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Cross Sections and Angular Distributions of Three-Jet Final States in e^+e^- Annihilation for Heavy Quarks

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Abstract

The cross sections $d\sigma_U$, $d\sigma_L$, $d\sigma_T$ and $d\sigma_I$ for $e^+e^- \rightarrow q\bar{q}g$ are calculated for heavy quarks, and the resultant angular distribution is discussed.

Recently, evidence has been reported for the existence of three-jet events in e^+e^- annihilation into hadrons [1)-3)]. Such events have been predicted by QCD on the basis of asymptotically free perturbation theory proceeding from hard gluon bremsstrahlung [4)].

The topological features of three-jet events such as orientation and correlation of jet axes have been extensively studied in the literature [5)-8)]. In this note we shall extend our previous work [6)] to the case of massive quarks. Clearly, the ever heavier quark masses cannot be neglected in the analysis of the data. The cross section for $e^+e^- \to \gamma^* \to q(p_1)\bar{q}(p_2)g(p_3)$ is given by:

$$(2\pi)\frac{d^4\sigma}{dx_1\, dx_2\, d\cos\theta\, d\chi} = \sigma^{(1)} \frac{3}{16\, q^2} \frac{1}{4} L_{\mu\nu} H^{\mu\nu} \tag{1}$$

where $x_i = 2E_i/\sqrt{q^2}$, $x_1 + x_2 + x_3 = 2$. The angle θ is the angle between the beam axis and the direction of the most energetic jet, i.e. the thrust axis, while χ is the azimuthal angle as defined in Fig. 1 which fixes the orientation of the $q\bar{q}g$ production plane with respect to the scattering plane. $\sigma^{(1)} = \frac{2}{3}\frac{\alpha_s}{\pi}\sigma_0$, σ_0 being the cross section for $e^+e^- \to q\bar{q}$:

$$\sigma_0 = \frac{4\pi\alpha^2}{q^2} \sum_f Q_f^2 \left(1 - \frac{4m^2}{q^2}\right)^{1/2} \left(1 + \frac{2m^2}{q^2}\right) \tag{2}$$

The lepton-tensor $L_{\mu\nu}$ is given as usual:

$$L_{\mu\nu} = 4\left\{q_1, q_2\right\}_{\mu\nu} = 4\left(q_{1\mu} q_{2\nu} + q_{1\nu} q_{2\mu} - q_1 \cdot q_2\, g_{\mu\nu}\right) \tag{3}$$

with $q_1(q_2)$ being the electron (positron) momentum. The hadronic tensor is given by

$$
\begin{aligned}
\frac{1}{4} H_{\mu\nu} = & \frac{1}{p_1\cdot p_3}\left[\{p_2,p_3\}_{\mu\nu} - \{p_1,p_1\}_{\mu\nu} + \{p_1,p_2\}_{\mu\nu}\right] \\
& + \frac{1}{p_2\cdot p_3}\left[\{p_1,p_3\}_{\mu\nu} - \{p_2,p_2\}_{\mu\nu} + \{p_1,p_2\}_{\mu\nu}\right] \\
& + \frac{m^2}{(p_1\cdot p_3)^2}\left[m^2 g_{\mu\nu} - \{p_1,p_2\}_{\mu\nu} - \{p_2,p_3\}_{\mu\nu}\right] \\
& + \frac{m^2}{(p_2\cdot p_3)^2}\left[m^2 g_{\mu\nu} - \{p_1,p_2\}_{\mu\nu} - \{p_1,p_3\}_{\mu\nu}\right] \qquad (4) \\
& - \frac{m^2}{(p_1\cdot p_3)(p_2\cdot p_3)}\left[2\, p_1\cdot p_2\, g_{\mu\nu} + \{p_3,p_3\}_{\mu\nu}\right] \\
& + \frac{p_1\cdot p_2}{(p_1\cdot p_3)(p_2\cdot p_3)}\left[2\{p_1,p_2\}_{\mu\nu} + \{p_1,p_3\}_{\mu\nu} + \{p_2,p_3\}_{\mu\nu}\right]
\end{aligned}
$$

m denotes the mass of the quark and antiquark.

