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RESPONSE OF SU(2) LATTICE GAUGE THEORY TO A GAUGE INVARIANT EXTERNAL FIELD

by

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Abstract:

Topologically determined Z(2) variables in pure SU(2) lattice gauge theory are discussed. They count the number of "vortex souls". The expectation value of the corresponding Z(2) loop and the dependence of the string tension on an external field h coupled to them is calculated to lowest order in the high temperature expansion. The result is in agreement with the conjecture that the probability distribution of vortex souls determines the string tension. A different formula for the string tension is found in the two limiting cases $0 \leq |h| \ll \beta \ll 1$ and $0 \leq \beta \ll h \ll 1$. This phenomenon is traced to the effect of short range interactions of the vortex souls which are mediated by the other excitations in the theory.

1. Introduction

We consider a pure SU(2) lattice gauge theory in four dimensions [1,2,3]. The random variables $U(b) \in SU(2)$ are attached to links b of a hypercubical lattice Λ and the Euclidean action is given by:

$$L(U) = \frac{\beta}{2} \sum_{p} \mathrm{tr}\,[\,U(\dot{p}) - 1\,] \tag{1.1}$$

Summation is over all unoriented plaquettes p of the lattice.
$U(\dot{p}) = U(b_1) \cdots U(b_4)$ is the parallel transporter around a plaquette p with boundary consisting of links $b_1 \dots b_4$.
The expectation value of an observable F(U) is given by:

$$\langle F \rangle = \frac{1}{Z} \int \prod_{b} dU(b)\; F(U)\; \exp L(U) \tag{1.2}$$

where Z is defined by the requirement $\langle 1 \rangle = 1$. dU is normalized Haar measure on SU(2). $\langle F \rangle$ will in general depend on the choice of boundary conditions. According to Wilson [1] the non-vanishing of the following quantity

$$\alpha(\beta) = \lim_{|\Xi| \to \infty} \frac{-1}{|\Xi|} \log \langle \mathrm{tr}\, U(C) \rangle \tag{1.3}$$

is a condition for the confinement of static quarks. In eq. (1.3) C is a large closed loop and Ξ is the minimal surface bounded by C. α is called the string tension.
It has been argued [4,...,11] that the special kind of disorder that is responsible for confinement of static quarks in pure lattice gauge theories is caused by topological excitations called vortices. They are associated with the center Γ of the simply connected gauge group G (or, alternatively and equivalently, with the first homotopy group $\pi_1(G/\Gamma) \approx \Gamma$ of the adjoint group G/Γ). More particularly, it has been conjectured that the probability distribution of vortex souls determines the string tension [6,12].
To define vortex souls [6,12] one introduces new variables $\sigma(b) = \pm 1$ and $\overline{W(b)} \in SO(3) \approx SU(2)/Z(2)$. They depend on the original variables $U(b')$ and have the following properties:
(a) locality: $\overline{W(b)}$ and $\sigma(b)$ depend only on $U(b')$ on links b' in a neighborhood of one lattice spacing of b and $\overline{W(b)}$ depends furthermore only on cosets
$\overline{U(b)} = U(b)\Gamma \in SU(2)/Z(2) \approx SO(3)$ $[\,\Gamma = Z(2)\,]$

(b) gauge invariance: $\overline{W(b)}$ is gauge invariant and $\sigma(b)$ transforms like $\sigma(b) \rightarrow v(x)\,\sigma(b)\,v(y)^{-1}$ for $b=(x,y)$ and with $v(x)=\pm 1$.

(c) completeness: there exists a gauge transformation $S(.)$ such that

$$W(b)\,\sigma(b) = S(x)^{-1}\,U(b)\,S(y) \qquad \text{for } b=(x,y) \tag{1.4}$$

($W(b)$ is uniquely determined by the requirements $W(b) \in SU(2)$, $\overline{W(b)} = W(b)\Gamma$, $\operatorname{tr} W(b) \geq 0$). $S(.)$ depends on U.
Because of (c) σ and $\overline{W}$ determine the variables $U(b)$ up to a gauge transformation.
We say that a "soul of a vortex" passes through the plaquette p if $\sigma(\dot{p}) = \prod_{b \in \partial p} \sigma(b) = -1$.
Vortex souls are counted modulo 2 in an SU(2) theory.
(a), (b), and (c) do not determine W and σ uniquely. To get explicit formulae one has to choose a local gauge [14]. This can be done in various ways.
A convenient one is defined as follows: consider the components of the electric field $E(x)$ arranged in a 3x3 matrix and decompose $E(x) = \overline{S(x)}\,P(x)$ with $\overline{S(x)} \in SO(3)$ and $P(x)$ has to have certain properties, for instance upper triangular form (Δ-gauge) or positive or negative definiteness (P-gauge).
Then define $S(x) \in SU(2)$ by $\overline{S(x)} = S(x)\Gamma$ and $\operatorname{tr} S(x) \geq 0$, $\overline{W(b)} = \overline{S(x)}^{-1} \cdot \overline{U(b)} \cdot \overline{S(y)}$ for $b=(x,y)$. For further details see appendix B.

We are interested in the behaviour of the Z(2) loop, i.e. $\langle \sigma(C) \rangle$ for a closed loop C, and its relation to the Wilson loop $\langle \operatorname{tr} U(C) \rangle$. From eq. (1.4) one obtains

$$\langle \operatorname{tr} U(C) \rangle = \langle \operatorname{tr} W(C) \prod_{p \in \Xi} \sigma(\dot{p}) \rangle \tag{1.5}$$

$$\operatorname{tr} W(C) = \sum_{\alpha_1 \cdots \alpha_n} W_{\alpha_1 \alpha_2}(b_1) \cdots W_{\alpha_n \alpha_1}(b_n)$$

if C consists of links $b_1, \ldots, b_n$.
$\operatorname{tr} W(C)$ is a sum of products of local gauge invariants. It has been conjectured [6] that

$$\langle \prod_{p \in \Xi} \sigma(\dot{p}) \rangle \equiv \langle \sigma(C) \rangle = K \cdot e^{-\alpha |\Xi|} \tag{1.6}$$

($|\Xi|$ denotes the area of the surface Ξ) with slowly varying K so that $\alpha = \limsup \frac{-1}{|\Xi|} \log \langle \sigma(C) \rangle$ and α coincide with the string tension (1.3). If the conjecture is true, this implies that the probability distribution of vortex souls determines α, since the behaviour of $\sigma(C)$ only depends on this probability distribution.
In this paper we compute the first term in the high temperature expansion of $\langle \sigma(C) \rangle$ (section 2). The result is in agreement with the conjecture.

To further test relation (1.6) we consider the more general model with an external field $h \sum_{p} \sigma(\dot{p})$ coupled to the action (1.1).
Relation (1.6) remains true to lowest order for $h \neq 0$. We study the change of the string tension under a change of h in section 3 and discuss the results in terms of the properties of an effective Z(2) theory of vortex souls in section 4. Some details of the computations are relegated to an appendix.

