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ABSORBED MUELLER-REGGE MODEL FOR BACKWARD INCLUSIVE PROTON PRODUCTION IN POSITIVE PION-PROTON COLLISIONS

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Abstract

The inclusive production of protons in the backward direction in the reaction $\pi^+ + p \rightarrow p + X$ is considered in a parameter-free Mueller-Regge model with absorption corrections. A large dip in the inclusive differential cross section, similar to that predicted by us for the inclusive reaction $\pi^- + p \rightarrow \pi^0 + X$ and later found experimentally, is predicted. The inclusive differential cross sections for fixed u and for fixed M_X^2/s are given.

1. Introduction

In the past, the Regge model with absorption corrections has provided a good description (1) of the differential cross sections for both forward and backward two-body exclusive reactions. Recently, it has been shown, in a series of papers (2,3,4,5), that the Regge model with absorption also provides an adequate description of forward and backward single-particle-inclusive reactions. In particular, in Ref. 5 we showed that the triple-Regge formalism with absorption provides a good representation of the backward process $\pi^- + p \rightarrow p + X$ which is mediated by $\Delta_\delta(1236)$ exchange. In the present paper, we extend the formalism to include $N_\alpha(938)$ exchange and, by applying this to a reaction which can mainly go by $N_\alpha(938)$ exchange, i.e. $\pi^+ + p \rightarrow p + X$ (see Fig. 1), we make an unambiguous and dramatic prediction for its differential cross section as a function of u for fixed M_X^2/s.

It is well known that MacDowell symmetry (6) forces baryon trajectories to occur in opposite parity pairs called parity doublets. The opposite parity partner of the $N_\alpha(938)$ is not well observed experimentally. Therefore, we follow the Carlitz-Kislinger cut formalism (7) which forces the unwanted partner poles onto the unphysical sheet. It is therefore natural to consider absorption corrections to amplitudes corresponding to processes mediated by baryon exchange.

The formalism for our absorbed Mueller-Regge model is given in Sec. 2. In Sec. 3, the results of the absorption model are discussed and predictions given.

2. Formalism

The Mueller generalized optical theorem [8] relates the inclusive cross section for the process $a + b \to c + X$ to the M_X^2 discontinuity in the forward $3 \to 3$ amplitude (see Fig. 2).

Using s-channel helicity amplitudes, we have the following general expression for the unpolarized cross section

$$\frac{s}{\pi}\frac{d^2\sigma}{du\, dM_X^2} = \frac{1}{64\pi^2 k^2}\,\frac{1}{(2s_a+1)}\,\frac{1}{(2s_b+1)}\sum_{\lambda_a,\lambda_b,\lambda_c} H^{\lambda_a\lambda_b\lambda_c}_{\lambda_a\lambda_b\lambda_c}(s,u,M_X^2)\ ,$$

where the λ_i are helicity labels, s_i are the spins and k is the c.m. three-momentum. In the case of the reaction $\pi^+ + p \to p + X$ ($s_a = 0$) we have

$$H^{\lambda_b\lambda_c}_{\lambda_b\lambda_c}(s,u,M_X^2) = \sum_{\kappa}(J_N^{\lambda_c}\,\nabla\Gamma_N^{\lambda_b\kappa})(J_N^{\lambda_c}\,\nabla\Gamma_N^{\lambda_b\kappa})^{\dagger},$$

where ∇ is the Reggeized propagator, $J_N^{\lambda_c}$ is the on-shell nucleon current at the particle-particle-Reggeon vertex, κ is the helicity of the missing-mass state, and $\Gamma_N^{\lambda_b\kappa}$ is the structure function at the target-proton-Reggeon vertex. The spin-$\frac{1}{2}$ propagator is given by

$$\nabla = \frac{(\not P + m)}{(u - m^2)}\ ,$$

where m is the mass of the proton and $P = p_a - p_c$.

Reggeization is carried out by the standard replacement [9]

$$\frac{1}{u-m^2} \to \frac{1}{2}\alpha_N'\,\Gamma[\tfrac{1}{2} - \alpha_N(u)](1 + i\,\tau_N \exp[-i\pi\alpha_N(u)])\left(\frac{s}{M_X^2}\right)^{\alpha_N(u)-\frac{1}{2}}\ ,$$

where we have used the Gell-Mann (nonsense) ghost-eliminating mechanism, α_o and α' are the intercept and slope of the $I = \frac{1}{2}$, $p = +$, $\tau = +(N_\alpha)$ baryon trajectory. The current is given by [10]

$$J_N^{\lambda_c} = g\,\bar{u}(p_c,\lambda_c)\gamma_5\,\phi_5\ ,$$

where u is the spin-$\frac{1}{2}$ wave function and ϕ_5 is the spin-0 wave function.

