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Abstract

We show that $|\epsilon'/\epsilon|$ is unacceptably large in $SU(2)_L \times U(1)$ models with spontaneous CP violation and natural flavor conservation in neutral Higgs-particle interactions if it is assumed that charmed quark intermediate states contribute significantly and that $M_W^2 \gg M_H^2 \gg M_c^2$. From the quark operator structure of box and penguin graphs involving Higgs-particle exchange, we demonstrate that pseudoscalar current matrix elements cannot be enhanced over axial-current matrix elements as has been previously assumed in similar calculations. Alternate ways to evaluate the matrix elements are employed.

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Introduction

It is well known that CP is automatically conserved in the $SU(2)_L \times U(1)$ gauge theory of weak and electromagnetic interactions with two left-handed doublets of quarks, four right-handed singlets of quarks and one Higgs scalar doublet [1]. More particles are needed - either more quarks or more Higgs particles or both - in order to allow for CP nonconservation. The six-quark option for generating CP nonconservation in the theory is very economical, introducing exactly one CP-violating phase. It accounts for known CP-violation phenomenology, and one of the predicted new flavors, b, has been discovered. [2]

Nonetheless, the option of Higgs-boson generated CP-violation is attractive and has been frequently discussed [3,4,5,6]. This method of producing CP violation has received attention in the literature for a variety of reasons. Recently, for example, it was shown that adding more Higgs particles to the minimal SU(5) grand unification model enables one to generate the observed cosmological baryon asymmetry [7,8]. The minimal model, which cannot produce a large enough baryon asymmetry, has only one $SU_L(2) \times U(1)$ weak Higgs boson doublet, and CP violation enters only via the Kobayashi-Maskawa phase in the quark mass matrix [1]. The expanded models [7,8] have extra $SU_L(2) \times U(1)$ Higgs doublets which lead to Higgs boson-mediated CP violation.

The most frequently used picture of CP violation as caused by Higgs bosons is one due to Weinberg [5] in a four-quark model. It has been studied in its original version [9] and its six-quark generalizations [10] by a number of authors. The distinguishing feature of these models is that flavor is naturally conserved in neutral Higgs couplings because quarks of a given charge couple to only one Higgs field. Two different Higgs doublets are needed, one for each quark charge value, plus at least one more Higgs doublet that does not couple to quarks but only to other Higgs fields, to the gauge bosons and, possibly to the leptons [11]. If CP-violating phases are absent in the original Yukawa couplings, then CP nonconservation can only occur via spontaneous breakdown, and only Higgs-boson interactions with the quarks violate the CP conservation in the quark sector [12]. This way of breaking CP conservation introduces a factor of light-quark to Higgs mass ratio M_q^2/M_H^2 in CP-violating amplitudes relative to the CP-conserving, W-boson mediated amplitudes. Lee [3] and Weinberg [5] emphasized the attractiveness of this simple explanation for the suppression of CP-violating effects relative to CP-conserving ones. We wish to preserve this feature in our discussion of the model.

Although the properties and physical consequences of this model have been discussed several times in the literature [9,10], there is no consensus on its success in accounting for the experimental facts [13]. In particular, it is important to clarify its predictions for the parameters ϵ and ϵ' which describe the neutral kaon CP nonconservation.

We reevaluate the standard $\Delta S = 2$ and $\Delta S = 1$ CP-violating amplitudes as generated in this scheme. We find that evaluating the CP-violating parts of the matrix elements of $\bar{K}^0$-K^0 mixing and $K^0 \to 2\pi$ according to the methods of Vainstein, Zakharov and Shifman [14], as done in previous work [13], is incorrect when applied to the $\Delta S = 2$ box graph with W boson and Higgs boson propagators and to the $\Delta S = 1$ Higgs boson-mediated penguin graph. The origin of the trouble is the explicit factors of external momentum which appear in four quark interactions. We show that it is incorrect to neglect vector (V) and axial-vector (A) current matrix elements in comparison to scalar (S) and

pseudoscalar (P) matrix elements. Since the dominance of S and P over V and A is used in existing literature on these processes [13], the values derived for relevant matrix elements are not correct.

In the next section we discuss the necessary couplings and the evaluation of the relevant Feynman diagrams. Following that, we show the source of the error described above. We carry through estimates of the matrix elements which avoid the contradictions. We conclude that the milliweak $\Delta S = 1$ CP nonconservation due to the penguin graph must be the dominant one if one insists that, for light quarks, $M_q^2/M_H^2 \ll 1$. The resulting estimate of $|\epsilon'/\epsilon| \simeq 0.05$ flatly disagrees with the experimental limit [15] $|\epsilon'/\epsilon|_{\text{expt.}} = 0.003 \pm 0.015$!