2. Relation between Z(2) loop and Wilson loop

The leading term in the high temperature expansion (small β) of $\langle \sigma(C) \rangle$ is given by the following cluster integral:

$$I_{\ell t}^{Z(2)} = \int \prod_{b} dU(b) \left(\frac{\beta}{2}\right)^{|\Xi|} \prod_{p \in \Xi} \operatorname{tr} U(\dot{p})\ \sigma(C) \tag{2.1}$$

The vanishing of terms of lower order is based on the fact that $\sigma(b) \rightarrow -\sigma(b)$ if $U(b) \rightarrow -U(b)$. The argument goes as follows [5] :
given a cluster Δ ,i.e. a set of plaquettes, which contains fewer plaquettes than $|\Xi|$ one can find a "thin vortex" X, i.e. a coclosed set of plaquettes, which winds around C and has no plaquettes in common with Δ. This implies the existence of a function $\tau(b) = \pm 1$ such that $\tau(\dot{p}) \equiv \prod_{b \in \partial p} \tau(b) = -1$ if $p \in X$ and otherwise $\tau(\dot{p}) = +1$. "X winding around C" means that there is an odd number of links $b \in C$ with $\tau(b) = -1$. For an illustration see figure 1.
The transformation $U(b) \rightarrow \tau(b) U(b)$ in the cluster integral I_Δ yields:

$$I_\Delta = \int \prod_{b} dU(b) \left(\frac{\beta}{2}\right)^{|\Delta|} \prod_{p \in \Delta} \operatorname{tr} U(\dot{p})\ \sigma(C) = -I_\Delta$$

because $\operatorname{tr} U(\dot{p})$ stays invariant for $p \in \Delta$ and $\sigma(C)$ changes its sign.
Therefore $I_\Delta = 0$.
The evaluation of $I_{\ell t}^{Z(2)}$ is not straightforward because of the complicated dependence of the Z(2) variables $\sigma(b)$ on U. In fact $I_{\ell t}^{Z(2)}$ depends on the local gauge chosen to define the Z(2) variables.
For the sake of simplicity we consider only the electric gauge. The situation for the magnetic gauge is completely analogous, although the visualization is more difficult. The technique used to evaluate the integrals is explained in the appendix C. Here we list only the results. For a rectangular loop C bounding the surface Ξ we have: ($|C|$ is the length of the path C)
(a) electric P-gauge:

$$I_{\ell t}^{Z(2)} = 4 \cdot \left(\frac{4}{3\pi}\right)^{|C|} \left(\frac{\beta}{4}\right)^{|\Xi|} \tag{2.2}$$

(b) electric Δ-gauge:

(i)

$$I_{\ell t}^{\Xi(2)} = 4 \cdot \left(\frac{4}{3\pi}\right)^{|C|} \cdot \left(\frac{\beta}{4}\right)^{|\Xi|} \qquad (2.3)$$

if the surface Ξ lies in a hyperplane x_0 = const. ("spacelike" Ξ)

or if Ξ lies in a (0,3)-plane

(ii)

$$I_{\ell t}^{\Xi(2)} = 4\left(\frac{4}{3\pi}\right)^{|C|} \left(\frac{\beta}{4}\right)^{|\Xi|} \mathrm{Re}\left[\left(1+i\frac{8}{3\pi}\right)^{\frac{1}{2}|C|-1}\right] \qquad (2.4)$$

if Ξ lies in a (0,1)-plane

(iii)

$$I_{\ell t}^{\Xi(2)} = 4\left(\frac{4}{3\pi}\right)^{|C|} \left(\frac{\beta}{4}\right)^{|\Xi|} \mathrm{Re}\left[\left(1+i\frac{1}{6}\right)^{\frac{1}{2}|C|-1}\right] \qquad (2.5)$$

if Ξ lies in a (0,2)-plane

(in the magnetic gauge the corresponding results can be obtained from (2.2),... (2.5) by a simple permutation).

For a single plaquette ($|\Xi|=1, |C|=4$) eqs.(2.2)...(2.5) give the same value.

This should be compared with the leading term for the Wilson loop $\langle \mathrm{tr}\, U(C)\rangle$:

$$I_{\ell t}^{W} = 2\left(\frac{\beta}{4}\right)^{|\Xi|} \qquad (2.6)$$

We see that the difference between (2.2) and (2.6) is given by a suppression factor dependent on the perimeter $|C|$. For $\Xi = p$ = one plaquette we can also compare the result with results of Monte Carlo computations by Mack and Pietarinen [13] (figure 4).

Since the cluster integrals in the (electric) P-gauge does not have the spatial anisotropy of the results in the Δ-gauges, it appears to be preferable for numerical computations (series expansions, Monte Carlo techniques, etc). Unfortunately, Monte Carlo computations in the P-gauge involve taking a square root of a positive 3x3 matric and are therefore more time-consuming than for the Δ-gauges.

The presence of perimeter law behaved suppression factors is not unexpected for theoretical reasons [6] . Also the Wilson loop $\langle \mathrm{tr}\, U(C)\rangle$ has perimeter law factors in higher orders. These have to be removed by renormalization before a continuum limit is considered [18] . The perimeter law behaved suppression factors are caused by vortices of small extension that wind tightly around the loop C.

The additional perimeter law factors depend on the choice of the local gauge that is used to define σ but the relation $\alpha = \limsup \frac{-1}{|\Xi|} \log\langle\sigma(C)\rangle$ (eq. (1.6)) is true for all the gauges that were investigated.

3. Change of string tension in response to an external field

It is of some interest to couple a constant external field h to $\sigma(\dot{p})$, i.e. one changes the action L(U) to $L'(U) = L(U) + h \sum_{p} \sigma(\dot{p})$ (3.1)

Expectation values with this action are denoted by $\langle \cdots \rangle_h$.

A positive h-field suppresses vortex souls and one expects therefore on the basis of the vortex condensation picture of quark confinement that it will lower the string tension. (Moreover, one expects that a sufficiently large h-field will suppress confinement altogether - but unfortunately this cannot be checked by high temperature expansions to low orders.).

We consider the leading term in the high temperature expansion (small β and h) of $\langle\sigma(C)\rangle_h$ for $C = \partial\Xi$ and spacelike Ξ . We choose an electric gauge.

The first non-vanishing term is of the form $I_{\ell t} = \sum_{X\subset\Xi} I_X$ (3.2)

with

$$I_X = \int \prod_b dU(b) \left(\frac{\beta}{2}\right)^{|\Xi - X|} \cdot (\tanh h)^{|X|} \cdot \prod_{p\in\Xi-X} \mathrm{tr}\, U(\dot{p}) \cdot \prod_{p\in X} \sigma(\dot{p})\ \sigma(C) \qquad (3.3)$$

Sum over X is over all subsets of plaquettes of Ξ .

Using the same techniques as in appendix C we get: (see eq.(2.1) and (2.2))

$$I_X = (\tanh h)^{|X|} \cdot 4\left(\frac{4}{3\pi}\right)^{|\partial(\Xi-X)|} \left(\frac{\beta}{4}\right)^{|\Xi-X|} \qquad (3.4)$$

To see whether relation (1.6) remains true for $h \neq 0$ to lowest order we consider $\langle \mathrm{tr}\, U(C)\rangle_h$. The leading term is again given by (3.2) and (3.3), in the latter one $\sigma(C)$ is replaced by $\mathrm{tr}\, U(C)$. A straightforward calculation yields:

$$I_X' = (\tanh h)^{|X|} \cdot \left(\frac{4}{3\pi}\right)^{|\partial X|} \cdot 2\left(\frac{\beta}{4}\right)^{|\Xi-X|} \qquad (3.5)$$

Eq. (3.2),(3.4) and (3.2),(3.5) are now investigated for two extremal situations: (a) $\beta \ll h$ and h small and (b) $h \ll \beta$ and β small .

