受入
80-7-199
高工研図書室

Erratum

to 'Estimate of the relation between scale parameters and the string tension by strong coupling methods'
DESY 8o/57, June 1980
by G.Münster and P.Weisz.

For the case of $\nu = 4$ dimensions a contribution $-16\ u^{12}$ resp. $-16\ x^{12}$ has to be added to the string tension series for SU(3) resp. Z_3 lattice gauge theory in the above paper, (Equ.5 and table 1). Any figures and numerical results remain unchanged.

DEUTSCHES ELEKTRONEN-SYNCHROTRON DESY

DESY 80/57
June 1980

受入
80-7-19
高エ研図書室

ESTIMATE OF THE RELATION BETWEEN SCALE PARAMETERS AND THE STRING TENSION BY STRONG COUPLING METHODS

by

Gernot Münster and Peter Weisz
II. Institut für Theoretische Physik der Universität Hamburg

NOTKESTRASSE 85 · 2 HAMBURG 52

DESY behält sich alle Rechte für den Fall der Schutzrechtserteilung und für die wirtschaftliche Verwertung der in diesem Bericht enthaltenen Informationen vor.

DESY reserves all rights for commercial use of information included in this report, especially in case of apply for or grant of patents.

To be sure that your preprints are promptly included in the
HIGH ENERGY PHYSICS INDEX ,
send them to the following address (if possible by air mail) :

DESY
Bibliothek
Notkestrasse 85
2 Hamburg 52
Germany

DESY 80/57
June 1980

ESTIMATE OF THE RELATION BETWEEN SCALE PARAMETERS AND THE STRING TENSION BY STRONG COUPLING METHODS

Gernot Münster and Peter Weisz

II. Institut für Theoretische Physik der Universität Hamburg

Abstract : We estimate the relation between the scale parameter Λ_L and the string tension α in pure SU(3) gauge theory using strong coupling expansions for Euclidean lattice gauge theory up to 12^{th} order. The result is $\Lambda_L = 3.7 \times 10^{-3} \sqrt{\alpha}$. For the more conventional scale parameter Λ^{MOM} this gives $\Lambda^{MOM} = .31 \sqrt{\alpha}$ by use of the proportionality factor of Hasenfratz. Results for the string tension of Z_3 lattice gauge theory are also discussed.

It is generally believed that in non-Abelian pure gauge theories far separated static quarks are confined by a linear potential $V(r) = \alpha \cdot r$. The string tension α is not computable by perturbation theory. Due to dimensional transmutation [1] there exists no free dimensionless parameter in asymptotically free gauge theory. The only parameter is a scale parameter Λ which has dimension of a mass. Every other physical quantity such as α is proportional to some power of Λ with a numerical factor which is fixed by the theory and can in principle be calculated by non-perturbative methods.

The natural framework for such problems is lattice gauge theory [2,3] . It provides us with a non-perturbative cut-off which respects local gauge symmetry . Strong coupling pure lattice gauge theories are known to confine static quarks [4] . The main question is wether the theory possesses a continuum limit with persisting confinement property. Due to asymptotic freedom [5] the continuum limit of SU(2) or SU(3) lattice gauge theory is supposed to be a weak coupling limit [6,7] , where the lattice spacing a and the bare coupling g simultaneously go to zero. Recent investigations suggest that this picture is correct [8 - 11] .

In this paper we use strong coupling expansions to calculate the relation between α and Λ . We consider Euclidean lattice gauge theories on a hypercubical lattice. Our main interest is the case of gauge group SU(3) in $\nu = 4$ dimensions, but we also investigate gauge group Z_3 and dimension $\nu = 3$. The gauge field $U(b) \in SU(3)$ is attached to links b of the lattice. The ordered product of the variables U(b) on the boundary of an elementary plaquette p is called U(p) . The action is

$$L = \frac{\beta}{3} \sum_p \mathrm{Re\,tr}\, U(p) \qquad (1)$$

The sum extends over all unoriented plaquettes of the lattice and β is related to the usual coupling constant g by

$$\beta = \frac{6}{g^2} \qquad (2)$$

We use the methods described in detail by one of us in ref. 12 , where SU(2) was considered, to calculate the cluster expansion for the string tension of pure SU(3) and Z_3 [13] lattice gauge theories up to 12^{th} order. The series for SU(3) has been calculated up to 10^{th} order previously by Kogut, Pearson and Shigemitsu [8] .