The cross section can be written in terms of four independent cross sections:

$$(2\pi)\frac{d^4\sigma}{d\cos\theta\, d\chi\, dx_1\, dx_2} = \frac{3}{8}(1+\cos^2\theta)\frac{d^2\sigma_u}{dx_1\, dx_2} + \frac{3}{4}\sin^2\theta\,\frac{d^2\sigma_L}{dx_1\, dx_2} + \frac{3}{4}\sin^2\theta\cos 2\chi\,\frac{d^2\sigma_T}{dx_1\, dx_2} - \frac{3}{2\sqrt{2}}\sin 2\theta\cos\chi\,\frac{d^2\sigma_I}{dx_1\, dx_2} \tag{5}$$

In the case of quarks with mass m thrust is given by

$$T = \max\left\{\sqrt{x_1^2 - \frac{4m^2}{q^2}}\,,\ \sqrt{x_2^2 - \frac{4m^2}{q^2}}\,,\ x_3\right\} \tag{6}$$

This leads us to distinguish between three kinematical regions as usual:

$$\mathrm{I}:\quad \sqrt{x_1^2 - \frac{4m^2}{q^2}} > \sqrt{x_2^2 - \frac{4m^2}{q^2}}\,,\ x_3$$

$$\mathrm{II}:\quad \sqrt{x_2^2 - \frac{4m^2}{q^2}} > \sqrt{x_1^2 - \frac{4m^2}{q^2}}\,,\ x_3$$

$$\mathrm{III}:\quad x_3 > \sqrt{x_1^2 - \frac{4m^2}{q^2}}\,,\ \sqrt{x_2^2 - \frac{4m^2}{q^2}}$$

In region I, II and III we obtain the cross sections:

I:

$$\frac{d^2\sigma_U}{dx_1\,dx_2} = \sigma^{(1)}\left\{ \frac{1}{(1-x_1)(1-x_2)} \left[x_1^2 + x_2^2\left(1 - \frac{1}{2}\sin^2\theta_{12}\right) - \frac{2m^2}{q^2}\left(3 + \cos^2\theta_{12} - x_1 - x_2\right)\right] \right.$$

$$\left. - \frac{1}{(1-x_1)^2}\,\frac{2m^2}{q^2}\,x_1 + \frac{1}{(1-x_2)^2}\,\frac{m^2}{q^2}\left[\left(x_2^2 - \frac{4m^2}{q^2}\right)\sin^2\theta_{12} - 2x_2\right]\right\}.$$

$$\frac{d^2\sigma_L}{dx_1\,dx_2} = \sigma^{(1)}\left\{ \frac{1}{(1-x_1)(1-x_2)} \left[\frac{x_2^2}{2}\sin^2\theta_{12} - \frac{2m^2}{q^2}\left(3 - \cos^2\theta_{12} - \frac{5}{2}(x_1 + x_2) \right.\right.\right.$$

$$\left.\left. + x_1 x_2 + \frac{4m^2}{q^2}\right)\right] + \frac{1}{(1-x_1)^2}\,\frac{m^2}{q^2}\left(x_1^2 - x_1 - \frac{4m^2}{q^2}\right)$$

$$\left. + \frac{1}{(1-x_2)^2}\,\frac{m^2}{q^2}\left[\left(x_2^2 - \frac{4m^2}{q^2}\right)\cos^2\theta_{12} - x_2\right]\right\}$$

$$\frac{d^2\sigma_T}{dx_1\,dx_2} = \sigma^{(1)}\left\{ \frac{1}{(1-x_1)(1-x_2)} \cdot \frac{1}{4}\left(x_2^2 - \frac{4m^2}{q^2}\right)\sin^2\theta_{12} \right.$$

$$\left. - \frac{1}{(1-x_1)^2}\,\frac{m^2}{q^2}\,\frac{1}{2}\left(x_2 - \frac{4m^2}{q^2}\right)\sin^2\theta_{12}\right\}$$

$$\frac{d^2\sigma_I}{dx_1\,dx_2} = \frac{\sqrt{2}}{4}\sigma^{(1)}\left\{ \frac{1}{(1-x_1)(1-x_2)} \left[\frac{1}{2}\left(x_2^2 - \frac{4m^2}{q^2}\right)\sin 2\theta_{12} \right.\right.$$

$$\left. + \frac{4m^2}{q^2}\left(1 - x_1 - x_2 + \frac{x_1 x_2}{2} + \frac{2m^2}{q^2}\right)\tan\theta_{12}\right]$$

$$\left. - \frac{1}{(1-x_2)^2}\,\frac{m^2}{q^2}\left(x_2^2 - \frac{4m^2}{q^2}\right)\sin 2\theta_{12}\right\} \tag{7}$$

II:

Same as under I but with $x_1 \leftrightarrow x_2$ (note that $\cos\theta_{12}$ does not change).