Relaxing the condition $M_q^2/M_H^2 \ll 1$ is possible, of course, but we do not address this question in the present work. We will work in lowest order in strong interactions, since the inclusion of renormalization group improved effective couplings is not expected to be large [16], and it would take order of magnitude changes due to strong interactions to change our conclusions.

Calculation of Box and Penguin Graphs

We turn now to the study of several consequences of an $SU(2)_L \times U(1)$ weak-electromagnetic gauge model with spontaneous CP violation [5]. In the kaon system, the CP violation occurs in lowest order perturbation theory only as a consequence of charged Higgs-particle exchange. Because the model has been discussed several times in the literature in both the four-quark and six-quark versions [9,10], we present only the final form of its key feature - the CP nonconserving charged Higgs-particle Yukawa couplings to quarks.

The Yukawa interaction between quarks and physical Higgs fields reads

$$L_y = (2\sqrt{2}\,G_F)^{1/2} \sum_{i=1}^{N-1} H_i^{+*}\left[\eta_i \bar{D}_R M(D) K U_L + \zeta_i \bar{D}_L K M(U) U_R\right] + H.c. \tag{1}$$

In Eq. 1 H_i^+, $i=1,..N-1$ denote the charged, physical Higgs fields ($N \geq 3$[5,12]) and

$$\bar{D}_{R,L} = (\bar{d}_{R,L}\ \bar{s}_{R,L}\ \bar{b}_{R,L}) \text{ and } \bar{U}_{R,L} = (\bar{u}_{R,L}\ \bar{c}_{R,L}\ \bar{t}_{R,L})$$

represent the physical charged -1/3 and charged +2/3 quark fields, respectively. The diagonal mass matrices are $M(D) = \mathrm{diag}\,(m_d, m_s, m_b)$ and $M(U) = \mathrm{diag}\,(m_u, m_c, m_t)$, while K is the three-by-three, real ($\delta = 0$) Kobayashi-Maskawa [1] matrix. K is the same matrix that appears in the quark interaction with the W-boson. G_F denotes the Fermi constant. The exchange of the i^{th} Higgs boson with momentum k will produce a complex, CP-violating four quark interaction proportional to $\eta_i \zeta_i^* (k^2 - M_{H_i}^2)^{-1}$. One can show quite generally that the $2(N-1)$ complex coefficients η_i and ζ_i that characterise the Higgs-particle quark couplings satisfy the constraint

$$\sum_i^{N-1} \eta_i \zeta_i^* = -1 \tag{2a}$$

This has the consequences that

(i)
$$Im \sum_{i=1}^{N-1} \eta_i \zeta_i^* \cdot \frac{1}{k^2 - M_{H_i}^2} \longrightarrow \frac{1}{k^4}\, Im \sum_{i=1}^{N-1} M_i^2 \eta_i \zeta_i^* . \tag{2b}$$

Eq. 2b shows that CP nonconserving amplitudes have faster convergence properties than the CP-conserving parts of Higgs exchange amplitude. This simply reflects the feature that CP nonconservation is due only to mixed Higgs propagators.

(ii) If the Higgs particle masses are degenerate, $M_i = M_H$, the CP-violating amplitude vanishes because

$$\frac{1}{k^2 - M_H^2}\, Im \sum_{i=1}^{N-1} \eta_i \zeta_i^* = 0 . \tag{2c}$$

This is a generalization of the observation * that the Yukawa couplings are real if there is only one physical Higgs field.

Eq. 2a and Eqs. 2b and 2c that are derived from it summarize the constraints that are obeyed by the Higgs-quark couplings in this model.

As proposed within a four-quark model [5], the advertised virtues of the Higgs exchange mechanism for CP violation were that:

(1) CP nonconservation can arise only from Higgs-particle exchange even for general complex Yukawa couplings in the original Lagrangian and

(2) the smallness of CP nonconservation is naturally explained by the smallness of M_q^2/M_H^2, where M_q^2 is the square (or product of any two) light quark masses M_c, M_s, M_u, M_d.

Feature (1) does not follow automatically in the six-quark model, so one must impose reality on the Yukawa coupling matrices in the original Lagrangian in order to implement purely spontaneous CP violation. Concerning feature (2), one would like to preserve it separately for each one of the virtual Higgs-

* McKay, Ref. [9] and Abbot, Sikivie and Wise, Ref. [10].

particle exchange contributions to a CP-violating process. This stays within the spirit of the suppression mechanism as originally proposed.

Let us re-examine [9,10] the four-light quark sector of the six quark model to see if the $\Delta S = 2$ and $\Delta S = 1$ CP-violating effects naturally explain the CP-violation parameters which have been measured in K decays. In the spirit of point (2) above, any one of the physical Higgs boson exchanges should give approximately the right magnitude of CP violation for a range of Higgs masses whose values are large compared to the largest quark mass, m_c. It should not be necessary to invoke cancellations among different Higgs-boson contributions, nor to require large values of $\eta_i^* \zeta_i$ if the description is to be regarded as natural.

