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EXPERIMENTAL LIMITS ON THE LOWEST POSSIBLE MASS HORIZONTAL GAUGE BOSON IN $SU(2)_H$

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Experimental limits on the lowest possible mass horizontal gauge boson in $SU(2)_H$.

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Abstract:
Experimental limits coming from μ^-, τ^- and K-decays are evaluated for the gauge boson mediating $SU(2)_H$ horizontal interactions and acting mainly (that is, appart from small mixing angles) between the second and third fermion generations.

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I. Introduction

The presently known basic fermions (leptons and quarks) are observed to belong to three generations: (ν_e, e, u, d); (ν_μ, μ, c, s); $(\nu_\tau, \tau, t^?, b)$. The two heavier generations are essentially identical copies of the lightest one. Within the standard $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ theory there seems to be no apparent reason for this "sequential" repetition (for recent reviews see e.g. [1]).

The most conventional explanation of the "generation puzzle" is that the existence of a new sort of presently unknown "horizontal" weak interactions requires the fermions to come in several generations (for the recently proposed simplest such schemes see [2,3]). The different generations then have different horizontal quantum numbers and they are spanning out some representation of the horizontal group in the same way as a single generation is a representation of the usual "vertical" group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. The only distinctive property of the new interaction is that the horizontal gauge bosons have large masses compared to the ordinary weak bosons W and Z. Experimental data require masses roughly at least in the 1o TeV region [2-4].

Qualitatively, the existence of horizontal weak interactions implies in the leptonic sector a small non-conservation of separate e-, μ- and τ -lepton numbers and a small breaking of universality of e-, μ - and τ -couplings. In the quark sector the Cabibbo-universality is broken similarly by the horizontal interactions and there are horizontal contributions also to the non-leptonic decays and flavour changing neutral currents. A general consequence of the horizontal interactions is the mixing of the members of the generations. The mixing can be characterized by Cabibbo - like angles for the e-, ν-, u- and d-type fermions separately. (A relative mixing of e.g. the quarks and leptons is also relevant!)

The purpose of the present paper is to investigate the lower limits for the masses of horizontal gauge bosons coming from known data on μ^-, τ^- and K-decays. I shall assume three generations within the anomaly free $SU(2)_H$ horizontal interaction [2].

There are three possibilities for the multiplet assignment within

$SU(2)_H$ for three generations:

A.) triplet,

B.) doublet + singlet,

C.) three singlets.

The third case is clearly uninteresting. The second one seems at the first sight perhapes also unusual but in reality there is nothing wrong about puting e.g.the lightest generation in a singlet just like the lightest members of the generations are the colourless, zero charge neutrinos. So I shall consider in this paper cases A und B. In both cases it is possible (and even natural) to arrange the spontaneous symmetry breaking in such a way that the horizontal gauge boson coupled mainly to the second and third generation has a much smaller mass than the other two. I shall concentrate in this paper on this interesting possibility. For definitness, I shall assume that the horizontal interactions are pure vector like. (This forbids some of the processes possible for, say, a V-A interaction, but does not alter the general conclusions) Neutrinos will be assumed purely left handed with a possible small non-zero Majorana mass. (The existence of four-component Dirac-neutrinos coupled to the horizontal gauge bosons would imply some additional factors 2 in a few places.)

The plan of the paper is as follows: in Section II the symmetry breaking pattern in $SU(2)_L \otimes U(1)_Y \otimes SU(2)_H$ will be shortly discussed and the angles characterizing the generation mixing will be introduced. In section III the general form of low energy effective four-fermion interactions will be derived in the above cases A and B The mass limits following from the data on μ^-, τ^- and K-decays are investigated in Section IV , whereas Section V contains some concluding remarks.

II. The symmetry breaking pattern in $SU(2)_L \otimes U(1)_Y \otimes SU(2)_H$- and generation mixing.

The spontaneous symmetry breaking responsible for the large masses of the horizontal gauge bosons involves the introduction of Higgs-field transforming like singlets under $SU(2)_L \otimes U(1)_Y$ [2]. Choosing the simplest possible sets of such fields it is easy to arrange that one of the three $SU(2)_H$ gauge bosons does not receive mass from the $SU(2)_L \otimes U(1)_Y$ scalar fields and, therefore, it is much lighter than the other two (its mass is in the order of the W and Z masses).

An example of such a Higgs-sector is to choose, besides the $SU(2)_L$-doublet $SU(2)_H$ triplet and quintet Higgs mesons (η and φ) introduced in [2] , an $SU(2)_L \otimes U(1)_Y$ singlet Higgs-scalar χ which is a triplet under $SU(2)_H$. The Higgs scalar χ has a large vacuum expectation value compared to the usual $SU(2)_L$ doublet Higgs'es, therefore the symmetry breaking pattern is

$$SU(2)_L \otimes U(1)_Y \otimes SU(2)_H \supset SU(2)_L \otimes U(1)_Y \otimes U(1)_H \supset U(1)_Q \quad (2.1)$$

In the first step one of the horizontal gauge bosons remains massless. This is the consequence of taking a single triplet χ in $SU(2)_H$ [5] . The second step is the usual symmetry breaking due to η and φ considered in [2] . This gives mass to the W- and Z-bosons and to the horizontal gauge boson left massless in the first step. In addition, unlike χ , the $SU(2)_L$- doublet Higgs-fields η and φ can couple also to the fermions. This is the origin of the mixing of generations [2] .

The known experimental limits exclude a light horizontal gauge boson if it is coupled mainly to the first two generations [2-4] . We shall see in the next Sections that the limits are much less restrictive if the light horizontal gauge boson acts mainly on the second and third generation. In addition, most of the existing limits are in this case sensitive to unknown mixing angles among the generations.