III:

$$\frac{d^2\sigma_u}{dx_1\,dx_2} = \sigma^{(1)}\left\{\frac{1}{(1-x_1)(1-x_2)}\left[x_1^2\left(1-\frac{1}{2}\sin^2\theta_{13}\right)+x_2^2\left(1-\frac{1}{2}\sin^2\theta_{23}\right)\right.\right.$$

$$-\frac{2m^2}{q^2}\left(2+\cos^2\theta_{13}+\cos^2\theta_{23}-x_1-x_2\right.$$

$$\left.\left.-\frac{1}{2}\left(x_1^2-\frac{4m^2}{q^2}\right)\sin^2\theta_{13}-\frac{1}{2}\left(x_2^2-\frac{4m^2}{q^2}\right)\sin^2\theta_{23}\right)\right]$$

$$+\frac{1}{(1-x_1)^2}\,\frac{m^2}{q^2}\left[\left(x_1^2-\frac{4m^2}{q^2}\right)\sin^2\theta_{13}-2x_1\right]$$

$$\left.+\frac{1}{(1-x_2)}\,\frac{m^2}{q^2}\left[\left(x_2^2-\frac{4m^2}{q^2}\right)\sin^2\theta_{23}-2x_2\right]\right\}$$

$$\frac{d^2\sigma_L}{dx_1\,dx_2} = \sigma^{(1)}\left\{\frac{1}{(1-x_1)(1-x_2)}\left[\frac{x_1^2}{2}\sin^2\theta_{13}+\frac{x_2^2}{2}\sin^2\theta_{23}\right.\right.$$

$$-\frac{2m^2}{q^2}\left(4-\cos^2\theta_{13}-\cos^2\theta_{23}-\frac{5}{2}(x_1+x_2)+x_1x_2+\frac{4m^2}{q^2}\right.$$

$$\left.\left.+\frac{1}{2}\left(x_1^2-\frac{4m^2}{q^2}\right)\sin^2\theta_{13}+\frac{1}{2}\left(x_2^2-\frac{4m^2}{q^2}\right)\sin^2\theta_{23}\right)\right]$$

$$+\frac{1}{(1-x_1)^2}\,\frac{m^2}{q^2}\left[\left(x_1^2-\frac{4m^2}{q^2}\right)\cos^2\theta_{13}-x_1\right]$$

$$\left.+\frac{1}{(1-x_2)^2}\,\frac{m^2}{q^2}\left[\left(x_2^2-\frac{4m^2}{q^2}\right)\cos^2\theta_{23}-x_2\right]\right\}$$

$$\frac{d^2\sigma_T}{dx_1 dx_2} = \sigma^{(1)} \Big\{ \frac{1}{(1-x_1)(1-x_2)} \Big[\frac{1}{4}\left(x_1^2 - \frac{4m^2}{q^2}\right)\sin^2\theta_{13} + \frac{1}{4}\left(x_2^2 - \frac{4m^2}{q^2}\right)\sin^2\theta_{23}$$

$$+ \frac{m^2}{q^2}\left(\frac{1}{2}\left(x_1^2 - \frac{4m^2}{q^2}\right)\sin^2\theta_{13} + \frac{1}{2}\left(x_2^2 - \frac{4m^2}{q^2}\right)\sin^2\theta_{23}\right)\Big] .$$

$$- \frac{1}{(1-x_1)^2}\frac{m^2}{q^2}\frac{1}{2}\left(x_1^2 - \frac{4m^2}{q^2}\right)\sin^2\theta_{13}$$

$$- \frac{1}{(1-x_2)^2}\frac{m^2}{q^2}\frac{1}{2}\left(x_2 - \frac{4m^2}{q^2}\right)\sin^2\theta_{23} \Big\}$$

$$\frac{d^2\sigma_I}{dx_1\,dx_2} = \frac{\sqrt{2}}{4}\sigma^{(1)} \Big\{ \frac{1}{(1-x_1)(1-x_2)} \Big[\frac{1}{2}\left(x_1^2 - \frac{4m^2}{q^2}\right)\sin 2\theta_{13}$$

$$+ \frac{1}{2}\left(x_2^2 - \frac{4m^2}{q^2}\right)\sin 2\theta_{23} - \frac{4m^2}{q^2}\left(1 - x_1 - x_2 + \frac{2m^2}{q^2}\right)\tan\theta_{12}\Big]$$

$$- \frac{1}{(1-x_1)^2}\frac{m^2}{q^2}\left(x_1^2 - \frac{4m^2}{q^2}\right)\sin 2\theta_{13}$$

$$- \frac{1}{(1-x_2)^2}\frac{m^2}{q^2}\left(x_2^2 - \frac{4m^2}{q^2}\right)\sin 2\theta_{23} \Big\}$$

