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On the Pomeranchuk Singularity in Massless Vector Theories

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Abstract

It is shown that the Pomeron in massless (abelian of nonabelian) vector theories, as derived from a perturbative high energy description which satisfies unitarity, comes as a diffusion problem in the logarithmic scale of transverse momentum. For a realistic theory there are reasons to expect that this diffusion should come to a stop: (a) the long range forces of the massless gluons should be screened, (b) the Pomeranchuk singularity in the j-plane should be t-dependent, and (c) there should not be a discontinuity in the zero mass limit at t=O or in the t O limit of the massless case. In the third part we outline a scheme for summing all diagrams which are required by unitarity. It uses reggeon field theory in zero transverse dimensions and leads to: (i) the diffusion comes to a stop (zero drift and zero diffusion constant); (ii) the total cross section is constant (up to powers of lns); (iii) in order to give a meaning to the divergent perturbation expansion, one has to add a nonperturbative term of the order $\exp(-\mathrm{const}/g^2)$.

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I. Introduction

The problem of determining the high energy behavior (nature of the Pomeron) of massless vector theories - abelian or nonabelian - has attracted the attention for many years. Almost all studies of this subject start from perturbation theory: order by order of the bare coupling constant one finds leading powers of lns and then takes their sum. It is well known that the leading-lns approximation alone does not make sense for this limit: it violates the Froissart bound and thus indicates that one has to go far beyond this approximation before one can hope to have a reliable high energy description. Attempts to specify such classes of terms in the perturbation expansion which go beyond this leading-lns approximation exist[1)-4)]. It is hoped that these contributions satisfy complete unitarity (asymptotically) and thus form the basis of a realistic high energy theory. One then is facing the next problem: how to find the sum of all these terms.

It is the purpose of this paper to formulate this summation problem, as it emerges within massless vector theories. All our discussions will be based upon the assumption that the problem of finding the perturbative high energy approximation which is in agreement with unitarity has already been solved. We shall try to formulate a summation technique which is capable to determine the leading behavior of all these diagrams; it applies, however, only to the nonabelian case. We start from an observation which has been made by Kuraev, Lipatov, and Fadin[5)] in the context of analysing the leading-lns approximation for the Pomeranchuk singularity. They demonstrated that in the vicinity of the leading j-plane singularity the summation of Feynmann diagrams can be seen as a diffusion problem in the logarithmic scale of the intrinsic transverse momentum. We first generalize this to the much larger class of diagrams which are expected to satisfy complete asymptotic unitarity. A qualitative discussion of possible solutions to this diffusion problem then adresses various physical aspects of high energy scattering: as long as the diffusion constants (drift and diffusion coefficient) remain different from zero, there is an instability with respect to transverse momentum. Although the transverse momentum integrations are convergent, on the average both large and

small values of momentum k become increasingly important, and the region of small k is responsible for making the extension of the hadron (in the transverse direction) very large. This is interpreted as a reflection of the fact that forces are mediated by massless particles. At the same time, the Pomeranchuk singularity stays t-independent (fixed cut in the j-plane), and the transition from the massive vector theory (spontaneously broken gauge theory) to the massless case cannot be smooth. All this indicates that, in nonabelian theories which are expected to describe the real world (absence of long range forces as a result of confinement, shrinkage of elastic scattering), the diffusion constant should be zero. We are then asked to explain which property of the gluon interaction is responsible for such a peculiar result. In abelian theories the situation may be different, and the diffusion may persist.

Guided by this general and qualitative discussion we will formulate a scheme for summing all the necessary Feynmann diagrams. It only holds for the nonabelian case and leads to the conclusion that the diffusion constant is zero (up to corrections of the order $\exp(-1/g^2)$). This then avoids all the undesirable features that we have listed above. The basic idea is this: we argue that the task of summing all the necessary terms of the perturbation expansion is equivalent to solving a certain reggeon field theory (RFT) with a negative mass (in zero transverse dimensions). The physical results of this procedure are the following: the Pomeron has intercept one (up to corrections of the order $\exp(-1/g^2)$), and the nonzero slope has both a nonperturbative piece (coming from the hadronic wave function) and another piece which is due to the gluon interactions and, hence, calculable. It is possible that this second contribution has the right size for explaining the smallness of the Pomeron slope. The hadron radius behaves, approximately, as in the multiperipheral model, i.e. it grows linearly with rapidity Y.

This scheme also indicates that the Regge limit, strictly speaking, lies outside the region where perturbation theory converges. This is most easily phrased in the language of reggeon field theory, whose phase structure has carefully been analysed during the last years[8)-10)]. Wthin our perturbative starting point we might, at first sight, think that the physical Pomeron is a bound state in the t-channel, made out of two, four.... gluons. After translating into the language of reggeon field theory it appears, however, that this is like expanding around the wrong vacuum: the physical Pomeron only appears after an infinite resummation. It even could be that an expansion of the physical Pomeron in terms of gluon states in the t-channel with any finite number of gluons is not convergent. We also learn from reggeon field theory that, when analytically continuing from the phase with positive mass to that with negative mass, one has to add a term of the order exp $(-2\Delta^2/\lambda^2)$. Translating this back to the underlying gauge theory, we see that in order to give a meaning to the perturbation expansion in the Regge limit, we have to add a certain term of the order exp $(-1/g^2)$. The form of this term is dictated by the structure of reggeon field theory, but we do not yet have a physical interpretation of it.

The discussion will be organized in three parts. We first (section II) describe the perturbative starting point, i.e. the Feynmann diagrams which we need to sum in order to have a unitary high energy theory. Since the program of finding such a unitary high energy approximation has not yet been completed, we have to make a few assumptions which we shall describe and motivate. We then formulate the mathematical problem which has to be solved, and we show that it is equivalent to a diffusion problem. In section III we present a qualitative discussion of some possible solutions, and section IV contains the calculational scheme which explains whay the diffusion constant, in nonabelian theories, vanishes. In the final section we present a brief summary.

II. Formulation of the Problem

All our following discussions will be based upon the perturbative calculations of the high energy behavior of massless vector theories which have been presented in other publications[1)-4)]. A common feature of all these calculations is the fact that all the vector particles have been made massive, either by hand (for QED) or by the Higgs mechanism (for the nonabelian case), hoping that at the end of all calculations the mass can be removed, and a correct description of the massless theory is reached in this way. At the stage where all particles of the theory are massive, it is possible to discuss unitarity properties of the high energy description: complete (asymtotic) unitarity in both s- and t-channel is considered to be the guideline in selecting those terms in the perturbation expansion which have to be retained for a reliable high energy description.

For the abelian case (QED) the most extensive attempt to achieve unitarity has been made in[1) 2)]: the high energy behavior of a $2\rightarrow 2$ scattering amplitude, e.g. electron-electron scattering, is described by all those diagrams (Fig.1) where - looked from the t-channel - any number of photon lines (carring two-dimensional transverse momentum) interact among themselves through closed fermion loops. As a result of any of these interactions, the number of photon lines may change, but in the vacuum exchange channel it must always be even. It has not yet been verified that these diagrams satisfy unitarity in all channels, but in the following we shall assume that the sum of all these diagrams represents, in the limit of small coupling constant, the correct high energy description of the (massive) theory.

For the nonabelian case a systematic attempt for achieving a fully unitary high energy description has been started[3)-4)6)7)]. In contrast to the abelian vector particle of QED, the nonabelian vector particle reggeizes and the high energy theory comes as a complete reggeon calculus. Its elements are dictated by the requirement of satisfying s-channel unitarity. The calculational program which has been described in Ref.6 allows to compute them in the weak coupling limit. Parts of this program have been carried out in Refs.3)4), and the form of the n-reggeon n-reggeon vertex functions has been derived in the second part of Ref 4. The general n-reggeon→m-reggeon vertex function is not yet completely known, but for the discussion in this paper it will be sufficient to have a few properties of them which follow from general considerations. Before the zero mass limit of this reggeon calculus can be taken, it is necessary to rewrite it in a form which is analogous to the high energy description of QED: by expanding each reggeon propagator in powers of the coupling constant g, we "undo" the reggeization and thus arrive at a description in terms of the elementary vector particle. The reggeon calculus is, therefore, equivalent to the sum of all diagrams of Fig.2, where any number of vector particles in the t-channel interact via (momentum dependent) vertex functions. Since in nonabelian theories vector particles have selfinteraction terms, it is not necessary to include fermions (in fact, it costs more energy to produce fermion pairs than vector particles). For the following discussion we will assume that the calculational program which has been started in Refs.4 will prove full (asymptotic) unitarity of the diagrams of Fig.2, and for this paper we are then left with the task of finding the sum of all these diagrams.

