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THE ROUGHENING TRANSITION IN LATTICE GAUGE THEORIES

by

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The roughening transition in lattice gauge theories

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Talk based on work with M. Lüscher and P. Weisz, given at the "International Symposium on the Statistical Mechanics of Quarks and Hadrons", Bielefeld (Germany), August 1980

I would like to talk about a phenomenon in lattice gauge theories, which has been recognized recently by several people [1-4], namely the roughening transition. Prof. Itzykson has already given a talk on this subject and there will necessarily be some overlap between us. On the other hand our starting points and approaches are different and therefore it might be justified to say some more words about roughening.

1. The width of a flux tube

It is generally believed that pure SU(2) or SU(3) gauge field theory in 4 dimensions confines static quarks. As Prof. Lee explained in his talk, there is a physical picture associated with the mechanism of quark confinement: the formation of chromo electric flux tubes between the quarks. In particular if we consider a static quark-antiquark pair at distance L,

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the energy density $\mathcal{E}(x)$ of the chromo electric field is supposed to be concentrated mainly in a tube like region (see fig. 1)

Fig. 1

An important physical quantity associated with the flux tube or string is the string tension α , i.e. the energy per unit length of a long string. It gives the slope of the approximately linearly rising potential

$$V(L) \sim \alpha \cdot L \tag{1}$$

between static quarks.

Another physical quantity of interest is the transversal width of a flux tube. A convenient measure for it is

$$\delta_L^2 = \frac{\int d^2x_\perp \; x_\perp^2 \; \mathcal{E}(x)}{\int d^2x_\perp \; \mathcal{E}(x)} \tag{2}$$

δ_L^2 is not necessarily an intrinsic width. It may come from fluctuations in the position of a 'thin' string. In general it depends on L. If L is large compared to natural length scales Λ^{-1} of QCD, perturbation theory is not applicable to calculate δ_L. The question arises: if the string is made longer and longer, is

$$\delta_\infty^2 = \lim_{L\to\infty} \delta_L^2 \tag{3}$$

finite?

2. Lattice calculations

In order to investigate δ_∞^2 we have to look for non-perturbative methods. The natural setting for such strong coupling problems is lattice gauge theory [5]. If SU(2) or SU(3) lattice gauge theory has a continuum limit (which is a weak coupling limit) with finite δ_∞^2 , then $\delta_\infty^2 m^2$ should extrapolate smoothly from strong to weak coupling, if m is any physical quantity with dimension of a mass. In particular this should be true for

$$\delta_\infty^2 \alpha \tag{4}$$

where α is the string tension.

Therefore let us consider strong coupling expansions for α [6,7] and δ^2 [1,4] in four-dimensional Euclidean lattice gauge theory. For the gauge group we take SU(2). Let me shortly explain the notations. The gauge field variables are group elements $U(b) \in SU(2)$ attached to links b of a hyper-cubical lattice. The ordered product of the four variables on the boundary of an elementary plaquette p is denoted U(p). The Lagrangean is

$$\mathcal{L} = \frac{\beta}{2} \sum_p \mathrm{tr}\, U(p) \quad , \qquad \beta = \frac{4}{g^2} \quad . \tag{5}$$

The path integral formula for expectation values of observables $\mathcal{O}$ reads

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \prod_b dU(b) \quad \mathcal{O}(\{U(b)\}) \exp \mathcal{L} \tag{6}$$

For a rectangular loop $\mathcal{C}$ of size L x T the Wegner-Wilson loop observable is

$$\mathrm{tr}\, U(\mathcal{C}) = \mathrm{tr} \prod_{b\in\mathcal{C}} U(b) \tag{7}$$

Its expectation value yields the static quark-antiquark potential V(L) via

$$\langle \mathrm{tr}\, U(\mathcal{C}) \rangle \underset{T\to\infty}{\sim} e^{-T\,V(L)} \approx e^{-\alpha T L} \tag{8}$$

The electric field strength is measured by time-like plaquettes p. Therefore the chromoelectric field energy density in the presence of a static quark-antiquark pair is

$$\mathcal{E}(x) = \frac{1}{2} \lim_{T\to\infty} \frac{\langle \mathrm{tr}\, U(\mathcal{C}) \;\; \mathrm{tr}\, U(p_x) \rangle_c}{\langle \mathrm{tr}\, U(\mathcal{C}) \rangle} \tag{9}$$

See fig. 2 for the geometrical situation.