(8)

$$\cos\theta_{12} = \left(x_1 x_2 - 2x_1 - 2x_2 + 2 + \frac{4m^2}{q^2}\right)\left(x_1^2 - \frac{4m^2}{q^2}\right)^{-1/2}\left(x_2^2 - \frac{4m^2}{q^2}\right)^{-1/2}$$

$$\cos\Theta_{13} = (x_1 x_3 - 2x_1 - 2x_3 + 2)\left(x_1^2 - \frac{4m^2}{q^2}\right)^{-1/2} x_3^{-1}$$

$$\cos\Theta_{23} = (x_2 x_3 - 2x_2 - 2x_3 + 2)\left(x_2^2 - \frac{4m^2}{q^2}\right)^{-1/2} x_3^{-1} \tag{9}$$

We notice that the relation $d\sigma_L = 2d\sigma_T$ which is valid for massless quarks, is not obeyed anymore. The cross section $\sigma = \sigma_u + \sigma_L$ is the same in all three regions. We find:

$$\begin{aligned}\frac{d^2\sigma}{dx_1 dx_2} = \sigma^{(1)} \Bigg\{ &\frac{1}{(1-x_1)(1-x_2)} \left[x_1^2 + x_2^2 - \frac{2m^2}{q^2}\left(6 - \frac{7}{2}(x_1+x_2) + x_1 x_2 - \frac{4m^2}{q^2}\right)\right] \\ &+ \frac{m^2}{q^2} \frac{1}{(1-x_1)^2}\left(x_1^2 - \frac{4m^2}{q^2} - 3x_1\right) \\ &+ \frac{m^2}{q^2} \frac{1}{(1-x_2)^2}\left(x_2^2 - \frac{4m^2}{q^2} - 3x_2\right)\Bigg\}\end{aligned} \tag{10}$$

which agrees with Ref. 9.

A good place to look for effects of gluon bremsstrahlung are single inclusive distributions, for example the thrust distribution. In Fig. 2 we have shown $1/\sigma_0 \; d\sigma/dT$ for various quark masses. The shape of the distribution is not changed very much, whereas the integrated cross section is reduced for massive quarks. We find

$$\frac{1}{\sigma_0}\int_{2/3}^{T_c} \frac{d\sigma_{q\bar{q}g}}{dT}\, dT = \begin{cases} 0.163 \text{ for } m = 0 \\ 0.135 \text{ for } m = 5 \text{ GeV} \\ 0.080 \text{ for } m = 10 \text{ GeV} \end{cases}$$

at $\sqrt{q^2} = 40$ GeV with a thrust cut-off $T_c = 0.90$.

In calculating the angular distribution of jet axes we shall proceed as in Ref. 7. The first step is to determine the average cosine of the angle ϑ between the thrust axis and the second most energetic jet for fixed T. In the case of massless quarks this was given by

$$\langle \cos\vartheta \rangle = 1 - A$$

$$\begin{aligned} A = 2\Big[& -3(2-T)(3T-2) + 6(2-T)(1-T)\ln\frac{2T}{2-T} \\ & + 2(2-3T+3T^2)\ln\frac{T^2}{(2-T)(1-T)}\Big] \\ * \Big[& -3T(2-T)(3T-2) + 2(2-3T+3T^2)\ln\frac{2T-1}{1-T}\Big]^{-1} \end{aligned} \qquad (11)$$

Note that the event is completely determined by T and $\cos\vartheta$. For massive quarks we obtain for a fixed T and for different masses at an energy $\sqrt{q^2} = 40$ GeV

$$T = 0.7 \qquad \langle \cos\vartheta \rangle = \begin{cases} -0.59 & m = 0 \\ -0.60 & m = 5 \text{ GeV} \\ -0.60 & m = 10 \text{ GeV} \end{cases}$$

$$T = 0.8 \qquad \langle \cos\vartheta \rangle = \begin{cases} -0.79 & m = 0 \\ -0.79 & m = 5 \text{ GeV} \\ -0.79 & m = 10 \text{ GeV} \end{cases}$$

$$T = 0.9 \qquad \langle \cos\vartheta \rangle = \begin{cases} -0.92 & m = 0 \\ -0.93 & m = 5 \text{ GeV} \\ -0.93 & m = 10 \text{ GeV} \end{cases}$$

As can be seen the mass of the quark has no influence on the shape of the 'typical event' for fixed thrust.