In order to take the zero mass limit (in either abelian or nonabelian case) one has to modify the coupling of the exchanged vector particles to external particles. The simplest model for this seems to be a closed fermion loop (Fig.3): in QED the photon annihilates into an e^+-e^- pair, and in the nonabelian case one may think of a composite meson which breaks up into quark-antiquark (the detailed form of the meson wave function is not relevant for what we have to say). For the simplest case where only two vector particles are exchanged (and interact in the t-channel) it has been shown[4) 11)] that the zero mass limit is finite, order by order perturbation theory. In Ref.4 this has been extended (for the nonabelian case) to the general class of diagrams where the number of vector particles in the t-channel is conserved. For the following discussion we make the assumption that the zero mass limit exists for all diagrams of Figs.1 and 2, once the external particles are chosen to be composite objects. This defines the (perturbatively derived) high energy description for massless vector theories.

The diagrams of Figs.1 and 2 are most easily described in terms of two dimensional transverse momentum k and the t-channel angular momentum $j=\omega+1$ (Fig.4): we then have a propagator $1/k^2$ for each vector particle line, a factor $1/\omega$ for each t-channel intermediate state, and a function $\hat{K}_{NN'}$ for the interaction vertex: N vectors $\rightarrow$ N' vectors. They depend

upon the momenta of the incoming and outgoing vector particles and contain the (unrenormalized) coupling constant of the theory (as we have said already, in case of QED, the $\hat{K}_{NN'}$ consist of closed fermion loops. We take the fermions to be massless. For the nonabelian case, the $\hat{K}_{NN'}$ come from certain tree diagrams of vector particles, as well as the trajectory function of the reggeizing vector particle (for details see Ref.4)). For the high energy behavior of the diagrams of Fig.4 one is interested in the leading (=right-most) singularity in the j-plane: one wants to know its position and the behavior of the scattering amplitude in its vicinity. At first sight this becomes the problem of solving a set of soupled integral equations. In order to see this in detail we organize the summation of the diagrams in the following way. First we divide in Fig.4 each propagator line into two pieces: one square root of the propagator $1/k^2$ goes into the upper end vertex, the other one is absorbed by the lower end vertex. We then obtain new vertex functions $K_{NN'}$ which differ from the old ones $\hat{K}_{NN'}$ by such a square root factor for each incoming and outgoing vector line. For the couplings of the exchanged vector lines to the external particles, we have functions $\hat{V}_N$, which depend upon the transverse mementa of the vector lines, but not on ω. After absorbing the appropiate number of square root factors of vector propagators, these functions are replaced by the new functions V_N. We now remove in Fig.4 the lower vertex parts V_N and denote the remainders by φ_N: these functions which depend upon $k_1, \ldots, k_N$ ($\sum_{i=1}^{N} k_i = q, q^2 = -t$) and represent the sum of all diagrams in Fig.4 with N lines at the lower end, satisfy the following integral equations:

$$\begin{pmatrix} \varphi_2 \\ \varphi_4 \\ \vdots \end{pmatrix} = \begin{pmatrix} V_2 \\ V_4 \\ \vdots \end{pmatrix} + \frac{1}{\omega} \begin{pmatrix} K_{22} & K_{24} & \cdot \\ K_{42} & K_{44} & \vdots \\ \cdot & \cdot & \cdot \end{pmatrix} \tag{1}$$

or, symbolically:

$$\varphi = V + \frac{1}{\omega} K \varphi \tag{2}$$

In (1) the $K_{NN'}$ stand for integral operators, depending on the N incoming and N' outgoing momenta. In terms of the φ, the scattering amplitude is:

$$T_{2\to 2} = \frac{i\pi s}{2\pi i}\int d\omega\, s^{\omega} \sum_{N=2}^{\infty} \int \prod_{i=1}^{N} d^2k_i\, \delta(\Sigma k_i - q) \cdot$$
$$\cdot V_N(k_1,\dots,k_N)\,\varphi_N(k_1,\dots,k_N;\omega) \qquad (3)$$

The functions φ (and the scattering amplitude) will have a singularity in ω, when the homogeneous version of (1) has a solution, i.e. singularites in ω are eigenvalues of the kernel operator K. For sufficiently large positive values of ω we do not expect such singularities. Consequently, the Neumann expansion in powers of $1/\omega$ should converge:

$$\varphi = \sum_{n=0}^{\infty} \varphi^{(n)} = \sum_{n=0}^{\infty} \left(\frac{1}{\omega}K\right)^n V \qquad (4)$$

When ω decreases, the convergence of (4) becomes more and more slowly, until we reach the right-most singularity where the series diverges.

To find an analytic solution to eq.(1) is, of course, beyond any hope. But a clever observation made first by Fadin, Kuraev, and Lipatov[5] allows to investigate the leading singularity of (1) and the behavior near this point without finding the exact solution to (1). We will from now on restrict ourselves to the massless case and to the point t=0. The observation of Ref.5 then is that near the leading singularity the dynamics of (1) can be formulated as a diffusion problem. To explain this in some detail we first note that the kernels $K_{NN'}$ in (1) are dimensionless, i.e. the expressions

$$K_{NN'}(k_1..k_N; k'_1,\dots,k'_{N'})\, \delta(\Sigma k_i)\, d^2k'_1 \dots d^2k'_{N'} \qquad (5)$$

are invariant under rescaling all momenta:

$$k_i \to \varsigma k_i \quad , \quad k'_i \to \varsigma' k'_i \qquad (6)$$

This is, of course, a consequence of the gauge coupling constant being of dimension zero. For the sets of momenta $k_1, \dots, k_N$ and $k'_1, \dots, k'_{N'}$ we define scale factors λ and λ':

$$\lambda = \frac{1}{N}\sqrt{\Sigma k_i^2} \qquad \lambda' = \frac{1}{N'}\sqrt{\Sigma k_i'^2} \qquad (7)$$

After the change of variables:

$$k_i \to \lambda k_i \quad , \quad k'_i \to \lambda' k'_i \qquad (8)$$

the momenta k_i and k'_i are constrained by $\Sigma k_i^2 = N$, $\Sigma k_i'^2 = N'$, and the kernels $K_{NN'}$ now also depend upon λ and λ'. Because of the scale invariance of (5), however, $K_{NN'}$ depends on λ and λ' only through the combination λ/λ'. It is therefore convenient to introduce the logarithms of the scale variables (in order to make the momenta dimensionless, we replace all k's by k/m, where m is some external mass):

$$\xi = \ln \lambda \ , \qquad \xi' = \ln \lambda' \qquad (9)$$

In terms of these logarithmic scale factors, the kernel only depends on the difference $\xi - \xi'$ but not on ξ and ξ' seperately. This fact has important implications as we shall discuss now.

Let us first, for simplicity, ignore all variables other than the logarithmic scale ξ. We than have a simple diffusion (or random walk) problem: The n-th term in the expansion (4) is obtained from the n-fold convolution of the kernel:

$$\varphi^{(n)}(\xi) = \frac{1}{\omega^n}\int d\xi_1 \dots d\xi_n\, K(\xi-\xi_1)\dots K(\xi_{n-1}-\xi_n)\, V(\xi_n) \qquad (10)$$

For large n we know from the central limit theorem[12] that K^n becomes a normal distribution whose parameters are determined by the first moments of K:

$$\frac{1}{\omega_0^n} K^n(\xi) \underset{n\to\infty}{\sim} \frac{h_0^n}{\sqrt{n h_2 \pi}} \exp\left[-\frac{(\xi - n h_1)^2}{n h_2}\right] \qquad (11)$$

where

$$h_0 = \frac{1}{\omega}\int d\xi\, K(\xi)$$
$$h_1 = \frac{1}{h_0}\frac{1}{\omega}\int d\xi\, \xi\, K(\xi) \qquad (12)$$
$$\frac{1}{2}h_2 = \frac{1}{h_0}\frac{1}{\omega}\int d\xi\, (\xi - h_1)^2 K(\xi)$$

and ω_0 has to be chosen such that $h_0 \approx 1$. From this it follows that after n steps of iterating (1), the expectation values of ξ and $(\xi - nh_1)^2$ grow linearly with n:

$$\langle \xi \rangle = \iint d\xi d\xi' \, \xi \, \frac{1}{\omega_0^n} K^n(\xi-\xi') V(\xi') \tag{13}$$

$$\underset{n\to\infty}{\sim} n h_1 h_0^n \int d\xi' V(\xi') + h_0^n \int d\xi' \xi' V(\xi')$$

$$\langle (\xi - nh_1)^2 \rangle = \iint d\xi d\xi' (\xi - nh_1)^2 \frac{1}{\omega_0^n} K^n(\xi-\xi') V(\xi') \tag{14}$$

$$\underset{n\to\infty}{\sim} \tfrac{1}{2} n h_2 h_0^n \int d\xi' V(\xi') + h_0^n \int d\xi' \xi'^2 V(\xi')$$

