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Abstract

We calculate the differential cross section for the process $e^+e^- \to \gamma, Z \to \tau^+\tau^- \to \nu_\tau X^+ \nu_\tau X^-$ for longitudinal or transversal polarization of the electron positron beams. The resulting formulae are evaluated using Monte Carlo methods to find measurable effects of the weak interaction, in particular those exhibiting, parity violation.

1. Introduction

The Salam-Weinberg model [1] of the unification of weak and electromagnetic interactions is now a generally accepted theory. Recent experiments in neutrino physics, atomic physics and polarized electron deuteron scattering [2] strongly support it. In view of the fundamental importance of this theory, however, more precise measurements and the investigation of different reactions are needed to establish it at the level of QED. Therefore many experiments at the new e^+e^- storage rings PETRA and PEP will try to find evidence for interference effects between weak and electromagnetic interactions in $e^+e^- \to \mu^+\mu^-$ or $e^+e^- \to \tau^+\tau^-$. These transitions involve only charged leptons. They therefore provide a very important independent test of the Weinberg-Salam model.

In principle the reactions $e^+e^- \to \mu^+\mu^-$ and $e^+e^- \to \tau^+\tau^-$ are equivalent if the τ-lepton is regarded as a sequential lepton [3]. Experimentally both reactions are easily separated from hadronic events at high energies. Because of the large decay lengths of the muons polarisation measurements require a major effort. However, at present energies the τ-leptons have an unmeasurably short decay length. Since the charged secondaries which are subject to measurement go very much in forward direction the angular distribution of the τ-leptons can be measured to practically the same precision as in the muonic case. Therefore, the τ-decay offers the advantage of a less difficult measurement of the τ-polarisation. Thus in the τ-pair production also parity violating effects can be observed even without using longitudinally polarized e^+e^-- beams

In this paper we present a calculation of the differential cross section for the reaction $e^+e^- \to \tau^+\tau^- \to$ secondaries. The one photon and heavy neutral vector meson exchange graphs Fig.1 have been included and the first order weak interaction has been retained. Arbitrary polarisations of the initial electron beams have been taken into account. The τ-decay channels $\tau \to \pi\nu_\tau$, $\tau \to K\nu_\tau$, $\tau \to \mu\nu_\tau \nu_\mu$, $\tau \to e\nu_\tau \nu_e$, $\tau \to A_1 \nu_\tau$ and $\tau \to K^* \nu_\tau$ have been considered. Results on unpolarized $e^+e^- \to \tau^+\tau^-$ have already been published [4,5].

The paper is organized as follows:
In section 2 we calculate the differential cross section for $\tau^+\tau^-$ creation taking one photon exchange and the interference of electromagnetic and weak interaction into account. The decay rate of a polarized heavy lepton is given in section 3 for various decay channels. In section 4 the reconstruction of the amplitude for the full process

$$e^+e^- \to \tau^+\tau^- \to \nu_\tau + \bar{\nu}_\tau + X + \bar{X}$$

from the production cross section and the decay rate is discussed. Section 5 then provides the formulae for the cross section of this process. Finally section 6 discusses the different features of the cross section in view of future experiments.

2. The differential cross section for the process $e^+(s^+)\ e^-(s^-) \to \gamma,\ Z \to \tau^+(\epsilon_b^+)\ \tau^-(\epsilon_a^-)$.

The process

$$e^+ e^- \to \gamma,\ Z \to \tau^+\tau^- \to \nu_\tau + \bar{\nu}_\tau + X + \bar{X} \tag{2.1}$$

with γ and Z as intermediate states is rather complicated. It is therefore very helpful for the clarification of the calculations if one breaks it up into the creation process of the heavy lepton pair $\tau^+\tau^-$ with general polarization vectors ϵ_b^+ and ϵ_a^- and their successive decay processes. Care has to be taken with respect to the coherence of the creation and the subsequent decay processes [6,7]. The method and the technical details of the appropriate factorization procedure will be discussed in section 4.

This section will be devoted to the calculation of the differential cross section of the creation of the heavy lepton pair:

$$e^+(p^+,s^+)\ e^-(p^-,s^-) \to \tau^+(q^+,n^+)\ \tau^-(q^-,n^-) \tag{2.2}$$

Here p and q denote the momenta of the particles, while $s^\pm$ and $n^\pm$ are the polarization four vectors. The four vector n is obtained by boosting the rest vector

$$n_R = \begin{pmatrix} 0 \\ \vec{\epsilon} \end{pmatrix} \rightarrow n = \begin{pmatrix} \frac{\vec{q}\vec{\epsilon}}{m} \\ \vec{\epsilon} + \frac{\vec{q}\cdot\vec{\epsilon}}{m(q_o+m)}\vec{q} \end{pmatrix} , \tag{2.3}$$

while $s^{\pm}$ results from a boost of

$$s_R^{\pm} = \begin{pmatrix} 0 \\ \pm\lambda^{\pm}\hat{e}_3 \pm \zeta^{\pm}\hat{e}_1 \end{pmatrix} . \tag{2.4}$$

In the coordinate system of the unit vectors $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ the momentum of the electron is supposed to have the representation

$$\vec{p}^{-} = p\,\hat{e}_3 . \tag{2.5}$$

Of course, we consider the process in the c.m.s frame so that the positron has the opposite momentum. The direction of $\hat{e}_1$ can in particular be put into the direction of the magnetic field in e^+e^- storage rings. In the final formulae $\lambda^{\pm}$ can be interpreted as the longitudinal $\zeta^{\pm}$ as the transverse polarizations of the electron- resp. positron beam.

In addition to the one photon exchange we consider the exchange of a neutral vector boson Z, assuming that the τ is a sequential lepton thus having the same coupling to the Z boson as the electron. The T matrix element for the Z exchange is given by

$$T^Z = \frac{1}{(2\pi)^3} F^Z \cdot (2\pi)^3 <\tau^+\tau^-|j_\rho^{(Z)}| o >(2\pi)^3 <o|j^{(Z)\rho}| e^+e^- >$$

$$= \frac{1}{(2\pi)^3} F^Z \bar{u}(q^-,n^-) \gamma_\rho[g_V+g_A\gamma_5] v(q^+,n^+)$$

$$\bar{v}(p^+,s^+) \gamma^\rho[g_V+g_A\gamma_5] u(p^-,s^-) , \tag{2.6}$$

where

$$F^Z = \frac{G^2}{M^2} \frac{M^2}{M^2-s} . \tag{2.7}$$

The quantities G, g_V and g_A characterize the coupling of the leptons to the Z boson. In the Weinberg-Salam model [1] one has

$$\frac{G^2}{M^2} = \sqrt{2}\, G_F , \tag{2.8}$$

$$g_V = \frac{1}{2} - 2\sin^2\Theta_W , \tag{2.9}$$

$$g_A = \frac{1}{2} . \tag{2.10}$$

The T-matrix element for the γ-exchange can be obtained from the expression (2.6) by the substitution

$$F^Z \rightarrow F^\gamma = -\frac{e^2}{s} , \tag{2.11}$$

$$g_V = 1 , \quad g_A = 0 . \tag{2.12}$$

The total transition matrix is

$$T = T^\gamma + T^Z .$$

The transition probability is proportional to the absolute square of T. Because of the smallness of F^Z compared to F^γ for $s \ll M^2$ we neglect the term $|T^Z|^2$ so that we are left with

$$|T|^2 = |T^\gamma|^2 + 2\,\mathrm{Re}\,\{T^{\gamma *} T^Z\} . \tag{2.14}$$

The differential cross section for the production of polarized τ leptons by polarized electrons and positrons is then given by

$$d\sigma(n^-,n^+) = \frac{8\pi^4}{s} \frac{d^3q^-}{2q_o^-} \frac{d^3q^+}{2q_o^+} \delta^4(p^+ + p^- - q^+ - q^-) |T(n^-,n^+)|^2 . \tag{2.15}$$

As we shall show in section 4 also for subsequent coherent decays of the τ-leptons, it is sufficient to calculate $|T(n^-,n^+)|^2$, if one carries the calculation out for general polarization vectors n^-,n^+ of the τ-leptons.

Obviously it suffices to evaluate the traces for $T^{\gamma *}\,T^Z$, since the result for $T^{\gamma *}\,T^\gamma$ can be obtained by the simple substitution (2.11), (2.12). We factorize $T^{\gamma *}\,T^Z$ into a product

$$L^{Z\gamma}_{\rho\sigma} \cdot \left(M^{\rho\sigma}_{Z\gamma}\right)^*$$

$$L^{Z\gamma}_{\rho\sigma} = (2\pi)^6 \langle e^+e^-|j^Z_\rho|o\rangle \langle o|j^\gamma_\sigma|e^+e^-\rangle ,$$

$$M^{\rho\sigma}_{Z\gamma} = (2\pi)^6 \langle \tau^+\tau^-|j^\rho_Z|o\rangle \langle o|j^\sigma_\gamma|\tau^+\tau^-\rangle . \qquad (2.16)$$

In terms of traces the tensor $M_{\gamma Z}$ has the form

$$M^{\rho\sigma}_{\gamma Z} = \frac{1}{4}\,\mathrm{tr}\,\{\gamma^\rho(g_V + g_A\gamma_5)\,(1-\gamma_5 \not n^+)(\not q^+ - m_\tau)$$
$$\cdot\gamma^\sigma\,(1-\gamma_5\not n^-)(\not q^- + m_\tau)\} . \qquad (2.17)$$

The electronic tensor $L^{Z\gamma}$ can be written down by making obvious changes in $M_{Z\gamma}$.

If we denote the $\tau^\pm$ polarization in their respective rest frames by $\vec{\epsilon}^{\,\pm}$, their total energy square by s, the differential cross section of τ-pair production by annihilation of polarized e^+e^- beams is finally given by

$$\frac{d\sigma}{d\Omega}\,(e^+e^- \to \tau^+\tau^-) =$$

$$\frac{\alpha^2}{16s}\,\beta\,\Big\{(1-\lambda\lambda')[(1-rg_V^2)t_1 + rg_Vg_A(t_3 - t_4) + rg_A^2\,t_2]$$
$$+(\lambda-\lambda')[(1-rg_V^2)t_3 + rg_Vg_A(t_1 - t_2) + rg_A^2\,t_4]$$
$$-\zeta\zeta'[(1-rg_V^2)\,t_5 + rg_Vg_A\,t_6 + rg_A^2\,t_7]\Big\} . \qquad (2.18)$$

Here

$$d\Omega = d\cos\theta\,d\varphi \qquad (2.19)$$

is the solid angle of the outgoing τ^-. The angles θ and φ are defined by

$$\vec{q} = q(\sin\theta\cos\varphi\,\hat{e}_1 + \sin\theta\sin\varphi\,\hat{e}_2 + \cos\theta\,\hat{e}_3) \qquad (2.20)$$

with $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ defined after eq. (2.4). Furthermore, we use the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \qquad \text{with} \qquad \beta = \frac{q}{q_o} . \qquad (2.21)$$

The symbols $\lambda^-(\lambda^+)$ resp. $\zeta^-(\zeta^+)$ denote the longitudinal resp. transversal polarizations of the electron (positron) beam. The quantity r stands for the following expression

$$r = \frac{G_F s}{\pi\sqrt{2}\alpha}\,\frac{1}{1-\frac{s}{M^2}} , \qquad (2.22)$$