Next we shall study the angular distribution for above thrust values and different masses. The general form of the cross section can be written as:

$$\frac{d^4\sigma}{dx_1 dx_2\, d\cos\theta\, d\chi} \sim 1 + a_1 \cos^2\theta + a_2 \sin^2\theta \cos 2\chi + a_3 \sin 2\theta \cos\chi \tag{12}$$

where $\vec{0x}$ (see Fig. 1) is now defined to point into the hemisphere of the second most energetic jet. This corresponds to case B in Ref. 6. The coefficients a_1, a_2, a_3 are given in the following table:

		a_1	a_2	a_3
T = 0.7	m = 0	0.29	0.18	0.03
	m = 5 GeV	0.29	0.17	0.03
	m = 10 GeV	0.32	0.14	0.02
T = 0.8	m = 0	0.59	0.10	0.09
	m = 5 GeV	0.55	0.09	0.08
	m = 10 GeV	0.58	0.05	0.09
T = 0.9	m = 0	0.85	0.04	0.10
	m = 5 GeV	0.79	0.03	0.10
	m = 10 GeV	0.70	0.0	0.12

Note that the angular distribution involves only the ratio m^2/q^2 so that this result can be scaled up into a whole lot of other energies and masses. We see that the influence of the quark mass in the angular distribution is quite small. Only for large $T = 0.9$ we obtain a visible 20 % effect for $m = 10$ GeV in the coefficient a_1. For heavy quarks we therefore expect a larger deviation from the $1 + \cos^2 \theta$ behaviour of the jet axis than for massless quarks.

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Figure Captions

Fig. 1. Definition of angles θ and χ. The thrust axis is along $\vec{Oz}$ while the q, $\bar{q}$ and g momenta lie in the plane (x, z). The (y, z) plane divides the final state into two hemispheres. $\vec{Ox}$ defines the hemisphere in which to find the antiquark (quark) in case of the thrust axis being given by the quark (antiquark) momentum. If the gluon is most energetic $\vec{Ox}$ defines the hemisphere in which to find the quark. The angles θ and χ vary between $0 \leq \theta \leq \pi$ and $0 \leq \chi \leq 2\pi$. When discussing the angular distribution of the three-jet events (Eq. (12)) $\vec{Ox}$ is defined to point into the hemisphere of the second most energetic jet.

Fig. 2. Thrust distribution for $e^+e^- \rightarrow q\bar{q}g$ for different quark masses at $\sqrt{q^2} = 40$ GeV
solid line: $m_q = 0$
dashed line: $m_q = 5$ GeV
dashed-dotted line: $m_q = 10$ GeV