For later purposes we also give the expectation value of $e^{\pm\xi}$:

$$\langle e^{\pm\xi} \rangle = \int d\xi d\xi' e^{\pm\xi} \frac{1}{\omega_0^n} K^n(\xi-\xi') V(\xi') \underset{n\to\infty}{\sim} h_0^n e^{\pm nh_1 + n\frac{h_2}{4}} \int d\xi' e^{\pm\xi'} V(\xi') \tag{15}$$

From this we see that all the essential information we need is contained in the moments of K. The condition $h_0=1$ or $\omega_0 = \int d\xi K(\xi)$ determines the position of the leading singularity: as long as $h_0 < 1$, the expansion (4) converges, and for $h_0=1$ we have reached that point, where (4) becomes singular. Near this singularity, i.e. for ω_0 such that $h_0=1$, we see from eqs.(13), (14), and (15) that the mean values of the transverse momenta change as a function of n: this defines the region of phase space which is responsible for generating the Pomeron singularity. The physical implications of this growth of transverse momenta will be discussed in the following section. Here we only note that in our problem h_1 vanishes: since the dynamics of the exchanged vector particles cannot change when we interchange target and projectile, the kernel $K(\xi)$ in (1o) must be symmetric in ξ, and its first moment is zero. This implies that the mean value of ξ, according to (13), does not increase as a function of n: it is détermined by the mean ξ-value of the vertex function V which plays the role of the "initial" value of $\langle\xi\rangle$ at "time" n=0. The mean value of ξ^2, on the other hand, will grow with n, unless some very special reason is found which makes $h_2=0$.

So far we have simplified the problem by ignoring that K in (1) not only depends upon ξ but also on N (number of vector paricles) and the k_i (transverse momenta with their overall scale being fixed). This makes K in (1o) being an operator: it comes as an infinite dimensional matrix whose elements are labelled by N (a discrete variable) and the k_i (continous variables). Similarly, the $\varphi^{(n)}$ and V are vectors. It is not difficult to generalize the central limit theorem to the case where the kernel is a matrix rather then just

a function: one shows that the limit

$$\lim_{n\to\infty} \left[\tilde{K}(q/\sqrt{n}) \right]^n \tag{16}$$

is finite (here $\tilde{K}(q)$ is the Fouriertransform operator of $K(\xi)$). Let us state the results which are relevant for our case. For simplicity we shall assume that $K(\xi)$ can be diagonalized:

$$K_d(\xi) = \int d\xi_1 d\xi_2 \Lambda^{-1}(\xi-\xi_1) K(\xi_1-\xi_2) \Lambda(\xi_2) \tag{17}$$

$$\int d\xi_1 \Lambda^{-1}(\xi-\xi_1)\Lambda(\xi_1) = \delta(\xi)$$

such that the operator $K_d(\xi)$ is diagonal. Let us further restrict ourselves to the case where the eigenvalues of the operator $h_0 = \frac{1}{\omega}\int d\xi K(\xi)$ are discrete and ordered

$$\mathrm{Re}\,\epsilon_0 > \mathrm{Re}\,\epsilon_1 > \ldots \tag{18}$$

Since in (1o) the kernel K comes with a factor $1/\omega$, we can chose ω such that the largest eigenvalue of K/ω takes the value one, and all others are smaller than one:

$$1 = E_0 > \mathrm{Re}\,E_1 > \ldots \tag{19}$$

For the n-fold convolution of K/ω we have, making use of (17):

$$\frac{1}{\omega^n} K^n(\xi-\xi') = \int d\xi_1 d\xi_2' \Lambda(\xi-\xi_1) \frac{1}{\omega^n} K_d^n(\xi_1-\xi_2) \Lambda^{-1}(\xi_2-\xi') \tag{2o}$$

Since K_d is diagonal we can use (11): for large n only the largest eigenvalue $E_0=1$ survives, and we find:

$$\frac{1}{\omega_0^n} K^n(\xi-\xi')_{ij} \underset{n\to\infty}{\sim} \int d\xi_1 d\xi_2 \Lambda(\xi-\xi_1)_{i0} \frac{E_0^n}{\sqrt{\pi n \sigma_0}} e^{-\frac{(\xi_1-\xi_2-na_0)^2}{n\sigma_0}} \Lambda^{-1}(\xi_2-\xi')_{0j} \tag{21}$$

where a_0 and σ_0 are the (0,0)-elements of the operators

$$h_1 = h_0^{-1} \frac{1}{\omega} \int d\xi \, \xi K_d(\xi) \tag{22}$$

and

$$\tfrac{1}{2} h_2 = h_0^{-1} \frac{1}{\omega} \int d\xi \, (\xi - a_0)^2 K_d(\xi) \tag{23}$$

respectively. After n steps of iterating (1o) only the state $|o\rangle$

which belongs to the largest eigenvalue E_0 survives. The mean values of $\xi, (\xi - na_0)^2$ and $e^{\pm\xi}$ in this state are:

$$\langle \xi \rangle_0 = \frac{1}{\omega_0^n} \iiint d\xi\, d\xi_1\, d\xi_2\, \xi\, K_d^n(\xi-\xi_1)_{0j}\, \Lambda^{-1}(\xi_1-\xi_2)_{0j}\, V(\xi_2)_j \tag{24}$$

$$\underset{n\to\infty}{\sim} E_0^n \Big\{ n a_0 \int d\xi_1\, \Lambda^{-1}(\xi_1)_{0j} \int d\xi_2 V(\xi_2)_j + \int d\xi_1 \xi_1\, \Lambda^{-1}(\xi_1)_{0j} \cdot$$

$$\cdot \int d\xi_2 V(\xi_2)_j + \int d\xi_1\, \Lambda^{-1}(\xi_1)_{0j} \int d\xi_2\, \xi_2 V(\xi_2)_j \Big\}$$

$$\langle (\xi - na_0)^2 \rangle_0 = \frac{1}{\omega_0^n} \iiint d\xi\, d\xi_1\, d\xi_2\, (\xi - na_0)^2 K_d^n(\xi-\xi_1)_{00}\, \Lambda^{-1}(\xi_1-\xi_2)_{0j}\, V(\xi_2)_j$$

$$\underset{n\to\infty}{\sim} E_0^n \Big\{ n\sigma_0 \int d\xi_1\, \Lambda^{-1}(\xi_1)_{0j} \int d\xi_2 V(\xi_2)_j + \int d\xi_1\, \xi_1^2\, \Lambda^{-1}(\xi_1)_{0j} \cdot$$

$$\cdot \int d\xi_2 V(\xi_2)_j + \int d\xi_1\, \Lambda^{-1}(\xi_1)_{0j} \int d\xi_2\, \xi_2^2\, V(\xi_2)_j + \tag{25}$$

$$+ 2 \int d\xi_1\, \xi_1\, \Lambda^{-1}(\xi_1)_{0j} \int d\xi_2\, \xi_2 V(\xi_2)_j \Big\}$$

$$\langle e^{\pm\xi} \rangle_0 = \frac{1}{\omega_0^n} \iiint d\xi\, d\xi_1\, d\xi_2\, e^{\pm\xi} K_d^n(\xi-\xi_1)_{00}\, \Lambda^{-1}(\xi_1-\xi_2)_{0j}\, V(\xi_2)_j$$

$$\underset{n\to\infty}{\sim} E_0^n\, e^{\pm n h_1 + n\sigma_0/4} \int d\xi_1\, e^{\pm\xi_1} \Lambda^{-1}(\xi_1)_{0j} \int d\xi_2\, e^{\pm\xi_2} V(\xi_2)_j \tag{26}$$

In eq. (24) we notice the appearance of a new term which was not present in (13): it is due to the operator Λ which takes us from the "perturbative" (N,k_i) basis to the basis of eigenstates of K_d. At time n=0, $\langle \xi \rangle_0$ is given by its initial value, namely the average scale of the vertex function V. Transforming to the basis of eigenstates of K_d, a new piece is added: $\int d\xi\, \xi\, \Lambda^{-1}(\xi)$. Then time starts to grow, and $\langle \xi \rangle_0$ increases linearly with n. In the following section we shall argue, that a_0 should be zero: then only the two constant pieces remain and determine $\langle \xi \rangle_0$.