The coupling constant g is the known $\bar{n}p\pi^+$ coupling constant given by

$$\frac{g^2_{\bar{n}p\pi^+}}{4\pi} = 29.8\ .$$

To examine the behaviour of the structure functions, we sum over κ, contract with spin-$\frac{1}{2}$ antiparticle wave functions denoted by v and apply the optical theorem. This gives

$$\bar{\sum_{\lambda_b,\kappa}}\sum_{\kappa}\bar{v}(P,r)\,\Gamma_N^{\lambda_b\kappa}\,\Gamma_N^{\lambda_b\kappa *}\,v(P,r) = M_X^2\,\sigma^{\bar{n}p}_{TOT}(M_X^2).$$

We also obtain an alternative, completely general expression for the $\bar{n}p$ total cross section. The elastic, no spin-flip $\bar{n}p$ amplitude is given by [11]

$$\begin{aligned}4\,T(\bar{N}N) = {} & A_S\,\bar{v}(P,r)\,v(P,r')\,\bar{u}(P,\lambda_b)\,u(P,\lambda_b)\\ & + \tfrac{1}{2}A_T\,\bar{v}(P,r)\,\sigma^\rho\,v(P,r')\,\bar{u}(P,\lambda_b)\,\sigma_\rho u(P,\lambda_b)\\ & + A_A\,\bar{v}(P,r)\,i\gamma_5\gamma^\rho\,v(P,r')\,\bar{u}(P,\lambda_b)\,i\gamma_5\gamma_\rho u(P,\lambda_b)\\ & + A_V\,\bar{v}(P,r)\,\gamma^\rho\,v(P,r')\,\bar{u}(P,\lambda_b)\,\gamma_\rho\,u(P,\lambda_b)\\ & + A_P\,\bar{v}(P,r)\,\gamma_5\,v(P,r')\,\bar{u}(P,\lambda_b)\,\gamma_5\,u(P,\lambda_b)\ .\end{aligned}$$

If we sum over target spins, i.e. κ, then only A_S and A_V survive giving

$$T(\bar{N}N) = 4m\,A_S + 4P\,p_b\,A_V\ .$$

Applying the optical theorem, this then gives

$$4m\,\mathrm{Im}\,A_S + 4P\,p_b\,\mathrm{Im}\,A_V = \Delta^{1/2}(M_X^2, m^2, m^2)\,\sigma^{\bar{n}p}_{TOT}(M_X^2)\ ,$$

which, in the triple-Regge limit, becomes

$$\mathrm{Im}\,A_V + \frac{m}{P\,p_b}\,\mathrm{Im}\,A_S = \frac{1}{2}\,\sigma^{\bar{n}p}_{TOT}(M_X^2).$$

giving, at high energy, $\mathrm{Im}\,A_V \sim \frac{1}{2}\,\sigma^{\bar{n}p}_{TOT}(M_X^2)$. Here $\Delta(a,b,c)$ denotes the Kibble function of arguments a, b and c.

The required $\sigma^{\bar{n}p}_{TOT}(M_X^2)$ is given by [12]

$$\sigma^{\bar{n}p}_{TOT}(M_X^2) = (37.5 + \frac{48.0}{M_X})\ \text{mb}.$$

The rescattering corrections for the process are taken into account (see Fig. 3) through the absorption prescription which is performed in impact parameter space

$$H^{\lambda_c}_{\lambda_c\,abs}(b',b) = S^{1/2}(b')H^{\lambda_c}_{\lambda_c}(b',b)S^{1/2}(b),$$

where b' and b are the impact parameters and S(b) is the elastic scattering matrix. Here S(b) assumes the usual Gaussian form

$$S(b) = 1 - C\exp(-\lambda b^2),$$

where C is the opacity and $\lambda = R^{-2}$ (R = the radius of interaction).

