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PHASE DIAGRAM OF SO(2) LATTICE GAUGE THEORY IN FOUR-DIMENSIONS

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PHASE DIAGRAM OF SO(2) LATTICE GAUGE THEORY IN FOUR-DIMENSIONS

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Abstract

Monte Carlo studies of SO(2) lattice gauge theory in four-dimensions are presented. The results include Wilson loop expectation values for a 6^4 lattice, the string tension, plaquette-plaquette and loop-plaquette correlations. The possibility of measuring the glueball mass is also discussed.

Wilson [1] and Polyakov [2] originally proposed the study of gauge theories on a lattice. Local gauge invariance survives in the lattice formulation of field theory and the space-time lattice regulates the ultraviolet divergences, providing a finite theory. Recently there has been a great surge of Monte Carlo simulation studies [3-12] of such theories. These calculations generate, by Monte Carlo simulation, field configurations on a finite lattice in such a manner that they simulate the partition function. Statistical equilibrium for the lattice, for any given value of the temperature, is, in general, achieved by means of the heat-bath method of Creutz [5].

Following Creutz [3], the system is defined in a hypercubical lattice of four Euclidean dimensions with fixed finite lattice spacing. With any link of the lattice joining nearest neighbor lattice sites labelled by i and j, we associate an element of the gauge group U_{ij}. We require that

$$U_{ji} = (U_{ij})^{-1} .$$

The partition function is defined by

$$Z(\beta) = \sum_{\{i,j\}} \exp[-\beta S(U)],$$

where β is the inverse temperature and S(U) is the action which is defined by

$$S(U) = \sum_{\square} S_{\square} ,$$

where $\square$ denotes a plaquette and

$$S_\square = 1 - \frac{1}{2}\,\mathrm{Tr}\,(U_{ij}U_{jk}U_{kl}U_{li}).$$

In the previous equation, the corners of the plaquette are labelled by the subscript i,j,k and l. The internal energy E is the average energy per plaquette, i.e.

$$E = \langle S_\square \rangle .$$

We have used a 6x6x6x6 lattice and periodic boundary conditions in our calculations.

In the method of Creutz [8], the lattice is brought to a state of equilibrium at a given value of the temperature. The expectation values of rectangular Wilson loops W(I,J), defined by

$$W(I,J) = \langle \frac{1}{2}\,\mathrm{Tr}\,U(C) \rangle ,$$

where C is a closed path of links and I and J are the linear dimensions of the loop $[\,1 \leqslant I,\ J \leqslant \frac{1}{2}$ (lattice length)$]$ are then calculated. Using these, the quantity $\chi(I,J)$, defined by

$$\chi(I,J) = -\ln\left[\frac{W(I,J)W(I-1,J-1)}{W(I,J-1)W(I-1,J)}\right],$$

was evaluated. In the case of $I \gg J \gg 1$, the quantity $\chi(I,J)$ is proportional to the force between a quark and an antiquark, separated by a distance Ja where a is the lattice spacing. In the strong coupling regime, where the

loops are dominated by the area law and I and J are both large

$$\ln W(I,J) \sim -KA$$

where A is the loop area and is given by $A = a^2IJ$, while K is the string tension. Then

$$\chi \longrightarrow a^2K,$$

so that $\chi(I,J)$ is equal to the coefficient of the area law. This is, strictly speaking, true only for asymptotically large loops. However, in the weak coupling regime, where the loops are dominated by the perimeter law and I and J are held fixed χ should have a perturbative expansion and small I and J should give a χ which differs from a^2K.

Following Refs. 13 and 14, the correlations between plaquettes of the form

$$C(r) = \langle \square_{x+r}\,\square_x \rangle - \langle \square \rangle^2,$$

where $r^2 = (I-1)^2 + (J-1)^2$ were also calculated. C(r) measures the effective potential between plaquettes. The exchanged object between a plaquette plaquette would be a color singlet, i.e. a glueball. If C(r) decays like exp(−mr), then the slope of ln C(r) plotted against r, gives a measure of the glueball mass. If C(r) decays like r^{-n}, then the glueball mass is zero. The correlations C(r) between a big loop and a plaquette were also evaluated.