What we learn from this is that the essential information is contained in the first moments of the operator K_d: the eigenvalues of

h_0, and the first elements of h_1 and h_2 +). For later convenience we rephrase this diffusion problem also in Fourier space. Let $\tilde{K}(q)$ be the Fourier transform of $K(\xi)$. Then eq. (17) reads:

$$\tilde{K}_d(q) = \tilde{\Lambda}^{-1}(q)\, \tilde{K}(q)\, \tilde{\Lambda}(q) \tag{27}$$

The operators h_0, h_1, and h_2 are related to the derivatives of $\tilde{K}_d(q)$:

$$h_0 = \frac{1}{\omega} \tilde{K}_d(0)$$
$$h_1 = \frac{1}{\omega} (-i)\, h_0^{-1} \tilde{K}_d'(0) \tag{28}$$
$$\tfrac{1}{2} h_2 = -\frac{1}{\omega} h_0^{-1} \tilde{K}_d''(0)$$

Since the limit $(\tilde{K}_d(q/\sqrt{n}))^n$, $n \to \infty$, is determined by the first two derivatives of $\tilde{K}_d(q)$ at q=0, we have the following recursion relation

$$\tilde{K}_d(q/\sqrt{n})^n = \Big[\tilde{K}_d(0) + \frac{q}{\sqrt{n}} \tilde{K}_d'(0) + \frac{q^2}{2n} \tilde{K}_d''(0)\Big] \tilde{K}_d(q/\sqrt{n})^{n-1} \tag{29}$$

For large n we can take n to be continous and obtain a differential equation for the operator $O(n,q) = \tilde{K}_d^n(q)/\omega^n$:

$$-\frac{\partial O(n,q)}{\partial n} = \frac{1}{\omega}\Big[\omega - \tilde{K}_d(0) - q\, \tilde{K}_d'(0) - \frac{q^2}{2} \tilde{K}_d''(0)\Big] O(n,q) \tag{30}$$

A similar equation holds for the time evolution of the state $\tilde{\varphi}_d(n,q)$:

$$\tilde{\varphi}_d(n,q) = \tilde{\Lambda}^{-1}(q)\, \tilde{\varphi}^{(n)}(q)$$

$$-\frac{\partial \tilde{\varphi}_d(n,q)}{\partial n} = \frac{1}{\omega}\Big[\omega - \tilde{K}_d(0) - q\, \tilde{K}_d'(0) - \frac{q^2}{2} \tilde{K}_d''(0)\Big] \tilde{\varphi}_d(n,q) \tag{31}$$

The last two equations are Schrödinger equations with n playing the role of (negative imaginary) time, and the square bracket term on the rhs acting as Hamilton operator in the space of states which are labelled by N and k_i. The variables q and ω act as parameters on which the Hamilton operator depends. The condition (19) then means that the energy spectrum should be bounded from below. The leading singularity in ω occurs when the lowest energy eigenvalue becomes zero.

+) The symmetry between target and projetile implies that $K(\xi) = K(-\xi)^T$. If the diagonalization matrix Λ would be unitary, we could conclude that $a_0=0$: in general, however, this will not be the case and we must consider the situation with $a_0 \neq 0$.

If near q=0 this lowest energy eigenvalue would be independent of q, then a_0 and σ_0 would be zero, and the expectation values of $\xi, \xi^2, e^{\pm\xi}$ would not grow as a function of n. In section IV we shall argue that exactly this happens in nonabelian theories.

III. The Physical Picture

Before we approach the question how to calculate in practice these parameters which contain all the dynamics of the Pomeron, we wish to discuss the physical implications of various possible solutions to the diffusion problem. Eq.(4) suggests to start from the well-known space time picture[13]: the functions φ_N (which, up to the integration in (3), contain all the information about the behavior of the scattering amplitude) are expanded in powers of $1/\omega \sim \ln s$, and each term in this expansion has a simple physical interpretation. In order that a fast incoming hadron can interact with a target at rest, its fast constituents (in our picture: the pair of fermions inside the vertex functions V) have to initiate cascading processes, at the end of which slow partons are created. The functions $\varphi_N^{(n)}$ then describe the situation after n steps of the cascade: the rapidity has decreased by n units, and the partons which are created at this step are seperated from the line of flight of the incoming hadron by some distance in impact parameter. In order to find this distance we have to Fourier transform from transverse momentum to impact parameter: if an exchanged vector particle which mediates the interaction between step "n" and "n+1" carries momentum k, then its two endpoints are seperated by the distance $|\Delta b| \sim \frac{1}{|k|}$. Sinc step "n" and step "n+1" are connected through N exchanged particles, we have to average over the length (in impact parameter) of all these N lines: this gives us the mean distance between the partons of step "n" and those of step "n+1". Finally, in order to find the total distance of partons of step "n" from the line of light of the incoming hadron, we have to add up all these mean distances between the n steps. To describe the situation in other words: the incoming fast fermion pair is surrounded by a cloud of partons - in the language of QCD: a cloud of gluons -, and the slowest ones which may be far out in impact parameter determine the extension of the incoming scattering object. In order to find out how large this hadron radius is, we have to investigate how the mean values of transverse momentum (and impact parameter distances) behave as a function of the number of steps.

What we see from eqs.(21) - (26) is that near the leading singularity in the ω-plane a diffusion takes place: the variable ξ plays the role of space, n acts as time, a_0 is the drift speed, and σ_0 is the diffusion constant. Let us fist assume that both a_0 and σ_0 are different from zero and see what this implies for the distribution in ξ of the wee partons. After sufficiently long time only one configuration of vector particles will survive, namely the eigenstate of the largest eigenvalue of h_0. All other states die out more or less rapidly. Let us then see how, within this configuration, the mean value of ξ changes as a function of time. At time=0 we are at the vertex function V, and the distribution in ξ is given by the dynamics of the fermion-anti-fermion pair. Because of the transformation Λ^{-1} in (24), the mean value of ξ then changes by some finite amount (which depends upon Λ, i.e. the dynamics of the vector particles and not the external particles). After this ξ starts to grow linearly with time n, and at the same time the probability distribution broadens (the width is proportional to $n \cdot \sigma_0$). At the final time (which depends on the total rapidity) we reach the other vertex V, and our distribution K^nV is folded into the ξ-distribution of this external particle. When the final time increases, both large positive and negative values of ξ (i.e. very small and very large values of transverse momentum) become increasingly important: from (21) we see that initial and final value of ξ (ξ and ξ', respectively) want to be seperated by the amount $n.a_0$. The initial value thus is pushed to large negative values, whereas the final value goes to large positive values or vice versa. Moreover, the spread around these mean values grows with n, and even at $t=t_{final}/2$ where the mean value of ξ may be finite, large positive and negative values of ξ are important because of the large <u>width</u> of the distribution in ξ. From this it follows that even if $a_0=0$ (but still $\sigma_0 \neq 0$)+) the dominance of large values $|\xi|$ would persist. In order to make all this more explicit, we have in (26) given the time evolution of the expectation value of $k=e^{\xi}$ and its inverse: they grow exponentially with time n. We conclude from this that, as long as a_0 or σ_0 are different from zero, the relevant scale of the intrinsic transverse momentum is highly unstable: with increasing rapidity, it wants to become very large and very small at the same time.

+) This holds for the leading-lns approximation in Ref.7: N is restricted to N=2, and the symmetry between target and projectile then leads to $a_0 = 0$.