The natural expansion parameters are the Fourier coefficients $0 \leq a_r(\beta) < 1$ in the series

$$\exp \frac{\beta}{3} \mathrm{Re}\,\mathrm{tr}\,U = N(\beta) \left(1 + \sum_{r \neq 1} d_r\, a_r(\beta) \chi_r(U) \right) \tag{3}$$

where the sum extends over all inequivalent nontrivial irreducible representations r of SU(3) with dimension d_r , and χ_r are the corresponding primitive characters. Representations are denoted in the usual way : $r = 1, 3, \bar{3}, \ldots$ N is an irrelevant normalization factor. For Z_3 the natural expansion parameter is [13]

$$x = \left(1 - \exp\left(-\frac{3}{2}\beta \right) \right) / \left(1 + 2\exp\left(-\frac{3}{2}\beta \right) \right) \tag{4}$$

Our results for the string tension are given in table 1 for $\nu = 3$ and $\nu = 4$ dimensions. Expanding the series for SU(3), $\nu = 4$ in terms of $u = a_3$ the result is

$$a^2 \alpha = -\ln u - 4u^4 - 12u^5 + 10u^6 + 36u^7 - \frac{391}{2} u^8 - \frac{1131}{10} u^9 - \frac{2550837}{5120} u^{10} + \frac{5218287}{2048} u^{11} - \frac{257043419}{61440} u^{12} \tag{5}$$

This is in agreement with the 10^{th} order result of Kogut, Pearson and Shigemitsu [8] .

The relation between α and Λ in the continuum limit can be estimated in the following way. If asymptotic freedom in the form predicted by perturbation theory is true for the physical theory and confinement persists in the continuum limit, the string tension should behave like [14,15]

$$\alpha \approx C\, a^{-2} \left(\beta_o g^2 \right)^{-(\beta_1/\beta_o^2)} \exp\left(-\beta_o^{-1} g^{-2} \right) \tag{6}$$

in the weak coupling limit of the lattice theory. β_o and β_1 are the lowest coefficients in the Gell-Mann-Low function

$$\beta_o = \frac{11}{16\pi^2} \quad , \quad \beta_1 = \frac{102}{(16\pi^2)^2} \quad \text{for SU(3)} \tag{7}$$

and C is a constant. The theory can be renormalized by holding the string tension fixed, while a and g go to zero [8] . Then a renormalization scale

parameter Λ_L is given by

$$\Lambda_L^2 = \alpha \cdot C^{-1} \tag{8}$$

Monte Carlo calculations [9,11,15] indicate that the above picture is correct. They show a rather small intermediate coupling region in which a changeover from strong coupling to weak coupling asymptotic freedom behaviour takes place. High temperature expansions for SU(2) string tension up to 12^{th} order [16,12] also show such an abrupt breakaway. They yield values for the string tension, which are in good agreement with the Monte Carlo data at strong and intermediate coupling. If one adjusts the constant C such that the weak coupling function (6) touches the high temperature curve, the obtained value for C agrees with the result of the Monte Carlo computation.