In Fig. 1 is shown the thermal cycle of the SO(2) model. Each point is the result of averaging the value of the internal energy over eleven sweeps through the lattice. As usual [4] the heated values are obtained starting with a totally ordered lattice and the cooled values are obtained with all the link variables randomly ordered. We note that this figure agrees nicely with Fig. 1 of Ref. 3, with a hysteresis effect clearly visible near $\beta = 0.99$. [In Ref. 4, Creutz found the exact value to be $\beta = 0.987 \pm 0.023$.]

In Fig. 2 we recalculate the SO(2) part of Fig. 2 of Ref. 4. Again the agreement is good with the figures showing that, if we start with an ordered and a disordered lattice, near the critical temperature, then large fluctuations are still present after 25 sweeps through the lattice and convergence is slow.

In Fig. 3 we have plotted the Wilson loops. For the Wilson loop on a single plaquette, we expect [5]

$$W(\square) = 1 - \langle S_\square \rangle .$$

This is confirmed exactly as can be seen by comparing Fig. 1 and Fig. 3 for I = 1. The sudden crossover from an area decay to a perimeter decay is evident near $\beta \simeq 1.0$. This change of the loop behavior is a clear indication of a phase transition. The 6^4 lattice was brought to equilibrium after approximately 15 sweeps through the lattice. However, 25 sweeps were calculated and the results of the last 5 sweeps were averaged.

Fig. 4 shows a plot of $\chi(I,I)$ for I = 1 and 2 against the inverse temperature. The strong coupling deviation from expected asymptotic behavior sets in at

$\beta \simeq 0.90$. All Wilson loops approach the limit 1 as $\beta \to \infty$, that is, at low temperatures and approach the limit zero as $\beta \to 0$, that is, at high temperatures.

Figs. 5 and 6 show C(r) against β for r = 0 and r = 1, respectively. More properly we should have plotted C(r) against r for fixed β. However, only C(r), for r = 0, shown in Fig. 5, appears to be statistically significant for the range of β we consider. For C(r), r = 1, shown in Fig. 6, the values are statistically insignificant in all regions of β except near the critical point. For $r \geqslant 2$ the value of C(r) is statistically not significant. Thus, it is impossible to plot a graph of ln C(r) against r and extract the slope of the curve. Therefore it is not possible to extract the gluevall mass.

The large loop-plaquette correlations C(r) were also measured and were found to be statistically insignificant for all values of r.

Acknowledgements

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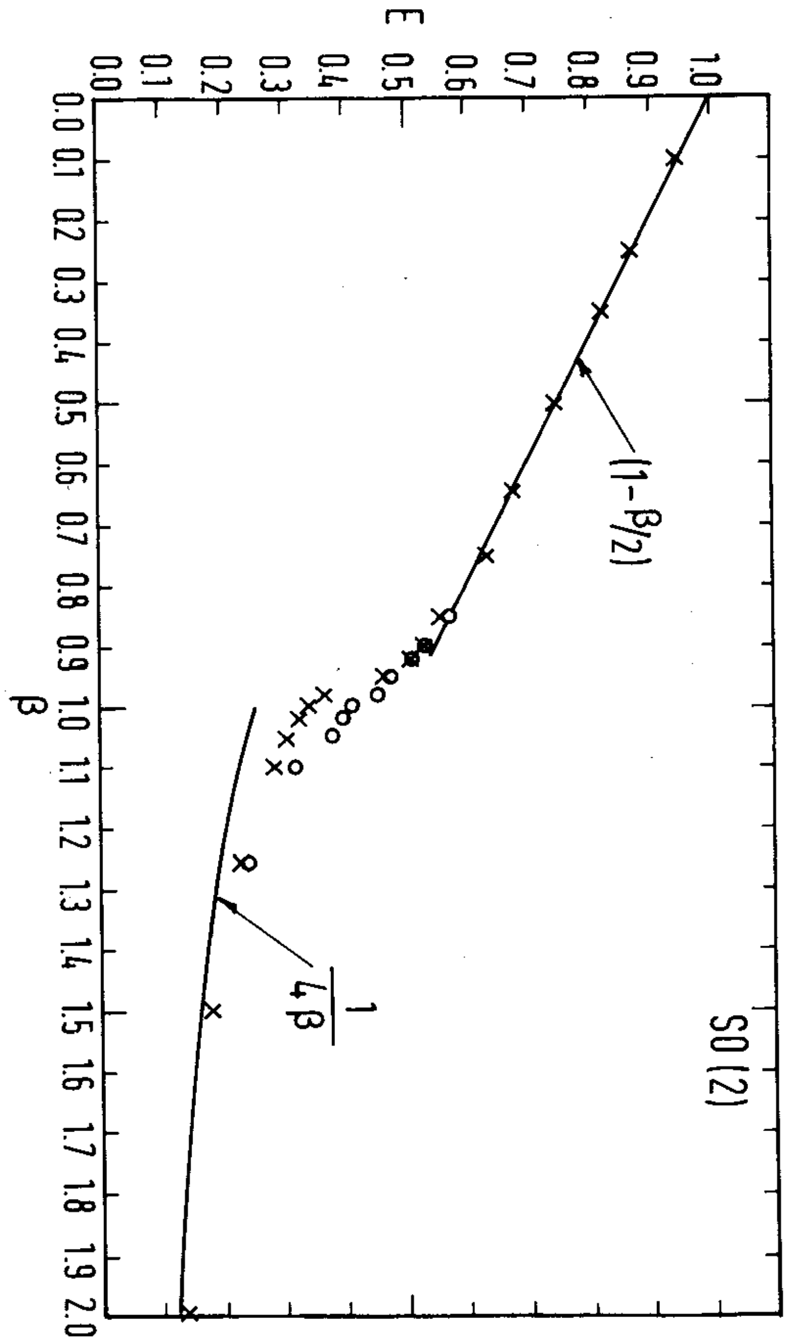