The contributions to the $\Delta S = 2$ transitions are shown in Fig. 1. The contribution of the u quark is not negligible because the Higgs boson-quark couplings involve a term proportional to $m_s \sin\theta_c \bar{s}_R u_L$ and the c plus u propagator graphs which are indicated in Fig.1 must be included.

Fig. 1 yields the $\Delta S = 2$ four quark amplitude * in momentum space. Dropping terms proportional to m_u and m_d, Fig. 1a gives, for example,

$$A^{\Delta S=2}_{(1a)} = -m_s m_c^2 \eta_i \zeta_i^* (2\sqrt{2} G_F) \frac{g^2}{2} \sin^2\theta_c \cos^2\theta_c (I_1 - I_2) \times \bar{s}_R^i(p_1') (\not{p}_1 - \not{p}_2') \gamma_\mu d_L^i(p_1) \cdot \bar{s}_L^j(p_2') \gamma^\mu d_L^j(p_2) \quad . \tag{3}$$

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* In order to keep the momentum factors explicit, we find it convenient to work with the $\Delta S = 2$ H-W exchange amplitude rather than the effective Hamiltonian. In Eq. 3 spinors $(u,\bar{u})$ and spinors $(v,\bar{v})$ are to be understood as associated with particle or antiparticle lines, respectively, in Fig. 1a.

where i and j are (summed) color labels. $L_I(W) = \frac{g}{\sqrt{2}} \bar{D}_L K W^- U_L$ gives the W-boson quark interaction. The values of the integrals I_1 and I_2 are

$$I_1 = \frac{i}{32\pi^2} \cdot \frac{1}{M_W^2 M_H^2} \cdot \left(\ln\left(\frac{M_H^2}{M_c^2}\right) - \frac{3}{2} \right)$$

$$I_2 = \frac{i}{32\pi^2} \cdot \frac{1}{M_W^2 M_H^2} \cdot \left(\ln\left(\frac{M_H^2}{M_c^2}\right) - 1 \right) \tag{4}$$

$$I_1 - I_2 = -\frac{i}{64\pi^2} \cdot \frac{1}{M_W^2 M_H^2}$$

when

$$M_W^2 >> M_H^2 >> M_c^2$$

The largest contribution to the $\Delta S = 1$ transition is the penguin graph, which is shown in Fig. 2. This graph plus two graphs with two gluons attached to the internal quark line lead to the effective Hamiltonian [17]

$$H^{\Delta S=1} = -i\, m_s m_c^2 \eta_i \zeta_i^* (2\sqrt{2} G_F) \cos\theta_c \sin\theta_c (2 M_W^2 I_1) \times g_s(m_c) \cdot \frac{i}{2} F^a_{\nu\mu} \bar{s}_R \sigma^{\nu\mu} \frac{\lambda^a}{2} d_L \quad , \tag{5}$$

where I_1 is defined in Eq. 4 and where λ^a is the a^{th} color SU(3) matrix, $F^a_{\nu\mu}$ is the gluon field and $g_s(m_c)$ is the strong, color SU(3) coupling constant evaluated at m_c.

Now we must evaluate the matrix elements of Eq. 3 between $\bar{K}^0$ and K^0 states and evaluate the matrix elements of Eq. 5 between K^0 and 2π states. A standard estimate of these matrix elements is obtained by using the vacuum saturation approximation [14]. Such an estimate can be easily made in the case at hand if momentum factors can be reduced to mass factors by using the Dirac equation. To this end, we write down the quark-quark scattering amplitudes corresponding to Fig. 1 and then rewrite them in a form where all momentum factors are replaced by quark mass factors in those terms which give non-vanishing contributions in the vacuum saturation approximation. We then show that the frequently used enhancement of pseudoscalar matrix elements relative to axial vector matrix elements leads to incorrect results. We proceed to evaluate the matrix elements in a consistent approximation and compare the results to experimental values of ϵ and ϵ'/ϵ .

Evaluation of $\Delta S = 2$ and $\Delta S = 1$ Amplitudes

In order to evaluate the contribution of Eq. 3 to the $\bar{K}^0$-K^0 mixing amplitude and the contribution of Eq. 5 to the $K^0 \to \pi^+\pi^-$ amplitude, it is convenient to picture the quarks as being effectively on-mass-shell, neglecting binding effects. We call their effective masses M_s, M_d and M_u, and we bring these mass factors out explicitly by using the free field equation of motion. For example, the term proportional to p_2' in Eq. 3 can immediately be expressed as

$$\text{constant} \times M_s\, \bar{s}^i d_L^i\, \bar{s}^j d_L^j \quad + \quad \sigma^{\mu\nu}\ term$$

We now attempt an evaluation of matrix elements by the methods of Vainstein, Zakharov and Shifman [14], and we show that their conjectured enhancement of pseudoscalar matrix elements compared to axial vector matrix elements between one pseudo-scalar meson and the vacuum leads to an incorrect result. We consider the $\Delta S = 2$ and $\Delta S = 1$ processes in turn.