We make the simplifying assumption that the rescattering effect in the $\bar{c}b$ channel is approximately equal to the rescattering effect in the ab channel so that $S_{ab} \simeq S_{\bar{c}b}$ and so, to first order, we have

$$S^{1/2}_{ab}\, S^{1/2}_{\bar{c}b} \simeq \frac{S_{ab} + S_{\bar{c}b}}{2} = S.$$

Then we have (see Fig. 3)

$$H^{\lambda_c}_{\lambda_c\,abs}(s,\tau,M_X^2) = \int_0^\infty \tau' d\tau' \int_0^\infty \tau_1' d\tau_1' \int_0^\infty b\,db\, J_\nu(b\tau)J_\nu(b\tau')S(b)$$
$$\times \int_0^\infty b_1 db_1\, J_{\nu'}(b_1\tau_1)J_{\nu'}(b_1\tau_1')S^*(b_1)\, H^{\lambda_c}_{\lambda_c}(s,\tau',\tau_1',M_X^2),$$

and finally

$$H^{\lambda_c}_{\lambda_c\,abs}(s,\tau,M_X^2) = \int_0^\infty \tau' d\tau' \int_0^\infty \tau_1' d\tau_1'\, H^{\lambda_c}_{\lambda_c}(s,\tau',\tau_1',M_X^2)$$
$$\times\left[\frac{1}{\tau}\delta(\tau-\tau') - \frac{C}{2\lambda}\exp\left(-\frac{\tau^2+\tau'^2}{4\lambda}\right)I_\nu\left(\frac{\tau\tau'}{2\lambda}\right)\right]$$
$$\times\left[\frac{1}{\tau}\delta(\tau-\tau') - \frac{C}{2\lambda}\exp\left(-\frac{\tau^2+\tau_1'^2}{4\lambda}\right)I_{\nu'}\left(\frac{\tau\tau_1'}{2\lambda}\right)\right],$$

with $\tau^2 q/k = u_{min} - u$, where k and q are the initial and final c.m. three-momentum and ν and ν' are the total helicity flip on each side of the M_X^2 discontinuity. $J_\nu(z)$ and $I_\nu(z)$ are the Bessel functions of the first and modified first kinds, respectively. Because we sum incoherently over all values of the helicity of the missing-mass state, we can treat the missing-mass state as a single spinless particle [13]. C and λ can be found from the elastic scattering data which in the present case leads to the values $C \simeq 0.70$ and $\lambda \simeq 0.068\ (\text{GeV/c})^{-2}$. For the evaluation of the absorbed amplitude, $\Gamma(1/2-\alpha_N(u))$ is approximated by a double exponential $\sum_{j=1}^{2} A_j \exp(B_j u)$, with a conventional $N_\alpha(938)$ trajectory $[\alpha_0 = -0.35$ and $\alpha' = 0.9\ (\text{GeV/c})^{-2}]$ which is obtained by assuming that the $N_\alpha(938)$ and the $N_\alpha(1692)$ lie on the same trajectory in the Chew-Frautschi plot. Full expressions for the helicity amplitude are given in the appendix.

It is well known that in the backward two-body exclusive process [8], $\pi^+ + p \to p + \pi^+$, which is mainly mediated by $N_\alpha(938)$ exchange, there is no difficulty in extrapolating to the baryon-exchange pole to determine the coupling constant. We make the assumption that this will also be the case with the backward inclusive process $\pi^+ + p \to p + X$ of the present calculation.

All the calculations of differential cross sections and total cross sections were carried out numerically [14]. All the figures were, for accuracy, plotted by computer [15].

3. Discussion

In Figs. 4 and 6 we present predictions for the backward inclusive reaction $\pi^+ + p \to p + X$ at P_{Lab} = 100 GeV/c $[s = 188.5\ GeV^2]$ for both fixed u and fixed M_X^2/s distributions. In all cases we have made allowances for the edge of phase space.

Fig. 4 shows the M_X^2/s distributions where each curve is labelled by a value of u which is the midpoint of a u bin 0.02 $(GeV/c)^2$ wide. The theoretical results are integrated by eight-point Gaussian-Legendre quadrature over this bin and this value is divided by the bin width. It can be seen that the results for fixed u are essentially straight lines, the effect of absorption being merely to lower the overall normalization of the curves by factors of between 1.4 and 2.7 in this region.

Since we are employing the Gell-Mann (nonsense) ghost eliminating mechanism (WSNZ), the $N_\alpha(938)$ pole amplitude will contain zeros at $\alpha_N(u) = -1/2, -5/2, \ldots$ The first zero occurs for the point u = $-0.17\ (GeV/c)^2$. This zero can be seen quite clearly in the dashed curve in Fig. 5. This zero is filled in by the absorption correction to produce a dip in the differential cross section at a point which is slightly closer to the backward direction. The continuous curve in Fig. 5 shows this effect clearly.