Fig.1

Figure Captions

Fig. 1. The internal energy as a function of β obtained when cooling and heating the SO(2) gauge system in four dimensions. The high and low temperature results of Ref. 4 are also indicated. [The crosses represent heating while the open circles represent cooling.]

Fig. 2. The internal energy as a function of the number of iterations at a fixed value of β .

Fig. 3. The Wilson loop W(I,I) for I = 1,2 and 3 as a function of β .

Fig. 4. The quantity χ(I,I) for SO(2) gauge theory as a function of β .

Fig. 5. The plaquette-plaquette correlation C(r), for r = 0, as a function of β .

Fig. 6. The plaquette-plaquette correlation C(r), for r = 1, as a function of β .

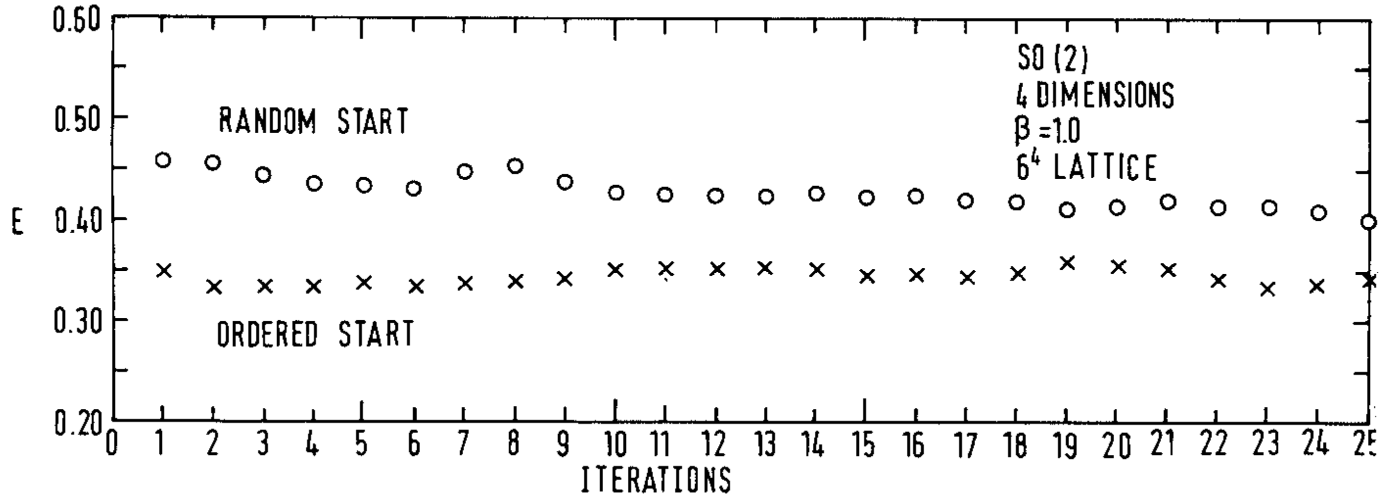

FIG.2.