The explicit calculation given in appendix D yields the following result:

The string tension α defined by (1.3) satisfies the relation (1.6) to lowest order and we have

(a)

$$\alpha = -\log\tanh h - \left(\frac{4}{3\pi}\right)^4 \frac{\beta}{4\tanh h} + \mathcal{O}\left(\left(\frac{\beta}{4\tanh h}\right)^2\right) \quad , \beta \ll h \qquad (3.6)$$

(b)

$$\alpha = -\log\frac{\beta}{4} - \left(\frac{4}{3\pi}\right)^4 \frac{4\tanh h}{\beta} + \mathcal{O}\left(\left(\frac{4\tanh h}{\beta}\right)^2\right) \quad , h \ll \beta \qquad (3.7)$$

If one starts with a timelike surface Ξ or chooses another local gauge the factors $\left(\frac{4}{3\pi}\right)^4$ are slightly modified.
Naively one might have expected that to lowest order:

$$\alpha = -\log\left(A \tanh h + B\beta\right) \tag{3.8}$$

Comparison with (3.6) and (3.7) yields:

$A = 1$ and $B = \frac{1}{4}\left(\frac{4}{3\pi}\right)^4$ for situation (a)

$A = \left(\frac{4}{3\pi}\right)^4$ and $B = \frac{1}{4}$ for situation (b)

Therefore (3.8) cannot hold with constant A and B because of the factors $\left(\frac{4}{3\pi}\right)^4 \neq 1$.
Further discussion of this effect will be presented in the next section.
Perimeter law behaved factors like $\left(\frac{4}{3\pi}\right)^{|C|}$ in the Z(2) loop can come from the effect of small vortices (vortices of small extension) which can wind tightly around C (whereas α is only dependent on the probability distribution of very large vortices).
The abundance of these small vortices apparently also leads to renormalization effects which have the consequence that switching on the h field is not simply equivalent to a change in β by a constant times h even if β and h are both small.

4. The effective Z(2) theory of interacting vortex souls

In this section we will show that deviations from the naive formula (3.8) arise from short range interactions between vortex souls that are mediated by the other excitations in the theory. To this end we rewrite the SU(2) lattice gauge theory model in the form of an effective Z(2) theory. The variables $\sigma(b)$ are the only fluctuating variables in this theory and the effective action is obtained by integrating over the other ones. Using (B7) we can write down the expectation value of any observable $F(\sigma)$ which depends only on the $\sigma(b)$ variables:

$$\langle F(\sigma)\rangle = \frac{1}{Z}\int \prod_b d\sigma(b) \prod_b d\overline{W(b)}\; F(\sigma) \prod_p e^{\frac{\beta}{2}\sigma(\dot p)\,\mathrm{tr}\,W(\dot p)} \prod_x \delta(\overline{S(x)}[\overline{W}]) \tag{4.1}$$

We want to change the integration from $\prod_b d\overline{W(b)}$ to $\prod_b dW(b)$ (Haar measure on SU(2)). To do this we define

$$k_W(\dot p) = \prod_{b\in\partial p} \mathrm{sign}\;\mathrm{tr}\,W(b) \tag{4.2}$$

for an arbitrary configuration $\{W(b)\}_{b\in\Lambda}$ (not necessary with positive trace $\mathrm{tr}\,W(b) \geq 0$).
With (4.2) we can rewrite (4.1) :

$$\langle F(\sigma)\rangle = \frac{1}{Z}\int \prod_b d\sigma(b) \prod_b dW(b)\; F(\sigma) \prod_p e^{\frac{\beta}{2}\sigma(\dot p)\,k_W(\dot p)\,\mathrm{tr}\,W(\dot p)} \prod_x \delta(\overline{S(x)}[\overline{W}]) \tag{4.3}$$

We note that the integrand depends on W only through cosets $\overline{W} = W\Gamma$.

Now we make a gauge transformation $W(b) \longrightarrow W_g(b) = g(x)\cdot W(b)\cdot g(y)^{-1}$ for $b=(x,y)$ (4.4)
and average over g. We use the formula $\int_G dg\, f(g) = \int_{G/\Gamma} d\overline{g} \int_\Gamma d\gamma\, f(\overline{g}\gamma)$
$d\overline{g}$ and $d\gamma$ are the Haar measures on SO(3) and Z(2) respectively.
The integrand in eq.(4.3) does not depend on $\gamma(x)$. This is true because $k_{W_g}(\dot p)$ depends on g(x) only via $\overline{g(x)}$. From the definition of $\overline{S}$ (appendix B) we get:

$$\overline{S(x)}[\overline{W}] \longrightarrow \overline{g(x)}\cdot\overline{S(x)}[\overline{W}] \tag{4.5}$$

under the gauge transformation (4.4).
With help of the δ-functions in (4.3) the $\overline{g(x)}$ integrations can be performed and we have finally:

$$\langle F(\sigma)\rangle = \frac{1}{Z}\int \prod_b d\sigma(b) \prod_b dW(b)\; F(\sigma) \prod_p e^{\frac{\beta}{2}\sigma(\dot p)\,K_W(\dot p)} \,, \tag{4.6}$$

where we have defined:

$$K_W(\dot p) = k_{W_{S^{-1}}}(\dot p)\cdot \mathrm{tr}\,W(\dot p) \tag{4.7}$$

(explicitly: $K_W(\dot p) = \mathrm{tr}\,W(\dot p)\cdot \prod_{b\in\partial p} \mathrm{sign}\;\mathrm{tr}\left(S(x)[W]^{-1}\cdot W(b)\cdot S(y)[W]\right)$)

The effective action of interacting vortex souls is now defined so that

$$\langle F(\sigma)\rangle = \frac{1}{Z}\int \prod_b d\sigma(b)\; F(\sigma)\, \exp L_{eff}(\sigma) \tag{4.8}$$

and is given by

$$L_{eff}(\sigma) = \log \int \prod_b dW(b) \prod_p \exp \frac{\beta}{2}\sigma(\dot p)\, K_W(\dot p) \tag{4.9}$$

We apply a cluster expansion [16,17] to (4.9). This yields a representation of $L_{eff}(\sigma)$ of the following form:

$$L_{eff}(\sigma) = \sum_{\mathcal{G}} \sum_{X\subset\mathcal{G}} \phi(\mathcal{G},X) \prod_{p\in X} \sigma(\dot p) \tag{4.10}$$