Fig. 2

The lattice version of equ. 2 is

$$\delta_L^2 = \frac{\sum_{x_\perp} x_\perp^2 \, \mathcal{E}(x)}{\sum_{x_\perp} \mathcal{E}(x)} \tag{10}$$

Strong coupling expansions for α suggested [6,7,8] that the expansions may still be reliable in the changeover region between strong and weak coupling behaviour [9]. Therefore the idea was to look for $\delta_\infty^2 \alpha$ in this region and to extract its continuum limit value if possible.

The strong coupling expansion for $\mathcal{E}(x)$ can be evaluated using the graphs for the expansion of the string tension α [7] by modifying the coupling $\beta \longrightarrow \tilde{\beta}$ on plaquettes p_x through

$$\mathcal{E}(x) = - \frac{\partial}{\partial \tilde{\beta}} \alpha(\beta, \tilde{\beta}, x) \Big|_{\tilde{\beta}=\beta} \tag{11}$$

It is convenient to expand in terms of the Fourier coefficient u of the fundamental representation in the character expansion of $\exp\left(\frac{\beta}{2} \mathrm{tr}\, U\right)$

$$u = \frac{\frac{1}{2}\int dU \; \mathrm{tr}\, U \; \exp\left(\frac{\beta}{2} \mathrm{tr}\, U\right)}{\int dU \; \exp\left(\frac{\beta}{2} \mathrm{tr}\, U\right)} \tag{12}$$

$$= \frac{I_2(\beta)}{I_1(\beta)} = \frac{\beta}{4} + \dots \qquad \text{for } SU(2)$$

where I_n is a modified Bessel function. We have

$$o \leq u < 1 \tag{13}$$

For small β resp. u the expansion for δ_∞^2 is expected to converge: $\delta_\infty^2 < \infty$. It reads [1]

$$\frac{1}{4}\delta_\infty^2 = u^4 + 2u^6 + \frac{32}{3}u^8 + \frac{37724}{405}u^{10} + \frac{1412551}{1215}u^{12} + \mathcal{O}(u^{14}) \tag{14}$$

In fig. 3 it is plotted up to 10th (a) and up to 12th (b) order as a function of β.

Fig. 3

We see that δ_∞^2 rises very fast, in fact faster than one would expect, if a limiting value for $\delta_\infty^2 \alpha$ existed. So let us have look at $(\delta_\infty^2 \alpha)^{-1}$ in the strong coupling expansion. The result is shown in fig. 4.

Fig. 4

Rather than approaching a constant, $(\delta_\infty^2 \alpha)^{-1}$ seems to vanish near $\beta \approx 1{,}8 \dots 1{,}9$. This is on the high temperature side of the crossover region and α is still large there. Therefore the result indicates that δ_∞^2 diverges above this point. Two questions arise naturally. Is this result reliable or an artefact of the procedure? What does it mean?

3. Roughening transition

The above observation reminds one of a phenomenon in the 3 dimensional Ising model, namely the roughening transition [10]. At low temperatures two phases of opposite magnetization may coexist [11]. They are separated by a domain wall with a well defined effective width δ^2.

Fig. 5

If the system is heated to $\beta_R^I \approx 0{,}39$ the width δ^2 diverges (in the infinite volume system) and the domain wall disappears. This happens below the critical temperature corresponding to $\beta_c^I = 0{,}2217$ and the surface tension is still non-zero. The interface roughening is associated with long-wavelength fluctuations of the domain wall.