It is not difficult to translate this into the geometrical picture in impact parameter space. Whenever the mean value of ξ between time n and time n+1 is positive or negative large, the corresponding $b=e^{-\xi}$ is small or large, i.e. the mean distance between partons of step n and step n+1 is small or large+). A crude estimate based on (26) then leads to a rapidly increasing hadron radius:

$$R^2 \sim \exp[\text{const.}\, Y] \qquad (32)$$

It is very instructive to compare this with the familiar random walk picture which is based upon the multiperipheral model. There the steplenght in impact parameter is constant (independent upon n), and the radius grows linearly with the total number of steps (or rapidity Y):

$$R^2 \sim \text{const.}\, Y \qquad (33)$$

It seems that, in order to reach such a situation within our framework, we need that both a_0 and σ_0 vanish: in this case the distribution in ξ stays stable as a function of time, i.e. the mean value stays constant and the width of the distribution around this mean value does not grow with n either. In the following we shall call the condition that both a_0 and σ_0 vanish the "stability condition". Among all possible solutions that our diffusion problem in massless vector theories may have, such a situation with $a_0=\sigma_0=0$ looks very exceptional. We therefore conclude that, if the experience from multiperipheral physics is taken seriously, there must be a special feature of the interaction of nonabelian gluons which is capable of satisfying these stability conditions.

The significance of this stability condition becomes clearer if we

+) If the steplength in impact parameter is large it is, strictly speaking, not correct to speak about "the position of partons at step n": they themselves have a distribution im impact parameter, and if the mean value of ξ before and after step n is negative large, this distribution is very broad.

remember that the situation (33) results from models where the forces are due to massive paricles in the t-channel (the constant on the rhs of (33) is inverse proportional to the mass square of this particle). In contrast to this, we have started from a theory where the particles in the t-channel are massless: we therefore have to expect that the forces which are mediated by the exchange of these massless vector particles are of long range type, i.e. the clouds of slow vector particles which surround the fast constituents of the incoming hadron are not much confined in impact parameter space. This suggests to identify the case (32) with a theory where the vector particles remain massless. If a massless vector theory, on the other hand, leads to a situation à la (33), then something must have happened in order to shield the long range nature of the forces, i.e. the massless vector particles no longer exist. Such an effect would be nonperturbative, i.e. our perturbative starting point alone would not be sufficient ot achieve this result. In the next section where we shall argue how nonabelian theories might be able to satisfy the stability condition, we shall see that, in fact, a nonperturbative contribution of the order $\exp(-1/g^2)$ has to be added to the diagrams of Fig.4. The constant on the rhs of (33) is the inverse of the Pomeron slope and has the meaning of the mean steplenght in impact parameter space. According to (26) it has two contributions: one comes from the vertex function V, i.e. the bound state dynamics of the fermion-antifermion pair, whereas the other part comes from the transition matrix Λ which takes us from the basis of gluon states (states with given N and k_i) to the eigenstate of h_O. Most naively, one could expect that the first contribution is relatively small: $|k| \sim$ few hundred MeV. In order to obtain a small Pomeron slope, the main contribution has to come from the transition matrix .

There is another aspect which distinguishes the case $a_O=0$ and/or $\sigma_O=0$ from the situation where the stability condition is satisfied: this is the nature of the Pomeranchuk singularity in the angular momentum plane. So far we have restricted ourselves to the point t=0: it is the behavior of the scattering amplitude near this point which determines the behavior at large values of impact parameter. If we take t away from zero in order to study the t-dependence of the power of s, the diffusion picture breaks down since the scale invariance of the kernel K no longer holds. Nevertheless, we can make the following qualitative statement. For the case t=0 we have seen

that unless the stability condition is satisfied both very small and very large values of transverse momentum become increasingly important when the total number of steps increases. If t is different from zero, then for the region of small k we only can say that, since it feels the presence of the scale breaking parameter t, no diffusion takes place and we do not have any mechanism which enhances the importance of this region as the number of steps increases. We therefore expect that at this end of the k-spectrum the distribution will stay constant as n varies. In the region of large k, on the other hand, the diffusion picture which lead to increasing importance of this region can still be used: for $k^2 \gg t$, we can neglect the parameter t in the kernel and our previous argument then tells us that in this part of the k-spectrum we have the same situation as before, i.e. large values of k come more and more into play when rapidity grows. After many steps, the system then "forgets" about the t-value, and the j-plane singularity is a fixed one (t-channel unitarity then requires that it is a cut and not a pole). The only way (within our framework) to stop this "loss of memory" is by preventing the diffusion for large k-values to take place, i.e. by satisfying the stability condition. Thus if we want the Pomeranchuk singularity to be t-dependent (e.g. because of shrinking of the elastic cross section), then we should look for a mechanism which, in nonabelian theories, leads to $a_O=\sigma_O=0$. The (very special) case of a moving Pomeron is found to be equivalent to the disappearance of the long range forces of the massless vector particles.

In order to complete this discussion about physical aspects of the different solution to the diffusion problem we say a few more words about the connection between the massless case and the massive one. In the very beginning of this paper we have put all masses of the vector particles equal to zero, assuming that no infrared problems arise (for a certain subset of diagrams of Fig.4 this has been studied in Ref.4), but a general proof is still missing). However, the derivation of the diagrams of Fig.4 has been done within the massive theory, and for a full understanding of the zero mass limit it is necessary to see how the diffusion dynamics changes as a function of the mass parameter. We first notice that the presence of a mass parameter in the kernel K spoils the scale invarance in the same way as a t-value different from zero. We therefore can repeat the same argument and conclude that for large n only the region of large k (and not the small k regime) dominates, giving rise to a fixed cut Pomeron singularity. Because of the presence of the mass parameter, however, the situation now does not depend upon whether t is equal to zero or not: even at t=0 we still have that only large k values are important for building the Pomeron. Correspondingly, the steps in impact parameter decrease as a function of n, and the resulting hadron radius grows more slowly than in (33). Taking now, still at t=0, the mass parameter to zero, the situation drastically changes. The diffusion now goes in both directions, and the emphasis of small k-values leads to the growth of the hadron radius according to (32) which is faster than (33). The zero mass limit, therefore, even if it is finite order by order perturbation theory, causes a jump in the dynamics of the wee partons, and the hadron radius is a discontinous function of the mass parameter. If, on the other hand, the constants a_0 and σ_0 vanish, then neither large nor small k-values come into play and the steplenght in impact parameter does not change with n. When the mass of the vector particle is taken to zero, there could be, at most, a finite change in the value of the steplenght, and the dynamics of the vector particles stays the same. We thus conclude that, only if the stability condition is satisfied, the zero mass limit has a good chance to be smooth.

To summarize the discussion of this section, we have seen that various aspects of high energy scattering are linked together: hadron radius (extension of the hadron in transverse direction) and the question whether there are long range forces, the nature of the Pomeron singularity in the j-plane, and the smoothness of the zero mass limit. Furthermore, we have given arguments that in nonabelian theories, which are expected to describe the real world, the diffusion should come to a stop. For this it is necessary that the stability conditions $a_0=\sigma_0$ are satisfied.

IV. Stability for the Nonabelian Case

We now turn to the most difficult part of our discussion: how could it happen that the constants a_0 and σ_0 vanish, and the transverse momenta remain stable? In this section we shall try to find, for the nonabelian case, an answer to this question. As we have said at the end of the first part of our discussion, what we have to show is that the largest eigenvalue of $\tilde{K}(q)$ in the vicinity of q does not depend on q at all (or so weakly that we can disregard this dependence).

Before we can start to calculate the eigenvalues of $\tilde{K}(q)$ we have to make approximations. Within our perturbative approach the operator K comes in the (N,k_i)-basis. The dependence upon N, as we shall discuss below, turns out to be crucial and cannot be disregarded. But for the k_i we have reasons to believe that their integration can be approximated by taking the integrands at some average value and thus replacing the integral by a constant multiplicative factor: since we have scaled out already the overall scale factor λ (eqs.(7) - (9)), the k_i are always of the order unity and cannot become arbitrarily large. As to the small k-region, it follows from our assumptions on the infrared finiteness that there are no divergencies. Each time, therefore, when in (1o) the kernel K acts on $\varphi^{(n)}$, the transverse momenta k_i are slightly rearranged but without any tendency of becoming very large or very small (for the diagonal elements K_{NN} we also know that they treat each vector particle line in the same way: this prevents the k_i to become very different from each other). We thus make the assumption that we are allowed to replace the k_i-integration by multiplication with the factor $K_{NN}(\xi)$. The operator K then becomes a matrix in N-space, and the matrix elements only depend on ξ (or, in Fourier space, on q).