The mixing of generations is rather important for horizontal interactions. It means that the mass eigenstates do not coincide with the states serving as basis for the definition of horizontal and vertical interactions (the "generation eigenstates"). After absorbing relative phases into the definition of states there are three real rotation angles and one phase for the ν-, e-, u- and d-type states, separately.

From the different possible parametrizations [6,7] I shall use the one given by Maiani [7]. For simplicity, I shall neglect the (CP- non - conserving) phases. Denoting the mass eigenstates by r_k (k=1,2,3 for the three generations and r= ν,e,u,d) and the corresponding generation eigenstates by R_k (R= N,E,U,D), we can write

$$R_k = C(\gamma_r, \beta_r, \Theta_r)_{kl}\, r_l \quad , \qquad (2.2)$$

where C is the ratation matrix

$$C(\gamma,\beta,\Theta) = \begin{pmatrix} \cos\beta\cos\Theta & \cos\beta\sin\Theta & \sin\beta \\ -\cos\gamma\sin\Theta-\sin\gamma\sin\beta\cos\Theta & \cos\gamma\cos\Theta-\sin\gamma\sin\beta\sin\Theta & \sin\gamma\cos\beta \\ \sin\gamma\sin\Theta-\cos\gamma\sin\beta\cos\Theta & -\sin\gamma\cos\Theta-\cos\gamma\sin\beta\sin\Theta & \cos\gamma\cos\beta \end{pmatrix} \qquad (2.3)$$

In most cases only relative mixing matters, therefore one can choose some horizontal set of mass eigenstates to coincide with the generation eigenstates. For instance, it is possible to put $\gamma_u = \beta_u = \Theta_u = 0$. In this case $\gamma_d \equiv \gamma$, $\beta_d \equiv \beta$ and $\Theta_d \equiv \Theta$ are the usual mixing angles [7] entering in the vertical weak interaction of quarks. Note that if we define a "generation number" G = k for the mass eigenstates r_k (k=1,2,3) then, due to the mixing, the usual weak interactions do also change G. (The other possibility, namely, $G = k$ for R_k seems perhapes theoretically more attractive because G is then changed only by the horizontal interactions but phenomenologically it is rather awkward.)

III. <u>The low energy effective four-fermion interaction.</u>

The exchange of horizontal gauge bosons leads at low energy to an effective four-fermion interaction similar to ordinary "vertical" weak interactions. As explained in the previous Section, it is assumed here that one of the horizontal gauge bosons has substantially lower mass (denoted by M_H) than the other two. The effective four-fermion coupling constant G_H is defined similarly to G (the Fermi coupling constant):

$$G_H = \frac{g_H^2}{4 M_H^2 \sqrt{2}} \quad . \qquad (3.1)$$

Here g_H is the dimensionless coupling constant of the horizontal gauge interaction. The vector currents appearing in the effective horizontal interaction are:

$$\mathcal{F}^{(r)}_{kl}(x)_\varrho \equiv \tilde{\psi}_{rk}(x)\,\gamma_\varrho\,\psi_{rl}(x) \quad , \qquad (3.2)$$

where r= ν,e,u,d stands for the different horizontal sets of fermions and k,l = 1,2,3 are indices for the three generations (sometimes k,l= u,c,t will also be used for labelling the generations). For r= e,u, d $\psi_r(x)$ is the full four-component Dirac field, whereas for the neutrinos (r= ν) ψ_r means the left-handed field:

$$\psi_{\nu L}(x) = \frac{1}{2}(1-\gamma_5)\,\psi_\nu(x) \quad . \qquad (3.3)$$

The current $\mathcal{F}^{(r)}_{kl}$ generates the transitions

$$r_l \to r_k \quad , \qquad 0 \to \bar{r}_l\, r_k \quad , \qquad (3.4)$$

$$\bar{r}_k \to \bar{r}_l \quad , \qquad \bar{r}_k\, r_l \to 0 \quad .$$

(The bar denotes, of course, the antiparticle.) In momentum space, e.g. for the first reaction, $\mathcal{F}^{(r)}_{kl}$ goes over into

$$j(r_l \to r_k)_\varrho = \tilde{u}_{rk}\,\gamma_\varrho\, u_{rl} \quad , \qquad (3.5)$$

where u_{rl} is the Dirac-spinor belonging to r_l. The whole amplitude for the effective horizontal four-fermion interaction is the following:

$$\frac{G_H}{\sqrt{2}}\Big\{\sum_r \sum_{k\ell} K(\gamma_r\beta_r\Theta_r)_{k\ell}\, j(r_\ell \to r_k)\,\varrho\Big\}\cdot$$
$$\cdot\Big\{\sum_{r'} \sum_{k'\ell'} K(\gamma_{r'}\beta_{r'}\Theta_{r'})_{k'\ell'}\, j(r'_{\ell'} \to r'_{k'})^{\varrho}\Big\}\ . \qquad (3.6)$$

Of course, for the other reactions in (3.4) both $j\,(r_1 \to r_k)$ and $j(r'_{\ell'} \longrightarrow r'_{k'})$ have to be replaced here by the corresponding expressions with ν Dirac-spinors. The matrix K takes into account the generation mixing. It will be given below for the two cases A and B.

A.) Generation triplet.