A similar dip was predicted by us [4] for the inclusive reaction $\pi^- + p \to \pi^0 + X$. In this case, the dip is due to a zero in the amplitude, resulting from the ρ-trajectory going to zero, being filled in by the absorption correction. This result is not common to all absorption models for single-particle-inclusive reactions. In the model of Pumplin [16], there are no wrong-signature-nonsense-zeros and hence no dip is predicted for the differential cross section for the reaction $\pi^- + p \to \pi^0 + X$. However, the differential cross section for $\pi^- + p \to \pi^0 + X$ has recently been measured experimentally [17] and this has dramatically confirmed our model (with WSNZ) and cast doubt on the model of Pumplin. On the basis of this success we firmly believe that a WSNZ dip will be found in the differential cross section for the backward reaction $\pi^+ + p \to p + X$ should it be measured experimentally.

In Fig. 6 the fixed M_X^2/s distribution, for four values of M_X^2/s, are presented as a function of u. The slow change is shape of the differential cross section curves for changes in M_X^2/s is apparent.

The total cross section has been calculated for $|u_{min}| \leq |u| \leq 1.0\ (GeV/c)^2$ and $0.02 \leq M_X^2/s \leq 0.2$ to be 0.213 mb.

Our absorption model is factorizable and has identically zero momentum transfer at the inclusive vertex. As a consequence, our model predicts zero polarization for the inclusively produced backward proton . This is the same result as a model with only the $N_\alpha(938)$ pole. The measurement of the proton polarization would be of great interest.

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We wish to thank Professor G. Kramer for communicating to us the work of Dr. R. Tegen and Dr. R.W.B. Ardill for providing the parameters for the Γ-function fit. The support and encouragement of Professor H.G. Eggleston in this work is gratefully acknowledged. One of the authors (K.J.M.M.) wishes to thank the DESY directorate for the award of a Visiting Fellowship to visit DESY where part of this work was carried out. The other author (L. McC.) wishes to thank the Science Research Council of Great Britain for financial support.

Appendix

The absorbed helicity amplitude in the c.m. is given by

$$H^{\lambda_c}_{\lambda_c\ abs}(\tau,\tau') = N(C'[f(\tau) - f_{abs}(\tau)][f(\tau') - f_{abs}(\tau')]^*$$

$$+ S[g(\tau) - g_{abs}(\tau)][g(\tau') - g_{abs}(\tau')]^*),$$

with $N = g^2_{\bar{n}p\pi} + m_b^2\, \sigma^{\bar{n}p}_{TOT}(M_x^2)$,

$$C' = 2(E_bE_c + kq)\,[(m + m_c)^2 + m_a^2]$$

$$+ 2\,(s - m_a^2 - m_b^2)[m_c(m + m_c) - (E_aE_c - kq)] ,$$

$$S = 2(E_bE_c - kq)[(m + m_c)^2 + m_a^2]$$

$$+ 2(s - m_a^2 - m_b^2)[m_c(m + m_c) - (E_aE_c + kq)],$$

and defining

$$M_o = \frac{1}{2}\alpha'\left(\frac{s}{M_x^2}\right)^{\alpha_o + \alpha' u_{min} - 0.5} ,$$

$$\phi_j = -\frac{q}{k}[B_j + \alpha' \ln\left(\frac{s}{M_x^2}\right)] ,$$

$$\xi_o = i\,\exp[-i\pi\alpha_o - i\pi\alpha' u_{min}] ,$$

$$\beta_o = i\,\pi\alpha'\frac{q}{k} ,$$

$$a_j = A_j\,\exp[B_j\, u_{min}] ,$$

and

$$\psi_j = \phi_j + \beta_o ,$$

then

$$f(\tau) = M_o \sum_j a_j(\exp[\phi_j\,\tau^2] + \xi_o\,\exp[\psi_j\,\tau^2])\sqrt{\left(1 - \frac{\tau^2}{4k^2}\right)} ,$$

$$g(\tau) = M_o \sum_j a_j (\exp[\phi_j \tau^2] + \xi_o \exp[\psi_j \tau^2]) \frac{\tau}{2k} ,$$

$$f_{abs}(\tau) = \frac{C}{2\lambda} \exp[- \frac{\tau^2}{4\lambda}] M_o \sum_j a_j (F[\phi_j,\tau] + \xi_o F[\psi_j, \tau]),$$

and

$$g_{abs} (\tau) = \frac{C}{2\lambda} \exp [- \frac{\tau^2}{4\lambda}] M_o \sum_j a_j(G[\phi_j,\tau] + \xi_o G[\psi_j,\tau]) .$$