G_F being the conventional Fermi coupling constant, α the fine structure constant. The quantities t_i are listed below

$$
\begin{aligned}
t_1 = {} & 1 + \cos^2\theta + \frac{1}{\gamma^2}\sin^2\theta \\
& + \epsilon_z^- \epsilon_z^+ [1 + \cos^2\theta - \frac{1}{\gamma^2}\sin^2\theta] \\
& + \epsilon_x^- \epsilon_x^+ (1 + \frac{1}{\gamma^2}) \sin^2\theta \\
& - \epsilon_y^- \epsilon_y^+ \beta^2 \sin^2\theta \\
& - (\epsilon_x^- \epsilon_z^+ + \epsilon_z^- \epsilon_x^+) \frac{1}{\gamma}\sin 2\theta \quad , \qquad (2.23)
\end{aligned}
$$

$$
\begin{aligned}
t_2 = {} & -2\beta\cos\theta - 2\epsilon_z^- \epsilon_z^+ \beta\cos\theta \\
& + (\epsilon_x^- \epsilon_z^+ + \epsilon_z^- \epsilon_x^+) \frac{\beta}{\gamma} \sin\theta \quad , \qquad (2.24)
\end{aligned}
$$

$$
t_3 = 2(\epsilon_z^- + \epsilon_z^+) \cos\theta - 2(\epsilon_x^- + \epsilon_x^+) \frac{1}{\gamma} \sin\theta \quad , \qquad (2.25)
$$

$$
t_4 = -(\epsilon_z^- + \epsilon_z^+) \beta (1 + \cos^2\theta) + (\epsilon_x^- + \epsilon_x^+) \frac{\beta}{2\gamma} \sin 2\theta , \qquad (2.26)
$$

$$
\begin{aligned}
t_5 = {} & \beta^2 \sin^2\theta \cos 2\varphi \\
& + \epsilon_z^- \epsilon_z^+ (2 - \beta^2) \sin^2\theta \cos 2\varphi \\
& + \epsilon_x^- \epsilon_x^+ (1 + \cos^2\theta - \frac{1}{\gamma^2}\sin^2\theta) \cos 2\varphi \\
& - \epsilon_y^- \epsilon_y^+ (1 + \cos^2\theta + \frac{1}{\gamma^2}\sin^2\theta) \cos 2\varphi \\
& - 2(\epsilon_x^- \epsilon_y^+ + \epsilon_y^- \epsilon_x^+) \cos\theta \sin 2\varphi \\
& + (\epsilon_x^- \epsilon_z^+ + \epsilon_z^- \epsilon_x^+) \frac{1}{\gamma} \sin 2\theta \cos 2\varphi \\
& - 2(\epsilon_y^- \epsilon_z^+ + \epsilon_z^- \epsilon_y^+) \frac{1}{\gamma} \sin\theta \sin 2\varphi \quad , \qquad (2.27)
\end{aligned}
$$

$$
\begin{aligned}
t_6 = {} & (\epsilon_z^- + \epsilon_z^+) \beta \sin^2\theta \cos 2\varphi \\
& + (\epsilon_x^- + \epsilon_x^+) \frac{\beta}{2\gamma} \sin 2\theta \cos 2\varphi \\
& - (\epsilon_y^- + \epsilon_y^+) \frac{\beta}{\gamma} \sin\theta \sin 2\varphi \quad , \qquad (2.28)
\end{aligned}
$$

$$
\begin{aligned}
t_7 = {} & -2(\epsilon_x^- \epsilon_x^+ - \epsilon_y^- \epsilon_y^+) \beta \cos\theta \cos 2\varphi \\
& + (\epsilon_x^- \epsilon_y^+ + \epsilon_y^- \epsilon_x^+) \beta (1 + \cos^2\theta) \sin 2\varphi
\end{aligned}
$$

$$-(\epsilon_x^- \epsilon_z^+ + \epsilon_z^- \epsilon_x^+)\ \frac{\beta}{\gamma}\ \sin\theta \cos 2\varphi$$

$$+(\epsilon_y^- \epsilon_z^+ + \epsilon_z^- \epsilon_y^+)\ \frac{\beta}{2\gamma}\ \sin 2\theta \sin 2\varphi \quad . \tag{2.29}$$

In the above expression terms of the order electron mass divided by electron energy have been neglected. The $\epsilon^{\pm}_{x,y,z}$ originate from the τ polarization four vectors $n^{\pm}$, compare eq. (2.3). Besides the laboratory system $\hat{e}_1$, $\hat{e}_2$, $\hat{e}_3$ here we have introduced another frame $\hat{e}_x$, $\hat{e}_y$, $\hat{e}_z$ particularly adapted to the τ-lepton.
To be specific

$$\hat{e}_z = \frac{1}{q}\vec{q} = \hat{q}, \qquad \hat{e}_y = \frac{1}{\sin\theta}\ \hat{e}_3 \times \hat{q} \quad ,$$

$$\hat{e}_x = \hat{e}_y \times \hat{e}_z = \frac{1}{\sin\theta}\ (\hat{q}\cos\theta - \hat{e}_3) \quad . \tag{2.30}$$

3. The differential rate for the decay of a polarized heavy lepton.

In this section we consider the decay

$$\tau^-(q^-,n^-) \rightarrow \nu_\tau(\ell^-) + X_i^-\ (K^-), \tag{3.1}$$

where X_i^- stands for the decay channels

$$X_1^- = \pi^- \ , \qquad X_5^- = \rho^- \ ,$$
$$X_2^- = K^- \ , \qquad X_6^- = A_1^- \ ,$$
$$X_3^- = e^-\bar{\nu}_e, \qquad X_7^- = K^{*-} \ ,$$
$$X_4^- = \mu^-\bar{\nu}_\mu, \tag{3.2}$$

and K denotes the total momentum of the state X_i. We assume that the decay of the τ-lepton proceeds through a local current-current interaction. The weak heavy lepton current, however, is allowed to be an arbitrary mixture of the vector- and axial currents. The invariant transition matrix element then reads:

$$T_i^- = \frac{G_F}{\sqrt{2}} \langle X_i^- \mid J_\alpha^- \mid o\rangle\ \bar{u}(\ell^-)\gamma^\alpha(a + b\gamma_5)\ u(q^-,n^-) \quad . \tag{3.3}$$

Here a and b determine the relative strength of the axial vector coupling, in particular

$$a = b = 1 \tag{3.4}$$

means pure V - A current.
The transition matrix elements yield a differential decay rate $d\Gamma^-$ which can be expressed in invariant form by

$$\frac{q_o^-}{m_\tau}\ k_o^-\ \frac{d^3\Gamma_i^-}{d^3k^-} = \frac{\pi}{2m_\tau} \int d(LIPS)_i \sum_{spin} |T_i^-|^2 \quad . \tag{3.5}$$

The four-vector k^- refers to the momentum of the negatively charged particle in X^-. The spin summation extends over the neutrino spin and the spins of the other outgoing particles. The integral over the Lorentz invariant phase space is in case of the semi-leptonic decays of eq. (3.2) given by

$$\int d(\mathrm{LIPS})_i = \int \frac{d^3\ell}{2\ell_o} \delta^4(\ell^- + K^- - q^-), \quad i = 1,2,5,6,7 \qquad (3.6)$$

whereas for the leptonic decays we have

$$K^- = k^- + \varkappa^- \qquad (3.7)$$

and

$$\int d(\mathrm{LIPS})_i = \int \frac{d^3\varkappa^-}{2\varkappa_o^-} \frac{d^3\ell}{2\ell_o} \delta^4(\varkappa^- + \ell^- + k^- - q^-), \quad i = 3,4 \qquad (3.8)$$

The other constituent in eq. (3.5) is composed of two tensors

$$\sum_{\mathrm{spins}} |T_i^-|^2 = \frac{G_F^2}{2} S^{\alpha\alpha'}(n^-)\, Y^{(i)}_{\alpha\alpha'} , \qquad (3.9)$$

where

$$S^{\alpha\alpha'}(n^-) = \sum_{\nu_\tau \mathrm{spin}} \bar{u}(\ell)\gamma^\alpha(a+b\gamma_5)u(q^-,n^-)\, \bar{u}(q^-,n^-)\gamma^{\alpha'}(a+b\gamma_5)u(\ell) \qquad (3.10)$$

depends on the τ, ν_τ variables only, and

$$Y^{(i)}_{\alpha\alpha'} = \sum_{X\text{-spin}} \langle X_i^- | J_\alpha^- | o\rangle \langle o | J^+_{\alpha'} | X_i^+\rangle \qquad (3.11)$$

is determined by the variables describing the final state X_i^-.

The polarization carried by the particles contained in the state X_i is summed over. In particular we have for the various semi-leptonic τ-decays

$$\langle\pi^-(k^-)| J_\alpha^- |o\rangle = \frac{-i}{(2\pi)^{3/2}} f_\pi \cos\theta_C k_\alpha^- , \qquad (3.12)$$

and similarly for the K^- with the replacement

$$f_\pi \cos\theta_C \rightarrow f_K \sin\theta_C , \qquad (3.13)$$

$$\langle\rho^-(k^-)| J_\alpha^- |o\rangle = \frac{1}{(2\pi)^{3/2}} f_\rho m_\rho^2 \cos\theta_C \epsilon_\alpha(k^-), \qquad (3.14)$$

and analogously for the A_1^- and the K^{*-}, in the latter case, however, with the substitution

$$f_\rho m_\rho^2 \cos\theta_C \rightarrow f_{K^*} m_{K^*}^2 \sin\theta_C . \qquad (3.15)$$

For the weak current of e^- and ν_e one gets

$$\langle e^-(k^-),\ \bar{\nu}_e(\varkappa^-) | J_\alpha^- | o\rangle = \frac{1}{(2\pi)^3} \bar{u}(k^-)\gamma_\alpha(1+\gamma_5)v(\varkappa^-) , \qquad (3.16)$$

and similarly for the decay into μ^-, $\bar{\nu}_\mu$. With these definitions one can calculate the differential decay rate for the various final states.

Equation (3.5) is a relation among Lorentz invariants which can be decomposed in two terms independent of resp. linear in n^-

$$\frac{q_o^-}{m_\tau} k_o^- \frac{d^3\Gamma_i^-}{d^3k^-} = \Gamma_i C_i^- \left[A_i^- + B_i^-(k^- \cdot n^-)\right] . \qquad (3.17)$$

For convenience, the partial decay rate Γ_i and a coefficient C_i^- have been factorized out. The linearity in n^- arises from the τ-spin projection operator which is linear in n^-. For the decay of τ^+ of course the arguments run completely parallel. Invariance under CP connects the two processes

$$A_i := A_i^- = A_i^+, \qquad B_i := B_i^- = -B_i^+, \qquad C_i := C_i^- = C_i^+ . \tag{3.18}$$

The values of the three types of coefficients are listed below.

$$A_\pi = \frac{m_\tau^2 - m_\pi^2}{2m_\tau} \delta(m_\tau^2 + m_\pi^2 - 2k^- \cdot q^-) ,$$

$$B_\pi = -\frac{2ab}{a^2 + b^2} \delta(m_\tau^2 + m_\pi^2 - 2k^- \cdot q^-) ,$$

$$C_\pi = \frac{2m_\tau^3}{\pi(m_\tau^2 - m_\pi^2)^2} . \tag{3.19}$$

The corresponding formulae for the decay into the kaon are obtained by substituting the index π by K. For the decay into the ρ-meson we get

$$A_\rho = \frac{m_\tau^2 - m_\rho^2}{2m_\tau} \delta(m_\tau^2 + m_\rho^2 - 2k^- \cdot q^-) ,$$

$$B_\rho = -\frac{2ab}{a^2+b^2} \frac{m_\tau^2 - 2m_\rho^2}{m_\tau^2 + 2m_\rho^2} \delta(m_\tau^2 + m_\rho^2 - 2k^- \cdot q^-) ,$$

$$C_\rho = \frac{2m_\tau^3}{\pi(m_\tau^2 - m_\rho^2)^2} , \tag{3.20}$$

the expressions for the τ-decay into A_1 resp. K^* are obtained by replacing ρ by A_1 resp. K^*.

