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PARTON TRANSVERSE MOMENTUM CORRECTIONS IN LEPTOPRODUCTION JET CROSS SECTIONS

by

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Parton Transverse Momentum Corrections in Leptoproduction Jet Cross Sections

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Abstract: We have calculated the unpolarized, the longitudinal polarized and other polarization dependent cross sections for jet production in deep inelastic eN scattering up to order α_s of the quark-gluon coupling and for finite transverse momentum of the initial partons. The primordial transverse momentum leads to additional structure which can be used to test the lowest order QCD jet production matrix element.

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I. Introduction

Recently several groups studied theoretically the properties of jets produced in deep inelastic lepton-nucleon scattering in the framework of perturbative Quantum Chromodynamics (QCD) [1,2,3] . However, to actually compare with experimental data one must calculate also the fragmentation of the final state quarks and gluons into hadrons as well as make some definite assumption on the extent of the intrinsic primordial transverse momentum of the partons which are emitted from the nucleon. For very high values of Q^2 and W^2 the effects caused by hadronization or primordial transverse momentum diminish and, as is well known, the perturbative QCD effects become dominant. However, at present available energies, which are still much lower than in high energy e^+e^- annihilation experiments, one must expect, that non-perturbative QCD effects are still very important and must be included if one wants to test meaningfully the perturbative QCD predictions. The final state hadronization effects have heen studied quite recently in some detail [2,3] whereas the primordial transverse momentum effects of the initial partons were left out so far.[(F1)] As was pointed out in [2], there exist jet properties, as for example, the increase of jet size, measured with respect to jet axis (as defined by thrust or sphericity) with energy, which are much less dependent on the initial parton transverse momentum, there are certainly other jet effects which depend rather strongly on the size of the primordial transverse momentum. Such properties are, for example, the transverse momentum of final state particles with respect to the current vector $\vec{q}$ or jet asymmetries measured with respect to the lepton plane. These jet asymmetries have contributions from lowest order diagrams, if the parton transverse momentum p_T is taken into account, which are of order $|p_T|/Q$ and p_T^2/Q^2 respectively [5] depending on which asymmetry is being studied, and from QCD diagrams which are of order $\alpha_s \sim \ln^{-1} Q^2/\Lambda^2$ [2]. These first order QCD contributions are also modified by a finite primordial transverse momentum. It is the purpose of this paper to calculate these effects and to see how much the order α_s asymmetries are modified by a finite p_T . It is clear that these modifications are also of order p_T/Q (or p_T/W) etc. multiplied by kinematical factors which may amplify these terms in special kinematical regions.

We employ a covariant parton formulation with on-mass-shell quarks and gluons, which can incorporate parton transverse momentum (p_T). The target is assumed to be massless. Finite p_T and non-zero target-mass effects are certainly related. So some effect of a non-vanishing target mass can be simulated by an increased average transverse momentum p_T . The quarks are taken mass-less in the initial and final state.

In the next section we calculate the various polarization dependent cross sections which determine the jet asymmetries. For completeness we also give a short account of the results for the zero-order parton model which had been already obtained in [2]. First we present the full angular correlation which gives the cross section in terms of the azimuthal angle of one of the current jets and the angle of the target jet. From this various angular distributions can be obtained by integration. In section 3 we study the correction terms for small transverse momentum of order $\langle p_T \rangle /W$ and $\langle p_T^2 \rangle /W^2$. Some concluding remarks are found in section 4.

2. Cross Sections and Asymmetries

We study only electromagnetically induced reactions of the kind $e^- + N \to e^- +$ jets. The calculations for neutrino or anti-neutrino induced processes are analogous and will not be presented here.

We start with the more complicated first order parton diagrams shown in Fig. 1 which lead to a three-jet final state. For example, in the case of Fig. 1a we have the parton process (γ = virtual photon)

$$\gamma(q) + q(p) \to q(p_1) + g(p_2) \tag{2.1}$$

which leads on a target N to the process

$$\gamma(q) + N(P) \to q(p_1) + g(p_2) + (qq)(p_3) \tag{2.2}$$

in the impulse approximation. Here the final state consists of three jets, a quark jet, a gluon jet and diquark jet with momenta p_1, p_2 and p_3 respectively. Analogous interpretations follow for the diagrams in Fig. 1b and c.

The electroproduction cross section is written as [2]:

$$d\sigma = 2\pi\Gamma \left\{ d\sigma_U + \frac{2(1-y)}{1+(1-y)^2}\, d\sigma_L + \frac{2(1-y)}{1+(1-y)^2}\, d\sigma_T + \frac{(1-y)^{1/2}(2-y)}{1+(1-y)^2}\, d\sigma_I \right\} \tag{2.3}$$

where Γ is the equivalent photon spectrum

$$\Gamma = \frac{\alpha W^2}{4\pi^2 Q^2 (W^2+Q^2)^2} \left(1+(1-y)^2\right) \tag{2.4}$$

with $Q^2 = -q^2$, $W^2 = (P+q)^2$ and $y = Pq/Pl$ with l being the incoming lepton momentum. $d\sigma_X$ ($X = U, L, T, I$) are the usual cross sections depending on the virtual photon polarization. These cross sections are calculated from

$$d\sigma_{ij} = \frac{(2\pi)^4}{2W^2}\, \delta^{(4)}(q + P - p_1 - p_2 - p_3)\, T_{ij}\, \mathcal{F}(\vec{p})\, \frac{d^3p}{(2\pi)^3 2p_0}\, \frac{d^3p_1}{(2\pi)^3 2p_{10}}\, \frac{d^3p_2}{(2\pi)^3 2p_{20}} \qquad (2.5)$$

with

$$T_{ij} = \epsilon(i)^*_\mu\, T^{\mu\nu}\, \epsilon(j)_\nu \qquad (2.6)$$

where $\epsilon(j)$ is the polarization vector of the incoming virtual photon and $T_{\mu\nu}$ is the hadronic tensor which is calculated from the diagrams in Fig. 1. $\mathcal{F}(\vec{p})$ describes the momentum distribution of the quark q in the nucleon N, over which we have to integrate later. Our normalization of $\mathcal{F}(\vec{p})$ is such that

$$\int \frac{d^2p_T}{2\,(2\pi)^3}\, \mathcal{F}(\vec{p}) = f(\eta_0) \qquad (2.7)$$