It is well known [12] that the three dimensional Ising model is related to the three dimensional Z_2 lattice gauge theory by a duality transformation. The couplings are related by

$$\exp(-2\beta^I) = \tanh\beta \qquad (15)$$

The duality transformation maps the Ising model surface tension onto the string tension of the Z_2 gauge theory. Roughening in the Ising model implies a divergence of δ^2 in the Z_2 gauge theory at

$$\beta_R \approx 0{,}50 \qquad (16)$$

while the deconfining phase transition is at $\beta_c = 0{,}76$. To check our

procedure we also applied it to the Z_2 case [1]. And indeed we found a divergence of δ_∞^2 at

$$Z_2: \quad \begin{array}{ll} \beta_R \approx 0{,}50\ ! & \nu = 3 \text{ dimensions} \\ \beta_R \approx 0{,}43 & \nu = 4 \text{ dimensions } (\beta_c = 0{,}44) \end{array} \qquad (17)$$

This supports the reliability of the method. Therefore the conclusion is: there is a roughening of the chromoelectric string in SU(2) lattice gauge theory, i.e. above β_R fluctuations of the confining string lead to a divergent effective width. The roughening transition is not deconfining.

5. Consequences

The observation of a roughening in lattice gauge theories has the following consequences.

a) If the lattice theory has a continuum limit with persisting confinement property, then the width δ_L^2 of a string of length L diverges as L goes to infinity in the continuum theory. This is contrary to what was believed previously.

From relativistic string model calculations [1] we expect that δ_L^2 diverges logarithmically with L.

b) The string tension α is expected to have a point of non-analyticity at β_R [2,3] as Prof. Itzkyson explained in his talk. If this is the case then β_R is a barrier for the strong coupling expansion of α . In particular some doubt is shed on any estimates of continuum quantities which rely on strong coupling expansions for the string tension [8]. On the other hand if the crossover from strong to weak coupling behaviour occurs rapidly, as it seems to be the case [9], and if the non-analyticity is mild, the mentioned estimates might perhaps be not much affected, because they mainly depend only on the location of the changeover.

6. Other gauge groups

We have also investigated the roughening transition for the case of other gauge groups [4,15]. As an indicator for roughening we consider

i) a zero of $(\sigma_\infty^2 \alpha)^{-1}$ in the strong coupling expansion.

ii) a pole of Padé approximates for the expansion of σ^2.

For the Abelian groups Z_n and U(1) in 3 dimensions we find a roughening transition in the confining phase, i.e. $\beta_R < \beta_c$. In 4 dimensions the roughening seems to coincide with the phase transition point β_c for Z_2, Z_3 and Z_4, while for Z_n, $n \geq 5$ and U(1) it is near the first phase transition point β_c and we are not able to decide whether they coincide or not. For details see [4].

In the case of non-Abelian groups SU(n), n=2,3 and n large [15] we find signals for a roughening transition in both 3 and 4 dimensions, while a deconfining phase transition is believed not to exist. The case of SU(3) (as well as Z_3) is a bit delicate, because the Padé table is not as stable as for the other groups.

In 3 and in 4 dimension one finds that the value of the expansion parameter u at the roughening point is nearly equal for all groups.

$$\nu = 3 : \quad u_R \approx 0{,}46$$
$$\nu = 4 : \quad u_R \approx 0{,}40 \qquad (18)$$

This can be interpreted as a consequence of the nature of the roughening phenomenon, which is thought to be due to long-wavelength fluctuations of the string, such that details of the dynamics are of minor importance [2,3]. This point and the related solid-on-solid (SOS) model approximation for roughening are discussed by Prof. Itzykson in his talk.

Furthermore one observes that β_R is always near the changeover from strong to weak coupling for the case of SU(n). Thus the changeover seems to have something to do with the roughening transition.

In conclusion there is evidence for a general occurence of a new kind of transition in lattice gauge theory, where a roughening of the confining string takes place.