For the leptonic decay of the τ the results are

$$A_e = \frac{(a+b)^2}{2(a^2+b^2)} \left[-2\eta_e^2 + 3(1+9\eta_e^2) \frac{k^-\cdot q^-}{m_\tau^2} - 4(1 + 8\eta_e^2)\left(\frac{k^-\cdot q^-}{m_\tau^2}\right)^2\right]$$
$$+ 3\frac{(a-b)^2}{a^2+b^2} \left[1 + 9\eta_e^2 - 2(1 + 8\eta_e^2) \frac{k^-\cdot q^-}{m_\tau^2}\right]\frac{k^-\cdot q^-}{m_\tau^2} ,$$

$$B_e = \frac{1}{m_\tau} \frac{(a+b)^2}{2(a^2+b^2)} \left[-1 - 11\eta_e^2 + 4(1 + 8\eta_e^2) \frac{k^-\cdot q^-}{m_\tau^2}\right]$$
$$+ \frac{3}{m_\tau} \frac{(a-b)^2}{a^2+b^2} \left[1 + 9\eta_e^2 - 2(1 + 8\eta_e^2) \frac{k^-\cdot q^-}{m_\tau^2}\right] ,$$

$$C_e = \frac{4}{\pi m_\tau^2} , \tag{3.21}$$

where $\eta_e = m_e/m_\tau$. The corresponding formulae for the decay into μ and $\bar{\nu}_\mu$ are obtained by the replacement $e \to \mu$. We remark, that in section 5 these formulae will be used in the respective rest frames of the τ^- and the τ^+. Therefore, we introduce the polarization rest vecotr $\vec{\xi}^-$ by eliminating n^- in eq. (3.17) with the help of eq. (2.3)

$$\frac{q_o^-}{m_\tau} k_o^- \frac{d^3\Gamma}{d^3k^-} = \Gamma_i C_i^- \left[A_i^- - B_i^- (\vec{h}^- \cdot \vec{\xi}^-)\right] . \tag{3.22}$$

Here $\vec{h}^-$ is the spatial part of the momentum vector of the charged particle of the τ-decay in the τ^- rest frame:

$$h^- = \begin{pmatrix} \frac{q_o}{m} k_o^- - \frac{q}{m} k_z^- \\ k_x^- \\ k_y^- \\ -\frac{q}{m} k_o^- + \frac{q_o}{m} k_z^- \end{pmatrix} . \tag{3.23}$$

The x, y, z components refer to the basis vectors (2.30) which have been particularly adapted to the τ-lepton. Of course, analogous expressions hold true for the τ^+ decay in terms of the τ^+ rest frame variables.

4. Factorization of the coherent $\tau^+\tau^-$-production and decay amplitude.

In this section we discuss the technical details of the correlated production and decay process leading to the final formula (4.27). The reader interested in the result only, may skip this section and continue directly with section 5.

The process under consideration consists of two stages: the production of the $\tau^+\tau^-$ pair and the subsequent coherent decays of the two leptons. In the production of the lepton pair an intermediate photon and the interference term with a neutral vector boson will be taken into account. The interaction Lagrangian for the coupling of the photon and the neutral vector boson to the leptons is

$$L_{int}(x) = e\, j_\rho^{(\gamma)}(x) \cdot A^\rho(x) + G j_\rho^{(z)}(x) \cdot Z^\rho(x) . \qquad (4.1)$$

Here it is understood, that both currents consist of the sum of the electron- and the τ-lepton current,

$$j_\rho^{(\gamma)} = j_\rho^{(\gamma e)} + j_\rho^{(\gamma\tau)} , \qquad j_\rho^{(Z)} = j_\rho^{(Ze)} + j_\rho^{(Z\tau)} . \qquad (4.2)$$

For the τ-decay we simply write the effective Lagrangian

$$L_{eff}(u) = \frac{G_F}{\sqrt{2}} J_\alpha^-(u) \cdot j^{+\alpha}(u) . \qquad (4.3)$$

The transition matrix element for the entire process

$$e^+ e^- \to \tau^+\tau^- \to \bar{\nu}_\tau X_k^+ \nu_\tau X_i^- , \qquad (4.4)$$

considering at first the Z as the intermediate boson, is in lowest order given by

$$(S-1)^Z = G^2 \frac{G_F^2}{2} \int d^4x\, d^4y\, d^4u\, d^4v$$

$$\langle\bar{\nu}_\tau\; X_k^+\; \nu_\tau\; X_i^- \left| T\left\{J^{+\beta}(v)\, j_\beta^-(v)\, J^{-\alpha}(u)\, j_\alpha^+(u)\;\; j_\sigma^{(Z\tau)}(y)\cdot Z^\sigma(y)\, j_\rho^{(Ze)}(x)\cdot Z^\rho(x)\right\}\right| e^+e^-\rangle . \tag{4.5}$$

Because of the commutativity of the operators representing different particle fields the T-product factorizes in lowest order into

$$(S-1)^Z = G^2 \frac{G_F^2}{2} \int d^4x\, d^4y\, d^4u\, d^4v$$

$$\langle\bar{\nu}_\tau\;\nu_\tau \left| T(j_\beta^-(v)\, j_\alpha^+(u)\, j_\sigma^{(Z\tau)}(y))\right| o\rangle\; \langle o \left| T(Z^\sigma(y) Z^\rho(x))\right| o\rangle$$

$$\langle X_k^+ \left| J^{+\beta}(v)\right| o\rangle\; \langle X_i^- \left| J^{-\alpha}(u)\right| o\rangle\; \langle o \left| j_\rho^{(Ze)}(x)\right| e^+e^-\rangle . \tag{4.6}$$

The T-product of the Z operators between vacuum states is the Feynman propagator $\Delta_F^{\sigma\rho}(y-x)$. In the high mass limit it can be approximated by

$$\langle o|T(Z^\sigma(y)\;\; Z^\rho(x))|o\rangle = i\frac{g^{\sigma\rho}}{M^2}\delta^4(y-x) . \tag{4.7}$$

Integrating over y with the help of (4.7) leads to an expression for $(S-1)^Z$, which after exploiting translation invariance contains among others the following matrix element

$$\int d^4u'\, d^4v'\, e^{i(K^+v' + K^-u')}\; \langle\bar{\nu}_\tau\;\; \nu_\tau | T(j_\beta^-(v')\, j_\alpha^+(u')\, j_\rho^{(Z\tau)}(o))|o\rangle$$

$$= -(2\pi)^6 \sum_{\tau\text{-spins}} \frac{1}{q^{-2}-\tilde{m}^2}\, \frac{1}{q^{+2}-\tilde{m}^2} \langle\bar{\nu}_\tau | j_\beta^-(o)|\tau^+\rangle\langle\nu_\tau|j_\alpha^+(o)|\tau^-\rangle\langle\tau^+\tau^-|j_\rho^{(Z\tau)}(o)|o\rangle . \tag{4.8}$$

The right hand side of this formula is obtained by approximating the contributions arising from the various time orderings of the T-product with suitable semi-disconnected intermediate states. The momenta q^- and q^+ have the values

$$q^\pm = K^\pm + \ell^\pm . \tag{4.9}$$

The quantity $\tilde{m}$ is the τ mass which will be supposed to acquire a small imaginary part later on.

With the definition

$$(S-1)^Z = 2\pi i\; \delta^4(K^+ + K^- + \ell^+ + \ell^- - p^+ - p^-)\; M^Z \tag{4.10}$$

we have

$$M^Z = -F^Z \frac{G_F^2}{2} (2\pi)^9 \frac{1}{(q^{-2}-\tilde{m}^2)(q^{+2}-\tilde{m}^2)} .$$

$$\sum_{\alpha,\alpha^+}^{\pm 1} \langle X_i^-(K^-)|J^{-\alpha}|o\rangle\langle\nu_\tau(\ell^-)|j_\alpha^+|\tau^-(q^-,\alpha^- n_3^-)\rangle$$

$$\cdot\langle X_k^+(K^+)|J^{+\beta}|o\rangle\langle\bar{\nu}_\tau(\ell^+)|j_\beta^-|\tau^+(q^+,\alpha^+ n_3^+)\rangle$$

$$\cdot\langle\tau^-(q^-,\alpha^- n_3^-)\tau^+(q^+,\alpha^+ n_3^+)|j_\rho^{(Z\tau)}|o\rangle\langle o|j^{(Ze)\rho}|e^+(p^+,s^+)e^-(p^-,s^-)\rangle . \tag{4.11}$$

This transition matrix element can be directly translated into the graph of fig. 1. (Comparing the expression (4.11) with (2.6) and (3.3) one has

$$M^Z = -\frac{1}{q_-^2-\tilde{m}^2}\,\frac{1}{q_+^2-\tilde{m}^2}\sum_{\alpha^-,\alpha^+}^{\pm 1} T_i^-(\alpha^- n_3^-)\; T^Z(\alpha^- n_3^-,\alpha^+ n_3^+)\; T_k^+(\alpha^+ n_3^+) \quad . \tag{4.12}$$

The matrix element for γ-exchange is again obtained from (4.12) by replacing T^Z by T^γ.

We now come to the factorization of the cross section of the overall process, fig. 1, into the τ production cross section and decay probabilities. For brevity we deal for the moment with the main contribution $\sim|M^\gamma|^2$, since the treatment of the interference term $M^{\gamma *}M^Z$ proceeds completely analogously. In equ. (4.11) we have introduced the complex τ-mass

$$\tilde{m} = m_\tau + \frac{i}{2}\Gamma \quad , \tag{4.13}$$

thereby taking into account the total width Γ of the τ-lepton. In taking the absolute square of M^γ one encounters the absolute squares of the poles, which for small total width $\Gamma \ll m_\tau$ can be approximated by

$$\left|\frac{1}{q_\pm^2-\tilde{m}^2}\right|^2 = \frac{\pi}{m_\tau\Gamma}\delta(q_\pm^2 - m_\tau^2) \quad . \tag{4.14}$$

This then leads to

$$|M^\gamma|^2 = \left(\frac{\pi}{m\Gamma}\right)^2 \delta(q_-^2-m^2)\;\delta(q_+^2-m^2)$$

$$\cdot \sum_{\alpha^-\alpha^+\beta^-\beta^+} T_i^-(\alpha^- n_3^-)\; T_i^{-*}(\beta^- n_3^-)$$

$$T^\gamma(\alpha^- n_3^-,\alpha^+ n_3^+)\; T^{\gamma *}(\beta^- n_3^-,\beta^+ n_3^+)$$

$$T_k^+(\alpha^+ n_3^+)\; T_k^{+*}(\beta^+ n_3^+) \quad . \tag{4.15}$$

Since this formula does not contain absolute squares of the various transition amplitudes, it is not yet suited for a factorization into production cross section and decay probabilities. This can be achieved, however, by using the following decomposition with the help of the Pauli matrices σ_i and the spin operators s_i

$$|\alpha n_3\rangle\langle\beta n_3| = \left(\frac{1}{2} + \vec{\sigma}\cdot\vec{s}\right)_{\beta\alpha} \sum_\xi^{\pm 1} |\xi n_3\rangle\langle\xi n_3| \tag{4.16}$$

which can directly be verified.