In this case the three generations transform under $SU(2)_H$ like ordinary three-vectors under rotations [2] . Let us denote the generators of $SU(2)_H$ in the Cartesian-basis by T_l (l=1,2,3):

$$T_1=\begin{pmatrix}0&0&0\\0&0&i\\0&-i&0\end{pmatrix}\qquad T_2=\begin{pmatrix}0&0&-i\\0&0&0\\i&0&0\end{pmatrix}\qquad T_3=\begin{pmatrix}0&i&0\\-i&0&0\\0&0&0\end{pmatrix} \qquad (3.7)$$

The "lightest" generator we shall consider here is T_1 acting on the second and third generation. (This belongs to the gauge boson with mass M_H.) The matrix C in Eq.(2.3) is a rotation matrix, therefore we have

$$C\,T_\ell\,C^{-1} = C_{\ell k}\,T_k\ . \qquad (3.8)$$

The matrix $K \equiv K_A$ in Eq.(3.6) can be obtained from Eqs.(3.8), (2.2-3) and (3.7):

$$K_A(\gamma\beta\Theta)=\begin{pmatrix}0 & \sin\beta & -\cos\beta\sin\Theta\\ -\sin\beta & 0 & \cos\beta\cos\Theta\\ \cos\beta\sin\Theta & -\cos\beta\cos\Theta & 0\end{pmatrix} \qquad (3.9)$$

This is antisymmetric, therefore the diagonal elements vanish and the diagonal ($\Delta G=0$)currents do not appear in the effective interaction (3.6). The processes given by Eq.(3.6) are $\Delta G=0,\pm 2$. This rule is a consequence of choosing the Cartesian-basis for the generation eigenstates. (For the possibility to use the canonical basis in the triplet see the remark at the end of case B .)

B.) Generation doublet + singlet.

In this case it is assumed that (appart from the mixing) the first generation is a singlet under $SU(2)_H$ and the second and third generation form a doublet. The light generator, we are interested in, is the following:

$$T_1 = \frac{1}{2}\begin{pmatrix}0&0&0\\0&1&0\\0&0&-1\end{pmatrix} \qquad (3.1o)$$

In this case the simple relation in Eq.(3.8) does not hold, therefore the matrix $K \equiv K_B$ in (3.6) is considerably more complicated than K_A:

$$K_B(\gamma\beta\Theta)=\begin{pmatrix}
\cos 2\gamma\sin^2\Theta - \cos 2\gamma\sin^2\beta\cos^2\Theta + \sin 2\gamma\sin\beta\sin 2\Theta & -\frac{1}{2}\cos 2\gamma\sin 2\Theta - \frac{1}{2}\cos 2\gamma\sin^2\beta\sin 2\Theta - \sin 2\gamma\sin\beta\cos 2\Theta & -\sin 2\gamma\cos\beta\sin\Theta + \frac{1}{2}\cos 2\gamma\sin 2\beta\cos\Theta\\
-\frac{1}{2}\cos 2\gamma\sin 2\Theta - \frac{1}{2}\cos 2\gamma\sin^2\beta\sin 2\Theta - \sin 2\gamma\sin\beta\cos 2\Theta & \cos 2\gamma\cos^2\Theta - \cos 2\gamma\sin^2\beta\sin^2\Theta - \sin 2\gamma\sin\beta\sin 2\Theta & \sin 2\gamma\cos\beta\cos\Theta + \frac{1}{2}\cos 2\gamma\sin 2\beta\sin\Theta\\
-\sin 2\gamma\cos\beta\sin\Theta + \frac{1}{2}\cos 2\gamma\sin 2\beta\cos\Theta & \sin 2\gamma\cos\beta\cos\Theta + \frac{1}{2}\cos 2\gamma\sin 2\beta\sin\Theta & -\cos 2\gamma\cos^2\beta
\end{pmatrix} \qquad (3.11)$$

This is symmetric and has also diagonal ($\Delta G=0$) elements. As a consequence the processes given by (3.6) have $G=0,\pm 1,\pm 2$.

A somewhat simpler expression for K_B could be obtained by a different parametrization of the mixing matrix C. The generator T_1 in (3.1o) is, namely, the same as 1/2-times the corresponding generator in the triplet representation if the canonical basis is taken. (The two heavier generators $T_{2,3}$ are, of course, different in the two cases.) Therefore, a relation analogous to Eq.(3.8) would hold if the mixing matrix C would be parametrized like a rotation matrix in the canonical basis. I shall not use this possibility here, because I would like to keep the conventional parametrization of C in Eq.(2.3).

The other consequence of the proportionality of T_1 in the doublet +

singlet and triplet representation (on the canonical basis) is that $2K_B$ in (3.11) gives also the matrix K for (3.6) if for the triplet assignment the canonical basis is used. The difference between K_A in (3.9) and $2K_B$ may be surprising at the first sight. The reason is, however, that in the mixing relative phases are neglected here. The transformation from the Cartesian- to the canonical basis involves a redefinition of relative phases followed by a real rotation like (2.3). This example shows that a complete treatment of the effective four-fermion interactions has to include the relative phases, too. For the moment, however, we are only interested in order of magnitude estimates and qualitative properties of the horizontal interaction processes. For this purpose it is enough to consider as prototypes cases A and B with real mixing.