This experience suggests applying the same procedure to the case of gauge group SU(3). In fig. 1 we have plotted the expansion of α according to table 1 up to 10^{th}, 11^{th} and 12^{th} order. Fitting the weak coupling function to the 12^{th} order curve we get

$$\Lambda_L = 3.7 \times 10^{-3} \sqrt{\alpha} \tag{9}$$

This is in agreement with Creutz's result for SU(3) [15] :

$$\text{Creutz :} \quad \Lambda_L = (5.0 \pm 1.5) \times 10^{-3} \sqrt{\alpha} . \tag{10}$$

We would like to add some critical remarks concerning our procedure. Above $\beta \approx 6$ the contributions of the higher orders are relatively large. In particular the 11^{th} order curve differs significantly from the curves representing the series up to 10^{th} resp. 12^{th} order and is situated even above the 8^{th} and 9^{th} order curves. Therefore one has to be cautious in using the series at these values of β. On the other hand the sequence of even order curves moves down uniformly and appears to converge even in the intermediate region. Remembering that for SU(2) only even orders contribute, one gets the impression that the sequence of even order curves behaves better than the odd ones, and we propose to use the even order curves to extract numerical results. For comparison we quote the numbers extracted from some lower order curves.

$$\Lambda_L \cdot \alpha^{-\frac{1}{2}} = \begin{cases} 2.4 \times 10^{-3} \text{ , up to } 8^{th} \text{ order} \\ 3.4 \times 10^{-3} \text{ , up to } 10^{th} \text{ order} \\ (1.7 \times 10^{-3} \text{ , up to } 11^{th} \text{ order}) \end{cases} \tag{11}$$

One might think of an extrapolation of the strong coupling series for α, e.g. by Padé methods. But the expected weak coupling behaviour is qualitatively very different from what Padé approximants produce. Furthermore from consideration of graphs we expect that in higher orders in the expansion there are relatively more contributions with positive sign than in the low orders, while Padé approximants merely extrapolate the low order terms. Therefore we do not see any reason why Padé approximants should be closer to the truth.

The relation between Λ_L and the better known scale parameter Λ^{MOM} [17] has been investigated by A. and P. Hasenfratz [14]. They find

$$\Lambda^{MOM} = 83.5 \ \Lambda_L \tag{12}$$

Using this relation one gets with formula (9)

$$\Lambda^{MOM} = .31 \ \sqrt{\alpha} \tag{13}$$

Inserting the 'experimental' value $\sqrt{\alpha} \approx 450$ MeV [18] one obtains

$$\Lambda^{MOM} \approx 140 \text{ MeV} \tag{14}$$

This number cannot yet be taken too seriously, because dynamical quarks have been neglected.

Finally we would like to discuss the expansion for the string tension of Z_3 lattice gauge theory in 4 dimensions. The theory is self-dual [13] and is supposed to undergo a first order phase transition at the point $\beta_c = .670$ [19]. For $\beta > \beta_c$ the string tension is zero. Because of the first order nature of the transition α may have a discontinuity at β_c. In fig. 2 we plot the high temperature series for α up to 9^{th} and 12^{th} order. At β_c it appears to converge to some finite value around .83. This supports the expected first order nature of the transition.

We would like to thank G. Mack, M. Lüscher and B. Berg for discussions. One of us (P.W.) thanks the Deutsche Forschungsgemeinschaft for financial support.