The matrix on the right hand side of (4.16) can be rearranged in the form

$$\left[-1 + (\tfrac{1}{2} + \sigma_1\, s_1) + (\tfrac{1}{2} + \sigma_2\, s_2) + (\tfrac{1}{2} + \sigma_3\, s_3)\right]_{\beta\alpha} \quad . \tag{4.17}$$

Obviously one has

$$(\tfrac{1}{2}+\sigma_r s_r)\sum_\xi^{\pm 1} |\xi n_3\rangle\langle\xi n_3| = (\tfrac{1}{2}+\sigma_r s_r)\sum_\xi^{\pm 1} |\xi n_r\rangle\langle\xi n_r| = \sum_\xi^{\pm 1}\frac{1}{2}(1+\xi\sigma_r)\,|\xi n_r\rangle\langle\xi n_r|$$

for the three unit vectors n_1, n_2, n_3 corresponding to the spin operators s_1, s_2, s_3. Because of the arbitrariness in the choice of the spin projection axis n_3 in (4.16) one finally gets [8]

$$|\alpha n_3\rangle\langle\beta n_3| = \sum_{\xi,r} \frac{1}{2}(\tfrac{1}{3} + \xi\sigma_r)_{\beta\alpha}\; |\xi n_r\rangle\langle\xi n_r| \quad . \tag{4.18}$$

Ket-bras of this form appear in (4.15) as is obvious from (4.11). Thus we can rewrite (4.15) in a factorized form

$$\sum_{spin}|M^\gamma|^2 = \left(\frac{\pi}{m\Gamma}\right)^2 \delta(q_-^2-m^2)\ \delta(q_+^2-m^2)$$

$$\Xi |T_i^-(\xi^- n_r^-)|^2 |T^\gamma(\eta^- n_s^-, \eta^+ n_u^+)|^2 |T_k^+(\xi^+ n_v^+)|^2 \ . \qquad (4.19)$$

where Ξ denotes the restricted summation

$$\Xi \triangleq \frac{1}{4} \sum_{\xi_-,\xi_+,\eta_-,\eta_+}^{\pm 1} \ \sum_{r,s,u,v}^{1,2,3} \left(\frac{1}{9} + \xi_-\eta_-\delta_{rs}\right)\left(\frac{1}{9} + \xi_+\eta_+\delta_{uv}\right) . \qquad (4.20)$$

The differential cross section for the τ-production with subsequent coherent decay is determined by

$$d\sigma = \frac{8\pi^4}{s} \int d^4q^-\ \theta(q_o^-) \int d^4q^+ \theta(q_o^+)\quad \delta^4(p_+ + p_- - q_+ - q_-)$$

$$\int d(LIPS)_i \int d(LIPS)_k \sum_{spins} |M^\gamma|^2 \frac{d^3k^-}{2k_o^-} \frac{d^3k^+}{2k_o^+} \qquad (4.21)$$

The integrations over Lorentz invariant phase space refer to the neutrino momenta, as given in equs. (3.6) and (3.8). The integration over q_+ and q_- has been introduced in order to separate the kinematics of the production from the one of the decays. With the help of the cross section (2.15) and the decay probability (3.5) we obtain the Lorentz invariant differential cross section

$$k_o^- k_o^+ \frac{d\sigma^\gamma}{d^3k^-\ d^3k^+} = \Xi \int \frac{d^3q^-}{2q_o^-} \delta\left(s-(p^+ + p^-)\cdot q^-\right)\left[2q_o^- \frac{d^3\sigma^\gamma(\eta^- n_s^-, \eta^+ n_u^+)}{d^3q^-}\right]$$

$$\left[\frac{q_o^-}{m} k_o^- \frac{1}{\Gamma} \frac{d^3\Gamma_i^-(\xi^- n_r^-)}{d^3k^-}\right]\left[\frac{q_o^+}{m} k_o^+ \frac{1}{\Gamma} \frac{d^3\Gamma_k^+(\xi^+ n_v^+)}{d^3k^+}\right] \qquad (4.22)$$

Exploiting the remaining δ-function one obtains the more common expression containing the differential cross section $d\sigma/d\Omega_{q^-}$.

$$k_o^- k_o^+ \frac{d\sigma^\gamma}{d^3k^- d^3k^+} = \Xi \int (d\Omega_{q^-}) \frac{d\sigma^\gamma(\eta^- n_s^-, \eta^+ n_u^+)}{d\Omega_{q^-}}$$

$$\cdot\left[\frac{q_o^-}{m} k_o^- \frac{1}{\Gamma} \frac{d^3\Gamma_i^-(\xi^- n_r^-)}{d^3k^-}\right]\left[\frac{q_o^+}{m} k_o^+ \frac{1}{\Gamma} \frac{d^3\Gamma_k^+(\xi^+ n_v^+)}{d^3k^+}\right] \ . \qquad (4.23)$$

The general structure of this cross section remains unchanged if one adds the γ-Z interference term in the $\tau^+\tau^-$ production cross section, compare (2.14). In the following we therefore drop the superscript γ. In order to clarify the restricted summation Ξ defined in (4.20) we study at first the subsum over τ^--indices:

$$\Xi^- \triangleq \frac{1}{2} \sum_{\xi_-\eta_-}^{\pm 1} \ \sum_{r,s}^{1,2,3} \left[\frac{1}{9} + (2\delta_{\xi_-\eta_-} - 1)\ \delta_{rs}\right] , \qquad (4.24)$$

where use has been made of

$$2\delta_{\xi_-\eta_-} = 1 + \xi_-\eta_-$$

The general structure of the terms summed over by Ξ^- in (4.22), is of the form

$$D^- = a + \xi^-(b\cdot n_r^-) + \eta^-(c\cdot n_s^-) + \xi^-\eta^-(d\cdot n_r^-)\ (f\cdot n_s^-) \ . \qquad (4.25)$$

One easily verifies that the result of the restricted summation Ξ^- is given by

$$\Xi^- D^- = 2a + 2\sum_r^{1,2,3} (d\cdot n_r^-)\ (f\cdot n_r^-) \ . \qquad (4.26)$$

The argument runs completely parallel for Ξ^+. The effect of the summation Ξ in equ. (4.22) can be summarized in the following formula

$$k_o^- k_o^+ \frac{d\sigma}{d^3k^- d^3k^+} = \sum_{\xi^- \xi^+}^{\pm 1} \mathop{S}_{r,u}^{1,2,3} \int d\Omega_{q^-} \frac{d\sigma(\xi^- n_r^-, \xi^+ n_u^+)}{d\Omega_{q^-}}$$

$$\left[\frac{q_o^-}{m} k_o^- \frac{1}{\Gamma} \frac{d^3\Gamma_i^-(\xi^- n_r^-)}{d^3k^-}\right]\left[\frac{q_o^+}{m} k_o^+ \frac{1}{\Gamma} \frac{d^3\Gamma_k^+(\xi^+ n_u^+)}{d^3k^+}\right] , \quad (4.27)$$

The sign S is a summation operating on terms bilinear in either n_r or n_u only [7]. The formula (4.27) has the nice feature that its integrand factorizes into the physical τ-production cross section and the relative decay probabilities of polarized τ-leptons. These quantities have been calculated in sections 2 and 3.

5. The differential cross section for the process $e^+ (s^+)\ e^- (s^-) \rightarrow \gamma,\ Z \rightarrow \tau^+\tau^- \rightarrow \bar{\nu}_\tau X^+\ \nu_\tau X^-$.

We are now in a situation where we can assemble the formulae for the decay probabilities and the production cross section according to (4.27) obtaining an expression suitable for a comparison with experiment. The summation over the three polarisation vectors $\xi^- n_r^- (\xi^+ n_u^+)$ in (4.27) carries over into a summation over $\xi^- \vec{\epsilon}_r^{\,-} (\xi^+ \vec{\epsilon}_u^{\,+})$ if one uses the representations (3.22) and (2.18) together with (2.23 - 2.29). The sum in ξ^+, ξ^-, extending over the values ± 1, yields a non vanishing result for those contributions only in which $\xi^- \vec{\epsilon}_r^{\,-}$ or $\xi^+ \vec{\epsilon}_u^{\,+}$ are either absent or occur quadratically. The restricted sum (S), explained at the end of section 4, can be carried out with the help of

$$\sum_{\xi^-}^{\pm 1} \sum_{r=1}^{3} \xi^- (\vec{h} \cdot \vec{\epsilon}_r^{\,-})\ \xi^- \vec{\epsilon}_r^{\,-} = 2\vec{h} \quad , \quad i,k, = 1,2,3, \quad (5.1)$$

and an analogous expression for $\xi^+ \vec{\epsilon}_u^{\,+}$. The differential cross section for the coherent τ pair production and decay process then comes out to be

$$k_o^- k_o^+ \frac{d\sigma}{d^3k^- d^3k^+} = h_o^- h_o^+ \frac{d\sigma}{d^3h^- d^3h^+} = \frac{\alpha^2}{4s} \beta \frac{\Gamma_i}{\Gamma} \frac{\Gamma_k}{\Gamma} C_i\ C_k \int d\Omega_\tau$$

$$\Big\{ (1 - \lambda\lambda') \left[(1 - rg_V^2)\ T_1 + rg_V g_A\ (T_3 - T_4) + rg_A^2\ T_2 \right]$$

$$+ (\lambda - \lambda') \left[(1 - rg_V^2)\ T_3 + rg_V g_A (T_1 - T_2) + rg_A^2\ T_4 \right]$$

$$- \zeta\zeta' \left[(1 - rg_V^2)\ T_5 + rg_V g_A\ T_6 + rg_A^2\ T_7 \right] \Big\}. \quad (5.2)$$

The coefficients T_i arising from the t_i (2.23-2.29) are

$$T_1 = A_i A_k (1 + \cos^2\theta + \frac{1}{\gamma^2} \sin^2\theta) - B_i B_k \Big[h_z^- h_z^+ (1 + \cos^2\theta - \frac{1}{\gamma^2}\sin^2\theta)$$

$$+ h_x^- h_x^+ (1 + \frac{1}{\gamma^2}) \sin^2\theta - h_y^- h_y^+ \beta^2 \sin^2\theta - (h_x^- h_z^+ + h_z^- h_z^+) \frac{1}{\gamma} \sin 2\theta \Big], \tag{5.3}$$

$$T_2 = -A_i A_k\, 2\beta \cos\theta$$

$$+ B_i B_k \Big[h_z^- h_z^+ 2\beta \cos\theta - (h_x^- h_z^+ + h_z^- h_x^+) \frac{\beta}{\gamma} \sin\theta \Big], \tag{5.4}$$

$$T_3 = (A_i B_k h_z^+ - A_k B_i h_z^-)\, 2\cos\theta - (A_i B_k h_x^+ - A_k B_i h_x^-) \frac{2}{\gamma} \sin\theta , \tag{5.5}$$

$$T_4 = -(A_i B_k h_z^+ - A_k B_i h_z^-)\, \beta (1 + \cos^2\theta)$$

$$+(A_i B_k h_x^+ - A_k B_i h_x^-) \frac{\beta}{2\gamma} \sin 2\theta , \tag{5.6}$$

$$T_5 = A_i A_k \beta^2 \sin^2\theta \cos 2\varphi - B_i B_k \Big[h_z^- h_z^+ (2 - \beta^2) \sin^2\theta \cos 2\varphi$$

$$+ h_x^- h_x^+ (1 + \cos^2\theta - \frac{1}{\gamma^2} \sin^2\theta) \cos 2\varphi$$

$$- h_y^- h_y^+ (1 + \cos^2\theta + \frac{1}{\gamma^2} \sin^2\theta) \cos 2\varphi$$

$$- (h_x^- h_y^+ + h_y^- h_x^+)\, 2\cos\theta \sin 2\varphi$$

$$+ (h_x^- h_z^+ + h_z^- h_x^+) \frac{1}{\gamma} \sin 2\theta \cos 2\varphi$$

$$- (h_y^- h_z^+ + h_z^- h_y^+) \frac{2}{\gamma} \sin\theta \sin 2\varphi \Big], \tag{5.7}$$

$$T_6 = (A_i B_k h_z^+ - A_k B_i h_z^-) \beta \sin^2\theta \cos 2\varphi$$

$$+(A_i B_k h_x^+ - A_k B_i h_x^-) \frac{\beta}{2\gamma} \sin 2\theta \cos 2\varphi$$

$$-(A_i B_k h_y^+ - A_k B_i h_y^-) \frac{\beta}{\gamma} \sin\theta \sin 2\varphi , \tag{5.8}$$

$$T_7 = B_i B_k \Big[(h_x^- h_x^+ - h_y^- h_y^+)\, 2\beta \cos\theta \cos 2\varphi$$

$$-(h_x^- h_y^+ + h_y^- h_x^+) \beta (1 + \cos^2\theta) \sin 2\varphi$$

$$+(h_x^- h_z^+ + h_z^- h_x^+) \frac{\beta}{\gamma} \sin\theta \cos 2\varphi$$

$$-(h_y^- h_z^+ + h_z^- h_y^+) \frac{\beta}{2\gamma} \sin 2\theta \sin 2\varphi \Big] . \tag{5.9}$$

Equation (5.2) together with the definition of the coefficients T_i serves as a starting point for a Monte Carlo calculation.

