18,52] to the coefficient functions C_i^n, allow now a detailed quantitative comparison of non-leading terms with experiment. As already discussed in Section 3.A, a moment analysis[15,16] of F_2 yields (by using n-dependent[18] Λ's) an equally good agreement with experiment as do the leading 1-loop terms γ_0^n with Λ held fixed. Moreover, studying[53] the explicit x- and Q^2-dependence of $F_2^{e(\mu)p}(x,Q^2)$ in the 2-loop approximation gives also an equally good agreement with experiment as does the 1-loop approximation, provided the scale Λ of the effective coupling constant is reduced from 0.5 GeV to about 0.3 GeV. Therefore, although further detailed analyses are certainly required before we can draw definite conclusions, it appears that non-leading terms do not significantly alter the successful quantitative results based on the 1-loop approximation; this gives us some additional confidence in the usefulness and validity of perturbative lowest order calculations in QCD.

For practical applications a very neat "trick" has been suggested[52] to study "finite" α_s terms in C_i^n without having to take into account the 2-loop contribution γ_1^n explicitly. This is achieved by defining effective Q^2-dependent parton distributions relative to F_2, i.e. by demanding that $F_2(x,Q^2)$ expressed in terms of them should have the same form as in the naive quark model (F_2 is given a special status because it satisfies the Adler sum rule):

$$\langle q(Q^2)\rangle_n \equiv C_2^n(\alpha_s(Q^2))\,\langle q_0\rangle_n \exp[\cdots] \tag{44}$$

instead of the usual definition $\langle \tilde{q}(Q^2)\rangle_n \equiv \langle q_0\rangle_n \exp[\cdots]$ where $\exp[\ldots]$ is our general RG exponent in Eq. (41). Similar definitions[52] apply to singlet (gluon) densities. Because of the inclusion of the coefficient function in the definition (44) of parton densities, the Q^2-evolution equations of these quantities are the same[52] as in Section 2 keeping only the 1-loop anomalous dimensions γ_0^n, with γ_1^n being suppressed by $O(\alpha_s)$. Thus, once the input parton distributions $q(x,Q_0^2)$ are fitted at $Q^2 = Q_0^2$ to $F_2(x,Q^2) = x\sum_q [q(x,Q^2) + \bar{q}(x,Q^2)]$, which evolute according to Eqs. (20), say, one can study the effects of finite terms in structure functions (or processes) <u>other</u> than F_2 since

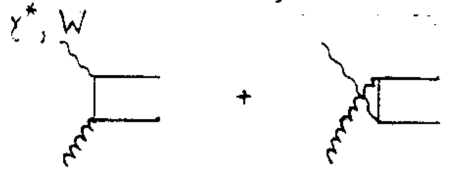
$$\langle F_i(Q^2)\rangle_n = \frac{C_i^n(\alpha_s(Q^2))}{C_2^n(\alpha_s(Q^2))}\; C_2^n(\alpha_s(Q^2))\,\langle q_0\rangle_n\, \exp[\cdots]$$
$$\equiv \frac{C_i^n(\alpha_s(Q^2))}{C_2^n(\alpha_s(Q^2))}\;\langle q(Q^2)\rangle_n \tag{45}$$

which is of course a fully gauge invariant and renormalization prescription independent procedure. Thus, expanding in α_s, we get always <u>differences</u> $c_i^n - c_2^n$ of "finite" terms contributing to F_i with $i \neq 2$:

$$\langle F_3(Q^2)\rangle_n = (-\langle q(Q^2)\rangle_n + \langle \bar{q}(Q^2)\rangle_n)\,[1+\alpha_s(Q^2)(c_{3,q}^n - c_{2,q}^n)]\;,\;\text{etc.} \tag{46}$$

where the fermionic Wilson coefficients $c_{i,q}^n$ result from[52]

and gluonic Wilson coefficients $c_{i,G}^n$ are calculated from



The importance of these "finite" $\alpha_s(Q^2)$ corrections for phenomenological applications are obvious. For example, the effect of gluon corrections on sea distributions to be determined from neutrino reactions are enormous:[52]

$$\sigma_{RH}^{\nu} \sim \frac{1}{2}\int_0^1 dx\,(2xF_1^{\nu} + xF_3^{\nu}) \tag{47}$$

$$= \underbrace{(\bar{U}+\bar{D})}_{\simeq\, 0.06}(1-0.16\alpha_s) + \underbrace{0.018\,\alpha_s(U+D+2S)}_{\simeq\, 0.003\ (\text{small})} - \underbrace{0.106\,\alpha_s G}_{\simeq\, 0.02}$$

$$\sigma^{\nu}(y=0) - \sigma^{\nu}(y=1) \sim \int_0^1 dx\,(F_2^{\nu} - xF_1^{\nu} + \tfrac{1}{2}xF_3^{\nu}) \tag{48}$$
$$= \underbrace{(\bar{U}+\bar{D})}_{\simeq 0.06}(1-0.018\alpha_s) + \underbrace{0.16\alpha_s(U+D+2S)}_{\simeq 0.025} + \underbrace{0.106\alpha_s G}_{\simeq 0.02}$$

where $U \equiv \langle xu(Q^2)\rangle_2$, etc. and we have neglected the small contributions from the charm sea. It is already clear from these equations that the gluon corrections cannot be neglected for a precise quantitative determination of sea densities; even quarks play a non-negligible role in determining the deviations from flatness in $\epsilon^{\nu}(y)$ as it is evident from Eq. (48).

Similarly, one can and has to check simultaneously, using the same parton distributions determined in deep inelastic reactions, the importance of "finite" terms in other reactions such as Drell-Yan dimuon production $(pp \to \mu^+\mu^- + X)$. Here "finite" α_s terms are obtained[52,54] from the same, but crossed diagrams which yielded the above quantities $c^n_{i,q}$ and $c^n_{i,G}$. The corrected Drell-Yan formula thus obtained is

$$\frac{d\sigma_{DY}^{p_1p_2\to\mu^+\mu^- X}}{dQ^2} = \frac{4\pi\alpha^2}{9sQ^2}\int_0^1\frac{dx_1}{x_1}\int_0^1\frac{dx_2}{x_2}\Big\{\Big[\sum_q e_q^2\, q^{(1)}(x_1,Q^2)\,\bar{q}^{(2)}(x_2,Q^2) + (1\leftrightarrow 2)\Big]$$
$$\times\Big[\delta(1-z) + \alpha_s\,\theta(1-z)\big(f_q^{DY}(z) - 2f_{2,q}(z)\big)\Big] \tag{49}$$
$$+\alpha_s\,\theta(1-z)\Big[\frac{1}{x_1}F_2^{(1)}(x_1,Q^2)\,G^{(2)}(x_2,Q^2) + (1\leftrightarrow 2)\Big]\Big[f_G^{DY}(z) - f_{2,G}(z)\Big]\Big\}$$

with $z = Q^2/x_1x_2s$ and where the two terms $f_q^{DY} - 2f_{2,q}$ and $f_G^{DY} - f_{2,G}$, with $c_i^n \equiv \int_0^1 dz\, z^{n-1} f_i(z)$, are given for example in Ref. 52. It is now generally agreed[55,56] upon that the quark-gluon correction in Eq. (49) is small (less than ~10 %), provided the full Q^2-dependence of the (nonscaling) parton distributions is taken into account! However, the existing analyses[56,57] concerning the size of the α_s correction to $q\bar{q}$ scattering in Eq. (49) yield entirely contradictory results.

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