IV. μ-, τ- and K-decays.

The general form of the low-energy four-fermion interaction due to the exchange of the lightest horizontal gauge boson is shown by Eq.(3.6). The matrix K reflecting generation mixing is given by Eqs.(3.9), (3.11) in the two cases A and B. It depends on the real mixing angles γ_r, β_r and θ_r for the four horizontal sets of fermions: $r = \nu, e, u, d$. According to the general philosophy, the mixing angles are small, typically of the order known from the relative mixing of u- and d-type quarks relevant also for vertical weak interactions. (This feature is built in the mixing schemes proposed in connection with horizontal symmetries. See, for instance, [2,3]. Nevertheless, I do not like to constrain the following discussion to any specific model for the mixing, therefore, the small mixing angles will be considered as phenomenological parameters.) Appart from the relative mixing of u and d quarks, there is nothing known experimentally about the magnitude of $\gamma_r, \beta_r, \theta_r$. As a guess, one can take as "normal" values in the range $\gamma_r, \theta_r \gtrsim 0.1$ and $\beta_r \cong 0.01$ (β_r is smaller because it is mixing the two non-neighbouring generations). In this case the matrices K_A and K_B can be approximated like

$$K_A \cong \begin{pmatrix} 0 & \beta & -\theta \\ -\beta & 0 & 1 \\ \theta & -1 & 0 \end{pmatrix} ; \tag{4.1}$$

$$K_B \cong \begin{pmatrix} \theta^2 - \beta^2 + 4\gamma\beta\theta & -\theta - 2\gamma\beta & \beta - 2\gamma\theta \\ -\theta - 2\gamma\beta & 1 & 2\gamma + \beta\theta \\ \beta - 2\gamma\theta & 2\gamma + \beta\theta & -1 \end{pmatrix} . \tag{4.2}$$

In the present Section only these simple forms will be used for K_A and K_B.

It is obvious that the most advantageous processes are generally those involving the least possible number of first generation fermions. A first generation fermion always implies a "penalty" in terms of the small mixing angles.

1.) $\mu \to e\nu\bar{\nu}$.

In the case of $\mu^- \to e^- \nu_\mu \bar{\nu}_e$ the horizontal amplitude interferes with the usual vertical weak interaction amplitude

$$T = \frac{G}{\sqrt{2}} \tilde{u}_{\nu_\mu} \gamma_\varrho (1-\gamma_5) u_{\mu^-} \tilde{u}_{e^-} \gamma^\varrho (1-\gamma_5) v_{\nu_e} + \qquad (4.3)$$

$$+ G_H(\mu^-\nu_e \to e^-\nu_\mu)\sqrt{2}\, \tilde{u}_{e^-} \gamma_\varrho u_{\mu^-} \tilde{u}_{\nu_\mu} \gamma^\varrho (1-\gamma_5) v_{\nu_e}.$$

The effective horizontal coupling G_H ($\mu^-\nu_e \to e^-\nu_\mu$) in the two cases A and B is, from Eqs. (4.1-2):

$$G_H(\mu^-\nu_e \to e^-\nu_\mu) = \begin{cases} G_H \beta_e \beta_\nu ; \\ G_H(\Theta_e + 2\gamma_e\beta_e)(\Theta_\nu + 2\gamma_\nu\beta_\nu) . \end{cases} \qquad (4.4)$$

For simplicity, the mass of the electron and of the neutrinos will be neglected here. Then, due to the Fierz-transformed form of the horizontal term in (4.3), the electron spectrum is the same as in the purely vertical ($G_H=0$) case. In other words, the measurement of the Michel parameter (ϱ =3/4) does not give information about the absence or presence of the horizontal interaction. The total width is

$$\Gamma(\mu^- \to e^-\nu_\mu\bar{\nu}_e) = \frac{m_\mu^5}{192\pi^3}\left[G^2 - 2G\,G_H(\mu^-\nu_e \to e^-\nu_\mu) + 2G_H(\mu^-\nu_e \to e^-\nu_\mu)^2\right] . \qquad (4.5)$$

For the "normal" values of mixing angles and for G_H=G (according to Eq. (3.1) this means, e.g. the same coupling strength and same mass for the lightest horizontal gauge boson as for the vertical W-boson) the interference term in (4.5) is $1o^{-4}$ in case A and $1o^{-2}$ in case B . These numbers are not particularly "dangerous" for a light (i.e. $M_H \cong$ 1oo GeV) horizontal gauge boson. They would only somewhat alter the value of the Cabibbo-angle deduced from the μ -lifetime compared to β - decay.

The other 8 processes: $\mu^- \to e^-\nu_e\bar{\nu}_\mu$, $e^-\nu_e\bar{\nu}_e$, $e^-\nu_\mu\bar{\nu}_\mu$, $e^-\nu_\tau\bar{\nu}_\mu$, $e^-\nu_\mu\bar{\nu}_\tau$, $e^-\nu_\tau\bar{\nu}_\tau$, $e^-\nu_\tau\bar{\nu}_e$, $e^-\nu_e\bar{\nu}_\tau$ have no vertical counterpart, therefore do not interfere. Experimentally it is, of course, rather difficult to identify them. A witness is the ridiculously weak limit $\mu^- \to e^-\nu_e\bar{\nu}_\mu < 0.25$ [8] . The effect on the total width is largest from $\mu^- \to e^-\nu_\mu\bar{\nu}_\mu$, $e^-\nu_\tau\bar{\nu}_\mu$, $e^-\nu_\mu\bar{\nu}_\tau$, $e^-\nu_\tau\bar{\nu}_\tau$. (Under the assumption $m_{\nu_\tau} \ll m_\mu$). The effective couplings involving the mixing angles are:

$$G_H(\mu^-\nu_\mu \to e^-\nu_\mu) = G_H(\mu^-\nu_\tau \to e^-\nu_\tau) = G_H(\Theta_e + 2\gamma_e\beta_e) ; \qquad (B)$$

$$G_H(\mu^-\nu_\mu \to e^-\nu_\tau) = -G_H(\mu^-\nu_\tau \to e^-\nu_\mu) = G_H\beta_e ; \qquad (A)$$

$$G_H(\mu^-\nu_\mu \to e^-\nu_\tau) = G(\mu^-\nu_\tau \to e^-\nu_\mu) = -G_H(\Theta_e + 2\gamma_e\beta_e)(2\gamma_\nu + \beta_\nu\Theta_\nu) . \qquad (B) \qquad (4.6)$$

These processes give again to the total width corrections of order $1o^{-4}$ in case A and $1o^{-2}$ in case B . For a summary see Tables A-B.