where η_0 is the longitudinal momentum fraction of the quark in the nucleon ($\eta = p_L/|\vec{P}|$) in the limit $p_T = 0$. The hadronic tensor for $\gamma + q \to q + g$ can be found in [2]. We decompose it in the form

$$T_{\mu\nu} = T_1 A_\mu A_\nu + T_2 A^1_\mu A^1_\nu + T_3 (A^1_\mu A_\nu + A_\mu A^1_\nu)$$
$$+ T_4 (A^1_\mu A_\nu - A_\mu A^1_\nu) + T_5 (g_{\mu\nu} + q_\mu q_\nu / Q^2) \qquad (2.8)$$

where

$$A_\mu = \left(p + \frac{pq}{Q^2} Q\right)_\mu$$

$$A^1_\mu = \left(p_1 + \frac{p_1 q}{Q^2} Q\right)_\mu$$

From [2] we read off the invariant functions (F2):

$$T_1 = T_2 = -2 e^2 c_2\, g^2 Q_f^2\, \frac{4Q^2}{st}$$
$$T_3 = T_4 = 0 \qquad (2.9)$$
$$T_5 = 2 e^2 c_2\, g^2 Q_f^2\, \frac{t^2 + s^2 - 2uQ^2}{st}$$

with $s = (p + q)^2$, $t = (q - p_1)^2$, $u = (q - p_2)^2$. g is the quark-gluon coupling, $c_2 = 4/3$ the colour factor and Q_f the charge of the quark with flavour f. It is clear that $T_4 = 0$ since $T_{\mu\nu}$ is symmetric in the order g. $T_3 = 0$ because $T_{\mu\nu}$ in [2] was calculated for massless quarks.

To define the photon polarization vectors we must fix a coordinate system. We choose a coordinate system xyz with z along the momentum of the incoming nucleon $z \parallel \vec{P}$ and go into the photon-nucleon center-of-mass frame $\vec{q} + \vec{P} = 0$. The lepton momenta l and l' are in the x-z-plane. Then $\epsilon^\mu(\pm) = \mp \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)$ and $\epsilon^\mu(0) = \frac{|\vec{q}|}{(Q^2)^{1/2}}(1, 0, 0, -\frac{q_0}{|\vec{q}|})$. The cross sections $d\sigma_U$, $d\sigma_L$, $d\sigma_T$ and $d\sigma_I$ are obtained from:

$$d\sigma_U = \frac{1}{2}\,(d\sigma_{++} + d\sigma_{--})$$
$$d\sigma_L = d\sigma_{00}$$
$$d\sigma_T = -\frac{1}{2}\,(d\sigma_{+-} + d\sigma_{-+}) \qquad (2.10)$$
$$d\sigma_I = \frac{1}{\sqrt{2}}\,(d\sigma_{-0} + d\sigma_{0-} - d\sigma_{+0} - d\sigma_{0+})$$

The cross sections defined by (2.10) are proportional to corresponding projections of the hadronic tensor T_U, T_L, T_T and T_I which have the following form:

$$T_U = \frac{T_1}{2}\left(p_T^2 + p_{1T}^2 + \frac{2(pq)^2}{Q^2} + \frac{2(p_1q)^2}{Q^2}\right)$$

$$T_L = T_1\left(p_T^2 + p_{1T}^2\right)$$

$$T_T = \frac{T_1}{2}\left(p_T^2 \cos 2\phi + p_{1T}^2 \cos 2\phi_1\right) \tag{2.11}$$

$$T_I = -\frac{2T_1}{\sqrt{Q^2}}\left(p_T\left(|\vec{q}|\, p_0 + q_0 p_L\right)\cos\phi + p_{1T}\left(|\vec{q}|\, p_{10} + q_0 p_{1L}\right)\cos\phi_1\right)$$

In (2.11) we introduced the azimuthal angles ϕ and ϕ_1 of p and p_1 in the coordinate system x,y,z and $\vec{p}_T$ ($\vec{p}_{1T}$) and $p_L(p_{1L})$, the transverse and longitudinal components of $p(p_1)$ with respect to the z axis ($p_T = |\vec{p}_T|$).

Instead of ϕ it is more appropriate to introduce the azimuthal angle ϕ_3 of the target jet. We have $\phi_3 = \phi + \pi$.

The results (2.11) can be converted into cross sections after the phase space reduction has been done starting from (2.5). For example, for $d\sigma_U$ we get:

$$\frac{d\sigma_U}{dp_{1L}\, dp_{1T}^2\, d\phi_1\, dp_T^2\, d\phi_3} = \frac{\mathcal{F}(\vec{p})\ T_U}{64(2\pi)^5\, p_{10}\, p_{20}\, W^2 p_L} \tag{2.12}$$

and similar exppressions for the other cross sections. Here

$$p_{10} = \left(p_{1L}^2 + p_{1T}^2\right)^{1/2}$$

$$p_{20} = \left\{ \left(p_{1L} + (1-\eta)P\right)^2 + p_{1T}^2 + p_T^2 + 2p_{1T}p_T\cos(\phi_3-\phi_1)\right\}^{1/2} \tag{2.13}$$

T_U and the other squared amplitudes T_L, T_T and T_I are given in terms of p_{1L}, p_{1T}^2, ϕ_1, p_T^2 and ϕ_3. Furthermore

$$P = |\vec{q}| = \frac{1}{2W}\left(W^2 + Q^2\right) \quad , \quad q_0 = \frac{1}{2W}\left(W^2 - Q^2\right)$$

$$2(p_1q)^2 = \frac{1}{2}\left(t + Q^2\right)^2 \quad , \quad t = (p_1 - q)^2 = -2p_{10}q_0 - 2p_{1L}P - Q^2 \tag{2.14}$$

In T_U we encounter also $2pq = s + Q^2$. This depends on p_{1L} and p_{1T}^2 in the following way:

$$s = W^2 - 2p_{30}W \tag{2.15}$$

where $\quad p_{30} = W - p_{10} - p_{20}$

and p_{20} is given by (2.13). Therefore the dependence of the cross sections $d\sigma_U$, $d\sigma_L$, $d\sigma_T$ and $d\sigma_I$ on the azimuthal angles ϕ_3 and ϕ_1 is rather complicated. In particular in $d\sigma_U$, because of s, and in $d\sigma_I$, because of p_L and $p_o = P - p_{30}$ we have ϕ_1 and ϕ_3 dependence in addition to the factor $(p_{20}p_L)^{-1}$ in the phase space factor. Although (2.12) and similar expressions for $d\sigma_L$, $d\sigma_T$ and $d\sigma_I$ together with (2.11) contain all the information about the azimuthal correlations induced by a finite transverse momentum of the quarks and the gluon emission we shall not pursue these cross section formulas any further. Of course, in the limit $p_T = 0$ the formulas simplify considerably, the dependence on ϕ_3 disappears and we recover the results given in [2].