References

1. S. Coleman, E. Weinberg, Phys.Rev. D 7, 1888 (1973)
D. Gross, A. Neveu, Phys.Rev. D 10, 3235 (1974)
2. K. Wilson, Phys.Rev. D 10, 2445 (1974)
3. J. Kogut, L. Susskind, Phys.Rev. D 11, 395 (1975)
4. K. Osterwalder, E. Seiler, Ann.Phys.(N.Y.) 110, 440 (1978)
5. G. 't Hooft, 1972, unpublished
H. Politzer, Phys.Rev.Lett. 30, 1346 (1973)
D. Gross, F. Wilczek, Phys.Rev.Lett. 30, 1343 (1973)
6. J. Kogut, Rev.Mod.Phys. 51, 659 (1979)
7. G. Mack, Properties of lattice gauge theory models at low temperatures. DESY-report 80/03 (Jan.1980), to appear in : Recent developments in gauge theories, G. 't Hooft et al., eds., Plenum Press, New York 1980
8. J. Kogut, R. Pearson, J. Shigemitsu, Phys.Rev.Lett. 43, 484 (1979)
9. M. Creutz, Solving quantized SU(2) gauge theory, Brookhaven preprint BNL-26847 (Sep. 1979)
10. G. Mack, Predictions of a theory of quark confinement, DESY 80/21 (March 1980)
11. G. Mack, E. Pietarinen, Test of some current ideas in quark confinement physics by Monte Carlo computations for finite lattices,DESY 80/51 (June 1980)
12. G. Münster, High temperature expansions for the free energy of vortices respectively the string tension in lattice gauge theories,DESY 80/44 (June 1980)
13. T. Yoneya, Nucl.Phys. B 144, 195 (1978)
C. Korthals-Altes, Nucl.Phys. B 142, 315 (1978)
14. A. and P. Hasenfratz, The connection between the Λ parameters of lattice and continuum QCD, preprint CERN TH 2827 (March 1980)
15. M. Creutz, Asymptotic freedom scales, Brookhaven preprint, (April 1980)
16. G. Münster, Vortex free energy and string tension at strong and intermediate coupling, DESY 80/23 (March 1980)
17. see A. Buras, Rev.Mod.Phys. 52, 199 (1980)
18. E. Eichten, K. Gottfried, T. Kinoshita, K.D. Lane, T.M. Yan, Phys.Rev. D 21,203 (1980)
P. Goddard, J. Goldstone, C. Rebbi, C.B. Thorn, Nucl.Phys. B 56, 109 (1973)
19. S. Elitzur, R. Pearson, J. Shigemitsu, Phys.Rev. D 19, 3698 (1979)
A. Ukawa, P. Windey, A. Guth, Phys.Rev. D 21, 1013 (1980)
M. Creutz, L. Jacobs, C. Rebbi, Phys.Rev. D 20, 1915 (1979)

Table 1

SU(3)

$$u = a_3 \;,\; v_1 = a_6 \;,\; v_2 = a_8 \;,\; w_1 = a_{10} \;,\; w_2 = a_{15_m}$$

$\nu = 3$:

$$\begin{aligned} a^2\alpha = &- \ln u - 2u^4 - 6u^5 + 32u^6 - 12u^4v_1 - 16u^4v_2 - 46u^8 \\ &+ 21u^9 + 48u^{10} + 48u^8v_1 - 8u^8v_2 - 72u^6v_1{}^2 - 96u^6v_2{}^2 \\ &- 12v_1{}^5 - 16v_2{}^5 + 414u^{11} + 90u^9v_1 + 144u^9v_2 - 432u^7v_1v_2 \\ &- 32v_1{}^5v_2u^{-1} - 32v_2{}^5v_1u^{-1} - \frac{8552}{3}u^{12} + 1164u^{10}v_1 \\ &+ 1552u^{10}v_2 - 30u^9w_1 - 135u^9w_2 + 64u^8v_2{}^2 + 384u^8v_1v_2 \\ &- 360u^6v_1{}^2v_2 - 224u^6v_2{}^3 - 48u^4v_1{}^4 - 64u^4v_2{}^4 \\ &+ 144v_1{}^6 + 128v_2{}^6 - 40v_1{}^5w_1u^{-1} - 80v_2{}^5w_2u^{-1} - 60v_1{}^5w_2u^{-1} \end{aligned}$$

$\nu = 4$:

$$\begin{aligned} a^2\alpha = &- \ln u - 4u^4 - 12u^5 + 64u^6 - 24u^4v_1 - 32u^4v_2 - 128u^8 \\ &- 186u^9 + 876u^{10} - 336u^8v_1 - 592u^8v_2 - 144u^6v_1{}^2 \\ &- 192u^6v_2{}^2 - 24v_1{}^5 - 32v_2{}^5 + 4836u^{11} - 1332u^9v_1 \\ &- 1632u^9v_2 - 864u^7v_1v_2 - 64v_1{}^5v_2u^{-1} - 64v_2{}^5v_1u^{-1} \\ &- \frac{58840}{3}u^{12} + 10344u^{10}v_1 + 13792u^{10}v_2 - 180u^9w_1 \\ &- 810u^9w_2 - 1296u^8v_1{}^2 - 2176u^8v_2{}^2 - 2688u^8v_1v_2 \\ &- 720u^6v_1{}^2v_2 - 448u^6v_2{}^3 - 96u^4v_1{}^4 - 128u^4v_2{}^4 + 288v_1{}^6 \\ &+ 256v_2{}^6 - 80v_1{}^5w_1u^{-1} - 160v_2{}^5w_2u^{-1} - 120v_1{}^5w_2u^{-1} \end{aligned}$$