In the relativistic limit where the τ mass is neglected, $\beta = 1$, the expressions for T_1 to T_7 simplify. With the abbreviations

$$K = A_i A_k - B_i B_k h_z^- h_z^+ \tag{5.10}$$

$$L = -A_k B_i h_z^- + A_i B_k h_z^+ \tag{5.11}$$

$$M = -B_i B_k (h_x^- h_x^+ - h_y^- h_y^+) \tag{5.12}$$

$$N = B_i B_k (h_x^- h_y^+ + h_y^- h_x^+) \tag{5.13}$$

one gets

$$k_o^- k_o^+ \frac{d\sigma}{d^3k^- d^3k^+} = h_o^- h_o^+ \frac{d\sigma}{d^3h^- d^3h^+} =$$

$$\frac{\alpha^2}{4s}\frac{\Gamma_i\Gamma_k}{\Gamma^2} C_i\, C_k \int d\,\Omega_\tau$$

$$\cdot\Big\{(1-rg_V^2)\Big[(1-\lambda\lambda')\{K(1+\cos^2\theta) + M\sin^2\theta\} + (\lambda-\lambda')\, L2\cos\theta\Big]$$

$$-rg_A^2\Big[(1-\lambda\lambda')\,K\cdot 2\cos\theta \qquad + (\lambda-\lambda')\,L(1+\cos^2\theta)\Big]$$

$$+rg_Vg_A\Big[(\lambda-\lambda')\{K(1+\cos\theta)^2 + M\sin^2\theta\} + (1-\lambda\lambda')\,L(1+\cos\theta)^2\Big]$$

$$-\zeta\zeta'\Big[(1-rg_V^2)\,\{K\sin^2\theta\cos 2\varphi + M(1+\cos^2\theta)\cos 2\varphi + N\,2\cos\theta\sin 2\varphi\}$$

$$+ rg_V\, g_A\, L\sin^2\theta\cos 2\varphi$$

$$- rg_A^2\,\{\qquad M\,2\cos\theta\cos 2\varphi + N(1+\cos^2\theta)\sin 2\varphi\}\Big]\Big\} \tag{5.14}$$

Experimentally it is difficult to identify in the reaction $e^+e^- \to \tau^+\tau^-$ the decay channels of both τ^+ and τ^- for all events unambiguously. Often only one of the τ-decays can be analysed. Integrating (5.2) over the τ^+ decay variables one obtains the differential cross section for the inclusive reaction

$$e^+\, e^- \to \tau^+\, \tau^- \;\; \lfloor\!\!\longrightarrow \nu_\tau \cdot X^- \tag{5.15}$$

$$k_o^-\frac{d\sigma}{d^3k^-} = h_o^-\frac{d\sigma}{d^3h^-} =$$

$$\int d\Omega_q\,\frac{\alpha^2}{4s}\,\beta\,\frac{\Gamma_k}{\Gamma}\,\frac{\Gamma_iC_i}{\Gamma}$$

$$\cdot\Big\{(1-\lambda\lambda')\Big[(1-rg_V^2)\,\hat{T}_1 + rg_Vg_A(\hat{T}_3-\hat{T}_4) + rg_A^2\,\hat{T}_2\Big]$$

$$+(\lambda-\lambda')\Big[(1-rg_V^2)\,\hat{T}_3 + rg_Vg_A(\hat{T}_1-\hat{T}_2) + rg_A^2\,\hat{T}_4\Big],$$

$$-\zeta\zeta'\Big[(1-rg_V^2)\,\hat{T}_5 - rg_Vg_A\,\hat{T}_6 - rg_A^2\,\hat{T}_7\Big]\Big\} \tag{5.16}$$

with

$$\hat{T}_1 = A_i\,(1+\cos^2\theta + \frac{1}{\gamma^2}\sin^2\theta) \tag{5.17}$$

$$\hat{T}_2 = -A_i\; 2\beta\;\cos\theta \tag{5.18}$$

$$\hat{T}_3 = -B_i(h_z^-\,2\cos\theta - h_x^-\frac{1}{\gamma}\sin\theta) \tag{5.19}$$

$$\hat{T}_4 = B_i\;\beta(h_z^-(1+\cos^2\theta) - h_x^-\frac{1}{2\gamma}\sin 2\theta) \tag{5.20}$$

$$\hat{T}_5 = A_i\,\beta^2\sin^2\theta\cos 2\varphi \tag{5.21}$$

$$\hat{T}_6 = B_i\,\beta\,(h_z^-\sin^2\theta\cos 2\varphi + h_x^-\frac{1}{2\gamma}\sin 2\theta\cos 2\varphi - h_y^-\frac{1}{\gamma}\sin\theta\sin 2\varphi) \tag{5.22}$$

$$\hat{T}_7 = 0 \tag{5.23}$$

In the relativistic limit we obtain for the inclusive differential cross section

$$k_o^- \frac{d\sigma}{d^3k^-} = h_o^- \frac{d\sigma}{d^3h^-} = \frac{\alpha^2}{4s}\frac{\Gamma_k}{\Gamma}\frac{A_i\Gamma_iC_i}{\Gamma}\int d\Omega$$

$$\cdot\Big\{(1-\lambda\lambda')\Big[(1-rg_V^2)\;(1+\cos^2\theta) - rg_Vg_A\frac{B_i}{A_i}h_z^-\;(1+\cos\theta)^2 - rg_A^2\;2\cos\theta\Big]$$

$$+(\lambda-\lambda')\Big[-(1-rg_V^2)\frac{B_i}{A_i}h_z^-\;2\cos\theta + rg_Vg_A(1+\cos\theta)^2 + rg_A^2\frac{B_i}{A_i}h_z^-\;(1+\cos^2\theta)\Big]$$

$$-\zeta\zeta'\Big[1-rg_V^2 - rg_Vg_A\frac{B_i}{A_i}h_z^-\Big]\sin^2\theta\;\cos2\varphi\Big\} \qquad (5.24)$$

In the next section we present the evaluation of these results by looking for measurable quantities which are most sensitive to the effects of the γ-Z-interference. In a Monte Carlo calculation we use the formulae (5.2) - (5.9).

6. Discussion of the results

In present experiments the τ trajectory cannot be observed. Only the laboratory momenta of k_ℓ^- and k_ℓ^+ of the charged secondaries can be measured. Equation (5.2) can be cast into the form

$$\frac{d\sigma}{d^3k_\ell^-d^3k_\ell^+} = \text{const}\;x(1-rg_V^2)\;\frac{k_{o\ell}^-k_{c\ell}^+}{s}\;u_o$$

$$\{1 + u_1\frac{rg_Ag_V}{1-r\,g_V^2} + u_2\frac{rg_A^2}{1-rg_V^2}\}\;, \qquad (6.1)$$

where the functions u_i depend only on $\vec{k}_\ell^-$ and $\vec{k}_\ell^+$ and the beam polarisation.

$$u_o = \int d\Omega_\tau\{(1-\lambda\lambda')\,T_1 + (\lambda-\lambda')\,T_3 - \zeta\zeta'\,T_5\} \qquad (6.2)$$

$$u_1 = \frac{1}{u_o}\int d\Omega_\tau\{(1-\lambda\lambda')\;(T_3 - T_4) + (\lambda-\lambda')\;(T_1 - T_2) - \zeta\zeta'\,T_6\} \qquad (6.3)$$

$$u_2 = \frac{1}{u_o}\int d\Omega_\tau\{(1-\lambda\lambda')\,T_2 + (\lambda-\lambda')\,T_4 - \zeta\zeta'\,T_7\} \qquad (6.4)$$

The parameters $\alpha_1 = rg_Ag_V/(1-rg_V^2)$ and $\alpha_2 = rg_A^2/(1-r\,g_V^2)$ can be determined by comparing $\frac{d\sigma}{du_1du_2}(\alpha_1\alpha_2)$ to the experimental distribution using Monte Carlo methods [9]. (g_V^2 can be measured only from the total rate because the electromagnetic interaction is also of the vector type).

This kind of analysis takes advantage of the spin correlation. However it has to be performed separately for each decay channel combination. In ref. [4] we briefly discussed the case where both τ^+ and τ^- decay into πν for unpolarized beams.

Here we turn to the discussion of polarization dependent effects. Firstly we look at the inclusive distributions of the reaction $e^+e^- \to \tau^+\tau^-$, $\tau^+ \to \ell^+\nu_\ell\nu_\tau$ where ℓ^+ is either a muon or a positron. In the inclusive cross section the spin correlation between the τ-leptons is lost and the distributions are therefore less sensitive to the weak interaction. On the other hand it is also much less sensitive to radiative corrections which primarily affect the acolinearity angle. About 0.7 events per τ pair belong to this class. In a Monte Carlo program we have generated such events for various sets of coupling constants g_V and g_A and for different beam polarizations. We used a beam energy of 15 GeV where the maximum luminosity is expected at the PETRA and PEP storage rings. For the computation we have used the exact formula (5.2) whereas for the discussion we prefer to refer to the simpler approximation (5.24). This formula shows that the cross section depends only on the solid angle (θ,φ) the energy of the lepton and its momentum component h_z

along the τ-direction in the τ-rest frame. At high energies as considered here, the τ decay products go very much in forward direction. Therefore the unknown τ-direction coincides nearly with the decay lepton direction. A still better approximation of the τ-solid angle is obtained from the solid angle of the vector $\vec{P}_D = \vec{h}_\ell^- - \vec{h}_\ell^+$. At 15 GeV beam energy the average of the cosine of the angle between $\vec{P}_D$ and $\vec{P}_\tau$ is about .995. Therefore θ and φ in (5.24) are reasonably well known.

This means that the reaction $e^+e^- \to \tau^+\tau^-$ is as suitable as $e^+e^- \to \mu^+\mu^-$ for the investigation of the angular distributions resulting from the γ-Z interference. In the case of τ production, however, additional information is available from the momenta of the decay particles.