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<u>Figure Captions</u>

Fig. 1: Flux tube between static quarks

Fig. 2: Illustration to equ. 9

Fig. 3: Plot of σ_∞^2 for SU(2) lattice gauge theory in 4 dimensions. a and b denote the 10th and 12th order curves respectively.

Fig. 4: Plot of $(\sigma_\infty^2\,\alpha)^{-1}$ for SU(2) in 4 dimensions. The curves a resp. b correspond to those of fig. 3.

Fig. 5: Part of a non-translation invariant Gibbs state in the three-dimensional Ising model on an infinite lattice. A well localized domain wall of finite width separates regions of opposite magnetization.

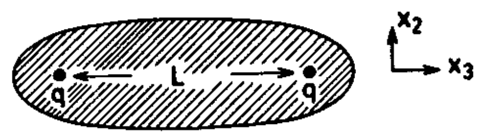


Fig.1

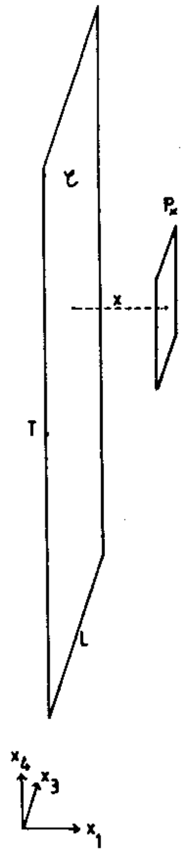
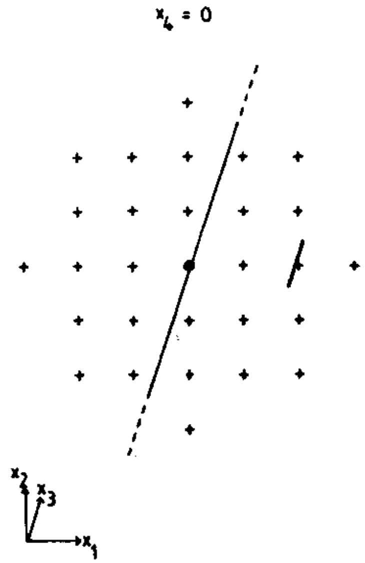