Z_3

$$x = \left(1 - \exp{-\tfrac{3}{2}\beta}\right) / \left(1 + 2\exp{-\tfrac{3}{2}\beta}\right)$$

$\nu = 3$:

$$a^2\alpha = - \ln x - 2x^4 - 2x^5 - 22x^8 + 7x^9 - 29x^{10} - 6x^{11} - \frac{428}{3}x^{12}$$

$\nu = 4$:

$$a^2\alpha = - \ln x - 4x^4 - 4x^5 - 80x^8 - 62x^9 - 130x^{10} - 20x^{11} - \frac{5728}{3}x^{12}$$

Figure captions

Fig. 1 The string tension α times the lattice spacing squared as a function of $\beta = 6/g^2$ for SU(3) lattice gauge theory in $\nu = 4$ dimensions. The solid lines represent results of strong coupling expansions up to 10^{th} (I), 11^{th} (II), and 12^{th} (III) order. The dashed lines are the lowest order strong coupling curve and the fitted weak coupling function (6).

Fig. 2 The string tension times the lattice spacing squared for Z_3 lattice gauge theory in $\nu = 4$ dimensions. The lines represent results of strong coupling expansions up to 9^{th} and 12^{th} order. The critical coupling β_c is indicated.

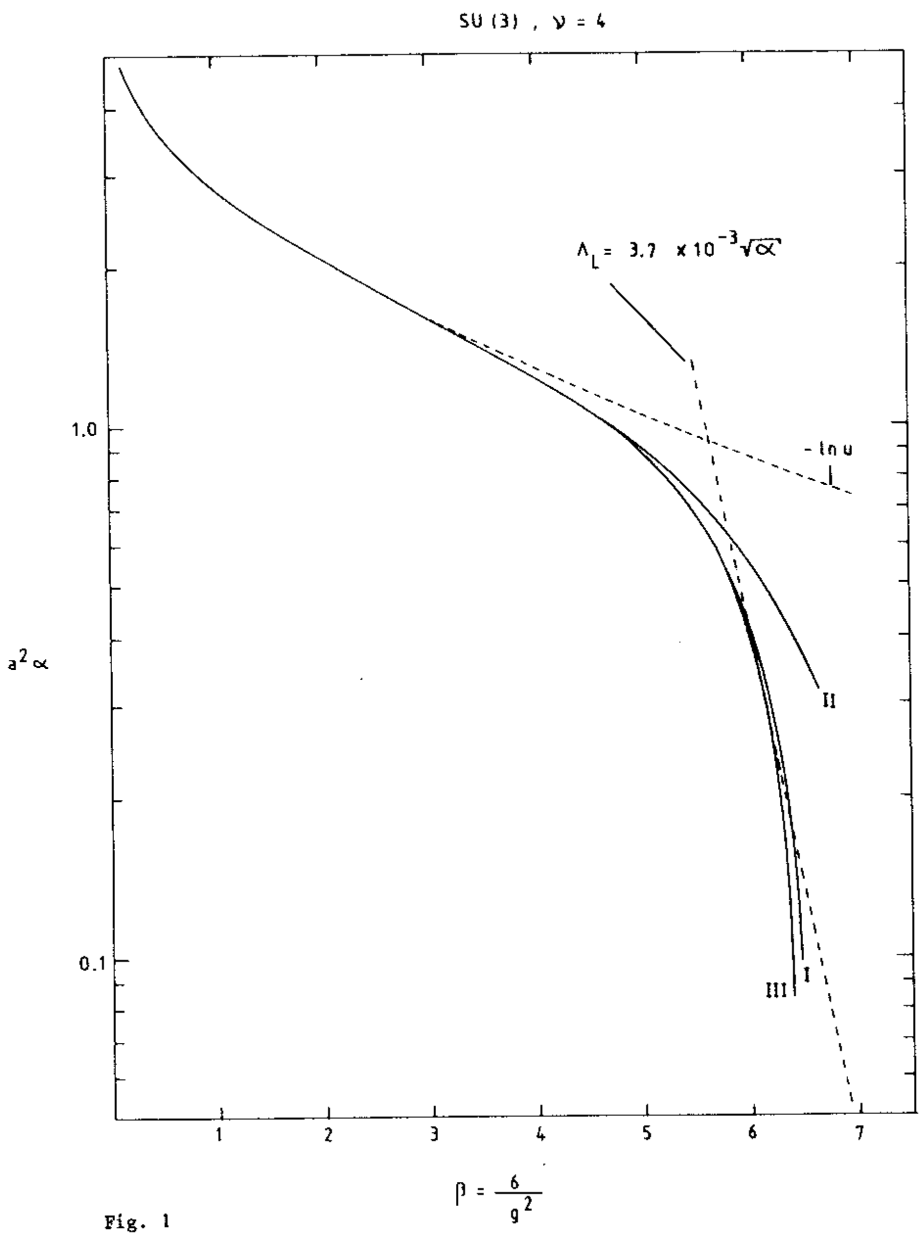
SU (3) , ν = 4
$\Lambda_L = 3.7 \times 10^{-3} \sqrt{\alpha}$
- ln u
I
II
III
1.0
0.1
$a^2 \alpha$
1
2
3
4
5
6
7
$\beta = \frac{6}{g^2}$