The functions $F[\alpha_j, \tau]$ and $G[\alpha_j, \tau]$ are given by

$$F[\alpha_j, \tau] = \frac{1}{4} (\frac{\tau}{\lambda})^{\frac{1}{2}} \frac{\Gamma(1.25)}{\Gamma(1.5)} (- E_j)^{-1.25}$$

$$\sum_{n=0}^{\infty} B(n) (\frac{1}{4k^2 E_j})^n \Phi (n + \frac{5}{4} , \frac{3}{2} ; Z) ,$$

$$G[\alpha_j, \tau] = \frac{1}{8k} (\frac{\tau}{\lambda})^{\frac{1}{2}} \frac{\Gamma(1.75)}{\Gamma(1.5)} (- E_j)^{1.75} \Phi(\frac{7}{4} , \frac{3}{2} ; Z),$$

with

$$E_j = \alpha_j - \frac{1}{4\lambda} ,$$

$$Z = - \frac{\tau^2}{16\lambda^2 E_j} ,$$

$$B(n) = (\frac{\frac{1}{2}}{n}) (n - 1 + \frac{5}{4}) (n - 2 + \frac{5}{4}) \dots (\frac{5}{4}),$$

and Φ is the degenerate hypergeometric function.

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Figure Captions

Fig. 1. The basic baryon-exchange process.

Fig. 2. The representation of the Mueller generalized optical theorem.

Fig. 3. The rescattering corrections to the helicity amplitude considered here.

Fig. 4. The differential cross section plotted against M_X^2/s for fixed values of u for $\pi^+ + p \to p + X$ at $s = 188.5$ GeV^2.

Fig. 5. The differential cross section plotted against u for a fixed value of M_X^2/s for $\pi^+ + p \to p + X$ at $s = 188.5$ GeV^2. (The dashed curve represents the pole-only result while the continuous curve represents the absorbed curve of the present model.)

Fig. 6. The differential cross section plotted against u for fixed values of M_X^2/s for $\pi^+ + p \to p + X$ at $s = 188.5$ GeV^2.

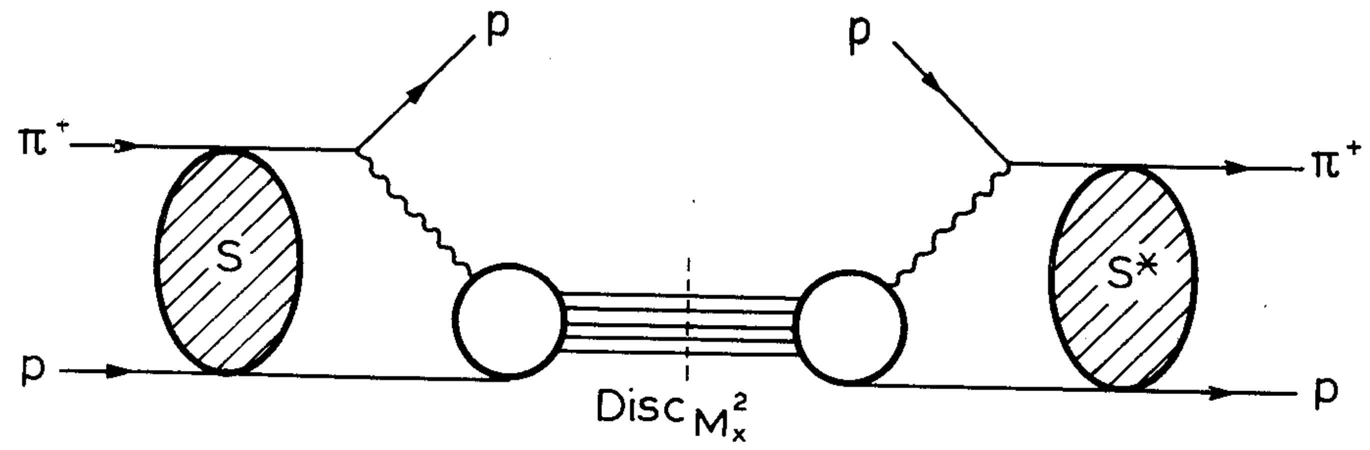

Fig. 3.

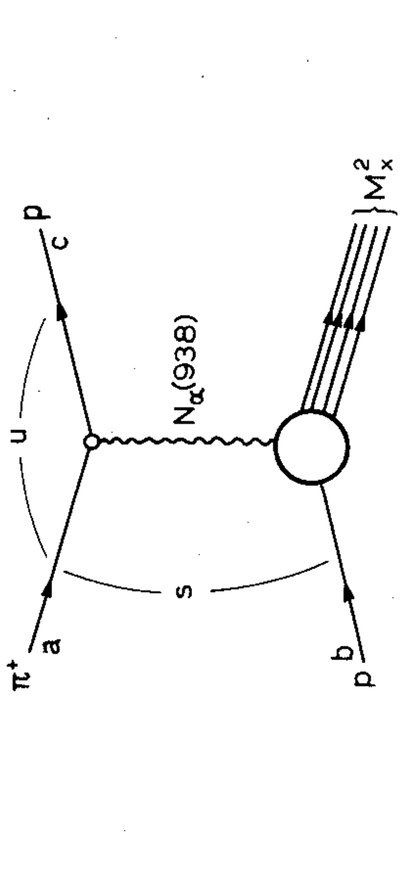

Fig.1.