The c.m. momentum component h_z is strongly correlated to the laboratory momentum k. The momentum and θ are the only relevant variables if the beams are not transversely polarized. Therefore the cross section can be presented as a two-dimensional histogram depending on $|k|$ and cos θ. This is done in figs. 2 to 4 for various parameter sets. We consider first unpolarized beams, then longitudinally and transversely polarized beams.

a) Unpolarized Beams

From equation (5.24) we see that a pure axial vector coupling leads to the well known asymmetry of the cos θ distribution, which is independent of the lepton energy (fig. 2a,b). The effect of the parity violating $g_A g_V$ term is strongest $(\sim(1+\cos\theta)^2)$ in the forward direction and for high or small lepton energies, where h_z reaches its positive and negative maximum values (fig.2c,d).

b) Londitudinally Polarized Beams

The cross section is obviously zero for fully antiparallel polarized beams, because γ and Z cannot couple to a spin zero state. The shape of the differential distributions for one beam fully polarized and the other beam unpolarized is the same as in the case where both beams have parallel polarizations. Figures 3a and 4a show the three dimensional distributions in the

absence of weak interactions. The g_A term again modifies only the cos θ-distribution independent of the lepton energy and the beam polarization (fig.3b,4b). (This is expected because it is a parity conserving term). The parity violation term $g_A g_V$ again is strongest in the forward direction (see (5.24). For maximum parity violation $g_A = \pm g_V$ and one beam fully polarized the weak interaction can be turned off if the "wrong" sign helicity is chosen i.e. $\lambda = 1$ $(\lambda'=-1)$ for $g_A = -g_V$ and $\lambda = -1(\lambda'=1)$ for $g_A = g_V$. The corresponding distributions are identical to the $g_A = g_V = 0$ distributions Figs. 3 c, 4d). For the "right" sign helicity the shape of the distributions is modified considerably (fig.3d,4c). We have estimated that $g_A\, g_V$ can be measured to a precision $\Delta g_A g_V \approx 0.03$ from a sample of 10^4 τ-pairs if the longitudinal polarization of one of the beams is 90%. For such strong polarizations μ-pairs are nearly equivalent to τ-pairs, because the cos θ-distribution is almost independent of the momenta of the decay particles. In ref. [4] we have calculated that $\Delta g_A\, g_V$ can be measured from unpolarized beams to $\Delta g_A\, g_V \approx 0.05$. Thus a longitudinal polarization is of considerable help for measuring the vector axial vector interference term.

c) Transverse Polarization

A transverse polarization of the beams is likely to be obtained in e^+e^- beams at discrete energies while for producing longitudinal polarizations additional technical equipment is needed. Formula (5.24) shows that terms proportional to g_A^2 do not depend on a transverse polarization of the beams and that the leading term $\sim(1-r\, g_V^2)$ has the same azimuthal dependence as the $g_A\, g_V$ term. Therefore at certain φ regions the g_A^2 term is enhanced relative to the leading term. Thus also a transverse beam polarization is useful for a measurement of γ-Z interference effects. In Fig.5 the distribution of the events in the (cosθ,φ)-plane is plotted. The effect of the transverse polarization is small compared to that of the longitudinal polarization. A quantitative analysis of the plots shows that about 10% less statistics are needed compared to the unpolarized case if $|\xi^\pm| = 0.9$ is assumed.

Secondly we investigate the momentum correlation of the pions in the process $e^+e^- \to \tau^+\tau^- \to \bar{\nu}_\tau\, \pi^+\, \nu_\tau \pi^-$ as another example. Figs. 6-8 show normalized distributions of events plotted as functions of the momentum difference $\vec{k}_\pi^- - \vec{k}_\pi^+$ and the cosine of its angle

against the electron beam direction. Here the shape of the distributions changes even more with the values of g_A^2 and $g_A g_V$ than in the inclusive distributions. Of course the evaluation of other τ-decay channels leads to similar effects in the corresponding distributions.
The results of the investigation carried out in this paper show that the τ-pair production in e^+e^- annihilation is well suited for a measurement of γ-Z interference effects. In many respects the exploitation of this process for the determination of weak contributions and parity violation is superior to the process of μ-pair production.

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Figure Captions

Fig.1 Graphical representation of the lowest order approximation of the process $e^+e^- \to \tau^+\tau^- \to \bar{\nu}_\tau X_k^+ \nu_\tau X_i^-$

Fig.2 Normalized distributions of events $e^+e^- \to \bar{\nu}_\tau \nu_e e^+ \nu_\tau \pi^-$, plotted as a function of the positron momentum $|k_e^+|$ and the cosine of the angle $\vartheta = \measuredangle(\vec{k}_\pi^- - \vec{k}_e^+, \vec{p}^-)$ for the vanishing polarization $\lambda^\pm = \zeta^\pm = 0$.
a) $g_V = g_A = 0$; b) $g_V = 0$, $g_A = 1$;
c) $g_V = g_A = 1$; d) $g_V = 1$ $g_A = -1$.

Fig.3 Distributions as in Fig. 2 for longitudinal polarization $\lambda^- = 1$ and $\lambda^+ = \zeta^\pm = 0$.

Fig.4 Distributions as in Fig. 2 for longitudinal polarization $\lambda^- = -1$ and $\lambda^+ = \zeta^\pm = 0$.

Fig.5 Normalized distribution of events plotted as a function of the cosine of the polar angle and the azimuthal angle φ for $g_V = 0$, $\lambda^\pm = 0$.
a) $g_A = 0$, $\zeta^+ = \zeta^- = 0$
b) $g_A = 1$, $\zeta^+ = \zeta^- = 0$
c) $g_A = 0$, $\zeta^+ = -\zeta^- = 1$
d) $g_A = 1$, $\zeta^+ = -\zeta^- = 1$

Fig.6 Normalized distributions of events $e^+e^- \to \nu_\tau \pi^+ \nu_\tau \pi^-$ plotted as a function of $|\vec{k}_\pi^- - \vec{k}_\pi^+|$ and the cosine of the angle $\vartheta = \measuredangle(\vec{k}_\pi^- - \vec{k}_\pi^+, \vec{p}^-)$ for vanishing polarization $\lambda^\pm = \zeta^\pm = 0$.
a) $g_V = g_A = 0$; b) $g_V = 0$, $g_A = 1$;
c) $g_V = g_A = 1$; d) $g_V = 1$ $g_A = -1$; .

Fig.7 Distributions as in Fig. 6 for longitudinal polarization $\lambda^- = 1$ and $\lambda^+ = \xi^\pm = 0$.

Fig.8 Distributions as in Fig. 6 for longitudinal polarization $\lambda^- = -1$ and $\lambda^+ = \xi^\pm = 0$.

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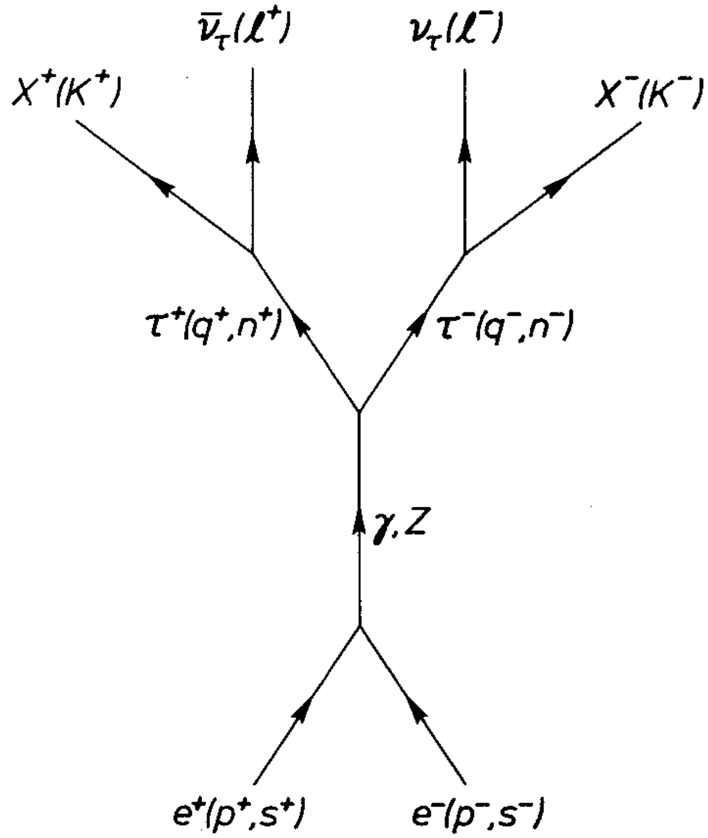