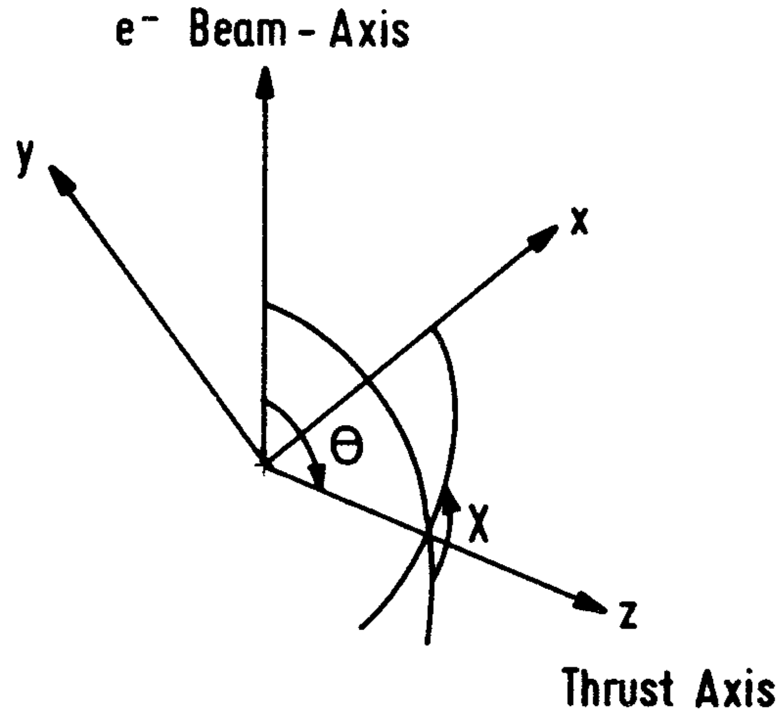

FIG. 1

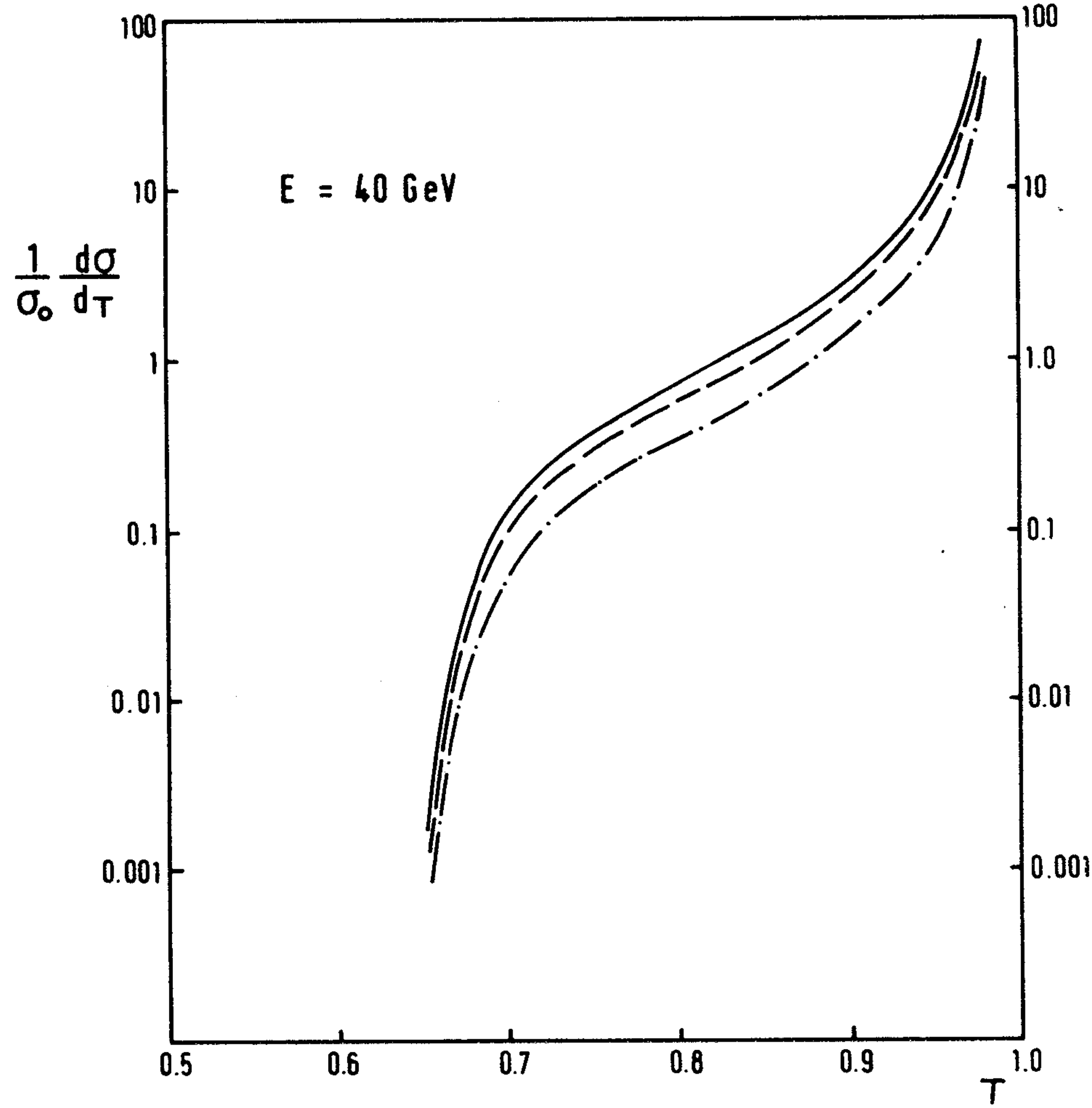
E = 40 GeV
$\frac{1}{\sigma_0}\frac{d\sigma}{d\tau}$
100
10
1
0.1
0.01
0.001
100
10
1.0
0.1
0.01
0.001
0.5
0.6
0.7
0.8
0.9
1.0
T

FIG. 2