Let us consider the $d^i(p_1) + \bar{s}^i(p_1') \to s^j(p_2') + \bar{d}^j(p_2)$ scattering amplitude. Equation 3 and its companion amplitudes give the spinor and momentum factors

$$A^{\Delta S=2}_{(1a+1c)} = \bar{s}^i(p_1')(\not{p}_1' - \not{p}_2')\gamma_\mu d_L^i(p_1) \cdot \bar{s}^j(p_2')\gamma^\mu d_L^j(p_2) \quad \text{(Fig.1a)}$$
$$+ \bar{s}^j(p_1')\gamma_\mu d_L^i(p_1) \cdot \bar{s}^i(p_2')(\not{p}_1 + \not{p}_1')\gamma^\mu d_L^j(p_2). \quad \text{(Fig.1c)} \qquad (6)$$

where i and j are color-SU(3) triplet labels. The contributions from Figs. 1b and 1d are obtained from those in Eq. 6 by interchanging $1 \leftrightarrow 2$. We shall only keep color singlet terms and, for notational convenience, drop explicit momentum arguments in bispinors which contain no momentum factors. By repeated use of the identity $\gamma^\mu\gamma^\nu = g^{\mu\nu} - i\sigma^{\mu\nu}$ one arrives at

$$A^{\Delta S=2}_{(1a+1c)} = -\frac{4}{3}M_s\, \bar{s}\, d_L\, \bar{s} d_L + \frac{1}{3}M_d\, \bar{s} d_R\, \bar{s} d_L$$
$$+ M_d\, \bar{s}\gamma_\mu d_R\, \bar{s}\gamma^\mu d_L + \bar{s}(p_1')(p_1 - p_2')^\nu \sigma_{\nu\mu} d_L(p_1)\, \bar{s}(p_2')\gamma^\mu d_L(p_2)$$
$$- \bar{s}(p_1')\gamma^\mu d_L(p_1)\bar{s}(p_2')p_1^\nu \sigma_{\nu\mu} d_L(p_2) \qquad (7)$$
$$+ \frac{1}{3}\bar{s}(p_1')\gamma^\mu d_L(p_1)\bar{s}(p_2')(p_1 + p_1')^\nu \sigma_{\nu\mu} d_L(p_2).$$

The $\sigma^{\mu\nu}$ terms give zero in the vacuum saturation approximation to the contribution of these quark processes in the $\bar{K}^0 \leftrightarrow K^0$ transition. We therefore write*

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* The terms obtained by the $1 \leftrightarrow 2$ interchange parallel those in 7 and do not affect our conclusions.

$$A^{\Delta S=2}_{(1a+1c)} \xrightarrow[\text{saturation}]{\text{vacuum}} -\frac{4}{3} M_s \bar{s} d_L \bar{s} d_L + \frac{1}{3} M_d \bar{s} d_R \bar{s} d_L + M_d \bar{s} \gamma^\mu d_R \bar{s} \gamma_\mu d_L \tag{8}$$

If the $\langle \bar{K}^0 | \bar{s}\gamma_5 d | 0 \rangle$ matrix elements are much bigger than $\langle \bar{K}^0 | \bar{s}\gamma_\mu \gamma_5 d | 0 \rangle$ matrix elements as argued by Vainstein, Zakharov and Shifman [14] and applied by Anselm and D'yakanov [9] and by Sanda [10] in the same calculation we are doing, one arrives at

$$A^{\Delta S=2}_{(1a+1c)} \xrightarrow[\bar{s}\gamma_5 d \text{ enhanced}]{} \frac{M_d}{3} \bar{s} d_R \bar{s} d_L - \frac{4}{3} M_s \bar{s} d_L \bar{s} d_L . \tag{9}$$