Figure 1

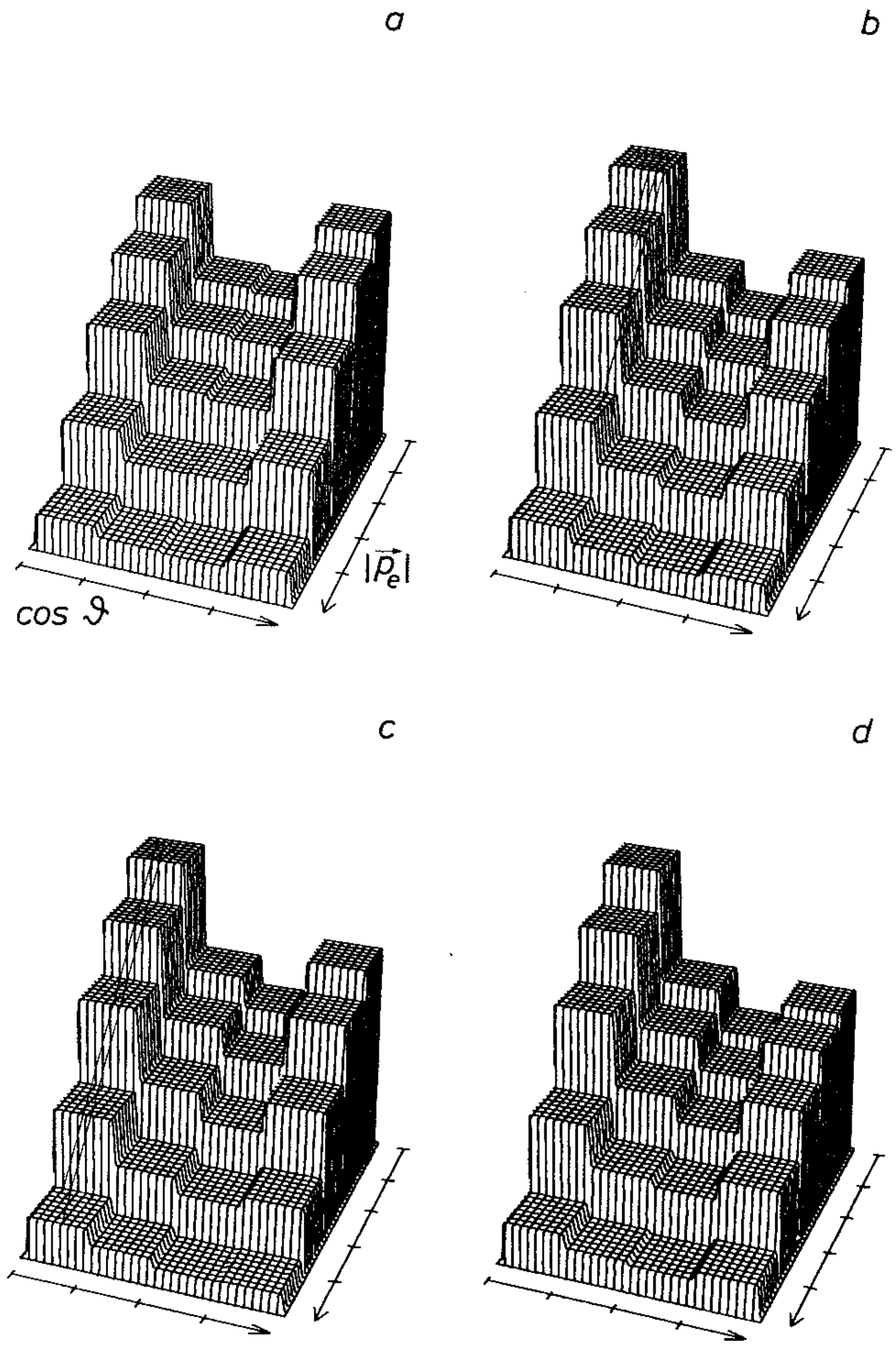
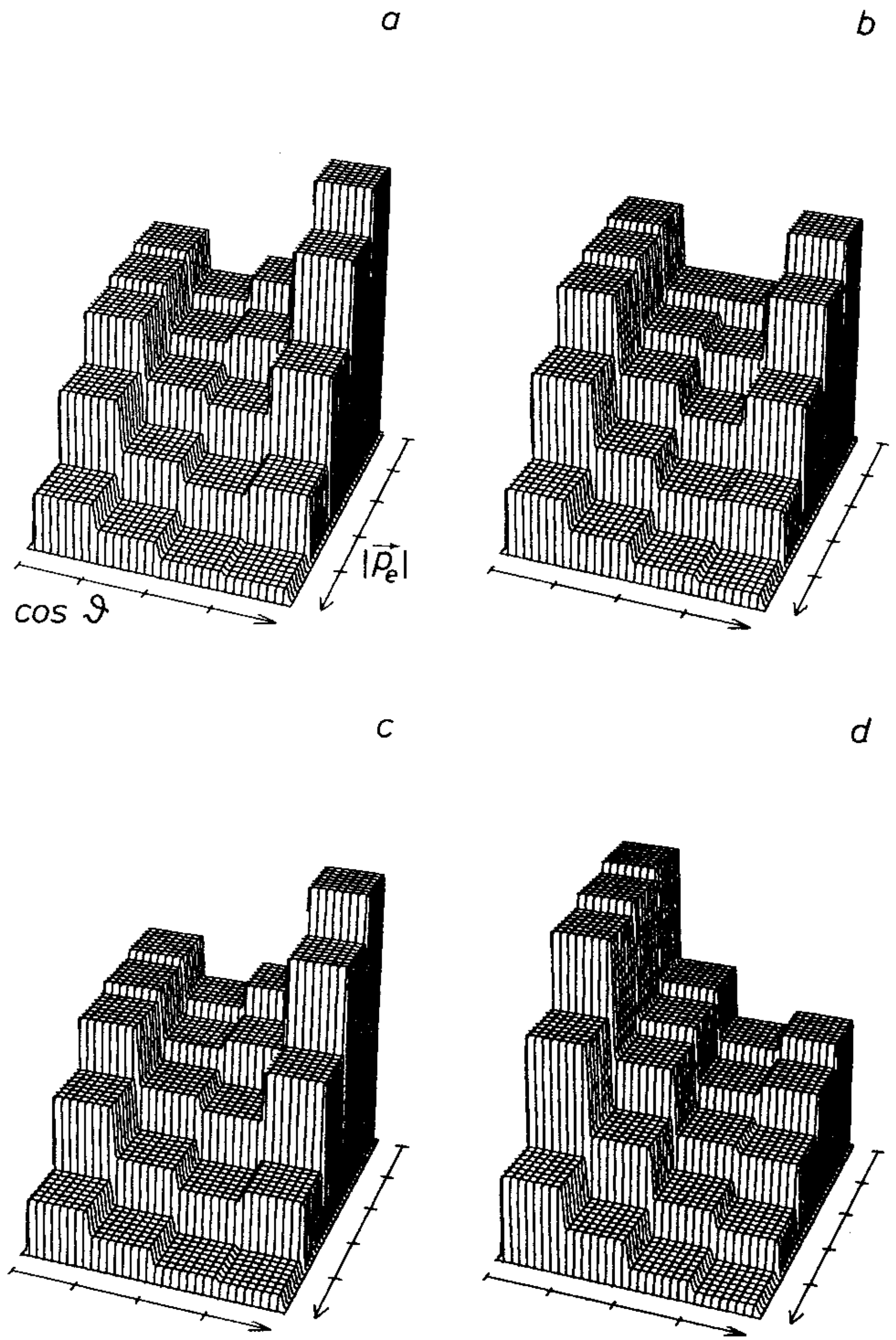


Figure 2



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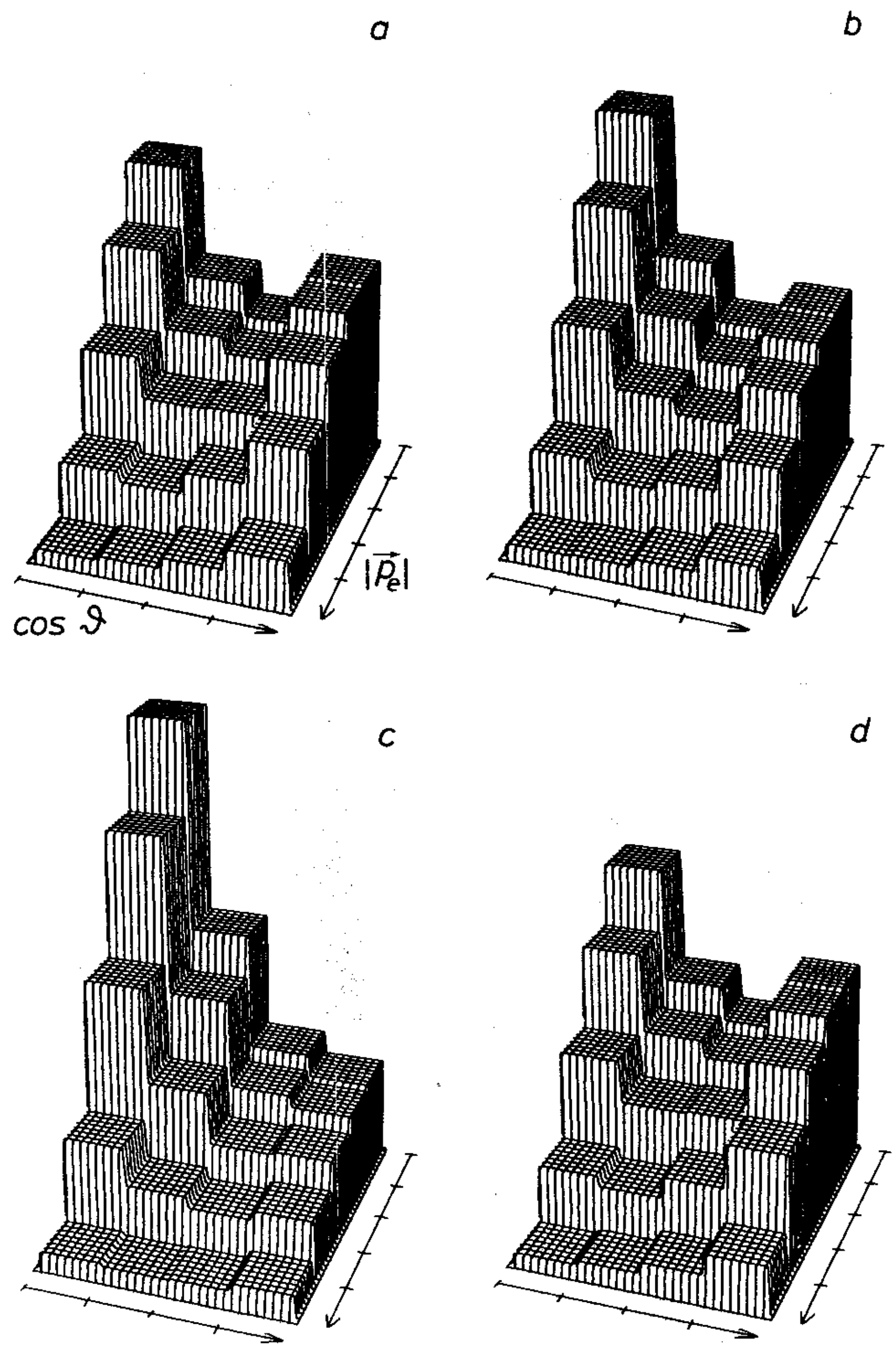

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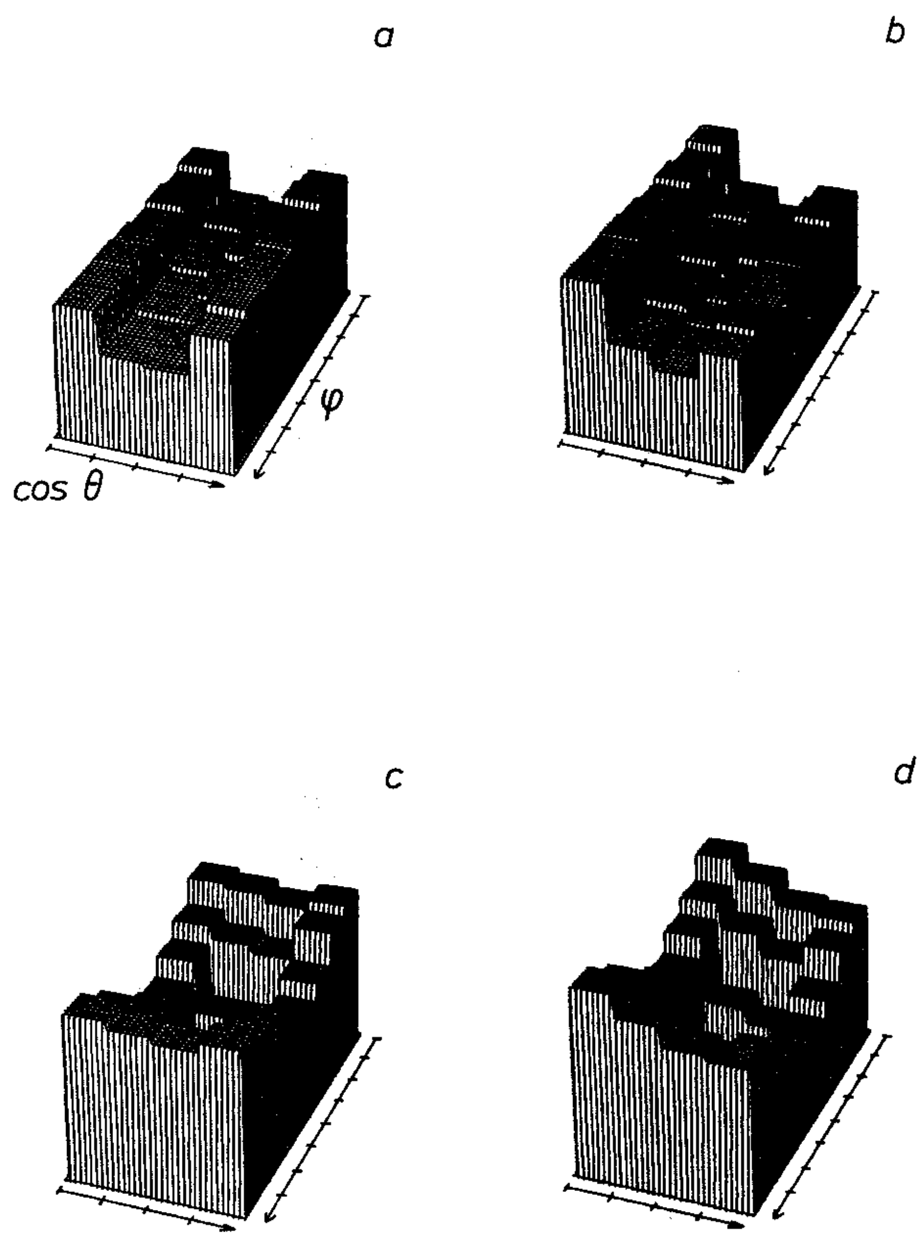