Summations over $\mathcal{G}$ and X are as follows. Clusters $\mathcal{G}$ are connected sets of polymers P_i counted with multiplicities n_i : $\mathcal{G} = (P_1^{n_1},\dots,P_k^{n_k})$. The polymers P_i are connected sets of plaquettes. Summation over X is over all subsets of plaquettes of $\mathcal{G}$. We have to use a modified definition of connectivity: two plaquettes are called connected if they are not farther away than one lattice spacing, e.g.
$\square\;\square$ is connected. The modification is due to the "next nearest" neighbor interactions included in $K_W(\dot p)$. The quantities $\phi(\mathcal{G},X)$ depend on the inverse temperature β and on the combinatorics and topology of the graph $\mathcal{G}$.
In particular we have $\phi(\mathcal{G},X) = \mathcal{O}(\beta^{|\mathcal{G}|})$ where $|\mathcal{G}|$ is the total "length" of $\mathcal{G}$: $|\mathcal{G}| = \sum_{i=1}^{k} n_i|P_i|$ for $\mathcal{G} = (P_1^{n_1},\dots,P_k^{n_k})$.
If we couple an external field h to the action in (4.1) the same arguments as above can be applied. The effective action $L'_{eff}(\sigma)$ is now $L'_{eff}(\sigma) = L_{eff}(\sigma) + h\cdot\sum_p \sigma(\dot p)$ with $L_{eff}(\sigma)$ given by (4.10).
The leading term in (4.10) has $\mathcal{G} = P$ = a single plaquette. Terms in which $\mathcal{G}$ contains more than one plaquette represent the effect of interactions of vortices that are

mediated by the fluctuations in the W-variables. The terms decay exponentially with the size $|\mathcal{G}|$ of the cluster. This indicates that the interaction is short range. Apart from the irrelevant, σ independent constant in $L'_{eff}(\sigma)$ we obtain for the leading term in $L'_{eff}(\sigma)$:

$$L'_{eff}(\sigma) = \sum_p \left[\left(\tfrac{4}{3\pi}\right)^4 \beta + h \right] \sigma(\dot{p}) + \mathcal{O}(\beta^2) \tag{4.11}$$

The simplest way to prove (4.11) is by expanding

$$\langle \sigma(\dot{p}) \rangle_h = \frac{1}{Z} \int \prod_b d\sigma(b)\, \sigma(\dot{p})\, \exp L'_{eff}(\sigma)$$

If we could neglect the correction terms in (4.11) we would get for the leading term in the high temperature expansion of $\langle \sigma(C) \rangle$:

$$I^{Z(2)}_{\ell t} = \left[\left(\tfrac{4}{3\pi}\right)^4 \beta + h \right]^{|\Xi|} \tag{4.12}$$

This gives a string tension $\alpha = -\log\left[\left(\tfrac{4}{3\pi}\right)^4 \beta + h \right]$ (4.13)

As discussed in section 3 (4.13) is not consistent with (3.6) and (3.7) except in the limit of extremely small β. To get correct results we have to take into account the higher order terms in (4.11) which represent effects of interactions of vortices. For instance, the term proportional to $\sigma(\dot{p}_1)\cdot\sigma(\dot{p}_2)$ for adjacent plaquettes p_1, p_2 is $\mathcal{O}(\beta^2)$. But we need only $\frac{1}{2}|\Xi|$ of them to fill the loop $C = \partial\Xi$. Therefore their effect on $\langle\sigma(C)\rangle$ is of the same order in β as the contribution from the leading simple plaquette term.

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Appendix

A. Character expansion of sign tr U

The character expansion of sign tr U is given by $\text{sign tr } U = \sum_{j=0,\frac{1}{2},1,\dots} c_j\, \chi_j(U)$

with $c_j = \int dU \text{ sign tr } U \cdot \chi_j(U)$

The transformation $U \to -U$ yields immediately $c_j = 0$ for any integer j.

In the following we consider only half-integer j.

The identity $\text{sign } X = 2\cdot\theta(X) - 1$ and the orthogonality of the characters, i.e.

$$\int dU\, \chi_r(AU)\, \chi_s(U^{-1}B) = \frac{\delta_{rs}}{2s+1}\, \chi_s(AB) \tag{A1}$$

gives:

$$c_j = \int dU\, \chi_j(U)\cdot 2\,\theta(\text{tr } U) = 2\cdot \int_{\text{tr } U \geq 0} dU\, \chi_j(U) \tag{A2}$$

To evaluate the integral (A2) we use canonical coordinates, i.e. we parametrize the elements of SU(2) by $U = e^{i\vec{s}\frac{\vec{\sigma}}{2}}$. The Haar measure and the characters are in these coordinates: $dU = \frac{1}{4\pi^2} d^3s\, \frac{\sin^2\frac{|\vec{s}|}{2}}{|\vec{s}|^2} \cdot \theta(2\pi - |\vec{s}|)$

$$\chi_j(U) = \frac{\sin(2j+1)\frac{|\vec{s}|}{2}}{\sin\frac{|\vec{s}|}{2}}, \quad \chi_{\frac{1}{2}}(U) = \text{tr } U = 2\cdot\cos\frac{|\vec{s}|}{2}$$

Computation of the integral (A2) is now straightforward and we get:

$$c_j = \frac{4}{\pi}\,\frac{2j+1}{4j(j+1)}\,(-1)^{j-\frac{1}{2}} \qquad \text{if j is half-integer.}$$

The character expansion is therefore

$$\text{sign tr } U = \sum_{j=\frac{1}{2},\frac{3}{2},\dots} (-1)^{j-\frac{1}{2}}\, \frac{4}{\pi}\, \frac{2j+1}{4j(j+1)}\, \chi_j(U) \tag{A3}$$

In particular we have

$$c_{\frac{1}{2}} = \frac{8}{3\pi}$$

B. Explicit construction of the gauge transformation S(.)

Given the matrix $U(b) \in SU(2)$ we define: $\overline{U(b)} = U(b)\cdot Z(2) \in SU(2)/Z(2) \approx SO(3)$.

$\overline{U(b)}$ may be regarded as a real 3x3 matrix $\left(\overline{U(b)}^{ac}\right)$ in SO(3).

The oriented plaquette with corner points x, $x+\hat{\mu}$, $x+\hat{\mu}+\hat{\nu}$, $x+\hat{\nu}$ is denoted by $p = (x;\mu\nu)$ ($\hat{\mu}$ = unit vector in μ-direction).

We define 3x3 matrices [6]: $E(x) = (E^a{}_k(x))$ (electric field)

and $B(x) = (\mathcal{B}^a{}_k(x))$ (magnetic field)

where
$$E^a{}_k(x) = \tfrac{1}{2}\varepsilon^{abc}\,\overline{U(\dot{p})}^{bc} = \tfrac{-i}{2}\,\text{tr } U(\dot{p})\cdot\text{tr }\sigma^a U(\dot{p}), \quad p = (x;0k) \tag{B1}$$

$$\mathcal{B}^a{}_k(x) = \tfrac{1}{2}\varepsilon^{abc}\,\overline{U(\dot{p})}^{bc}, \quad p = (x;ij),\ ijk = 123 \text{ or cyclic permutation} \tag{B2}$$

The configurations U such that det B(x)=0 or det E(x)=0 for some site x form a set of measure zero. In the following we restrict ourselves to the electric field, i.e. we consider only the local electric gauge. The situation for the local magnetic gauge is completely analogous. One has to substitute everywhere B(x) instead of E(x).

Every nonsingular E(x) admits a unique decomposition: $E(x) = \overline{S(x)}\cdot P(x)$ with $\overline{S(x)} \in SO(3)$ and P(x) positive or negative definite.