However, the choice of momentum factors in Eq. 6 is not unique, since momentum conservation can be used to replace $p_1 - p_2'$ by $-p_1' + p_2$ and $p_1 + p_1'$ by $p_2' + p_2$. The resulting expression is equivalent to Eq. 6, and proceeding as in Eqs. 7, 8 and 9 we find

$$\begin{aligned} A^{\Delta S=2}_{(1a+1c)} = &- M_d \bar{s} d_L \bar{s} d_R - \frac{1}{3} M_d \bar{s} \gamma_\mu d_L \bar{s} \gamma^\mu d_R + \frac{4}{3} M_s \bar{s} \gamma_\mu d_L \bar{s} \gamma^\mu d_L \\ &+ \frac{1}{3} \bar{s}(p_2') \gamma^\mu d_L(p_1) \cdot \bar{s}(p_1') p_2^\nu \sigma_{\nu\mu} d_L(p_1) \\ &- \frac{1}{3} \bar{s}(p_1') \gamma_\mu d_L(p_1) \cdot \bar{s}(p_2') p_2^\nu \sigma_{\nu\mu} d_L(p_2) \\ &+ \bar{s}(p_1') p_2^\nu \sigma_{\nu\mu} d_L(p_1) \cdot \bar{s}(p_2') \gamma^\mu d_L(p_2) , \end{aligned} \tag{10}$$

$$\begin{aligned} A^{\Delta S=2}_{(1a+1c)} \xrightarrow[\text{saturation}]{\text{vacuum}} &\frac{4}{3} M_s \bar{s} \gamma^\mu d_L \bar{s} \gamma_\mu d_L - M_d \bar{s} d_L \bar{s} d_R \\ &- \frac{1}{3} M_d \bar{s} \gamma_\mu d_L \bar{s} \gamma^\mu d_R , \end{aligned} \tag{11}$$

and finally [17]

$$A^{\Delta S=2}_{(1a+1c)} \xrightarrow[\bar{s}\gamma_5 d]{\text{enhanced}} - M_d \bar{s} d_L \bar{s} d_R . \tag{12}$$

Comparing Eqs. 9 and 12 we find a contradiction. The source of the trouble is in the last step, where V, A terms are dropped compared to S, P terms because the latter are supposed to be enhanced by current algebra masses in the denominator. This usual estimate of matrix elements gives the result

$$\langle \bar{K}^0 | H_w | K^0 \rangle \propto \frac{M_d}{(m_s+m_d)^2} \cdot \frac{M_K^4 f_K^2}{4} + \frac{4}{3} \frac{M_s M_K^4 f_K^2}{(m_s+m_d)^2} \cdot \frac{1}{4} \tag{13a}$$

from Eq. 9, whereas Eq. 12 yields

$$\langle \bar{K}^0 | H_w | K \rangle \propto - \frac{M_d}{(m_s+m_d)^2} \cdot \frac{M_K^4 f_K^2}{4} \tag{13b}$$

Both sign and magnitude of (13b) are different from (13a), but the quantities which they are supposed to approximate are identical. Evidently, the above approximation leaves the matrix elements completely undetermined.

Before presenting internally consistent approximations to the above matrix elements, let us discuss the amplitude derived from the $\Delta S = 1$ penguin graphs, Eq. 5, in the same manner as above, and show that the same difficulty is present here.

The $\Delta S = 1$ quark-quark scattering process which is relevant to our discussion is shown in Fig. 3. Fig. 3 yields the amplitude *

$$A^{\Delta S=1} = -i\,2\sqrt{2}\,G_F\,\eta_i\,\zeta_i^{*}\,g_s(m_c^2)\,g_s(\mu^2)\,m_s m_c^2 \sin\theta_c \cos\theta_c\,(2M_W I_1)$$

$$\times\left[\bar{s}(p_s)\sigma_{\nu\mu}\,q^{\nu}\frac{\lambda^a}{2}d_L(p_d)\cdot\sum_{Q=u,d}\bar{Q}\gamma^{\mu}\frac{\lambda^a}{2}Q\right], \qquad (14)$$

where $q = p_d - p_s$ and where we have assumed a simple coupling $g(\mu^2)\bar{Q}\frac{\lambda^a}{2}\gamma_\mu Q$ at the lower vertex, which is adequate for the purpose of the following discussion. μ is a low energy hadronic mass scale (i.e. less than the kaon mass). I_1 is the integral given in Eq. 4.

Denoting the quantity in brackets in Eq. 14 by $\mathcal{P}$, we observe that

$$\mathcal{P} = i\,\bar{s}(p_s)\,\not{q}\,\gamma_\mu\frac{\lambda^a}{2}d_L(p_d)\sum_Q\bar{Q}(k')\gamma^{\mu}\frac{\lambda^a}{2}Q(k) \qquad (15a)$$

by using $\bar{Q}\not{q}Q = 0$. The color triplet indices are suppressed. Only the Fierz reordered form contains the color singlet piece relevant to our purposes, so we write

$$\mathcal{P} = \frac{16}{9}\sum_Q\left[2i\bar{s}^i(p_s)\not{q}\,Q_R^i(k)\,\bar{Q}^j(k')\,d_L^j(p_d) - i\bar{s}^i(p_s)\not{q}\,\gamma_\mu Q_L^i\,\bar{Q}^j\gamma^\mu d_L^j(p_d)\right] \qquad (16)$$

* As before, particle (antiparticle) spinors are to be associated with particle (antiparticle) lines in the graph.