Instead we shall introduce the normalized jet energies as in [2]

$$x_i = \frac{2p_{i0}}{W} \quad , \quad i = 1,2,3 \tag{2.16}$$

with the constraint $x_1 + x_2 + x_3 = 2$ and shall describe the final state in terms of x_1 and x_2 and appropriately defined azimuthal angles. For this purpose we must be able to express the longitudinal and transverse components of p_1, which appear in (2.11), by these jet energies x_i. We do this by introducing a new coordinate system X, Y, Z defined in such a way that the momentum of the target jet is in the direction of the Z axis. In this system we have simple relations between p_{1L}, p_{1T} and x_1 and x_2. The momentum vectors $\vec{p}_i$ in this new system are denoted by $\vec{\hat{p}}_i$. Then $\vec{\hat{p}}_3 = |\vec{p}_3|(0,0,1)$ and the relation between vectors p_i in the system x,y,z and vectors $\hat{p}_i$ is obtained by the following rotations:

$$\hat{p}_i = R_{\tilde{y}}(\theta_3)\, R_z(\phi_3)\, p_i \qquad (2.17)$$

Then $\vec{p}_3 = |\vec{p}_3|(\cos\phi_3 \sin\theta_3, \sin\phi_3 \sin\theta_3, \cos\theta_3)$ so that θ_3 and ϕ_3 are just the polar and azimuthal angles of $\vec{p}_3$ in the system x,y,z. To express the components of p and p_1 which appear in (2.11) by the components of $\hat{p}$ and $\hat{p}_1$ we need the inverse relation of (2.17). For example, for p_1, this relation has the following explicit form:

$$p_{1T}\cos\phi_1 = \left(\cos\theta_3 \cos\phi_3 \cos\hat{\phi}_1 - \sin\phi_3 \sin\hat{\phi}_1\right)\hat{p}_{1T} + \sin\theta_3 \cos\phi_3\, \hat{p}_{1L}$$

$$p_{1T}\sin\phi_1 = \left(\cos\theta_3 \sin\phi_3 \cos\hat{\phi}_1 + \cos\phi_3 \sin\hat{\phi}_1\right)\hat{p}_{1T} + \sin\theta_3 \sin\phi_3\, \hat{p}_{1L}$$

$$p_{1L} = -\sin\theta_3 \cos\hat{\phi}_1\, \hat{p}_{1T} + \cos\theta_3\, \hat{p}_{1L} \qquad (2.18)$$

In (2.18) ϕ_1 is the azimuthal angle of p_1 in the rotated system X,Y,Z. The corresponding relations for p are similar. Since $p_{3T} + p_T = 0$ we have $\hat{p}_y = 0$ and

$$p_T\cos\phi = \sin\theta_3 \cos\phi_3 \left(\hat{p}_L - \cos\theta_3\, P\right)$$

$$p_T\sin\phi = \sin\theta_3 \sin\phi_3 \left(\hat{p}_L - \cos\theta_3\, P\right)$$

$$p_L = \cos\theta_3\, \hat{p}_L + \sin^2\theta_3\, P \qquad (2.19)$$

where $P = |\vec{q}|$. Actually $\hat{p}_L - \cos\theta_3\, P = -\hat{p}_{3L} = -|\vec{p}_3|$, so that the last equation is equivalent with

$$p_L = -\cos\theta_3 |\vec{p}_3| + P \qquad (2.20)$$

Similarly the first two equations in (2.19) yield $p_x + p_{3x} = p_y + p_{3y} = 0$. We notice from (2.18) that in the limit $\theta_3 = 0$: $\phi_1 = \phi_3 + \hat{\phi}_1$ and $p_{1L} = \hat{p}_{1L}$. $\hat{p}_{1T}$ and $\hat{p}_{1L}$ can be expressed by the normalized jet energies x_i:

$$\hat{p}_{1T}^2 = \frac{W^2}{x_3^2}\, x_{11}\, x_{12}\, x_{13}$$

$$\hat{p}_{1L} = \frac{W}{4x_3}\left(x_2^2 - x_1^2 - x_3^2\right) \qquad (2.21)$$

with $x_{1i} = 1 - x_i$.

The projections T_U, T_L, T_T and T_I of the hadronic tensor can be expressed by $\hat{p}_{1T} = |\vec{\hat{p}}_{1T}|$, p_{1L}, $|\vec{p}_3|$ and the azimuthal angles $\hat{\phi}_1$, ϕ_3 and the polar angle θ_3 using (2.18) and (2.19). The result is:

$$T_U = \frac{T_1}{2}\left(\alpha_{1U} + \alpha_{2U}\cos 2\hat{\phi}_1 + \alpha_{3U}\cos\hat{\phi}_1\right)$$

$$T_L = T_1\left(\alpha_{1L} + \alpha_{2L}\cos 2\hat{\phi}_1 + \alpha_{3L}\cos\hat{\phi}_1\right)$$

$$T_T = \frac{T_1}{2}\Big(\cos 2\phi_3\left[\alpha_{1T} + \alpha_{2T}\cos 2\hat{\phi}_1 + \alpha_{3T}\cos\hat{\phi}_1\right] + \sin 2\phi_3\left[\beta_{2T}\sin 2\hat{\phi}_1 + \beta_{3T}\sin\hat{\phi}_1\right]\Big)$$

$$T_I = \frac{-2T_1}{\sqrt{Q^2}}\Big(\cos\phi_3\left[\alpha_{1I} + \alpha_{2I}\cos 2\hat{\phi}_1 + \alpha_{3I}\cos\hat{\phi}_1\right] + \sin\phi_3\left[\beta_{2I}\sin 2\hat{\phi}_1 + \beta_{3I}\sin\hat{\phi}_1\right]\Big) \qquad (2.22)$$