The crucial point of our argument now is the following: if this assumption is correct, then the N-dependence of the diagonal elements $\tilde{K}_{NN}(q)$:a) distinguishes the nonabelian case from the abelian one and b) allows, for the nonabelian case, for a method of calculating the lowest eigenvalue of $\tilde{K}(q)$: it is found to be zero and independent of q (both results are correct up to terms of the order exp $(-1/g^2)$). In order to explain this in more detail, we first return to the simplest case where N is restricted to N=2: there the integral equation (1) has been solved exactly: for both the nonabelian[5)] [14)] and the abelian [15)] case the leading singularities in the ω-plane have been found to be positive: $\omega_2 = \frac{g^2}{\pi^2} N \ln 2$ for SU(N), and $\omega_2 = \frac{11}{32}\pi\alpha^2$ for QED. Within our approximation scheme we therefore put

$$\langle k_1' k_2' | K | k_1 k_2 \rangle \longrightarrow K_{22}(\xi - \xi') \qquad (34)$$

such that

$$\int d\xi\, K_{22}(\xi) = \omega_2 > 0 \qquad (35)$$

In Fourier space we have $\tilde{K}_{22}(0) = \omega_2$, and $\tilde{K}_{22}(q) > 0$ for sufficiently small values of q. The general diagonal element K_{NN}, for the SU(2) case, has been presented in Ref.3. Rather than giving here its explicit representation we only list those features which are needed for our discussion. The $N \to N$ transition is the sum of interactions K_{22} between any two of the N lines (Fig.5) and, hence, of the order g^2. Furthermore, there is an overall group weight factor $2/(N-1)$. The origin of this factor is easily understood: since each vector particle carries a quantum number, the total quantum number of N lines can take many different values, out of which we consider only one, namely the vacuum quantum number. The generalization to SU(L) is straightforward: the group weight factor becomes $C/(N-1)$ where C is a Casimir operator of the group:

$$f^{abc} f^{a'bc} = C\, \delta^{aa'} \qquad (36)$$

In order to estimate, for large N, the leading singularity in the ω-plane of K_{NN} we use the Hartee-Fock approximation. This leads to:

$$\langle k_1', .., k_N' | K | k_1, .. k_N \rangle \rightarrow K_{NN}(\xi - \xi')$$
$$\tilde{K}_{NN}(0) = \omega_N \sim N \frac{C}{2} \cdot \text{positive constant} \qquad (37)$$

Within our approximation scheme we therefore put:

$$\tilde{K}_{NN}(q) \sim \frac{N}{2} \Delta(q) \qquad (38)$$

with:

$$\frac{N}{2} \Delta(0) = \int d\xi\, K_{NN}(\xi) \qquad (N \text{ large}) \qquad (39)$$

For the abelian case we have already mentioned that the leading eigenvalue of K_{22} is positive, too. But when we come to the other diagonal elements of K, we see a difference between nonabelian and abelian theories. For the abelian case: there exists only one quantum number configuration in the t-channel, the vacuum exchange channel. Hence there is no group factor which suppresses the large N behavior, and the analogue to (37) is:

$$\omega_N \underset{N \to \infty}{\sim} \frac{1}{2} N(N-1) \cdot \text{positive constant} \qquad (40)$$

The fact that in both cases the diagonal elements of K are positive and grow when N becomes large indicates that perturbation theory diverges. We can think of summing the diagrams in Fig.4 in the following order: first we take only those diagrams where always N=2 and obtain the leading singularity to the right of j=1. The Froissart bound then is violated, i.e. our result is already too large. Then we include all diagrams with $N \leq 4$: the coupled set of integral equations is governed by the diagonal elements of K, because they are of order g^2 whereas the nondiagonal ones are of higher order. As a result, the leading singularity comes from K_{44} which, by (37), is larger than K_{22}. The amplitude thus grows even faster than in the previous case. Continuing in this way, the leading j-plane singularity moves arbitrarily far to the right. Consequently, a meaningful result for the sum of the diagrams of Fig.4 can be obtained only after an infinite summation of all terms.

Before we start to do this, we once more return to eqs.(39) and (40). Obviously, we now have reached the point where a distinction between abelian and nonabelian theories has to be made. So far, our discussion had been applicable to both cases: what went into the derivation of the diffusion picture was only the dimensionality of the coupling constant. Furthermore, the leading eigenvalue of K_{22} came out to be positive in both cases. Now, when we try to include the other matrixelements of K, the two theories require different methods. In this paper, we shall restrict ourselves to the nonabelian case and leave the abelian one to future studies.

It now will be useful to switch to the language of the Schrödinger equation which we have introduced at the end of section II. The behavior (37) then means that, for any finite and fixed value of ω, there is (at least around q=0) an infinite number of negative eigenvalues, and the spectrum is unbounded from below. Since (at least for large N) the eigenvalues are proportional to N, the spectrum is the same as that of an harmonic oscillator where the oscillator constant has the wrong sign. The N-vector particle state plays the role of the N-th excitation of the oscillator.

We now shall extend this analogy and argue that, when off-diagonal elements are taken into account, the problem of finding the eigenvalues of $\tilde{K}(q)$ can be reduced to reggeon field theory (RFT) in zero transverse dimension with a negative mass term. These theories have been investigated very thoroughly during the last years[8)-10)], and we can make use of the results. First we have to say what we know about non-diagonal elements of the matrix $\tilde{K}(q)$. Off-diagonal elements of $\tilde{K}$ are due to transitions in the t-channel where the number N of vector particles is not conserved. They are always of higher order in g^2 than the diagonal terms. The next-to-diagonal elements, for example, K_{NN+2}, are of the order g^3. The most important feature, however, of these elements is that some of them carry factors of i, just as in RFT. In order to see the reason for this we have to remember how the diagrams of Fig.4 were derived: starting from a massive (spontaneously broken) gauge theory, we first had to calculate a unitary high energy S-matrix which came in the form of a complete reggeon calculus. This we then reexpressed in terms of the diagrams â la Fig.4 where the lines denote elementary vector particles, and finally we took the zero mass limit $M^2 \to 0$. At the stage where the high energy behavior is still expressed in terms of the reggeon calculus we know from angular momentum theory that each vertex: 2n-reggeon→2m- reggeon is accompanied by signature factors:

$$\left[(-)^{n+1}\cos\frac{\pi}{2}\left(\sum_{i=1}^{2n}\alpha_i-2n\right)\right]^{1/2}\cdot\left[(-)^{m+1}\cos\frac{\pi}{2}\left(\sum_{j=1}^{2m}\alpha_j-2m\right)\right]^{1/2}\underset{\alpha\to 1}{\sim}\sqrt{(-1)^{n+m}} \tag{41}$$

This factor does not depend upon M^2 and hence remains in the limit $M^2\to 0$; after reexpressing everything in terms of the diagrams â la Fig.4, these factors stay with the vertex: 2n-vectors→2m-vectors, i.e. with $\tilde{K}_{2n\ 2m}$. In particular, the off-diagonal elements $\tilde{K}_{N\ N\pm 2}$ come with a factor i. In the language of the harmonic oscillator,

these elements denote transitions between neighboured energy states; in the language of field theory, the vertex K_{NN+2} (K_{NN-2}) creates (destroys) a quantum (note that N is always even, and only changes by two are allowed). In a Lagrangian field theory, this corresponds to terms of the form:

$$r\,(\psi^+)^p\,\psi^{p+1}\ ,\qquad r\,\psi^p\,(\psi^+)^{p+1} \tag{42}$$

The value of p depends upon the large N behavior of these vertices, which unfortunately is not known yet: for $K_{NN+2}\sim N^{3/2}$, we have p=1, for $K_{NN+2}\sim N^{5/2}$, p=1 or p=2 (in general, it will be a linear combination of both), etc. Very similar arguments apply to the other offdiagonal elements of $\tilde{K}$: changing the number N of vector particles is to be identified with creating or destroying field quanta:

$$\tilde{K}_{N\ N\pm 2k} \longleftrightarrow (\psi^+)^p\,(\psi)^{p+k'}\ ,\quad (\psi)^p\,(\psi^+)^{p+k'} \tag{43}$$

Factors of i work out in the same way as in RFT: vertices which change the number of field quanta by an odd number carry a factor i. In the absence of more precise information on the N-dependence of $\tilde{K}_{NN+2k'}$ we do not know the relation between k and k' in (43): in general we will have linear combinations of terms with different k' up to some maximal value which depends upon k: $k' \leq k'_{max}(k)$.