2.) $\tau \to \mu\nu\bar{\nu}$ and $\tau \to e\nu\bar{\nu}$.

These processes are in principle very similar to $\mu \to e\nu\bar{\nu}$ only the effect of mixing is different. From this point of view the τ-state is in general more advantageous than the μ. In particular, we have here the only "clean" processes in case A independent of the value of small mixing angles, for which there are experimental data. The effective couplings for $\tau^- \to \mu^-\nu_\tau\bar{\nu}_\mu$ and $\tau^- \to \mu^-\nu_\mu\bar{\nu}_\tau$ are, namely

$$G_H(\tau^-\nu_\mu \to \mu^-\nu_\tau) = -G_H(\tau^-\nu_\tau \to \mu^-\nu_\mu) = G_H \quad . \qquad (A) \qquad (4.7)$$

Hence, no suppression due to mixing! The similar processes with an electron have the effective couplings

$$G_H(\tau^-\nu_\mu \to e^-\nu_\tau) = -G_H(\tau^-\nu_\tau \to e^-\nu_\mu) = -G_H\Theta_e . \qquad (A) \qquad (4.8)$$

and for the processes $\tau^- \to e^-\nu_\tau\bar{\nu}_e$ (also possible "vertically") and $\tau^- \to e^-\nu_e\bar{\nu}_\tau$:

$$G_H(\tau^-\nu_e \to e^-\nu_\tau) = -G_H(\tau^-\nu_\tau \to e^-\nu_e) = G_H\Theta_e\Theta_\nu \quad . \qquad (A) \qquad (4.9)$$

These contain suppression factors from the mixing which give only $1o^{-2}$ corrections in the width for normal mixing and G=G_H. Therefore, the effect of horizontal interactions is negligible for $\tau \to e\nu\bar{\nu}$ but it can be substantial for $\tau \to \mu\nu\bar{\nu}$ resulting in

$$\Gamma(\tau^- \to \mu^-\nu\bar{\nu}) \neq \Gamma(\tau^- \to e^-\nu\bar{\nu}) . \qquad (4.1o)$$

From Eqs. (4.5) and (4.7) the ratio is (neglecting small phase-space effects and the change in $\tau \to e\nu\bar{\nu}$ due to horizontal interactions):

$$\frac{B_\mu}{B_e} = \frac{\Gamma(\tau \to \mu\nu\bar{\nu})}{\Gamma(\tau \to e\nu\bar{\nu})} = 1 - 2\frac{G_H}{G} + 4\frac{G_H^2}{G^2} . \qquad (4.11)$$

This is a relatively rapidly changing function near G_H=G (where its value is actually 3). For small $G_H \ll G$ it is somewhat smaller than 1.

The experimental numbers are, within large errors, consistent with equal branching ratios for μ and e decays [9-11] . An example of world average is $B_\mu/B_e = 1.13 \pm 0.16$ (G. Feldmann, quoted in [9]) compared to the purely vertical expectation $B_\mu/B_e = 0.98$ (involving the small phase space corrections due to $m_\mu > m_e$). The present value of B_μ/B_e can be easily accomodated with a value G_H near G. The importance of horizontal weak interactions would certainly justify a precize measurement of this ratio at least up to 1% accuracy. If, for instance, it would turn out to be equal to 1 ± 0.01, then it would push up the lower limit for the mass M_H of the lightest horizontal gauge boson to $M_H > 1$ TeV (independently from the small mixing angles and taking the gauge couplings equal: $g_H = g$).

3.) $\mu \to eee$ and $\tau \to \mu\mu\mu$.

These processes are possible only in case B . The amplitude for $\mu^- \to e^- e^- e^+$ is the following:

$$T(\mu^- \to e^- e^- e^+) = 2\sqrt{2}\, G_H(\mu^- e^- \to e^- e^-)\left[\tilde{u}_{e_1^-}\gamma_\varrho u_{\mu^-}\, \tilde{u}_{e_2^-}\gamma^\varrho v_{e^+} - (e_1^- \leftrightarrow e_2^-)\right], \quad (4.12)$$

where the effective coupling from (4.2) is:

$$G_H(\mu^- e^- \to e^- e^-) = -G_H(\Theta_e + 2\gamma_e\beta_e)(\Theta_e^2 - \beta_e^2 + 4\gamma_e\beta_e\Theta_e). \quad (4.13)$$

In the case of neglecting electron masses the kinematics is the same as for $\mu^- \to e^- \nu \bar{\nu}$ (the positron spectrum has $\varrho = 3/4$ like the electron spectrum there). The branching ratio is:

$$B(\mu \to eee) \equiv \frac{\Gamma(\mu^- \to e^- e^- e^+)}{\Gamma(\mu^- \to e^- \nu \bar{\nu})} = 2\,\frac{G_H(\mu^- e^- \to e^- e^-)^2}{G^2}. \quad (4.14)$$

For normal values of the mixing angles and $G_H = G$ this is 10^{-6} compared to the experimental limit $B(\mu \to eee) < 1.9 \cdot 10^{-9}$ [8] .This means either $M_H > 10\, M_W$ or (see Table B) the mixing angle $\Theta_e < 0.03$.