where the various coefficients α_{iX} and β_{iX} are given by

$$\alpha_{1U} = \hat{p}_{1T}^{2}\left(1-\tfrac{1}{2}\sin^2\theta_3\right) + \frac{t_1^2}{4Q^2} + \sin^2\theta_3\left(|\vec{p}_3|^2 + \hat{p}_{1L}^{2}\right) + \frac{1}{2Q^2}\left(\left(W^2x_{13}+Q^2\right)^2 + \left(t_0+Q^2\right)^2\right)$$

$$\alpha_{2U} = \tfrac{1}{2}\,\hat{p}_{1T}^{2}\sin^2\theta_3 + \frac{t_1^2}{2Q^2}$$

$$\alpha_{3U} = \sin 2\theta_3\,\hat{p}_{1L}\hat{p}_{1T} + \frac{t_1}{Q^2}\left(t_0+Q^2\right) \tag{2.23}$$

$$\alpha_{1L} = \hat{p}_{1T}^{2} + \sin^2\theta_3\left(|\vec{p}_3|^2 + \hat{p}_{1L}^{2} - \tfrac{1}{2}\hat{p}_{1T}^{2}\right)$$

$$\alpha_{2L} = -\tfrac{1}{4}\,\hat{p}_{1T}^{2}\sin^2\theta_3$$

$$\alpha_{3L} = \sin 2\theta_3\,\hat{p}_{1T}\hat{p}_{1L} \tag{2.24}$$

$$\alpha_{1T} = \sin^2\theta_3\left(|\vec{p}_3|^2 + \hat{p}_{1L}^{2} - \tfrac{1}{2}\hat{p}_{1T}^{2}\right), \quad \beta_{2T} = -\cos\theta_3\,\hat{p}_{1T}^{2}$$

$$\alpha_{2T} = \hat{p}_{1T}^{2}\left(1-\tfrac{1}{2}\sin^2\theta_3\right), \quad \beta_{3T} = -2\sin\theta_3\,\hat{p}_{1T}\hat{p}_{1L}$$

$$\alpha_{3T} = \sin 2\theta_3\,\hat{p}_{1T}\hat{p}_{1L} \tag{2.25}$$

$$\alpha_{1I} = \sin\theta_3\left(-\left(\mathcal{P}p_0 + q_0p_L\right)|\vec{p}_3| + \mathcal{P}p_{10}\hat{p}_{1L} - \tfrac{1}{2}q_0\hat{p}_{1T}^{2}\cos\theta_3 + q_0\hat{p}_{1L}^{2}\cos\theta_3\right)$$

$$\alpha_{2I} = -\tfrac{1}{2}\sin\theta_3\cos\theta_3\,q_0\hat{p}_{1T}^{2}$$

$$\alpha_{3I} = \mathcal{P}p_{10}\hat{p}_{1T}\cos\theta_3 + q_0\hat{p}_{1L}\hat{p}_{1T}\cos 2\theta_3$$

$$\beta_{2I} = \tfrac{1}{2}\sin\theta_3\,q_0\hat{p}_{1T}^{2}$$

$$\beta_{3I} = -\mathcal{P}p_{10}\hat{p}_{1T} - q_0\hat{p}_{1L}\hat{p}_{1T}\cos\theta_3 \tag{2.26}$$

where $\mathcal{P}$, q_0, p_0, p_L and p_{10} are determined from

$$\mathcal{P} = |\vec{q}| = \frac{1}{2W}\left(W^2+Q^2\right), \quad q_0 = \frac{1}{2W}\left(W^2-Q^2\right)$$

$$p_0 = \left(\sin^2\theta_3|\vec{p}_3|^2 + \left(\mathcal{P}-\cos\theta_3|\vec{p}_3|\right)^2\right)^{1/2}$$

$$p_{10} = \left(\hat{p}_{1T}^{2} + \hat{p}_{1L}^{2}\right)^{1/2}, \quad p_L = \mathcal{P} - \cos\theta_3|\vec{p}_3| \tag{2.27}$$

We remark that in the limit $\theta_3 = 0$ the squared amplitudes $T_T \sim \cos 2\phi_1$ and $T_I \sim \cos\phi_1$. The T_X are complicated functions of the two azimuth angles ϕ_3 and $\hat{\phi}_1$. Concerning $\hat{\phi}_1$ we must also take into account the structure of T_1. It has the following form (see (2.9))

$$T_1 = -\frac{8e^2c_\alpha\, g^2 Q_f^2\, Q^2}{W^2x_{13}\left(t_0 + t_1\cos\hat{\phi}_1\right)} \tag{2.28}$$

where

$$t_0 = -x_1Wq_0 - 2\mathcal{P}\hat{p}_{1L}\cos\theta_3 - Q^2$$

$$t_1 = 2\hat{p}_{1T}\mathcal{P}\sin\theta_3 \tag{2.29}$$

We notice that T_U and T_L and therefore also $d\sigma_U$ and $d\sigma_L$ depend on the azimuthal angle $\hat{\phi}_1$ but not on ϕ_3. The ϕ_3 dependence is only in $d\sigma_T$ and in $d\sigma_I$ as at should be.

The complete cross sections are obtained after calculation of the phase space as defined in (2.5). For example, for $d\sigma_U$ we get

$$\frac{d\sigma_U}{dx_1\,dx_2\,d\phi_3\,d\hat{\phi}_1\,dp_T^2} = \frac{\mathcal{F}(\vec{p})\,T_U}{32(2\pi)^5\,\eta\,x_3\left(W^2+Q^2\right)} \tag{2.30}$$

and analogous equations for the other cross sections. Here η is defined by $\eta = p_{L/P}$. The structure function $F(\vec{p})$ of the nucleon depends on p_L and p_T^2 which must be replaced (see (2.19)) by

$$
\begin{aligned}
p_T^2 &= \sin^2\theta_3 \, |\vec{p}_3|^2 \\
p_L &= P - \cos\theta_3 \, |\vec{p}_3| \\
\eta &= 1 - \frac{x_3 W^2}{W^2 + Q^2} \left(1 - \frac{4 p_T^2}{x_3^2 W^2}\right)^{1/2}
\end{aligned}
\tag{2.31}
$$

Of course the integration over p_T^2 is replaced by an integration over θ_3. $|\vec{p}_3| = x_3 W/2$. The comparison with our old results, where $p_T = 0$ [2], is achieved by using (2.7) with $f(\eta)$ defined as in [2].