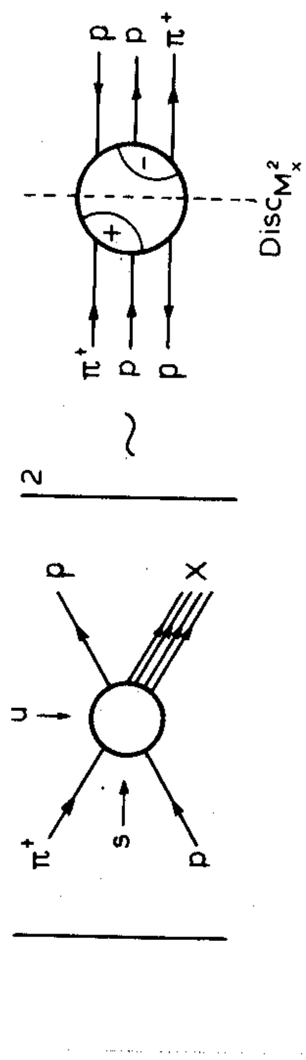

Fig.2.

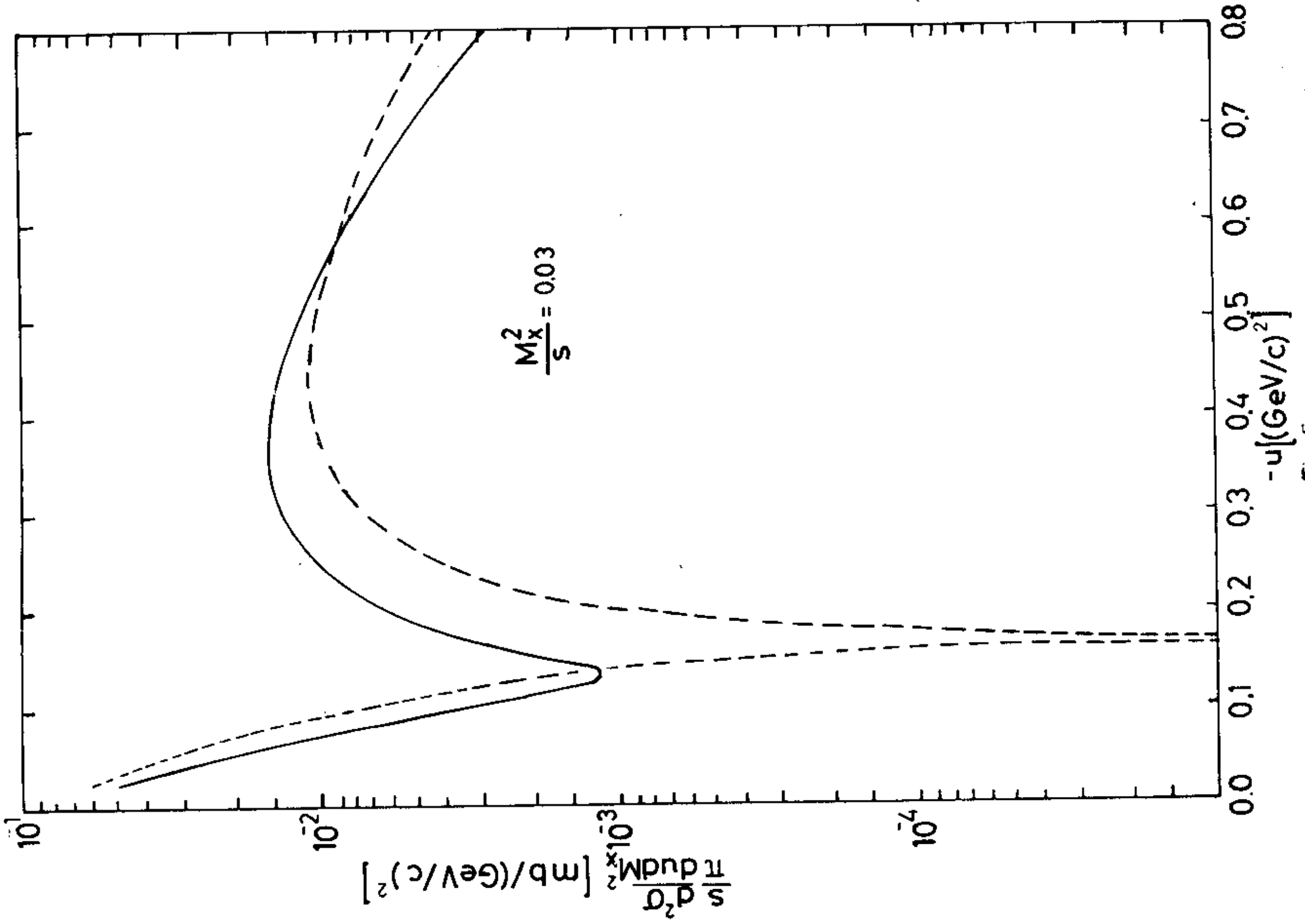

Fig. 5