The processes $\tau \to \mu\mu\mu, \mu\mu e, \mu ee, eee$ are very similar to $\mu \to eee$. The most "dangerous" process for a light horizontal boson is $\tau^- \to \mu^- \mu^+ \mu^-$ with

$$G_H(\tau^- \mu^- \to \mu^- \mu^-) = G_H(2\gamma_e + \beta_e\Theta_e). \quad (4.15)$$

This gives a branching ratio 10^{-2} for normal mixing angles and

$G_H = G$. The experimental limit is [9-11] $B(\tau \to 3$ charged leptons$) < 0.006$. (See also Table B.)

4.) $\tau \to \mu$ or e + vector meson.

A typical process is $\tau^- \to \mu^- K^{0*}$ or $\mu^- \bar{K}^{0*}$. The amplitude is of the form

$$T(\tau^- \to \mu^- K^{0*}) = 2\sqrt{2}\, G_H(\tau^- s \to \mu^- d)\,\tilde{u}_{\mu^-}\gamma_\varrho u_{\tau^-}\langle K^{0*}|\mathcal{F}^\varrho_{ds}(0)|0\rangle. \quad (4.16)$$

Here the effective coupling is given by

$$G_H(\tau^- s \to \mu^- d) = -G_H(\tau^- d \to \mu^- s) = -G_H\beta_d \; ; \qquad \text{(A)} \quad (4.17)$$

$$G_H(\tau^- s \to \mu^- d) = G_H(\tau^- d \to \mu^- s) = -G_H(2\gamma_e + \beta_e\Theta_e)(\Theta_d + 2\gamma_d\beta_d) \qquad \text{(B)}$$

in cases A and B , respectively. For an estimate we can neglect the difference between m_μ/m_τ and m_{ν_τ}/m_τ and then the kinematics is the same as for $\tau^- \to \nu_\tau K^{-*}$, therefore the branching ratios are

$$B(\tau^- \to \mu^- K^{0*}) = B(\tau^- \to \mu^- \bar{K}^{0*}) =$$

$$= B(\tau^- \to \nu_\tau K^{-*})\,\frac{2\,G_H(\tau^- s \to \mu^- d)^2}{0.05\,G^2} \cong 0.6\,\frac{G_H(\tau^- s \to \mu^- d)^2}{G^2}. \quad (4.18)$$

The factor 0.05 in the denominator comes from the Cabibbo suppression of $\tau^- \to \nu_\tau K^{-*}$ .The estimate for $B(\tau^- \to \nu_\tau K^{-*})$ is the standard one (see e.g. [12]) obtained from $V_{\ell 2}$ decays assuming SU(3) symmetry for matrix elements like $\langle K^{-*}|\mathcal{F}_{su}(0)_\varrho|0\rangle$.

For normal mixing angles and $G_H = G$ Eqs. (4.17-18) give a rather small branching ratio: 10^{-4} for $\tau^- \to \mu^- K^{0*}$. A less suppressed process exists in case B , namely, $\tau^- \to \mu^- \phi$ where the effective coupling is

$$G_H(\tau^- s \to \mu^- s) = G_H(2\gamma_e + \beta_e\Theta_e). \qquad \text{(B)} \quad (4.19)$$

This gives 10^{-2}. (See also Tables A , B) Generally, processes with an electron instead of the μ-meson involve an extra suppression factor Θ_e and $\beta_e - 2\gamma_e\Theta_e$ in cases A and B , respectively).

5.) $K \to \pi\mu e$.

A good candidate for imposing stringent limits on the mass and coupling of horizontal gauge bosons is the extremely well measured K-meson decay. The basic quark process, present also in the vertical interaction due to the generation mixing, is a generation changing one: $s \to d$. In our case the quark annihilation processes like e.g. $K^0 \to \mu e$ or the K^0 mass difference are not present due to the pure vector character of the horizontal interactions. The process $K \to \pi\mu e$, however, tests the vector currents and has a very low upper limit: the experimental branching ratio for $K^- \to \pi^-\mu^- e^+$ is $< 5.10^{-9}$ [8].

The amplitude for $K^- \to \pi^-\mu^- e^+$ is

$$T(K^- \to \pi^-\mu^- e^+) = 2\sqrt{2}\, G_H(se^- \to d\mu^-)\,\tilde{u}_{\mu^-}\gamma_\varrho v_{e^+}\langle\pi^-|\mathcal{F}_{ds}(0)^\varrho|K^-\rangle, \quad (4.20)$$

where the effective coupling constant from Eqs. (4.1-2), is

$$G_H(se^- \to d\mu^-) = -G_H(s\mu^- \to de^-) = G_H\beta_d\beta_e \; ; \qquad (A) \quad (4.21)$$

$$G_H(se^- \to d\mu^-) = G_H(s\mu^- \to de^-) = G_H(\Theta_d + 2\gamma_d\beta_d)(\Theta_e + 2\gamma_e\beta_e). \quad (B)$$

The vector current matrix element appearing in (4.20) can be expressed by the well-known $K_{\ell 3}$ form factors $F_\pm(t)$ $(t = (p_K - p_\pi)^2)$:

$$\langle\pi^-|\mathcal{F}_{ds}(0)_\varrho|K^-\rangle = (p_K + p_\pi)_\varrho F_+(t) + (p_K - p_\pi)_\varrho F_-(t). \quad (4.22)$$

Neglecting the electron mass (and ν_μ mass) the kinematics is the same as for $K^- \to \pi^0\mu^-\bar{\nu}_\mu$, therefore the branching ratio is

$$B(K^- \to \pi^-\mu^- e^+) = B(K^- \to \pi^0\mu^-\bar{\nu}_\mu)\,\frac{16\, G_H(se^- \to d\mu^-)^2}{0.05\, G^2} \cong$$
$$\cong 16\,\frac{G_H(se^- \to d\mu^-)^2}{G^2}. \quad (4.23)$$

For $G_H = G$ and the normal values of mixing angles this gives $\sim 10^{-7}$ and 10^{-3} in cases A and B, respectively, compared to the experimental upper limit 5.10^{-9}. This has a rather stringent consequence in case B: either $M_H > 24 M_W \cong 2$ TeV or very small mixing angles $\Theta_e\Theta_d \cong 1.8.10^{-5}$ (see Table B). In case A the corresponding requirement $\beta_e\beta_d \cong 1.8 10^{-5}$ is less unnatural since β is responsible for the mixing of the two

non-neighbouring generations.