Alternatively we may decompose $E(x) = \overline{S(x)}'\Delta(x)$ with $\overline{S(x)}' \in SO(3)$ and $\Delta(x)$ an upper triangular matrix whose first entry on the diagonal is positive.

Explicitly:
$$\overline{S(x)} = E(x)\cdot\left(E(x)^T E(x)\right)^{-\frac{1}{2}}\cdot \text{sign det } E(x) \tag{B3}$$

and $\overline{S(x)}' = (\vec{s}_1, \vec{s}_2, \vec{s}_3)$ with
$$\vec{s}_1 = \frac{\vec{E}_1}{|\vec{E}_1|}, \quad \vec{s}_2 = \frac{\vec{E}_1^2\vec{E}_2 - \vec{E}_1(\vec{E}_1\cdot\vec{E}_2)}{|\vec{E}_1|\cdot\left(\vec{E}_1^2\vec{E}_2^2 - (\vec{E}_1\cdot\vec{E}_2)^2\right)^{1/2}} \tag{B4}$$

$$\vec{s}_3 = \frac{\vec{E}_1\times\vec{E}_2}{\left(\vec{E}_1^2\vec{E}_2^2 - (\vec{E}_1\cdot\vec{E}_2)^2\right)^{1/2}}$$

if $E(x) = (\vec{E}_1, \vec{E}_2, \vec{E}_3)$

The first gauge will be called (electric) P-gauge for short, and the second one the (electric) Δ-gauge.

We define $\overline{W(b)} = \overline{S(x)}^{-1} \cdot \overline{U(b)} \cdot \overline{S(y)}$ for $b=(x,y)$ (B5)

To every $\overline{W}$ and $\overline{S}$ out of SO(3) there correspond two matrices $\pm W$ resp. $\pm S$ in SU(2). We choose the element with positive trace and call them W(b) resp. S(x). The Z(2) variable $\sigma(b)$ is now defined by the formula

$$W(b)\cdot\sigma(b) = S(x)^{-1}\cdot U(b)\cdot S(y) \qquad (B6)$$

Equivalently

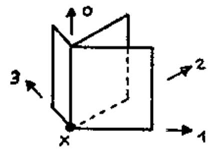
$$\sigma(b) = \text{sign tr } S(x)^{-1}\cdot U(b)\cdot S(y) \qquad \text{for } b=(x,y)$$

It is easy to check that this choice of W(b), S(x), and $\sigma(b)$ satisfies all requirements given in the introduction.

The local electric gauge is defined by the requirement $\overline{S(x)} = 1$.

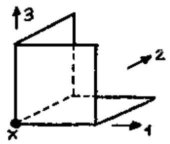
It is instructive to visualize the locality properties of $\overline{S(x)}$, i.e. to draw those links b such that $\overline{S(x)}$ depends only on gauge fields U(b) attached to them.

We get the following picture:



for the electric gauge

and



for the magnetic gauge

If we want to show explicitly the dependence on the configuration U we write $\overline{S(x)}[U]$, $W(b)[U]$, ...

It is easy to prove for any gauge invariant function $F(U) = \hat{F}(\overline{W},\sigma)$:

$$\int \prod_b dU(b)\, F(U) = \int \prod_b dU(b)\, F(U) \cdot \prod_x \delta(\overline{S(x)}[U]) \qquad (B7)$$

$$= \int \prod_b d\overline{W(b)} \prod_b d\sigma(b)\, \hat{F}(\overline{W},\sigma)\cdot \prod_x \delta(\overline{S(x)}[\overline{W}])$$

$d\overline{W(b)}$ and $d\sigma(b)$ are the Haar measures on SO(3) and Z(2) respectively.

(B7) is the basic relation when working explicitly in the local electric (magnetic) gauge. The gauge counterterm or Faddeev-Popov determinant is unity here [15].

C. Evaluation of the leading term for $\langle\sigma(C)\rangle$ in different gauges

We want to prove eq.(2.2)...(2.5). For doing this we consider first the situation for a spacelike surface Ξ (in the electric gauge). Afterwards we generalize to timelike Ξ . We have to compute the following integral:

$$I = \int \prod_b dU(b) \prod_{p\in\Xi} \text{tr}\, U(\dot{p})\ \sigma(C) \qquad (C1)$$

To each site x on the loop C we consider the oriented plaquettes lying in (o,i) direction (i=1,2,3) and having x as their initial corner point. In the following they will typically be denoted by $\overline{p}$. They build a wreath O around C. These are the plaquettes which appear in the definition of $\overline{S(x)}$ ($x\in C$).

We make the following insertion in (C1):

$$1 = \int \prod_{\overline{p}} dV(\overline{p}) \prod_{\overline{p}} \delta(V(\overline{p})\cdot U(\dot{\overline{p}})^{-1}) \qquad (C2)$$

Using the definition of $\sigma(b)$(eq.(B6)) and changing the order of the integrations we get:

$$I = \int \prod_{\overline{p}} dV(\overline{p}) \prod_{b\in\Xi} dU(b) \prod_{p\in\Xi} \text{tr}\, U(\dot{p}) \prod_{\substack{b\in C\\ b=(x,y)}} \text{sign tr}\left(S(x)^{-1}[V]\, U(b)\, S(y)[V]\right)\ \varphi(U,V) \qquad (C3)$$

with $$\varphi(U,V) = \int \prod_{b\notin\Xi} dU(b) \prod_{\overline{p}} \delta(V(\overline{p})\cdot U(\dot{\overline{p}})^{-1}) \qquad (C4)$$

Not all of the U variables which appear in the integrand of (C4) are integrated over (e.g. no integration over U(b) with $b\in C$) and we have to take into account a possible U dependence of φ .

To every plaquette $\overline{p}$ we select a link $b\in\partial\overline{p}$ such that b lies in the boundary of exactly one of the $\overline{p}$'s and furthermore $b\notin\Xi$. Here it is crucial that Ξ is spacelike and none of the $\overline{p}$'s lies on the surface Ξ . It is easy to check that this selection is always possible.

Integrating in (C4) first over these selected links we get rid of the δ-functions. The remaining integrations are trivial and we find that $\varphi(U,V) = 1$ (C5)

For fixed V we perform in (C3) a gauge transformation $U(b) \longrightarrow g(x)\cdot U(b)\cdot g(y)^{-1}$ with $g(x) = S(x)[V]$ if $x\in C$ and otherwise $g(x) = 1$.

After this the V dependence of the integrand has disappeared and we get finally:

$$I = \int \prod_b dU(b) \prod_{p\in\Xi} \operatorname{tr} U(\dot p) \prod_{b\in C} \operatorname{sign} \operatorname{tr} U(b) \tag{C6}$$

(In view of eq.(B7) this means that we can use in (C1) the explicit representation of the Z(2) variables in the local gauge $\sigma(b)=\operatorname{sign}\operatorname{tr} U(b)$ without the gauge fixing term)

With the help of the orthogonality relation of the characters (A1) and the character expansion (A3) the evaluation of (C6) is now straightforward. We get:

$$I = 4\left(\frac{1}{2}\right)^{|\Xi|}\cdot\left(\frac{4}{3\pi}\right)^{|C|} \tag{C7}$$

This proves (2.2) and part of (2.3).

The situation for timelike surfaces Ξ is much more difficult. Now we need the explicit representation of $\overline{S(x)}$ (eq.(B3) and (B4)). The geometry of the situation is given in figure 2.