The factor of 16/9 comes from projecting the color singlet part of $\lambda^a\cdot\lambda^a$. Using $q = p_d - p_s$ and the identity $\gamma^{\mu\nu} = g^{\mu\nu} - i\sigma^{\mu\nu}$ as before, we arrive at

$$\mathcal{P} \xrightarrow[\substack{\text{vacuum sat.}\\ \text{and enhance}\\ \bar{s}\gamma_5 Q}]{} \frac{16}{9}\sum_Q\left[-2iM_s\bar{s}Q_R\bar{Q}d_L - iM_d\bar{s}Q_L\bar{Q}d_R\right] \qquad (17a)$$

suppressing color indices from this point on.

Now instead of 15a, we can use the equivalent form

$$\mathcal{P} = -i\,\bar{s}(p_s)\gamma_\mu\not{q}\frac{\lambda^a}{2}d_L(p_d)\sum_Q\bar{Q}\gamma^{\mu}\frac{\lambda^a}{2}Q \qquad (15b)$$

and following the same steps as before we arrive at

$$\mathcal{P} \xrightarrow[\substack{\text{vacuum sat.}\\ \text{and enhance}\\ \bar{s}\gamma_5 Q}]{} \frac{16}{9}\sum_Q\left[-2iM_d\bar{s}Q_L\bar{Q}d_R - iM_s\bar{s}Q_R\bar{Q}d_L\right]. \qquad (17b)$$

Comparing Eq. 17a and Eq. 17b, one notes an inconsistency. As before, the source of the inconsistency is the last step, where one neglects $\bar{q}\gamma_5 q$ terms relative to $\bar{q}\gamma_5\gamma_\mu q$ terms.

Returning to Eq. 6, we find that a <u>consistent</u> evaluation of the $\langle\bar{K}^0|H_W^{\Delta S=2}|K^0\rangle$ matrix element follows in the vacuum saturation approximation if we keep the V,A terms on the same footing as the S,P terms in the expansion of the matrix elements of Eq. 3. That is, we make no distinction between constituent and current algebra mass values. With a static quark approximation $M_d + M_s \cong M_K$, we find that Eq. 6 and Eq. 10 both yield the value

$$\langle \bar{K}^0 | H^{\Delta S=2} | K^0 \rangle = \eta_i \zeta_i^* \frac{m_s M_s}{M_{H_i}^2} \left(\frac{M_K}{M_s}\right)\left(\frac{1}{3} + \frac{M_s - M_d}{M_K}\right) \frac{m_c^2}{16\pi^2} G_F^2 M_K^2 f_K^2 \cos^2\theta_c \sin^2\theta_c \qquad (18)$$

for the W-boson plus Higgs boson $\Delta S = 2$ contribution (Fig. 1). The light mass value, m_s, which came from Higgs couplings Eq. 1, is kept distinct from the static, constituent value, M_s, in Eq. 18.

Eq. 18 provides us with a means of estimating $\epsilon_m = Im \langle \bar{K}^0 | H_W | K^0 \rangle / (2\sqrt{2}\, Re \langle \bar{K}^0 | H_W | K^0 \rangle)$ that is free of internal inconsistency. We use the classic two W-boson box graph estimate of the real part of the $\bar{K}^0 \leftrightarrow K^0$ mass mixing matrix

$$Re \langle \bar{K}^0 | H_W^{\Delta S=2} | K^0 \rangle \cong \frac{G_F^2}{6\pi^2} \sin^2\theta_c \cos^2\theta_c m_c^2 M_K^2 |f_K|^2 \qquad (19)$$

Combining Eq. 18 and 19 we find that the i^{th} physical, charged Higgs boson contributes

$$\epsilon_M = \frac{3}{16\sqrt{2}} Im\, \eta_i \zeta_i^* \cdot \frac{m_s M_s}{M_{H_i}^2} \left(\frac{M_K}{M_s}\right)\left(\frac{1}{3} + \frac{M_s - M_d}{M_K}\right) \qquad (20)$$

to ϵ_m. Regarding the parameters $Im\, \eta_i \zeta_i^*$ and M_{H_i} which appear in Eq. 20, we show in Table 1 the values of $|Im\, \eta_i \zeta_i^*|$ which are needed to satisfy $|\epsilon_m| = |\epsilon|$ ($Expt. \sim 2.3 \times 10^{-3}$) for various M_{H_i} values. The point

that we want to stress is that, term by term, one can only satisfy $|\epsilon_m| = |\epsilon|$ for rather large values of $|Im\, \eta_i \zeta_i^*| \gtrsim 5$. In Table 1 we are requiring values $M_{H_i}^2 >> m_c^2$. Recall that $M_W^2 >> M_{H_i}^2 >> M_c^2$ is assumed in evaluating the integrals.