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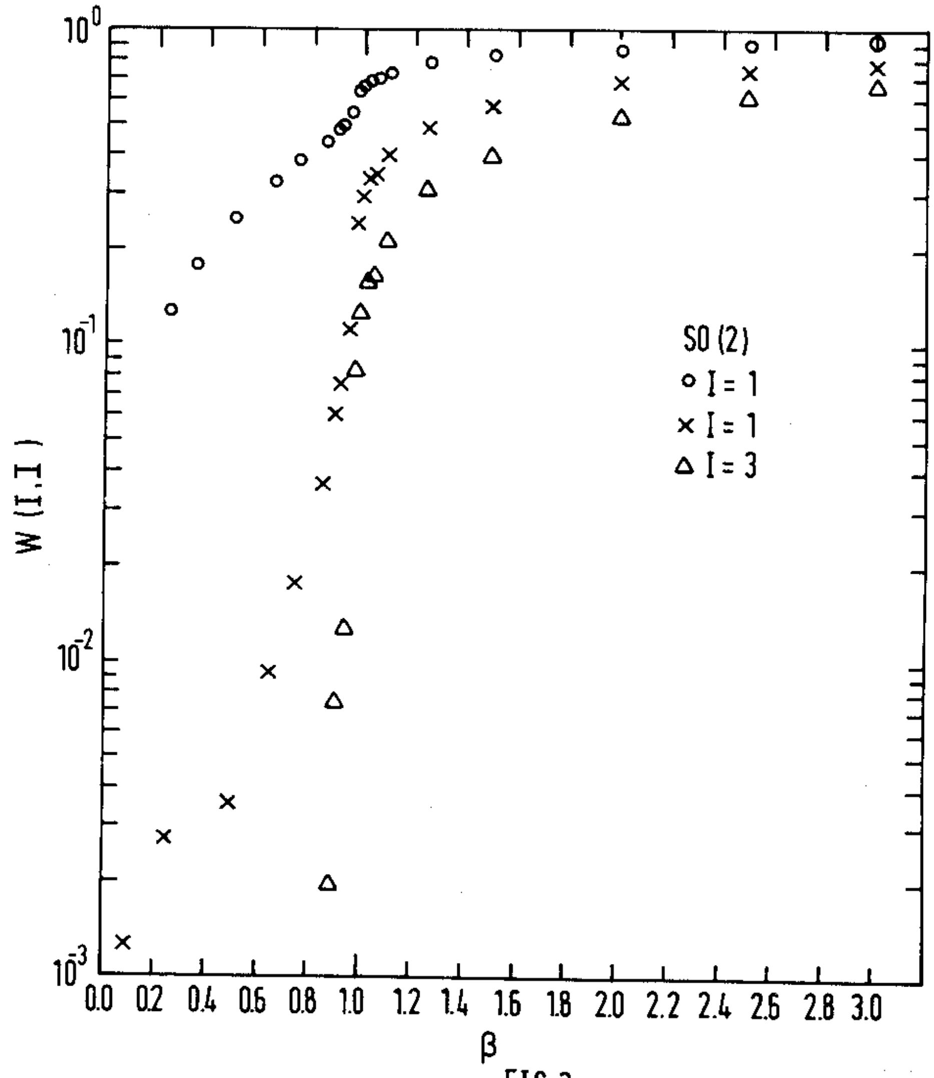

FIG.3.