We use the same manipulations as above up to eq.(C4). Instead of (C5) we have now:

$$\varphi(U,V) = \prod_{\bar p:\bar p\in\Xi} \delta\left(V(\bar p)\cdot U(\dot{\bar p})^{-1}\right) \tag{C8}$$

So we obtain: (C9)

$$I = \int \prod_{\bar p} dV(\bar p) \prod_{b\in\Xi} dU(b) \prod_{\substack{p\in\Xi\\ p\notin\{\bar p\}}} \operatorname{tr} U(\dot p) \prod_{\bar p\in\Xi} \operatorname{tr} V(\bar p) \prod_{\substack{b\in C\\ b=(x,y)}} \operatorname{sign}\operatorname{tr}\left(S(x)^{-1}[V]\, U(b)\, S(y)[V]\right) * \prod_{\bar p\in\Xi} \delta\left(V(\bar p)\cdot U(\dot{\bar p})^{-1}\right)$$

Our strategy to evaluate (C9) is as follows: first we integrate in three steps over all variables U(b) and secondly we do the V integrations with help of the explicit representations of $\overline{S(x)}$.

(i) Consider the links $b\in\Xi$ which lie on the boundary of exactly two plaquettes $p\in\Xi$. They will typically be denoted by $\bar b$. Performing the integrations over these variables yield:

$$\int \prod_{\bar b} dU(\bar b) \prod_{\bar p\in\Xi} \delta\left(V(\bar p)\cdot U(\dot{\bar p})^{-1}\right) = \delta\Big(\prod_{\substack{b\in\tilde C\\ b=(x,y)}} V(\bar p_x)\cdot U(b)\Big) \tag{C10}$$

For the definition of $\tilde C$, $V(\bar p_x)$, C_1, Ξ' see figure 3. In (C10) we have defined $V(\bar p_x) = \mathbb{1}$ if $x\notin C_1$.

(ii) Next we do the integrations over the remaining links b in the interior of Ξ , i.e. $b\notin C$ and $b\neq\bar b$. Again using the orthogonality relation of characters (A1) we obtain:

$$\int \prod_{\substack{b\in\Xi'\\ b\notin C}} dU(b) \prod_{p\in\Xi'} \operatorname{tr} U(\dot p)\, \delta\Big(\prod_{\substack{b\in\tilde C\\ b=(x,y)}} V(\bar p_x)\cdot U(b)\Big) = \left(\tfrac{1}{2}\right)^{|\Xi|-\frac{1}{2}|C|}\cdot \operatorname{tr}\Big(\prod_{\substack{b\in C\\ b=(x,y)}} V(\bar p_x)\cdot U(b)\Big) \tag{C11}$$

(iii) The integrations over the U(b)'s with $b\in C$ can now be done with help of the formula $\int dU\, \operatorname{sign}\operatorname{tr}(AU)\cdot\operatorname{tr}(BU) = \frac{1}{2}c_{\frac{1}{2}}\cdot\operatorname{tr}(BA^{-1}) = \frac{4}{3\pi}\operatorname{tr}(BA^{-1})$

Eq.(A1) and (A3) are used here.

Putting everything together we get:

$$I = \int \prod_{\bar p} dV(\bar p) \prod_{\bar p\in\Xi} \operatorname{tr} V(\bar p)\cdot\left(\frac{4}{3\pi}\right)^{|C|}\cdot\left(\frac{1}{2}\right)^{|\Xi|-\frac{1}{2}|C|}\cdot \operatorname{tr}\prod_{b\in C} K(b) \tag{C12}$$

with $K(b) = V(\bar p_x)\cdot S(x)\cdot S(y)^{-1}$ for $b = (x,y)$.

Remembering that $V(\bar p_x) = \mathbb{1}$ if $x\notin C_1$ we have:

$$\operatorname{tr}\prod_{b\in C} K(b) = \operatorname{tr}\Big(\prod_{x\in C_1} S(x)^{-1} V(\bar p_x)\, S(x)\Big) \tag{C13}$$

We will have succeeded if we know the following integral:

$$\tilde I = \int \prod_{\bar p} dV(\bar p) \prod_{x\in C_1} \operatorname{tr} V(\bar p_x)\cdot \operatorname{tr}\Big(\prod_{x\in C_1} S(x)^{-1} V(\bar p_x)\, S(x)\Big) \tag{C14}$$

S(x) depends only on those $V(\bar p)$ which live on plaquettes p having x as their initial corner point. We assume that Ξ lies in (0,i) direction and V_1, V_2, V_3 denotes the three V variables on which S(x) depends.

Defining $A = \int dV_1\, dV_2\, dV_3\, \operatorname{tr} V_i\; S(x)[V_1 V_2 V_3]^{-1}\, V_i\; S(x)[V_1 V_2 V_3]$

we obtain finally:

$$\tilde I = \operatorname{tr} A^{\frac{1}{2}|C|-1} \tag{C15}$$

To arrive at this formula we have used factorization properties of the integral in (C14) and $\frac{1}{2}|C|-1 = |C_1|$.

In the following we compute the complex 2x2 matrix A.

Using the representation $V_i = V_i^4 + i\sigma^a V_i^a \in SU(2)$, $(V_i^4)^2 + (\vec V_i)^2 = 1$

we get:

$$S(x)^{-1} V_i\; S(x) = V_i^4 + i\sigma^a\left(\overline{S(x)}^{-1}\vec V_i\right)_a \tag{C16}$$

From (B1) we have: $E(x) = (2V_1^4\vec V_1,\, 2V_2^4\vec V_2,\, 2V_3^4\vec V_3) = \overline{S(x)}\cdot\mathbb{P}[V_1 V_2 V_3]$

with $\mathbb{P}[V_1 V_2 V_3] = \mathcal{P}(x)$ or $\Delta(x)$ in P- or Δ-gauge respectively.

Therefore

$$A = \int dV_1\, dV_2\, dV_3\, \tfrac{1}{2}(\operatorname{tr} V_i)^2 + i\sigma^a \int dV_1\, dV_2\, dV_3\, \left(\mathbb{P}[V_1 V_2 V_3]\right)_{ai} =: A^{(1)} + i\sigma^a A^{(2)}_{ai} \tag{C17}$$

(1) $A^{(1)} = \frac{1}{2}\mathbb{1}$ (C18)

(2) (a) <u>electric P-gauge:</u>

In the expression for $A^{(2)}_{ai}$ we make the transformation $(V_1 V_2 V_3) \longrightarrow (V_1^{-1} V_2^{-1} V_3^{-1})$. This implies $E(x) \to -E(x)$ and $\mathbb{P}[V_1 V_2 V_3] \to -\mathbb{P}[V_1 V_2 V_3]$ and therefore $A^{(2)}_{ai}$ vanishes. This together with (C12),(C15),(C17),(C18) and (2.1) yields eq.(2.2).