Fig.2



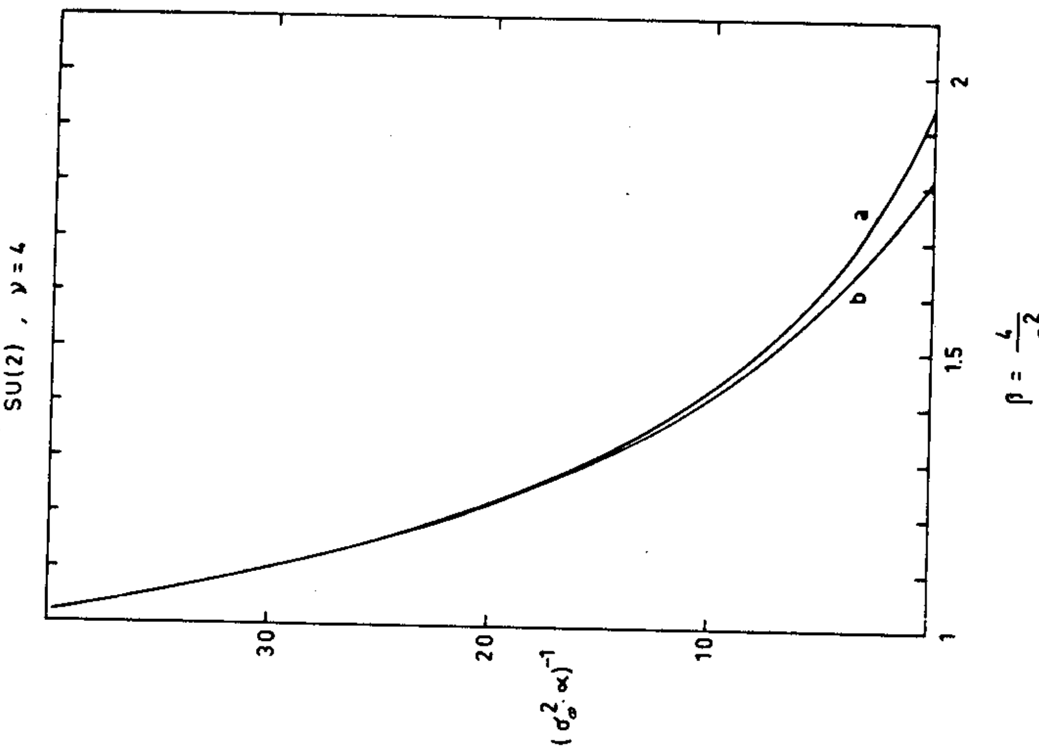


Fig. 4

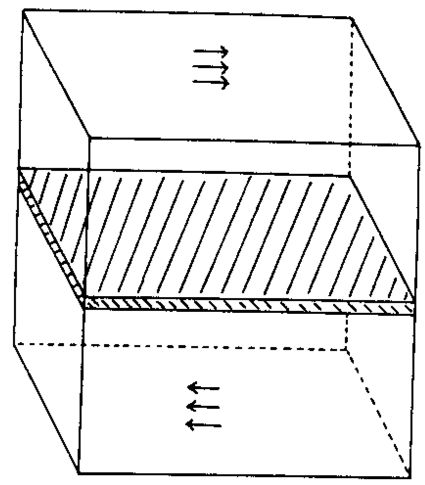


Fig. 5

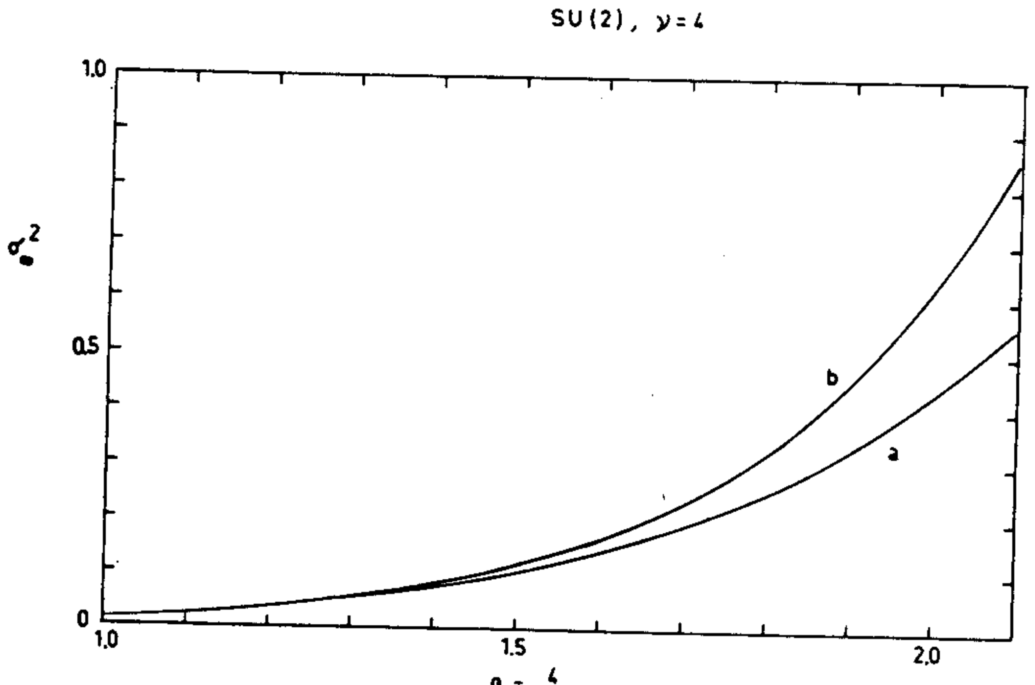


Fig. 3