The result (2.18) together with (2.23) - (2.29) can be used to study various distributions. For example, if we integrate over $\hat{\phi}_1$ and $\vec{p}_T$ we obtain the two-dimensional distribution of jet energies

$$
\frac{d\sigma_u}{dx_1\,dx_2} = \frac{\alpha_s}{2\pi} C_2 Q_f^2 e^2 \frac{Q^2}{W^2 (W^2+Q^2)\, \eta\, x_3\, x_{13}} \int \frac{dp_T^2}{4(2\pi)^2} \mathcal{F}(\vec{p}) \left\{ \alpha_{1u} I_1 + \alpha_{2u} I_2 + \alpha_{3u} I_3 \right\}
\tag{2.32}
$$

and

$$
\frac{d\sigma_L}{dx_1\,dx_2} = \frac{\alpha_s}{2\pi} C_2 Q_f^2 e^2 \frac{Q^2}{W^2 (W^2+Q^2)\, \eta\, x_3\, x_{13}} \int \frac{dp_T^2}{4(2\pi)^2} \mathcal{F}(\vec{p}) \left\{ \alpha_{1L} I_1 + \alpha_{2L} I_2 + \alpha_{3L} I_3 \right\}
\tag{2.33}
$$

The two other cross sections: $d\sigma_T$ and $d\sigma_I$ vanish because of the ϕ integration. Of course, we assume, that the structure function $F(\vec{p})$ does not depend on ϕ, only on p_T^2. (2.32) and (2.33) depend on the integrals I_1, I_2 and I_3 which are:

$$
\begin{aligned}
I_1 &= \int_0^{2\pi} d\hat{\phi}_1 \frac{1}{|t_0| - t_1 \cos\hat{\phi}_1} = \frac{2\pi}{\sqrt{t_0^2 - t_1^2}} \\
I_2 &= \int_0^{2\pi} d\hat{\phi}_1 \frac{\cos 2\hat{\phi}_1}{|t_0| - t_1 \cos\hat{\phi}_1} = \frac{2\pi}{t_1^2 \sqrt{t_0^2 - t_1^2}} \left(2t_0^2 - 2t_0\sqrt{t_0^2 - t_1^2} - t_1^2\right) \\
I_3 &= \int_0^{2\pi} d\hat{\phi}_1 \frac{\cos\hat{\phi}_1}{|t_0| - t_1 \cos\hat{\phi}_1} = \frac{2\pi}{t_1 \sqrt{t_0^2 - t_1^2}} \left(t_0 - \sqrt{t_0^2 - t_1^2}\right)
\end{aligned}
\tag{2.34}
$$

Of course, the contributions proportional to I_2 and I_3 are small corrections since $I_2 \sim t_1^2$ and $I_3 \sim t_1$ for small $t_1^2 \sim \sin^2\theta_3 \sim p_T^2/p_3^2$.

In case that also the direction of the target jet is observed, but not the directions of the current jets, a characteristic angular distribution in the angle ϕ_3 evolves

$$
\frac{d\sigma}{d\phi_3\, dx_1\, dx_2} \sim \left(a + b \cos 2\phi_3 + c \cos\phi_3\right)
\tag{2.35}
$$

a is obtained from (2.28) and (2.29). b is obtained by integrating $\alpha_{1T} I_1 + \alpha_{2T} I_2 + \alpha_{3T} I_3$ over p_T^2. It is of the order of $\sin^2\theta_3$, whereas c, obtained from $\alpha_{1I} I_1 + \alpha_{2I} I_2 + \alpha_{3I} I_3$, is of order $\sin\theta_3$. Therefore the $\cos\phi_3$ term might be observable also for higher energies. We notice, that the terms $\sim \sin 2\phi_3$ and $\sim \sin\phi_3$ can be detected only if also the direction of the current jet with respect to the target jet is measured.

The opposite case, that the azimuthal dependence of the current jet is observed but not the azimuth ϕ_3 of the target jet cannot be obtained from (2.22) since ϕ_1 was defined with respect to the vector $\vec{p}_3$. For this case we must go back to (2.12) (and to similar expressions for $d\sigma_L$, $d\sigma_T$ and $d\sigma_I$) and must integrate these expressions over ϕ_3. Since this can be done only numerically it will not be pursued any further here.

The treatment of the parton process with the gluon in the initial state (Fig. 1b)

$$\gamma(q) + g(p) \rightarrow q(p_1) + \bar{q}(p_2) \tag{2.36}$$

which produces on the nucleon N(P) the reaction

$$\gamma(q) + N(P) \rightarrow q(p_1) + \bar{q}(p_2) + (qqq)(p_3) \tag{2.37}$$

is completely analogous. The tensor amplitude $T_{\mu\nu}$ for (2.36) was calculated in [2] [F3]. This implies for the invariant amplitudes T_1^g, T_2^g etc. defined by (2.8):

$$T_1^g = \frac{1}{2} T_2^g = - T_3^g = \frac{8e^2 c_3 g^2 Q_f^2 Q^2}{ut}$$
$$T_4^g = T_5^g = 0 \tag{2.38}$$

Then the contribution of the gluon induced process to the squared amplitudes T_U, T_L, T_T and T_I have the same form as that of the quark induced process, namely the one given in (2.22). The amplitude T_1 is now given by (2.38) above and the various constants appearing in (2.22) are written down in the appendix with the notation α_{iU}^g etc.

The corresponding cross sections are calculated from (2.30) with the replacement $F(\vec{p}) \rightarrow G(\vec{p})$ where $G(\vec{p})$ is the gluon structure function of the nucleon. In the limit $p_T = 0$ we recover our old results [2].

The third contribution coming from Fig. 1c is identical in structure to the contribution coming from the production on quarks (Fig. 1a)

since the basic tensors $T_{\mu\nu}$ are identical. Only in the final cross section formula (2.30) (and the corresponding ones for $d\sigma_L$, $d\sigma_T$ and $d\sigma_I$) we must replace the quark structure function $\mathcal{F}(\vec{p})$ by the antiquark structure function $\bar{\mathcal{F}}(\vec{p})$ of the nucleon.