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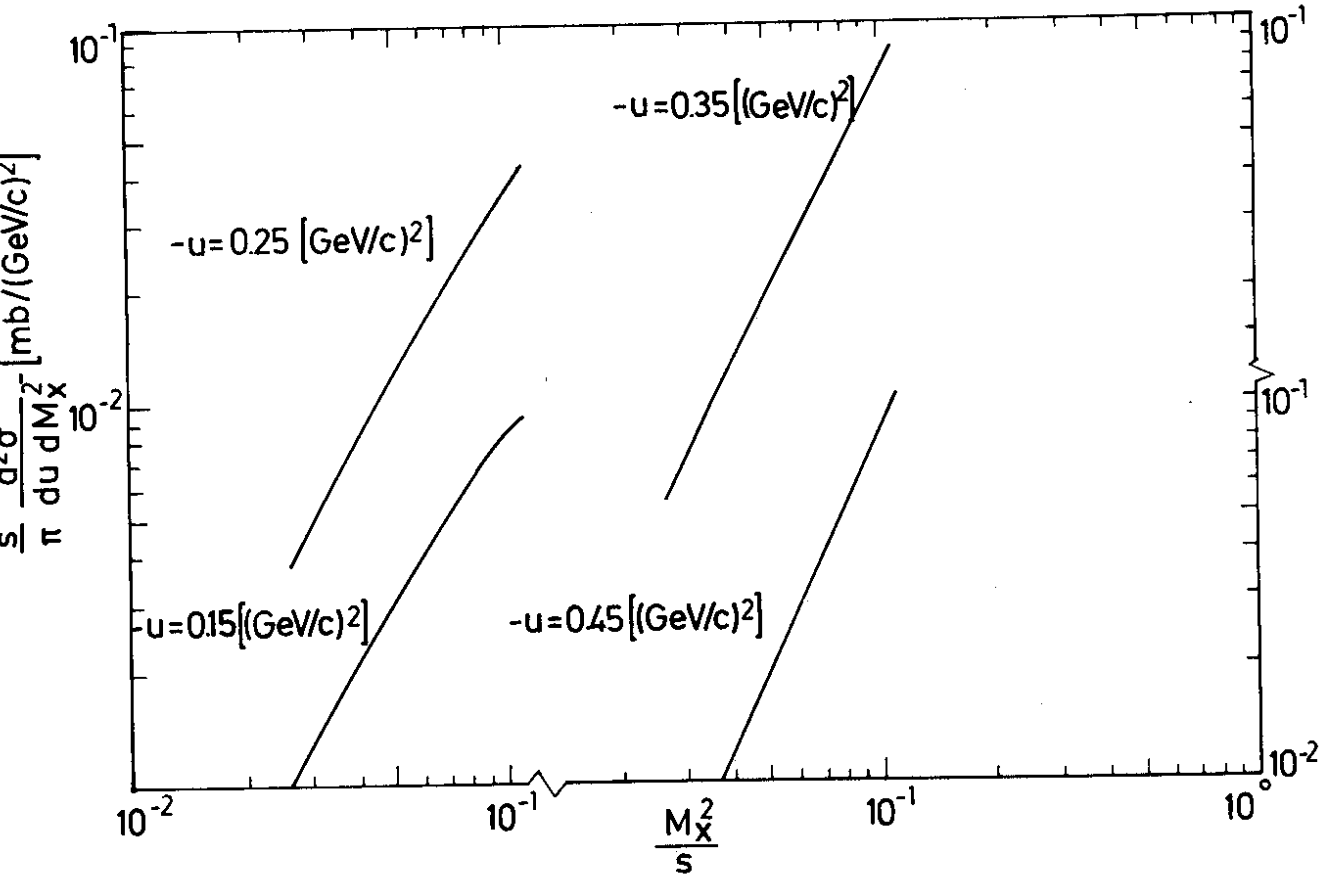