6.) $K \to \pi ee$, $\pi\nu\bar{\nu}$.

These processes are also mediated by vertical weak interactions through one-loop diagrams [13]. The observed branching ratios $B(K^- \to \pi^- e^- e^+) = 2.6.10^{-7}$, $B(K^- \to \pi^-\mu^-\mu^+) < 2.4.10^{-6}$, $B(K^- \to \pi^-\nu\bar{\nu}) < 0.6.10^{-6}$ [8] are consistent with the theoretical calculations giving, for instance, $B(K^- \to \pi^-\nu\bar{\nu}) \cong 10^{-10}$.

Neglecting the electron and neutrino masses the kinematics of the processes $K \to \pi ee$, $\pi\nu\bar{\nu}$ is the same as for $K^- \to \pi^0 e^-\bar{\nu}_e$. (The process $K \to \pi\mu\mu$ has an extra phase space suppression compared to $K \to \pi ee$.) The amplitude is of the form (4.20), for instance:

$$T(K^- \to \pi^-\nu_\tau\bar{\nu}_\mu) = \sqrt{2}\, G_H(s\nu_\mu \to d\nu_\tau)\,\tilde{u}_{\nu_\tau}\gamma_\varrho(1-\gamma_5)v_{\nu_\mu}\langle\pi^-|\mathcal{F}_{ds}(0)^\varrho|K^-\rangle \quad (4.24)$$

The effective couplings can be read off from Eqs.(4.1-2). The processes least sensitive to the generation mixing are those with ν_μ and ν_τ. We have in these cases

$$G_H(s\nu_\mu \to d\nu_\tau) = -G_H(s\nu_\tau \to d\nu_\mu) = G_H\beta_d \; ; \qquad (A)$$

$$G_H(s\nu_\mu \to d\nu_\tau) = G_H(s\nu_\tau \to d\nu_\mu) = -G_H(\Theta_d - 2\gamma_d\beta_d)(2\gamma_e + \beta_e\Theta_e); \quad (B)$$

$$G_H(s\nu_\tau \to d\nu_\tau) = -G_H(s\nu_\mu \to d\nu_\mu) = G_H(\Theta_d - 2\gamma_d\beta_d). \qquad (B) \quad (4.25)$$

Processes $K \to \pi\nu\bar{\nu}$ with electron neutrinos involve an extra suppression. The branching ratio for $K^- \to \pi^-\nu_\tau\bar{\nu}_\mu$ is, for instance:

$$B(K^- \to \pi^-\nu_\tau\bar{\nu}_\mu) = B(K^- \to \pi^0 e^-\bar{\nu}_e)\,\frac{8\, G_H(s\nu_\mu \to d\nu_\tau)^2}{0.05\, G^2} \cong$$
$$\cong 8\,\frac{G_H(s\nu_\mu \to d\nu_\tau)^2}{G^2}. \quad (4.26)$$

For $G_H = G$ and normal mixing this gives $\sim 10^{-3}$ in both cases A and B, compared to the experimental $< 0.6.10^{-6}$. This would mean for the mass $M_H > 6 M_W \cong 0.5$ TeV. Concerning the restriction for mixing angles assuming $G_H = G$ see Table A.

As it can be seen from Eq.(4.25), the most stringent limit for the horizontal interactions comes from $K \to \pi \nu_\mu \bar{\nu}_\mu$ or $\pi \nu_\tau \bar{\nu}_\tau$ in the case B . Normal mixing and $G_H = G$ involves a branching ratio ~ 0.1 in Eq. (4.26). The experimental upper limit $B(K^- \to \pi^- \nu \bar{\nu}) < 0.6 \cdot 10^{-6}$ means $M_H > 42 M_W \cong 3.5$ TeV for "normal" mixing or a mixing angle $\Theta_d < 2 \cdot 10^{-4}$ for $G_H = G$.

V. Concluding remarks.

Summarizing the previous Section: a horizontal gauge boson with low energy effective coupling constant G_H equal to the Fermi coupling constant G is roughly consistent with present experimental data if it acts mainly on the second and third generation. Almost all experimental limits depend on unknown mixing angles among the generations and choosing these mixing angles small enough it is possible to suppress the unobserved processes. (See Tables A and B for the two different multiplet assignments of the three generations considered in the present paper.) It has to be noted, however, that sometimes (especially in case B) the required values of mixing angles seem to be unnaturally small (e.g. much smaller than the relative mixing of u- and d-type quarks known from vertical weak interactions).

In this situation there is a great interest in finding processes which give limits independent of unknown mixing angles. As shown in the previous Section, at present there is only a single such process: $\tau \to \mu \nu_\tau \bar{\nu}_\mu$ in case A. The present experimental information on $\tau \to \mu \nu \bar{\nu}$ and $\tau \to e \nu \bar{\nu}$ is, however, insufficient to constrain seriously the magnitude of G_H. (See part 2 of Section III.)