All this leads us to the following identification. The matrixelements $\tilde{K}_{NN'}$ are to be identified with matrixelements of a RFT-Hamiltonian H with a negative mass term: $-\Delta(q)$ (in zero transverse dimensions). The basis in which the elements of this Hamiltonian H are given to us is formed by the eigenstates of the free Hamiltonian H_0:

$$-\tilde{K}_{NN'}(q) = \langle \tfrac{N}{2}\,|\,H\,|\,\tfrac{N'}{2}\rangle \tag{44}$$

As to the interaction part ot this Hamiltonian, we expect that all n→m transition vertices will be present. In particular, there will be a triple coupling which is purely imaginary. All constants which appear in this Hamiltonian should, in principle, be determined from the large N behavior of the $\tilde{K}_{NN'}$. They are functions of q, and around q=0 this dependence should be smooth.

We now can use the results which have been obtained for supercritical RFT[8)-10)]. Studies of such RFT models have usually been restricted to the case where only the imaginary triple interaction vertex is present, and it is commonly believed that higher order interaction terms do not alter the situation too much.
The negative mass term is treated by analytic continuation: one first solves the problem for a positive reggeon mass (e.g. the energy spectrum) and then continues down to negative mass values. Let us list those results which are the most interesting ones for our problem:
a) The spectrum of H is discrete. By a similarity transformation, the Hamiltonian is equivalent to a hermiten operator and, hence, can be diagonalized (there exists also a spectral decomposition for matrix-elements of exp(-HY)).
b) The lowest energy eigenvalue lies at $\epsilon_o = -\sqrt{\frac{2}{\pi}}\,\frac{\Delta^2}{r}\cdot\exp(-2(\Delta/r)^2)$.

c) When analytically continuing the Greens function $\langle\psi(E-H)^{-1}\psi^{+}\rangle$ from positive to negative mass values, the perturbation expansion for Greens function picks up a term of the order $\exp(-2(\Delta(0)/r(0))^2)$.

Property a) justifies the assumptions which we have made in (17) and (18). In b) and c) the combination Δ/r appears: as we have said before, Δ is order g^2 and r of order g^3. Hence, Δ/r is of the order $1/g$. We therefore find that the lowest eigenvalue of the operator K which determines the leading singularity in the - plane is negative and of the order $\exp(-1/g^2)$. Since all our calculations are based on the weak coupling expansion, we interpret this term to be very small and

$$\sigma_{tot} \sim const\cdot\left[s^{O(e^{-1/g^2})}\right] \sim const \qquad (45)$$

Both Δ and r depend upon q, and we are also interested in this q-dependence near q=0. The first two derivatives are also of the order $\exp(-1/g^2)$:

$$a_o \sim \sigma_o \sim O(e^{-1/g^2}) \sim 0 \qquad (46)$$

For small g we can again neglect these terms and conclude that both a_O and the diffusion constant σ_o vanish This shows that the stabilit conditions are satisfied: as we discussed in the previous section,

the intrinsic transverse momentum does not grow with increasing number of steps and we have the physical results:
- the hadron radius grows approximately linearly with rapidity Y
- the Pomeranchuk singularity is t-dependent
- the zero mass limit has a good chance of being smooth.

This together with (45) are the most important physical implications if the identification (44) is correct.

Result c) has a very interesting meaning. It says that the scattering amplitude (note that the solution to (1) can, formally, be expressed in terms of the Greens function $\langle\psi(E-H)^{-1}\psi^{+}\rangle$, when expanded in powers of g^2, has a contribution of the form $\exp(-1/g^2)$. We have already said that the Regge limit, strictly speaking, lies outside the region where perturbation theory can be applied safely. This is to be expected on general grounds:when continuing from the perturbative regime to the Regge limit (start from the deep-inelastic Bjorken limit with $x\neq 0$ and continue down in x to x=0), perturbation theory becomes worse and worse. Since the Regge limit is also sensitive to long distance physics, we somewhere must cross the boarder line between the perturbative region and the nonperturbative one.
As explained before, we also have seen explicitly (eq.(37)). that perturbation theory does not converge. By mapping our summation problem into reggeon field theory and using result c) of Ref.8), we now learn that, in order to give a meaning to the divergent g^2 expansion, we have to add a term of the type $\exp(-1/g^2)$. The form of this nonperturbative contribution is completely specified by the manner in which the divergence of perturbation theory occurs. The next logical step then would be to find a physical interpretation of this term, e.g. to determine teh field configuration which in the functional integral would give such a contribution. At the moment we only conclude that the number N of gluons in the t-channel seems not to be a good expansion parameter: expanding the physical Pomeron state into two, four,.... gluon bound states is like expanding, in a spontaneously broken theory, around the wrong vacuum.

We have to remember, however, that in our formulae of section II the transformation matrix Λ appears which describes the transition between the (N,k_i)-basis and the eigenstates of K_d. We even have argued that, in order to obtain a realistic scale of transverse momenta (i.e. a mean value of ξ which explains the smallness of the Pomeron slope) this transformation matrix must give a nonneglegible contributi It is, therefore, necessary to investigate this matrix in more detail: hopefully, it gives a large but finite contribution to $\langle \varrho \rangle$.

We conclude this section with two comments on the relation between nonabelian gauge theories and reggeon field theory. Within our approach the Pomeron is defined as the lowest energy eigenstate of the operator $\tilde{K}(q)$. The connection with gluon states in the t-channel is nonperturbative since, as we have argued before, it is meaningless to ask how a gluon configuration with a certain number of gluons in the t-channel contributes to the Pomeron state. This is somewhat different from conventional ideas where the Pomeron has always been defined to represent a certain subset of Feynmann diagrams (e.g. ladders in φ^3-theory). It therefore seems that earlier attempts to describe the high energy behavior of nonabelian gauge theories in terms of a reggeon field theory (e.g.Refs.16, 17) may have chosen a bad starting point: in all these cases the Pomeron was defined to be a bound state of a finite number of gluons whereas the discussion in this paper indicates that the use of reggeon field theory should set in at a somewhat later stage.

Finally, the results of this paper also shed some light on the phase structure of reggeon field theory. When studying supercritical reggeon field theory the underlying question has been: what happens to the total cross section, when the bare intercept of the Pomeron is pushed above a certain critical value whereas the (bare) slope is kept fixed different from zero? Now the following picture seems to emerge: when the Pomeron mass becomes negative, one should at the same time put the slope equal to zero (the Pomeron then becomes a fixed cut). Diffusion no longer takes place in b-space but in the variable $\ln k^2$. But this only holds if we insist on expanding around the perturbative vacuum; if we expand around the true vaccum, we again have a nonzero slope and the intercept is equal to one. It should, however, be clear that the ideas that we have outlined in this section are not yet sufficiently refined in order to describe this new phase of RFT in enough detail. For example, we do not yet know the nature of the Pomeron singularity: at t=o it could be a (moving) cut as in critical reggeon field theory[18] or a pure pole (Gribov's pole solution[19]). Consequently, the total cross section which we have obtained:

$$\sigma_{tot} = \text{const} \cdot \left[s^0 + O\left(\ln s \; e^{-\text{const}/g^2} \right) \right] \qquad (47)$$

could still have powers of lns which we cannot determine yet. The point where a more detailed analysis has to set in is, as we believe, the k-approximation of the matrix K.

V. Summary and Conclusions

In this paper we have presented a fairly general discussion of the Pomeron in massless vector theories. We made the assumption that the perturbative high energy description (Fig.4) which we take from earlier studies satisfies complete unitarity (in an asymptotic sense) and, when the mass of the vector particle is taken to zero, stays finite and represents the high energy behavior of massless theories. We then focussed our attention to the question how one could hope to find the sum of all these terms in the perturbation expansion.

We first argued that, as a consequence of the gauge coupling being dimensionless, a diffusion takes place in the logarithmic scale of the intrinsic transverse momentum. As long as the diffusion constants a_0 and σ_0 are different from zero, this has several implications:
a) the hadron radius in transverse direction grows fast as a function of rapidity. This is interpreted as being a reflection of the long range forces due to the massless vector particles;
b) the Pomeranchuk singularity is a fixed cut;
c) the limit $t \to 0$ in the massless case, and the zero mass limit at $t=0$ cannot be smooth.
We take this as an indication that, in nonabelian theories which are expected to describe the real world, these two diffusion constants should be zero (stability condition).