Fig. 1

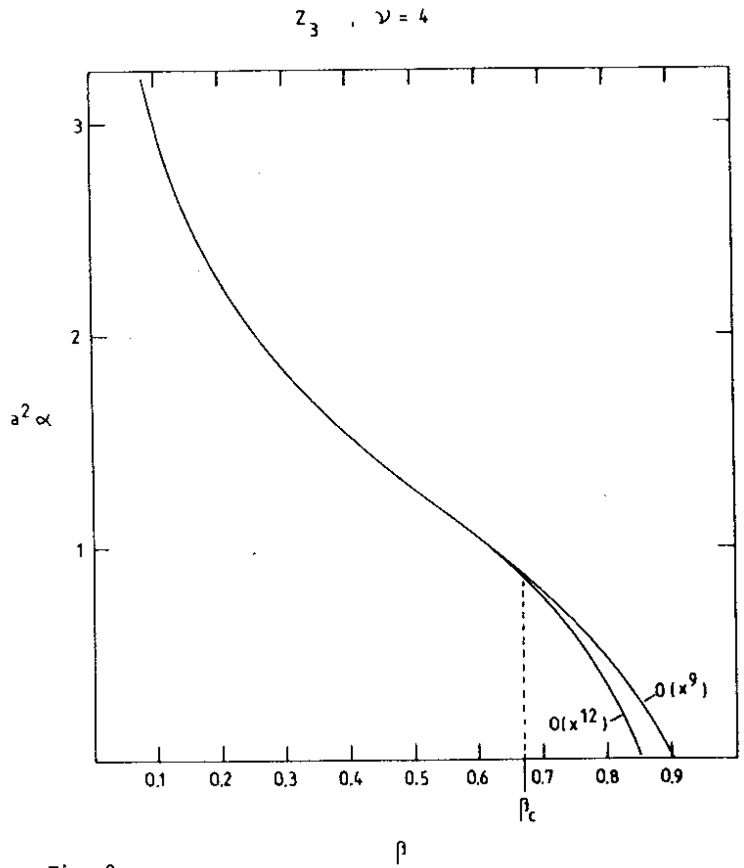
Z_3 , ν = 4
$O(x^9)$
$O(x^{12})$
3
2
1
$a^2 \alpha$
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
β_c
β

Fig. 2