fig. 4

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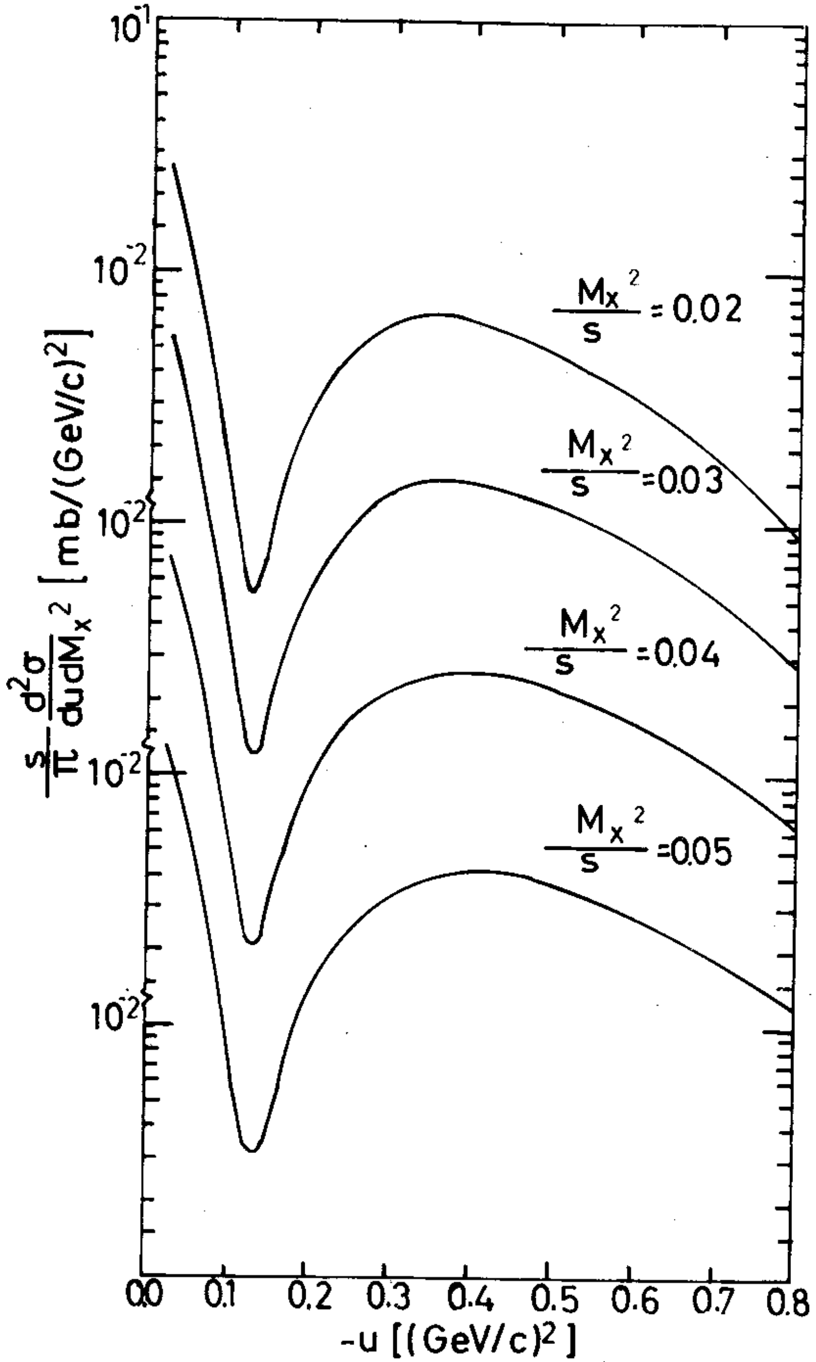
10^{-1}
10^{-2}
10^{-2}
10^{-2}
10^{-2}
$\frac{s}{\pi}\frac{d^2\sigma}{du\,dM_x^2}$ [mb/(GeV/c)^2]
$\frac{M_x^2}{s} = 0.02$
$\frac{M_x^2}{s} = 0.03$
$\frac{M_x^2}{s} = 0.04$
$\frac{M_x^2}{s} = 0.05$
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8
-u [(GeV/c)^2]

Fig.6

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