For completeness we also report the formulas for the two-jet contributions generated from the diagram in Fig. 1d. The cross section has the following form:

$$\frac{d\sigma_X}{dx_1\, dx_3\, d\phi_3\, d\hat{\phi}_1\, dp_T^2} = \frac{\mathcal{F}(\vec{p})\; T_X}{8(2\pi)^2 \eta (W^2+Q^2) W^2}\, \delta(1-x_1)\,\delta(1-x_3)\,\delta(\hat{\phi}_1) \tag{2.39}$$

where X stands for U, L, T and I and $\eta = {}^{p_L}/{}_{p}$ is now

$$\eta = 1 - \frac{W^2}{W^2+Q^2}\left(1 - \frac{4p_T^2}{W^2}\right)^{1/2}$$

The corresponding squared matrix elements are:

$$\begin{aligned}
T_U &= e^2 Q_f^2 \,(Q^2 + p_T^2) \\
T_L &= 4e^2 Q_f^2\, p_T^2 \\
T_T &= 2e^2 Q_f^2\, p_T^2 \cos 2\phi_3 \\
T_I &= -2e^2 Q_f^2 \frac{p_T}{\sqrt{Q^2}} \left\{ Q^2 + W^2 + (Q^2 - W^2)\left(1 - \frac{4p_T^2}{W^2}\right)^{1/2} \right\} \cos\phi_3
\end{aligned} \tag{2.40}$$

These formulas agree with [2] if in (2.40) only the lowest order terms in p_T^2/W^2 are kept.

3. Some Results for Small Transverse Momentum.

In actual applications the average parton transverse momentum $\langle p_T \rangle$, which is of the order of 1 GeV, will be small compared to W, which should be above 10 GeV, to see multi-jet effects. Therefore it is reasonale to try an expansion in $\langle p_T \rangle / W$ to see the main effects of the primordial transverse momentum of the partons. We do this for some of our results, namely $d\sigma_U$ and $d\sigma_L$. For the other partial cross sections the analysis is analogous.

We start from (2.30) together with (2.22), (2.23) and (2.24) and expand $(\alpha_{1,2,3})_{U,L}$, t^{-1} and $\frac{1}{\eta}\, F(\eta p_T^2)$ in powers of p_T/W. $F(\eta, p_T^2)$ is written for the quark structure function $F(\vec{p})$ introduced earlier. We assume for $F(\eta, p_T^2)$ the following factorization concerning the dependence on η and p_T^2:

$$F(\eta, p_T^2) = 16\pi^2 f(\eta)\, g(p_T^2) \tag{3.1}$$

$$= 16\pi^2 \left\{ f(\eta_0) + \frac{2p_T^2}{x_3 (W^2+Q^2)} f'(\eta_0) + \dots \right\} g(p_T^2)$$

where

$$\eta_0 = (\eta)_{p_T=0} = \frac{Q^2 + x_{13} W^2}{Q^2 + W^2} \tag{3.2}$$

$g(p_T^2)$ is normalized to

$$\int dp_T^2\, g(p_T^2) = 1 \tag{3.3}$$

and the averages of p_T and p_T^2 are defined with $g(p_T^2)$ only. They are the "true" averages over p_T and p_T^2 only if one neglects the p_T dependence of η. In general they differ from the true averages, for example

$$\langle p_T \rangle = \frac{\int dp_T^2\, p_T\, F(\eta, p_T^2)}{\int dp_T^2\, F(\eta, p_T^2)} \tag{3.4}$$

by terms of $\langle p_T \rangle^2 / (W^2 + Q^2)$. In the limit $p_T \to 0$ the integral over $F(\eta, p^2)$ is replaced by $f(\eta_0)$ (see (2.7)). One should notice, since η depends also on p_T^2 (see (2.31)), it is important to realize, that the structure function used in the 3-jet contribution is the same as in the 2-jet term (2.39) which has the p_T^2 dependence of η also. Then the corrections from finite p_T to the cross sections $\frac{d\sigma_{U,L}}{dx_1\, dx_2\, d\hat{\phi}_1}$ have the following form:

$$\frac{d\sigma_{U,L}}{dx_1\, dx_2\, d\hat{\phi}_1} = \left(\frac{d\sigma_{U,L}}{dx_1\, dx_2\, d\hat{\phi}_1}\right)_0 \left\{ 1 + \frac{\langle p_T \rangle}{W} \cos\hat{\phi}_1\, B_{U,L} \right.$$

$$\left. + \frac{\langle p_T^2 \rangle}{W^2} \cos 2\hat{\phi}_1\, C_{U,L} + \frac{\langle p_T^2 \rangle}{W^2} \left(A_{U,L} + \frac{W^2}{W^2+Q^2} A'_{U,L} \right) + O\!\left(\frac{\langle p_T^3 \rangle}{W^3}\right) \right\} \tag{3.5}$$

In (3.5) the cross sections with index zero are the limits for $p_T = 0$ given in [2]. They are in our notation:

$$\left(\frac{d\sigma_U}{dx_1\, dx_2\, d\hat{\phi}_1}\right)_0 = \frac{\alpha_S C_2 e^2 Q_f^2 f(\eta_0)}{4\pi\, \eta_0 (W^2+Q^2) x_3^2 x_{11} x_{13}} \left\{ \frac{W^2}{\eta_0 (W^2+Q^2)} x_{13}^2 (x_{11}^2 + x_{12}^2) \right.$$

$$\left. + x_{11} x_{13} (x_{12} + x_3) + \frac{Q^2}{W^2} (x_{12}^2 + x_3^2) \right\} \tag{3.6}$$

$$\left(\frac{d\sigma_L}{dx_1\, dx_2\, d\hat{\phi}_1}\right)_0 = \frac{\alpha_S C_2 e^2 Q_f^2 f(\eta_0)\, Q^2 x_{12}}{\eta_0^2\, x_3^2 (W^2 + Q^2)^2} \tag{3.7}$$