In the third part of this paper we then outlined a scheme which, for the nonabelian case, leads to precisely this result. We argue that the problem of finding the leading j-plane singularity of the sum of all diagrams of Fig.4. is equivalent to solving a certain reggeon field theory with a negative mass (in zero transverse dimensions). The resulting physical Pomeron then has the following properties:
a) it has intercept one (up to corrections of the order $\exp(-1/g^2)$);
b) it has a nonzero slope which could be small;
c) the hadron radius grows, approximately, linearly in rapidity Y. This means that the long range forces of the massless vector particles have disappeared.
d) the connection between the physical Pomeron and gluon configurations with finite numbers of gluons in the t-channel is nonperturbative, i.e. the Pomeron only emerges after an infinite resummation of all these gluon configurations. At the same time, one has to add

to the diagrams of Fig.4 a contribution of the type $\exp(-1/g^2)$: this signals that the Regge limit lies outside the region where perturbation theory converges. But from the way in which this divergence occurs we know what nonperturbative term we have to add in order to give a meaning to the divergent g-expansion.

In course of this discussion we have made several assumptions which we have tried to state as clearly as possible. Clearly, more work is needed in order to verify these assumptions. In particular, we feel that the nature of the Pomeron singularity in the j-plane requires a more detailed investigation: at present we cannot yet see whether at $t=0$ it is a pole or a (moving) cut. As a consequence we do not yet know possible powers of $\ln s$ in the behavior of the total cross section. Finally it would be very interesting if one could give a meaning to the nonperturbative term $\exp(-\text{const}/g^2)$.

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Figure captions:

Fig.1: QED diagrams which are expected to give a unitary high energy description. The wavy lines stand for photon exchanges, the straight lines denote fermions. The photon interaction vertices are closed fermion loops, as indicated for the four-photon vertex.

Fig.2: High energy description of nonabelian theories. It is indicated that the interaction of vector particles (wavy lines) is due to the production of vector particles, i.e. no fermions are included. Many more Feynmann diagrams, however, are needed in order to cancel ultraviolet divergencies (and, in the zero mass limit, also infrared divergencies).

Fig.3: Simplest model for a coupling of exchanged vector particles to a composite external object (fermion-antifermion bound state).

Fig.4: Symbolic notation for the structure of all diagrams (QED or nonabelian) that have to be summed: the straight lines carry two-dimensional transverse momentum, for each t-channel intermediate state there is a factor $1/\omega$. The interaction vertices are explained in Figs.1-3.

Fig.5: The $N \rightarrow N$ vector particle vertex: it is given by the sum of all possible pairwise interactions.

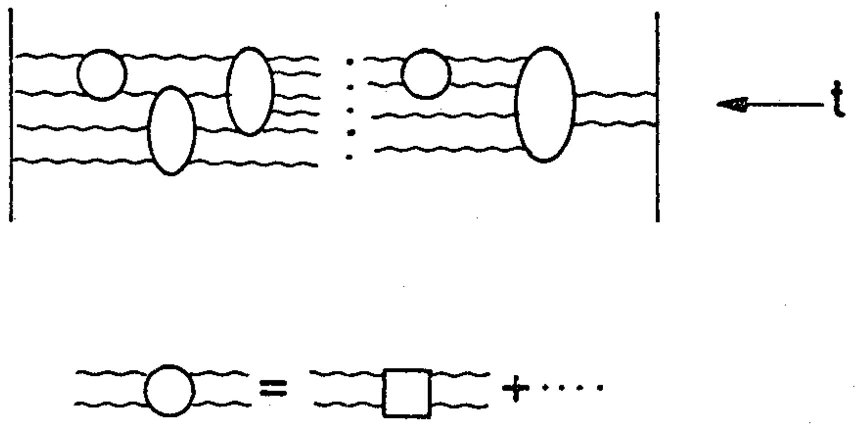

Fig.1

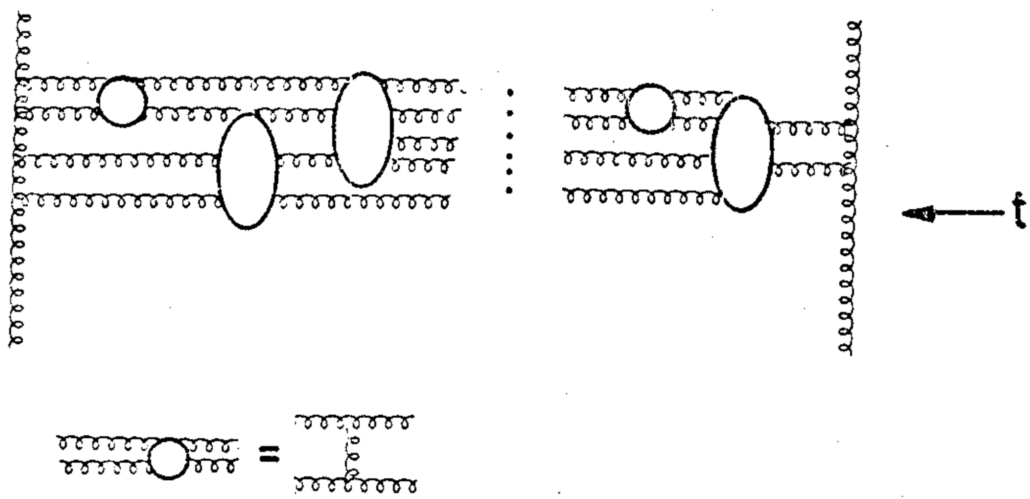

Fig.2

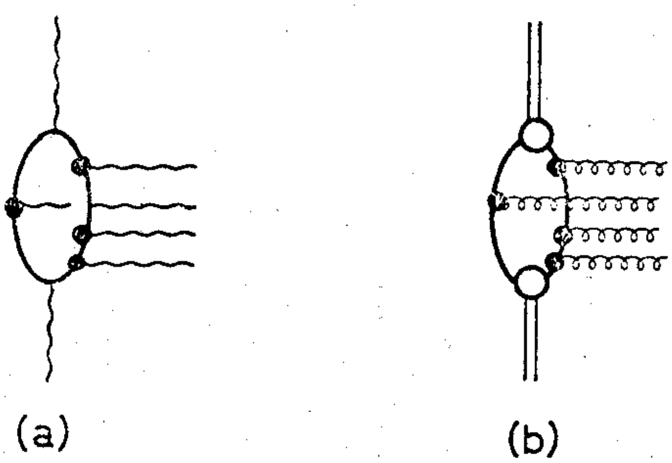

Fig.3

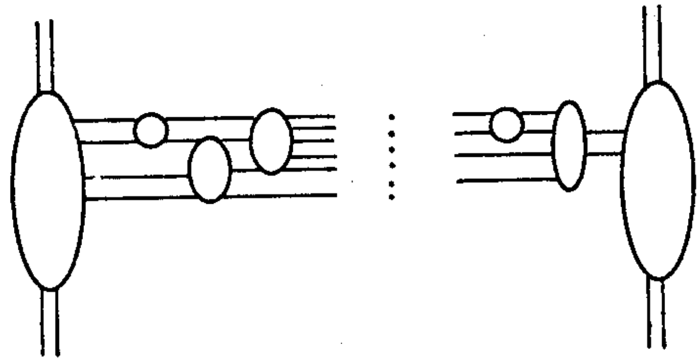

Fig.4

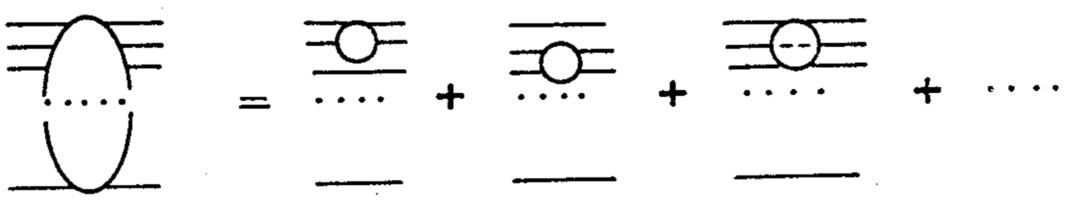

Fig.5