(b) <u>electric Δ-gauge:</u>

(i) <u>i=1:</u> Using (B4) we get $\overline{S(x)}^{-1}\vec{V}_1 = (\mathrm{sign}\, V_1^4 \cdot |\vec{V}_1|, 0, 0)^T$

and so $A = \frac{1}{2}\mathbb{1} + i\,\sigma^1 \int dV_1\, \mathrm{tr} V_1 \cdot \mathrm{sign}\, V_1^4 \cdot |\vec{V}_1| = \frac{1}{2}\mathbb{1} + i\,\sigma^1 \frac{8}{6\pi}$ (C19)

As above this yields eq.(2.4)

(ii) <u>i=2:</u> Using (B4) we get

$\overline{S(x)}^{-1}\vec{V}_2 = \left(\mathrm{sign}\, V_1^4 \cdot \frac{\vec{V}_1 \cdot \vec{V}_2}{|\vec{V}_1|}, \mathrm{sign}\, V_2^4 \cdot \frac{[\vec{V}_1^2 \vec{V}_2^2 - (\vec{V}_1 \cdot \vec{V}_2)^2]^{1/2}}{|\vec{V}_1|}, 0\right)$

After a straightforward integration we get:

$A = \frac{1}{2}\mathbb{1} + i\,\sigma^2 \frac{1}{3}$ (C20)

This implies (2.5).

(iii) <u>i=3:</u> $\overline{S(x)}$ does not depend on V_3 and so the transformation $V_3 \to V_3^{-1}$ maps $\mathrm{tr}\, V_3 \to \mathrm{tr}\, V_3$ and $\overline{S(x)}^{-1}\vec{V}_3 \to -\overline{S(x)}^{-1}\vec{V}_3$. Therefore $A^{(2)}_{a3}$ vanishes and we obtain eq.(2.3).

D. Analysis of eq.(3.2)...(3.5)

We have to evaluate the following sums (dropping the irrelevant factors 4 and 2):

$$I_1(\beta, h) = \sum_{X \subset \Xi} \left(\frac{\beta}{4}\right)^{|\Xi - X|} \cdot (\tanh h)^{|X|} \cdot \left(\frac{4}{3\pi}\right)^{|\partial(\Xi - X)|} \quad \text{(D1)}$$

and

$$I_2(\beta, h) = \sum_{X \subset \Xi} \left(\frac{\beta}{4}\right)^{|\Xi - X|} \cdot (\tanh h)^{|X|} \cdot \left(\frac{4}{3\pi}\right)^{|\partial X|} \quad \text{(D2)}$$

in the cases (a) $\beta \ll h$ and h small and (b) $h \ll \beta$ and β small.

We are interested in lim sup$\frac{-1}{|\Xi|}\log I_i(\beta,h)$ (i=1,2) and so it is natural to interprete (D1) and (D2) as a polymer system and to expand the logarithm via a linked-cluster expansion. For details and notations see [16] and [17]. The polymers are connected subsets of plaquettes $X \subset \Xi$ (two plaquettes are called connected if they share a link).

<u>First case:</u> $\beta \ll h$, h small

We extract a factor $(\tanh h)^{|\Xi|}$ and define the activities $\phi_i(X)$ (i=1,2) of a polymer X by

$$\phi_1(X) = \left(\frac{\beta}{4\tanh h}\right)^{|X|} \cdot \left(\frac{4}{3\pi}\right)^{|\partial X|} \quad \text{(D3.1)}$$

and

$$\phi_2(X) = \left(\frac{\beta}{4\tanh h}\right)^{|X|} \cdot \left(\frac{4}{3\pi}\right)^{|\partial X| - 2\cdot|\partial\Xi \cap \partial X|} \quad \text{(D3.2)}$$

In this notation $I_1(\beta, h) = (\tanh h)^{|\Xi|} \cdot \sum_{\Xi = \sum_j X_j} \prod_j \phi_1(X_j)$

$$I_2(\beta, h) = (\tanh h)^{|\Xi|} \cdot \left(\frac{4}{3\pi}\right)^{|\partial\Xi|} \cdot \sum_{\Xi = \sum_j X_j} \prod_j \phi_2(X_j) \quad \text{(D4)}$$

and

$$\log \sum_{\Xi = \sum_j X_j} \prod_j \phi_i(X_j) = \sum_{\substack{\text{connected} \\ \text{graphs } \mathcal{G}}} c(\mathcal{G})\, \phi_i(\mathcal{G})$$ ($c(\mathcal{G})$ certain combinatorial factors)

The graphs $\mathcal{G}$ with unequal activities $\phi_1(\mathcal{G})$ and $\phi_2(\mathcal{G})$ are associated with the boundary $\partial\Xi$ and do not contribute after dividing by $|\Xi|$ and taking the limit $|\Xi| \to \infty$. Finally we get:

$$\alpha = \lim\sup \frac{-1}{|\Xi|} \log I_2(\beta, h) = \lim\sup \frac{-1}{|\Xi|} \log I_1(\beta, h) \quad \text{(D5)}$$

$$= -\log \tanh h - \sum_{\mathcal{G}: \mathcal{G} \ni p_0} c(\mathcal{G})\, \phi_1(\mathcal{G})$$

($\mathcal{G} \ni p_0$ means that all graphs contain a fixed plaquette p_0)

The first correction term is given by the graph $\mathcal{G} = p_0$. It has $c(\mathcal{G}) = 1$ and

$\phi(\mathcal{G}) = \frac{\beta}{4\tanh h}\left(\frac{4}{3\pi}\right)^4$. This yields eq.(3.6).

<u>Second case:</u> $h \ll \beta$, β small

Now we extract a factor $\left(\frac{\beta}{4}\right)^{|\Xi|}$ and we define activities

$$\phi_1'(X) = \left(\frac{4\tanh h}{\beta}\right)^{|X|} \cdot \left(\frac{4}{3\pi}\right)^{|\partial X| - 2\cdot|\partial\Xi \cap \partial X|} \quad \text{(D6.1)}$$

and

$$\phi_2'(X) = \left(\frac{4\tanh h}{\beta}\right)^{|X|} \cdot \left(\frac{4}{3\pi}\right)^{|\partial X|} \quad \text{(D6.2)}$$

In this notation $I_1(\beta, h) = \left(\frac{\beta}{4}\right)^{|\Xi|} \cdot \left(\frac{4}{3\pi}\right)^{|\partial\Xi|} \cdot \sum_{\Xi = \sum_j X_j} \prod_j \phi_1'(X_j)$

$$I_2(\beta, h) = \left(\frac{\beta}{4}\right)^{|\Xi|} \cdot \sum_{\Xi = \sum_j X_j} \prod_j \phi_2'(X_j)$$

Applying the same arguments as above we get for the string tension

$$\alpha = -\log \frac{\beta}{4} - \sum_{\mathcal{G}: \mathcal{G} \ni p_0} c(\mathcal{G})\, \phi_2'(\mathcal{G}) \quad \text{(D7)}$$

which implies eq.(3.7).