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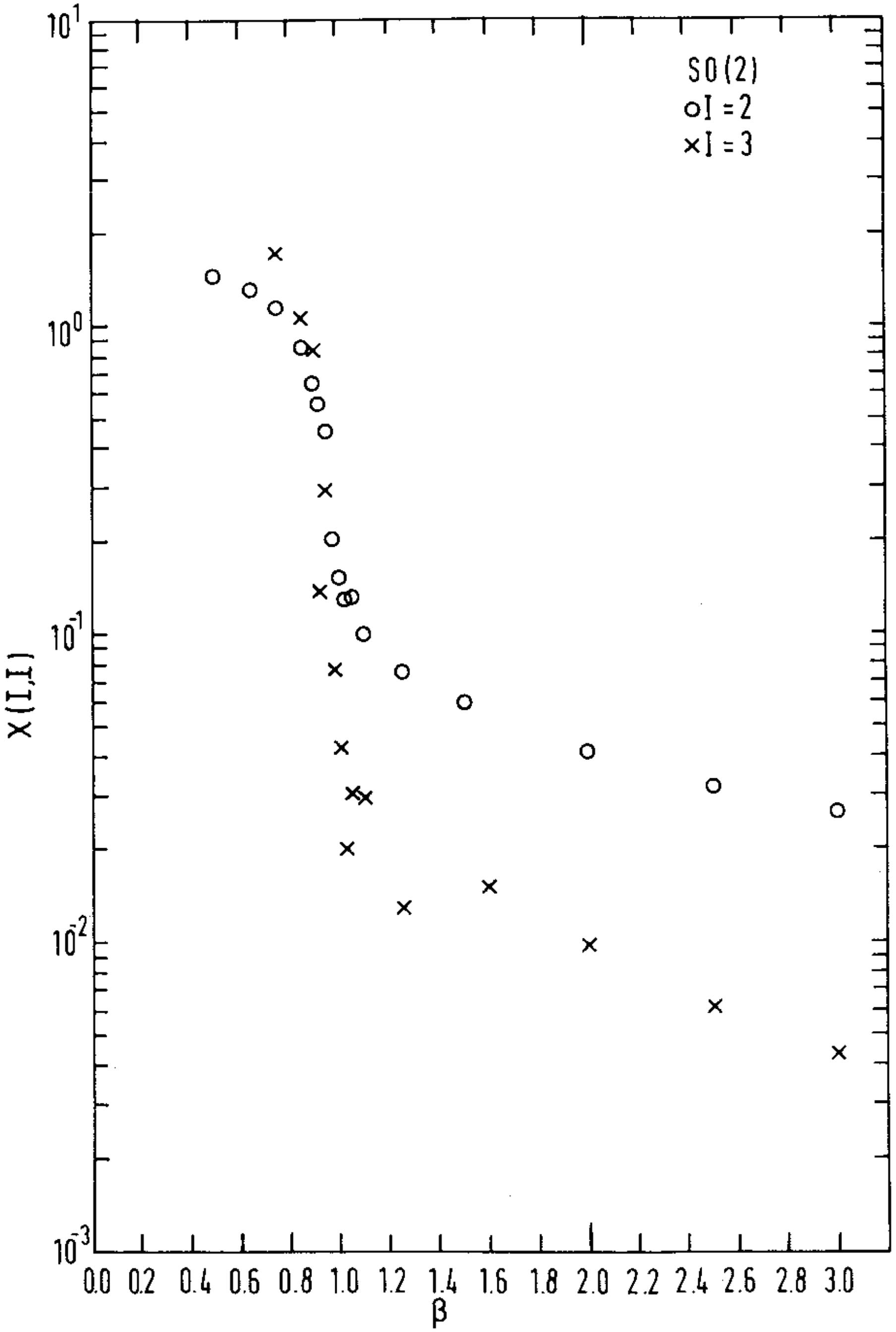


FIG.4.