Figure 5

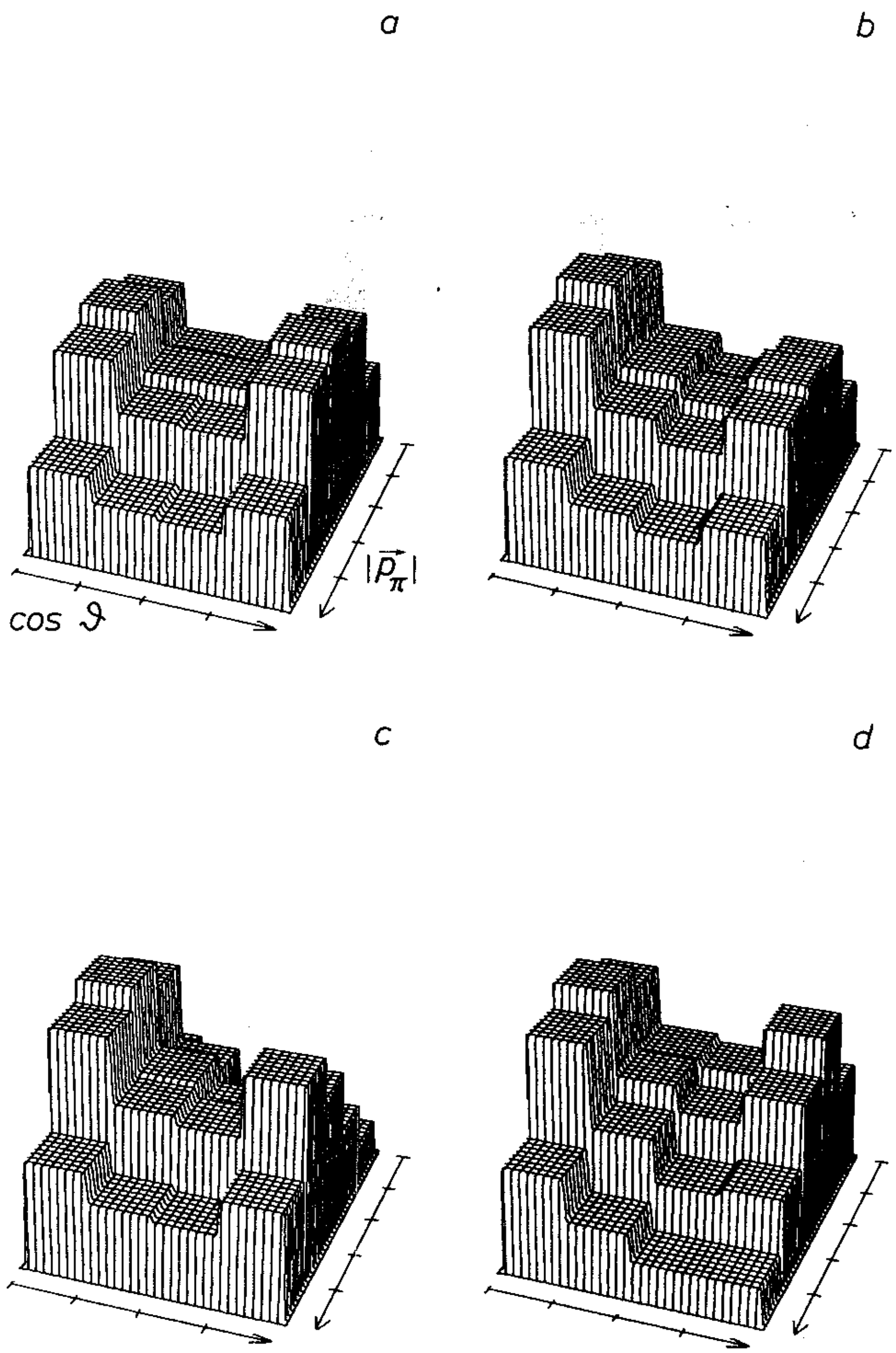
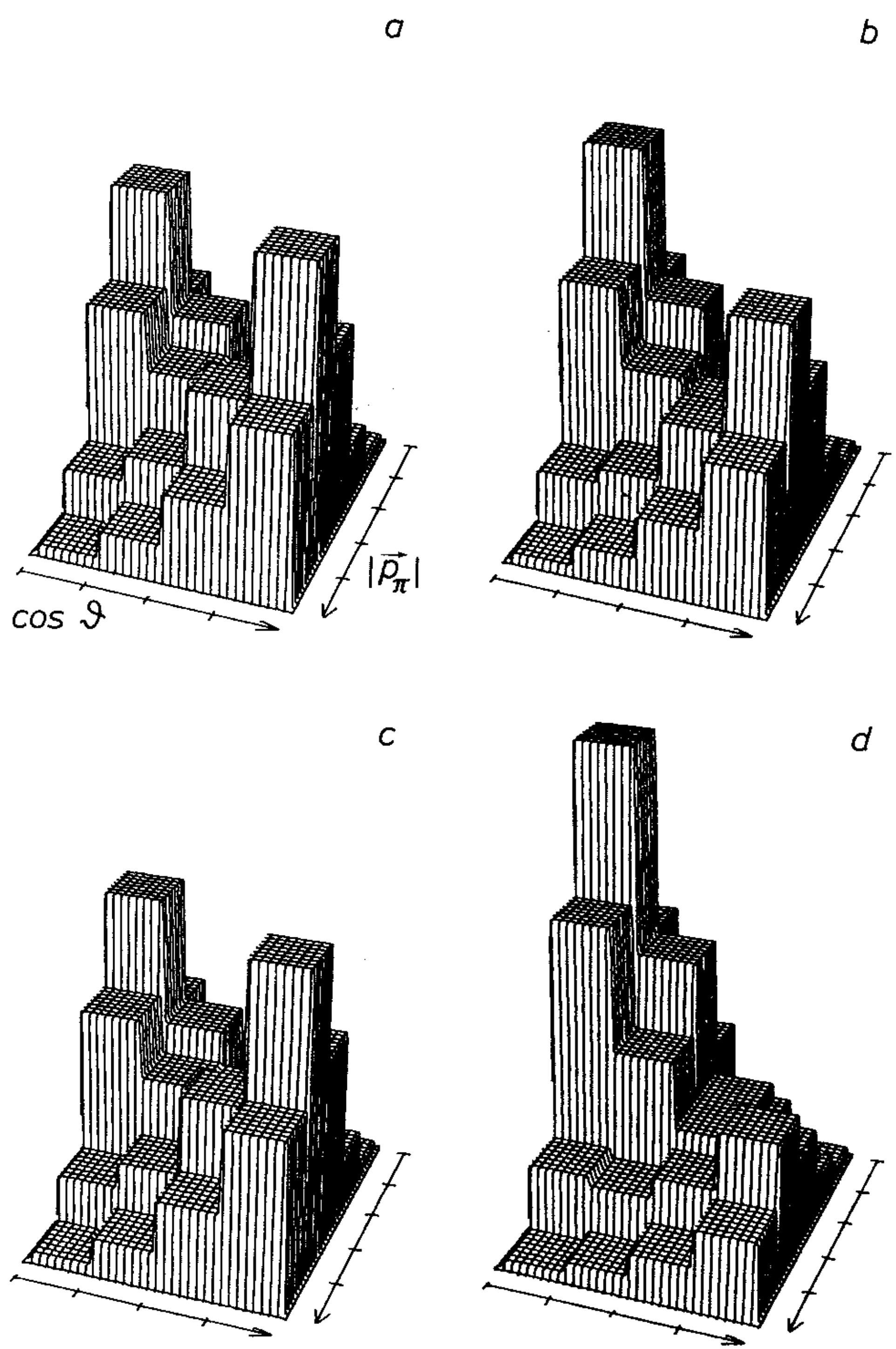


Figure 6



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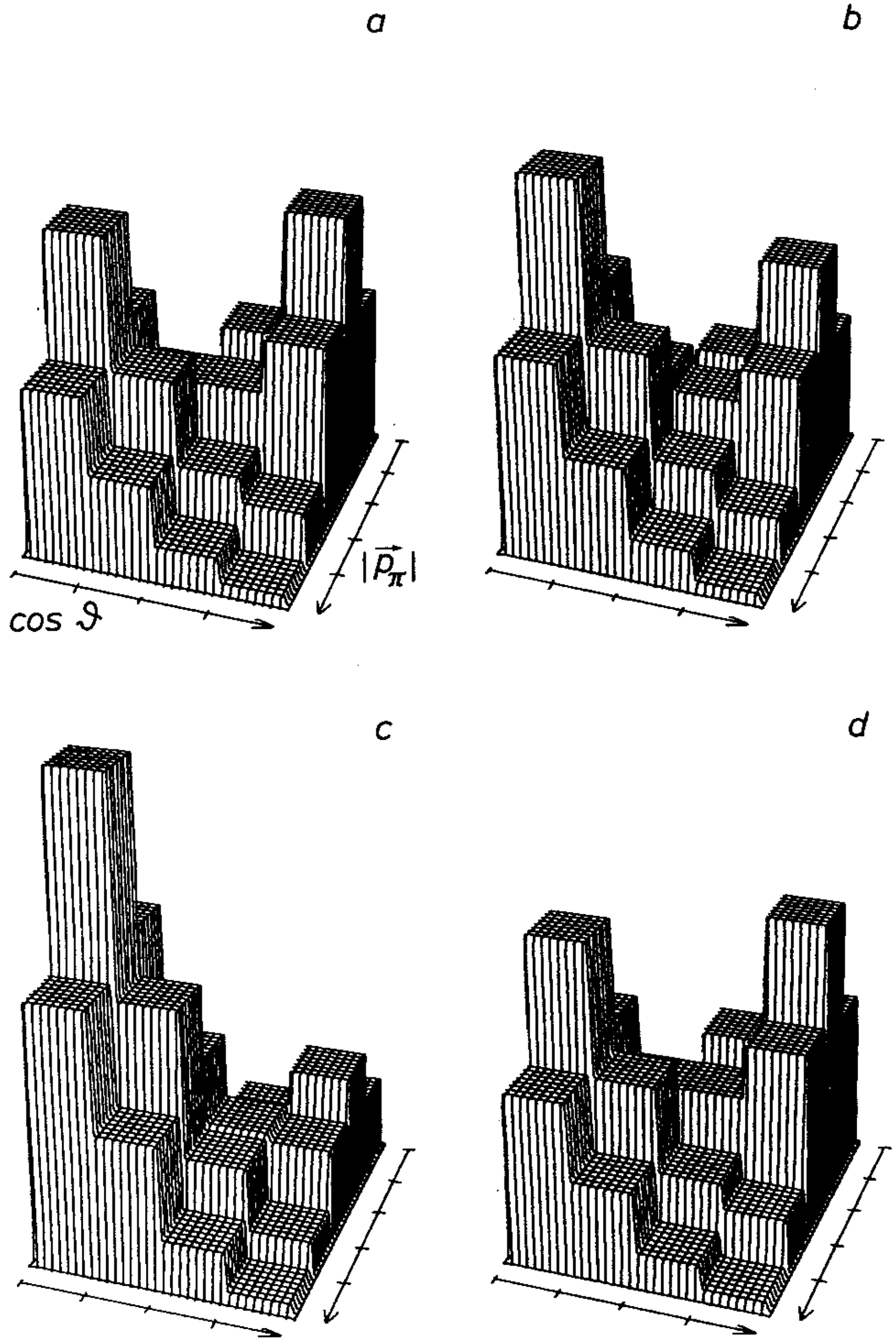

Figure 8