From Table 1 we see that unless one invokes values of $|Im\, \eta_i \zeta_i^*|$ on the order of 10, there must be at least one light, charged Higgs boson, $M_{H_i} \lesssim 5$ GeV, in order to have $|\epsilon_m| = |\epsilon|$. The small values of M_{H_i} and/or large values of $|Im\, \eta_i \zeta_i^*|$ which are needed to obtain $|\epsilon_m| = 2.3 \times 10^{-3}$ are a reflection of the sharp suppression factor $M_s m_s / M_{H_i}^2$ which appears in Eqs. 18 and 19.*

Next we estimate the contribution of the penguin graph, Fig. 2, to the imaginary part of the $K \rightarrow 2\pi$ matrix elements. This $\Delta S = 1$ (milliweak) mechanism must then be compared to the foregoing ("superweak") $\Delta S = 2$ mechanism to see if the experimental limits on ϵ'/ϵ are obeyed.

We use the results of Donoghue, Golowich and Holstein [19] to evaluate the $K - \pi$ matrix element of the operator

$$\bar{s}_R \frac{\lambda^a}{2} \sigma_{\nu\mu} d_L F^{a,\nu\mu}$$

which appears in the penguin graph contribution, Eq. 5. Using their results

* The suppression factor results from the LLLR helicity structure of $Im\, H_W^{Higg\text{-}W}$, which requires an external momentum factor (dimension seven four-quark operator) to balance angular momentum.

directly in evaluating $Im\langle \bar{K}^0 | H_w | 2\pi \rangle$, we find that the i^{th} Higgs boson gives

$$\xi \equiv \frac{Im\langle \pi\pi | H_w | K \rangle}{Re\langle \pi\pi | H_w | K \rangle} \simeq \frac{\cos\theta_c}{2} \frac{m_s m_c}{M_{H_i}^2} \left(\ln \frac{M_{H_i}^2}{m_c^2} - \frac{3}{2} \right) Im\, \eta_i \zeta_i^* . \quad (21)$$

Comparing Eq. 21 to Eq. 20, we are led by our estimate to conclude that the milliweak $\Delta S = 1$ CP violation effects are much bigger than the $\Delta S = 2$ K^0 mass mixing effects.*

The comparison between Eq. 21 and Eq. 20 is directly related to the measured parameter ratio ϵ'/ϵ. With A_0 and A_2 the $\Delta I = 1/2$ and $3/2$ amplitudes, respectively, we have

$$\frac{\epsilon'}{\epsilon} = \frac{Re A_2}{A_0} \cdot \frac{1}{\frac{\epsilon_m}{2\xi} + 1} \quad (22a)$$

when $Im A_2 << Re A_2 \cdot Im A_0 / A_0$, which is true in the model under discussion.** Now substituting the expression for ϵ_m and ξ

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* This is not true in the standard six quark model. For estimates of this effect see [20].

** The one Higgs-particle exchange process s+u → d+u is much smaller than the penguin graph process according to our estimate leading to Eq. 20 and our demonstration that $\bar{s}\gamma_5 d$ matrix elements cannot be enhanced.

from Eqs. 20 and 21, respectively, we obtain our principal result

$$\left|\frac{\epsilon'}{\epsilon}\right| = \left|\frac{Re A_2}{A_0}\right| \cdot \left[\frac{1}{\frac{M_s}{M_c} \cdot \frac{3}{16\sqrt{2}} \frac{\left(\frac{M_K}{M_s}\right)\left(\frac{1}{3} + \frac{M_s - M_d}{M_K}\right)}{\left(\frac{3}{2} + 2\ln\frac{M_H}{M_c}\right)} + 1} \right] \quad (22b)$$

As seen in table 1, for values of M_{H_i} such that $M_{H_i}^2/M_c^2 \gg 1$, one has

$$\left|\frac{\epsilon'}{\epsilon}\right| \simeq \left|\frac{Re A_2}{A_0}\right| \simeq 0.05 \quad . \quad (23)$$

Experimentally [15] $|\epsilon'/\epsilon| \simeq 0.003 \pm 0.015$, in sharp disagreement with Eq. 23. We conclude that the foregoing, simple treatment of the light-quark-mass sector of the Higgs-particle-generated CP nonconservation is untenable in this model. This treatment was patterned on the idea that the experimentally observed suppression is due simply to factors of M_q^2/M_H^2.[3,5]

Our conclusion that the prediction of ϵ'/ϵ in the model is too large is made on the basis of matrix element estimates that are free of inconsistencies. We showed that those estimates which enhance matrix elements of pseudoscalar quark bilinears relative to matrix elements of axial vector bilinears give incorrect results. Such an enhancement gave different answers for the $\Delta S = 2$ box graph, Fig. 2, depending on whether one choses momentum factors shown in Eq. 6, which yields Eq. 9, or equivalent ones $p_1 - p_2' \to -p_1' + p_2$ and $p_1 + p_1' \to p_2' + p_2$ which yields Eq. 12. Comparing Eqs. 9 and 12 (or 13a and 13b) one sees that the amplitudes and the $K^0 \leftrightarrow \bar{K}^0$ mixing derived from it are badly mangled. A similar discrepancy, Eq. 17a vs. Eq. 17b, was shown to result when evaluating the penguin graph Fig. 3.