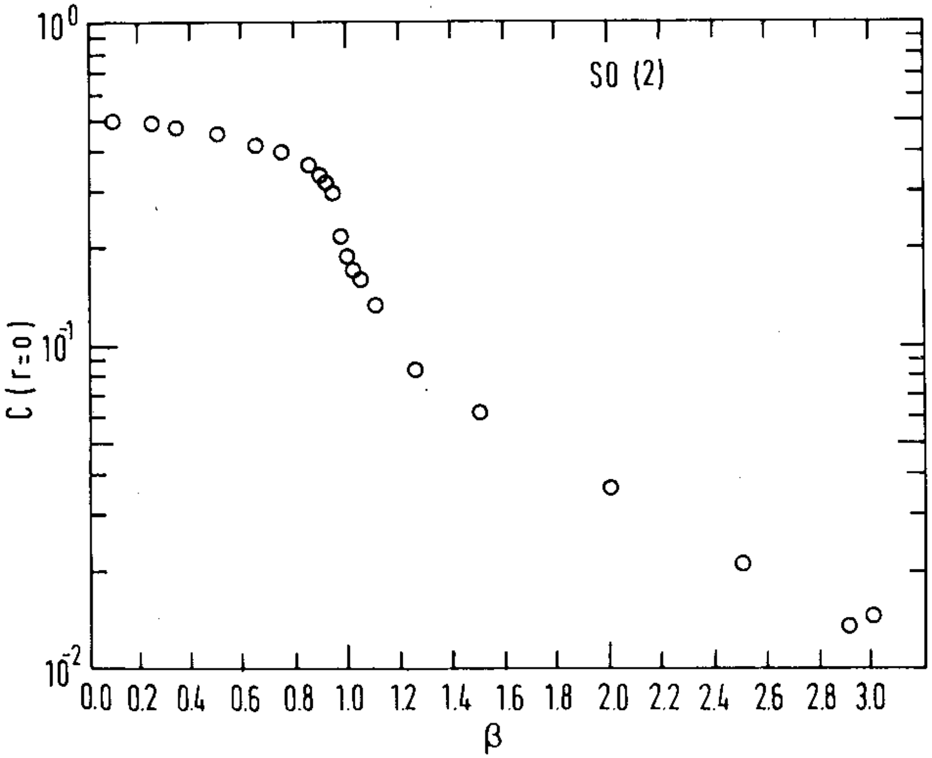
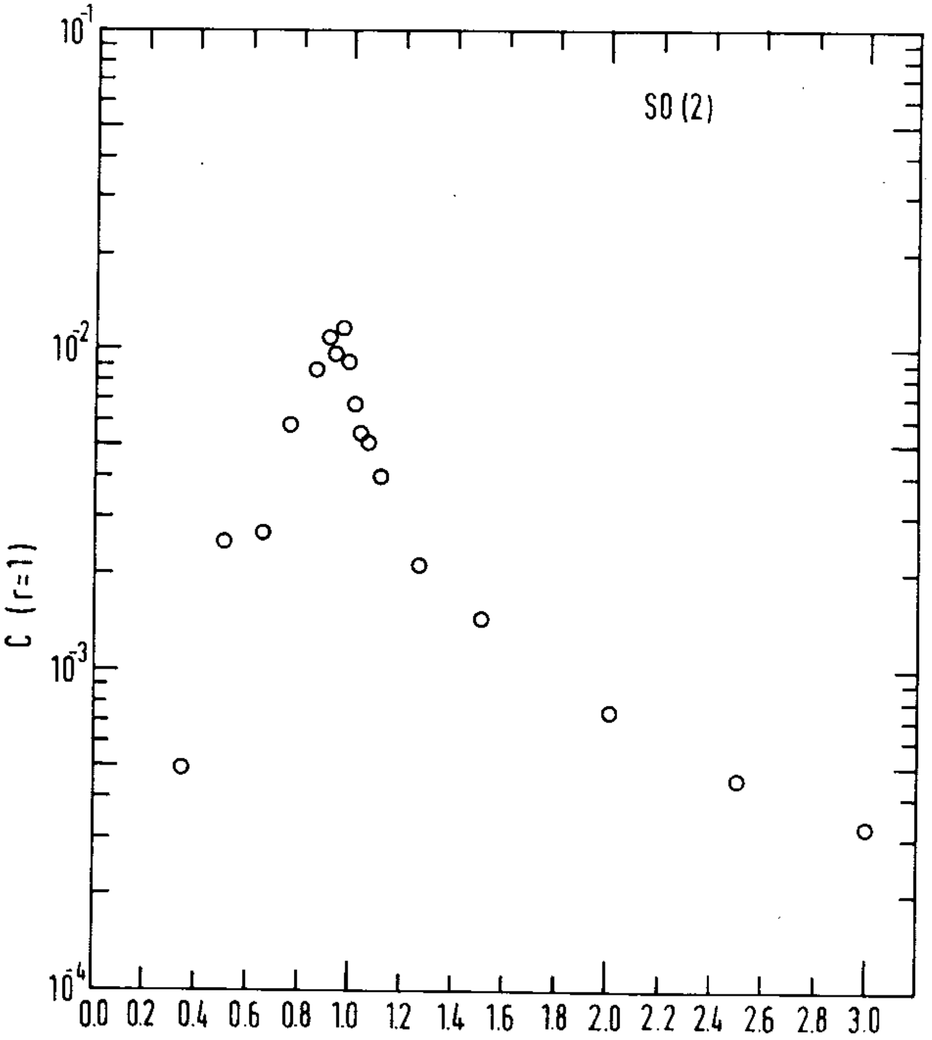


FIG.5.

30076



SO (2)
10^{-1}
10^{-2}
10^{-3}
10^{-4}
C (r=1)
0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 1.6 1.8 2.0 2.2 2.4 2.6 2.8 3.0
β

FIG.6.

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