The various coefficients in (3.5) are:

$$A_L = \frac{2\hat{p}_{1T}^2}{\eta_0^2 W^2 x_{11}^2} + \frac{6\hat{p}_{1L}}{\eta_0 W x_3 x_{11}} + \frac{x_1^2 + x_3^2}{x_{11} x_{12} x_{13}} - \frac{5}{x_3^2}$$

$$A_L' = \frac{2}{x_3}\left(\frac{f'(\eta_0)}{f(\eta_0)} - \frac{1}{\eta_0}\right)$$

$$B_L = \frac{2\hat{p}_{1T}}{\eta_0 W x_{11}} + \frac{4\hat{p}_{1L}}{\hat{p}_{1T}\, x_3} \tag{3.8}$$

$$C_L = \frac{2\hat{p}_{1T}^2}{\eta_0^2 W^2 x_{11}} + \frac{4\hat{p}_{1L}}{\eta_0 W x_{11} x_3} - \frac{2}{x_3^2}$$

$$A_u = \frac{1}{m_0 + \frac{1}{2}\hat{p}_{1T}^2}\Big\{\tfrac{1}{2}A_L\hat{p}_{1T}^2 + \frac{\hat{p}_{1T}^2(W^2+Q^2)}{W^2\eta_0 x_{11} x_3} + \frac{\hat{p}_{1L}(W^2+Q^2)}{Q^2 W x_3^3}(Q^2 x_{12} - W^2 x_{11} x_{13}) + \frac{2m_0\hat{p}_{1L}}{\eta_0 W x_{11} x_3} + \frac{2m_0\hat{p}_{1T}^2}{\eta_0^2 W^2 x_{11}^2}\Big\}$$

$$A_u' = A_L'$$

$$B_u = \frac{1}{m_0 + \frac{1}{2}\hat{p}_{1T}^2}\Big\{\tfrac{1}{2}B_L\hat{p}_{1T}^2 + \frac{2m_0\hat{p}_{1T}}{\eta_0 x_{11} W} + \frac{\hat{p}_{1T}(W^2+Q^2)}{W Q^2 x_3^2}(Q^2 x_{12} - W^2 x_{11} x_{13})\Big\} \tag{3.9}$$

$$C_u = \frac{1}{m_0 + \frac{1}{2}\hat{p}_{1T}^2}\Big\{\tfrac{1}{2}C_L\hat{p}_{1T}^2 + \frac{2m_0\hat{p}_{1T}^2}{\eta_0^2 x_{11} W^2} + \frac{\hat{p}_{1T}^2(W^2+Q^2)^2}{2W^2 Q^2 x_3^2} + \frac{\hat{p}_{1T}^2(W^2+Q^2)}{W^2\eta_0 x_{11} x_3}\Big\}$$

where

$$m_0 = \frac{1}{4Q^2 x_3^2}\Big((Q^2 x_{12} - W^2 x_{11} x_{13})^2 + x_3^2(W^2 x_{13} + Q^2)^2\Big) \tag{3.10}$$

The coefficients which determine the dependence on $\langle p_T\rangle$ have a rather complicated structure and depend only on the normalized jet energies x_1 and x_2 after substituting (2.21) for $\hat{p}_{1T}$ and $\hat{p}_{1L}$.

It is well known that the longitudinal cross section $\frac{d\sigma_L}{dx_1 dx_2}$ for $p_T = 0$ is free of mass singularities. This is also true for the coefficients A_L, A_L', B_L and C_L which multiply $\langle p_T\rangle$ and $\langle p_T^2\rangle$. The azimuthal dependence on $\hat{\phi}_1$ is of the order $\langle p_T\rangle/W$ for the $\cos\hat{\phi}_1$ and $\langle p_T^2\rangle/W^2$ for the $\cos 2\hat{\phi}_1$ term. Of course the coefficients $A_{U,L}$, $B_{U,L}$ etc. may be large in particular kinematical regious.

Similar formulas can be derived for the corrections of the other cross sections. We remark that the corrections are in general proportional to $\langle p_T\rangle/W$ (or $\langle p_T^2\rangle/W^2$) and not proportional to $\langle p_T\rangle/\sqrt{Q^2}$ (or $\langle p_T^2\rangle/Q^2$) as the corrections to the lowest order process are as can be seen from (2.40). An exception, of course, is the kinematic region $x_3 \to 1$, where the 2-jet limit is approached.

4. Concluding Remarks.

We have shown in this paper that non-zero transverse momenta of the partons in the nucleon lead to additional structure of the various jet cross sections in deep inelastic eN scattering. This additional structure is of the order $\langle p_T \rangle /W$ (or $\langle p_T^2 \rangle /W^2$) and therefore is in general small for large enough total energy of the outgoing hadrons. In this sense it was justified to neglect these effects in [2] where predictions for large energies in the HERA energy range have been worked out.

The results presented in section 2 and 3 can be used in two ways. First they can be applied to the calculation of angular asymmetries of singly produced hadrons relative to the lepton scattering plane. This had been done by Mendez [6] neglecting the transverse momentum of the initial parton. Actually the interpretation of recent measurements of cos φ and cos 2φ terms in the angular distribution of produced hadrons in muon-proton scattering was hampered by the unknown effect of the primordial transverse parton momentum [7] . Second the derived formulas could be directly applied to measurement of the angular distributions of jets, either the target or the current jets.

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Appendix.

In this appendix we list the coefficients a_{iX}^g which appear in (2.22) for the gluon induced 3-jet process (2.37):

$$\alpha_{1L}^g = 2\cos^2\theta_3/2\ \hat{p}_{1T}^2 + \sin^2\theta_3 \left((|\vec{p}_3| + \hat{p}_{1L})^2 + \hat{p}_{1L}^2 \right)$$

$$\alpha_{2L}^g = -\sin^2\theta_3\ \hat{p}_{1T}^2$$

$$\alpha_{3L}^g = \sin 2\theta_3\ \hat{p}_{1T} \left(2\hat{p}_{1L} + |\vec{p}_3| \right)$$

$$\alpha_{1u}^g = \alpha_{1L}^g + \frac{1}{2Q^2} \left(t_1^2 + s^2 + Q^2 - 2u_0 t_0 \right)$$

$$\alpha_{2u}^g = \alpha_{2L}^g - \frac{t_1^2}{2Q^2}$$

$$\alpha_{3u}^g = \alpha_{3L}^g + \frac{t_1 (t_0 - u_0)}{Q^2}$$

$$\alpha_{1T}^g = \sin^2\theta_3 \left((\hat{p}_{1L} + |p_3|)^2 + \hat{p}_{1L}^2 - \hat{p}_{1T}^2 \right)$$