There exists no such process in case B. A possibility would be to use the existing information on the relative mixing of u- and d-type quarks together with the good limit on $K \to \pi \nu \bar{\nu}$ involving $|\Theta_d + 2\gamma_d \beta_d| \cong |\Theta_d| < 2 \cdot 10^{-4}$. (See Table B.) One has to compare this e.g. with the branching ratio for $D^+ \to \pi^+ \mu^- \mu^+$,where the effective coupling involves the mixing angle combination $|\Theta_u + 2\gamma_\mu \beta_u| \cong |\Theta_u|$. Due to the known value of the Cabibbo-angle $|\Theta_c| \cong 0.23$, it is impossible to choose both $|\Theta_d|$ and $|\Theta_u|$ very small: from the above number for $|\Theta_d|$ we have $|\Theta_u| \cong 0.23$. This gives a branching ratio $\cong 0.05$ for $D^+ \to \pi^+ \mu^- \mu^+$ if $G_H = G$ (assuming $B(D^+ \to e^+ \nu_e K^0) \cong 0.12$ [10-11]). It would be of great interest to measure this branching ratio to the best possible accuracy. E.g., an upper limit $B(D^+ \to \pi^+ \mu^- \mu^+) < 0.005$ would push up the mass of the lightest horizontal gauge boson $M_H > 2M_W \cong 160$ GeV (in the singlet + doublet case B), independently of the value of unknown mixing angles.

An optimistic point of view in favour of horizontal interactions is, that the lightest horizontal gauge boson acts mainly on the second and third generation and therefore the smallness of mixing explaines the present absence of any positive evidence on horizontal weak interactions. Thereby, the strength of the horizontal interaction could be comparable to the strength of the usual vertical weak interactions. A more pessimistic (and perhapes also more realistic) attitude is to exclude unnaturally small mixing angles. In this case the data, based mainly on the very precise numbers on K-meson decays, roughly say that M_H is above 1 TeV.

The only possible way to diminish the uncertainty is to look experimentally for the possible processes where the generation number is not conserved. The most sensitive tests come presumably from decay processes, but in the case of not very large M_H valuable information can be obtained also from high energy scattering processes like e.g. $eN \to \mu N$, $e^+e^- \to e^+\mu^-, \tau^+\mu^-$ etc.. Among the decay processes a prominent role is played by τ and D -meson decays (and later on perhapes also the decays of b- and t-quarks) testing immediately horizontal couplings of second and third generation fermions.

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Table A.

Experimental limits on the lowest possible horizontal gauge boson in case A (triplet of generations) formulated in terms of the small mixing angles, assuming an effective horizontal coupling constant $G_H = G$.

Process	Exp. limit from Refs. [8-11]
$\mu^- \to e^- \nu_\mu \bar{\nu}_e$	$\lvert\beta_e\rvert\lvert\beta_\nu\rvert < 0.05$
$\to e^- \nu_\mu \bar{\nu}_\tau + e^- \nu_\tau \bar{\nu}_\mu$	$\lvert\beta_e\rvert < 0.05$
$\tau^- \to \mu^- e^- \mu^+ + \mu^- \mu^- e^+$	$\lvert\beta_e\rvert < 0.03$
$\to \mu^- K^{o*} + \mu^- \bar{K}^{o*}$	$\lvert\beta_d\rvert$
$K^- \to \pi^- \mu^- e^+$	$\lvert\beta_e\rvert\lvert\beta_d\rvert < 2\cdot 10^{-5}$
$\to \pi^- \nu_\mu \bar{\nu}_\tau + \pi^- \nu_\tau \bar{\nu}_\mu$	$\lvert\beta_d\rvert < 2\cdot 10^{-4}$

Table B.

The same as Table A in case B (singlet + doublet of generations).

Process	Exp. limit from Refs. [8-11]
$\mu^- \to e^- \nu_\mu \bar{\nu}_e$	$\lvert\Theta_e + 2\gamma_e\beta_e\rvert\lvert\Theta_\nu + 2\gamma_\nu\beta_\nu\rvert < 0.05$
$\to e^- \nu_\mu \bar{\nu}_\mu + e^- \nu_\tau \bar{\nu}_\tau$	$\lvert\Theta_e + 2\gamma_e\beta_e\rvert < 0.05$
$\to e^- e^- e^+$	$\lvert\Theta_e + 2\gamma_e\beta_e\rvert\lvert\Theta_e^2 - \beta_e^2 + 4\gamma_e\beta_e\Theta_e\rvert < 3\cdot 10^{-5}$
$\tau^- \to \mu^- \mu^- \mu^+$	$\lvert\gamma_e + \frac{1}{2}\beta_e\Theta_e\rvert < 0.04$
$\to e^- \mu^- \mu^+$	$\lvert\beta_e - 4\gamma_e\Theta_e\rvert < 0.04$
$\to \mu^- \phi$	$\lvert 2\gamma_e + \beta_e\Theta_e\rvert$
$\to \mu^- K^{o*} + \mu^- \bar{K}^{o*}$	$\lvert 2\gamma_e + \beta_e\Theta_e\rvert\lvert\Theta_d + 2\gamma_d\beta_d\rvert$
$K^- \to \pi^- \mu^- e^+$	$\lvert\Theta_e + 2\gamma_e\beta_e\rvert\lvert\Theta_d + 2\gamma_d\beta_d\rvert < 2\cdot 10^{-5}$
$\to \pi^- e^- e^+$	$\lvert\Theta_d + 2\gamma_d\beta_d\rvert\lvert\Theta_e^2 - \beta_e^2 + 4\gamma_e\beta_e\Theta_e\rvert < 10^{-4}$
$\to \pi^- \nu_\mu \bar{\nu}_\mu + \pi^- \nu_\tau \bar{\nu}_\tau$	$\lvert\Theta_d + 2\gamma_d\beta_d\rvert < 2\cdot 10^{-4}$