In view of the error inherent to the pseudoscalar-operator-enhancement mechanism in the Higgs-mediated processes treated in this paper, we feel that such an enhancement mechanism applied to W-boson mediated processes [14,20] should be viewed with caution.

Note added in proof: After completion of this manuscript, we received the paper " A Difficulty for Weinberg Model of CP-Nonconservation Through Higgs Exchange " by N.G. Deshpande OITS 160 (1981) . Just as the authors of Ref. 13, Deshpande uses the matrix element evaluations of Vainstein et al., Ref. 14, which we have argued are incorrect in the application to the CP-violating processes in the model under discussion.

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M_H/M_c	$\lvert Im(\eta_i \xi_i^*)\rvert$	$\lvert \epsilon'/\epsilon \rvert$
3	3.5	0.0473
5	9.8	0.0492
7	19.2	0.0493
9	31.7	0.0495

Table 1 . Values of the CP nonconservation factor $Im(\eta_i \xi_i^*)$ which appears in the model, and predicted values of ϵ'/ϵ (Exp. $\epsilon'/\epsilon = 0.003 \pm 0.015$) for several values of M_{H_i}/M_c satisfying $M_{H_i}^2/M_c^2 >> 1$. Values $M_s + M_d = M_K$, $M_K/M_s = 3/2$, $(M_s - M_d)/M_K = 1/3$ have been chosen.

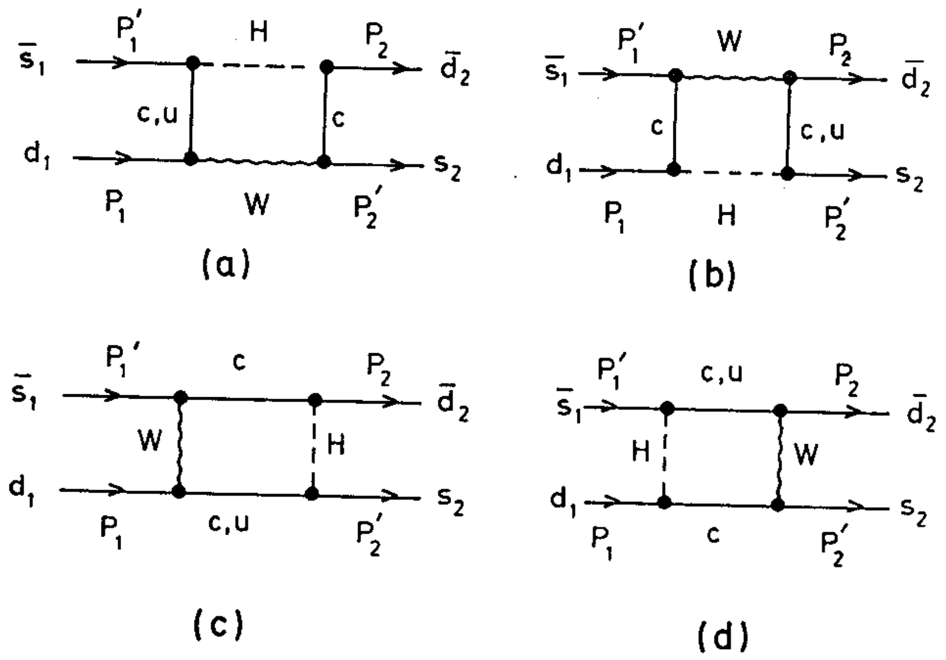

Fig. 1

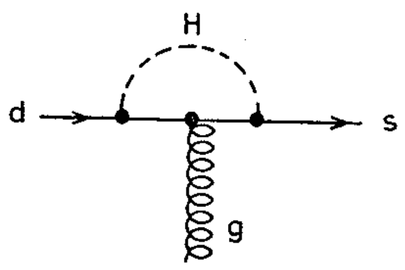

Fig. 2

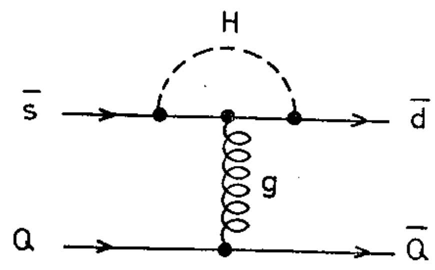

Fig. 3