$$\alpha_{2T}^g = \hat{p}_{1T}^2 \left(1 + \cos^2\theta_3 \right)$$

$$\alpha_{3T}^g = \alpha_{3L}^g$$

$$\beta_{2T}^g = -2\cos\theta_3\ \hat{p}_{1L}^2$$

$$\beta_{3T}^g = -2\sin\theta_3\ \hat{p}_{1T} \left(2\hat{p}_{1L} + |\vec{p}_3| \right)$$

$$\alpha_{1I}^g = -\sin\theta_3 \Big(q_0 \hat{p}_{1T}^2 \cos\theta_3 - (2\hat{p}_{1L} + |\vec{p}_3|)(\mathcal{P} p_{10} + q_0 \hat{p}_{1L} \cos\theta_3) + (\hat{p}_{1L} + |\vec{p}_3|)(\mathcal{P} p_0 + q_0 p_L) \Big)$$

$$\alpha_{2I}^g = -\frac{1}{2} \sin 2\theta_3\ q_0\ \hat{p}_{1L}^2$$

$$\alpha_{3I}^{q} = \hat{p}_{1T}\left(-\cos\theta_3\left(\mathcal{P}p_0 + q_0 p_L - 2\mathcal{P}p_{10}\right) + 2\cos 2\theta_3\, q_0 \hat{p}_{1L} - q_0 |\vec{p}_3| \sin^2\theta_3\right)$$

$$\beta_{2I}^{q} = \sin\theta_3\, q_0\, \hat{p}_{1T}^{2}$$

$$\beta_{3I}^{q} = -\hat{p}_{1T}\left(\mathcal{P}p_0 + q_0 p_L + 2\mathcal{P}p_{10} + 2q_0\hat{p}_{1L}\right)$$

u_o appearing above is defined in analogy with t_o, i.e.
$u = -s - t - Q^2 = u_o - t_1 \cos\hat{\phi}_1$
The rest of the quantities are defined in the main text.

Footnotes:

(F1) An exception is the work of Andersson et al. [3] and recent analysis of experimental data [4] with the models developped in [2] and [3].

(F2) We multiplied by $\frac{1}{2}$ to account for spin averaging of the initial nucleon (or quark).

(F3) Unfortunately the $T_{\mu\nu}$ as given in [2] contains some misprints. The correct expression in our normalization is:

$$T_{\mu\nu} = 2g^2e^2Q_f^2c_3\Big\{\frac{1}{pp_2}\left[\{p,p_1\}_{\mu\nu} - \{p_1,p_2\}_{\mu\nu} + \{p_2,p_2\}_{\mu\nu}\right]$$

$$+\frac{1}{pp_1}\left[\{p,p_2\}_{\mu\nu} - \{p_1,p_2\}_{\mu\nu} + \{p_1,p_1\}_{\mu\nu}\right]$$

$$+\frac{p_1p_2}{pp_1\,pp_2}\left[2\{p_1,p_2\}_{\mu\nu} - \{p,p_1\}_{\mu\nu} - \{p,p_2\}_{\mu\nu}\right]\Big\}$$

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Figure Caption:

Fig. 1: (a,b,c) First order QCD parton diagrams for lepton-parton scattering.
(d) Zeroth order diagram for lepton-parton scattering.

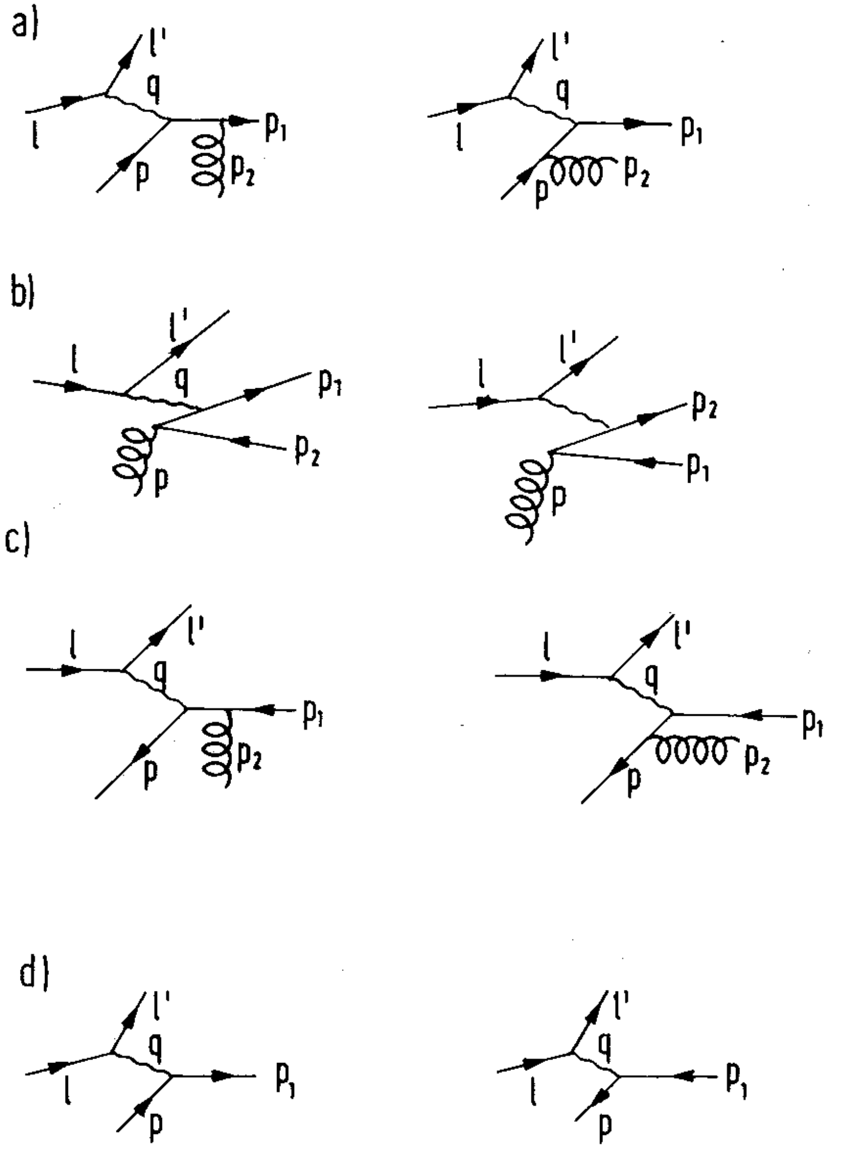

FIG. 1