References

1. K.Wilson, Phys.Rev. D10, 2445 (1974)
2. R.Balian, J.Drouffe, C.Itzykson, Phys.Rev. D10, 3376 (1974); D11, 2098 (1975); D11, 2104 (1975)
3. K.Osterwalder, E.Seiler, Ann.Phys. 110, 440 (1978)
4. G.'t Hooft, Nucl.Phys.B138, 1 (1978); B153, 141 (1979)
5. G.Mack, V.B.Petkova, Ann.Phys.123, 442 (1979); 125, 117 (1980)
6. G.Mack, Properties of lattice gauge theory models at low temperatures, DESY 80/03 (january 1980) and to appear in: G.'t Hooft et al. (eds.), Recent developments in gauge field theories, Plenum Press, New York
7. T.Yoneya, Nucl.Phys.B144, 195 (1978)
8. D.Foerster, Phys.Lett. 76B, 597 (1978); 77B, 211 (1978); 78B, 473 (1978)
9. H.B.Nielsen, P.Olesen, Nucl.Phys.B61, 45 (1973)
10. H.C.Tze, Z.F.Ezawa, Phys.Rev.D14, 1006 (1976)
11. J.Glimm, A.Jaffe, Nucl.Phys.B149, 49 (1979)
12. G.Mack, Quark confinement in lattice gauge theories, Schladming lectures 1980
13. G.Mack, E.Pietarinen (unpublished)
14. G.Mack, Physical principles, geometrical aspects, and locality properties of gauge field theories
    preprint Max Planck Institut München, MPI-PAE/PTh 41/79 (october 1979)
15. B.E.Baaquie, Phys.Rev.D16, 2612 (1977)
16. C.Gruber, H.Kunz, Commun.Math.Phys.22, 133 (1971)
17. M.Göpfert, Peierls argument for Z(2) lattice gauge theory, Diplomarbeit, Hamburg 1980 and DESY report (in preparation)
18. I.Ya.Aref'eva, Phys.Lett.93B, 347 (1980)

Figure captions

figure 1: thin vortex winding around the path C. Drawing for three dimensions. The heavy lines are the links b where $\tau(b)=-1$. The hatched plaquette lies on the surface Ξ and does not belong to the cluster Δ .

figure 2: illustration to appendix C for timelike Ξ . The plaquettes $\bar{p}\in\Xi$ are hatched. The surface Ξ lies in (0,1) direction. The wreath O around the loop C is also shown.

figure 3: illustration and explanation to eq.(C10). The full line is the curve $\tilde{C}$. C_1 is represented by a double line (end points included) $C_1 \subset \tilde{C}$. For $x \in C_1$ $\bar{p}_x$ is that plaquette $\bar{p}\in\Xi$ which has x as its initial corner point. $|\Xi'| = 1 + |\Xi| - \frac{1}{2}|C|$

figure 4: the local order parameter $\langle\sigma(\dot{p})\rangle$. The black dots represent the Monte Carlo data of Mack and Pietarinen. The straight line is the lowest order high temperature expansion eq.(2.2).

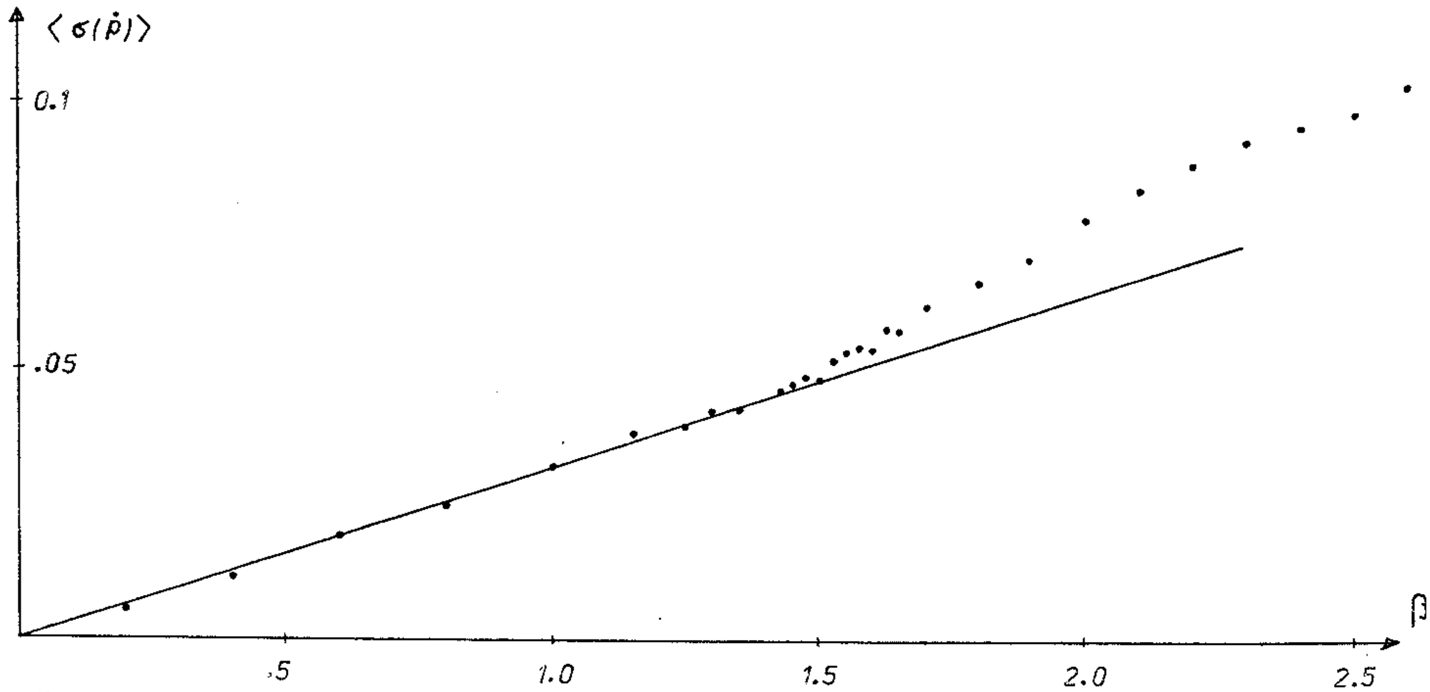


figure 4

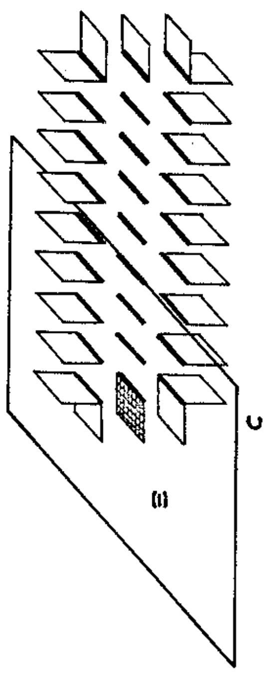


figure 1

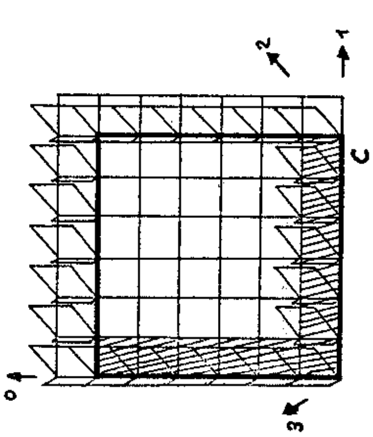


figure 2

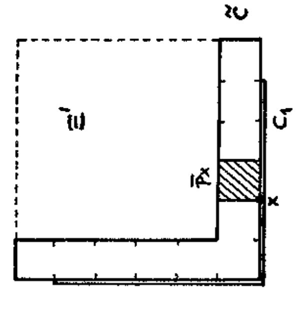


figure 3