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PERTURBATIVE CORRECTIONS TO MONTE CARLO Λ-PARAMETER DETERMINATIONS

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Perturbative Corrections to Monte Carlo Λ-Parameter Determinations.

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Abstract: We compute certain contributions to the lattice β_2 coefficient. These imply possibly, up to 30% corrections on m/Λ determinations from Monte Carlo calculations with Wilson, Manton and Villain actions. Other modifications of the standard Wilson action may have large corrections which may completely obscure the expected asymptotic freedom behaviour for moderate values of the bare coupling.

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Since Creutz's[1] results indicating the existence of a phase in lattice Yang-Mills theory with static quark confinement and asymptotic freedom there has been a rapid development and enthusiasm in performing Monte Carlo (MC) experiments. The stage has been reached where some groups[2] are determining QCD hadron spectra using MC methods and claiming good agreement with experiment, even for the m_π/m_ρ and m_π/m_N ratios. This state of affairs is very encouraging. However, to ensure that the MC experiments are giving us reliable results for the continuum limit, which are more significant than mere strong coupling expansions plus some extrapolation procedure incorporating asymptotic freedom, various improvements must be made. These include

i) proper account of finite size effects[3]

ii) estimates of lattice cutoff a^{-1} effects (a = lattice spacing) and their reduction by working with systematically improved lattice actions[4]

iii) a better treatment of lattice fermions than the methods at present available

iv) an estimate of effects of working at finite bare coupling g.

It is the latter perturbative effects with which this letter is concerned.

A physical mass m (i.e. having a finite continuum limit without multiplicative renormalization) behaves in the limit $a \to 0$, $g \to 0$, Λ_L finite as

$$m = c\Lambda_L \left(1 + O(a^2\Lambda_L^2)\right) \tag{1}$$

with

$$\Lambda_L = a^{-1} e^{-\frac{1}{2\beta_0 g^2}} (\beta_0 g^2)^{-\frac{\beta_1}{2\beta_0^2}} \left[1 + g^2 f(g^2)\right] \tag{2}$$

with

$$f(g^2) = \frac{1}{2\beta_0^3}(\beta_1^2 - \beta_0\beta_2) + O(g^2) \tag{3}$$

Here β_0, β_1 are the universal first two coefficients of the Callan-Symanzik β-function, which for SU(N) Y.M. are given by[5]

$$\beta_0 = \frac{11}{3}\frac{N}{16\pi^2} \quad , \quad \beta_1 = \frac{34}{3}\left(\frac{N}{16\pi^2}\right)^2 \tag{4}$$

If we have two masses m_1, m_2 we can get an idea of lattice cutoff effects by studying the ratio m_1/m_2 , which should be a constant with exponentially, damped corrections. Frequently, however, we have only one sufficiently well measured massive quantity, (eg. string tension) at our disposal and we use MC data to determine c. In practice, since as yet not even the lowest order correction f(0) has been calculated the MC data has been fitted to

$$m = c_f a^{-1} e^{-\frac{1}{2\beta_0 g^2}} (\beta_0 g^2)^{-\frac{\beta_1}{2\beta_0^2}} \tag{5}$$

comparing with (1), (2) we see

$$c_f \approx c\,(1 + g_f^2 + (g_f^2)) \; ; \; g_f \text{ in "fitting region"} \tag{6}$$

Since on a finite lattice we work in a region of finite bare coupling where the correlation length is not so large that finite size effects dominate, the correction factors in (6) may be significant. Indeed this appears to be the case in analogous calculations in the CP^{N-1} models[6], first pointed out by Martinelli, Parisi and Petronzio.

We should therefore indeed check for the lattice action under consideration, that the correction factor in (6) is not too large. An impression of the corrections involved comes from comparing measurements of a physical quantity using different lattice actions. Defining

$$\left(\Lambda_L/\Lambda_{L'}\right)_{exp} = c_{f'}/c_f \tag{7}$$

Lang, Rebbi and Salomonson[7] find $\left(\Lambda_L/\Lambda_{L'}\right)_{exp}$ differing by 40% from the theoretical value in the comparison of standard Wilson and Manton actions (see Table 1 taken from ref. 7). It would be satisfying if the perturbative corrections described above could account for these differences. In principle one should determine the relation between g and g' in the physical limit and compare the data in terms of one coupling. Without the raw data at hand we make the rough approximation $g_f \approx g_{f'}$ in (6) and thereby estimate

$$\Lambda_L/\Lambda_{L'} \simeq \left(\Lambda_L/\Lambda_{L'}\right)_{exp} \left(1 - g_f^2\, \delta_{LL'}\right) \tag{8}$$

with

$$\delta_{LL'} = \frac{\beta_{2L} - \beta_{2L'}}{2\beta_0^2} \tag{9}$$

One way to determine the relation between g, g', and thereby determine δ, is to compute the effective action $\Gamma(F)$ in the small coupling, slowly varying weak F approximation

$$\Gamma(F) = Q(F)\left[\frac{1}{g^2} + (2\beta_0 \ln a\lambda + l_1) + g^2(2\beta_1 \ln a\lambda + l_2) + \right. \\ \left. + O(a)\right] + \tag{10}$$

with λ an IR cutoff and

$$Q(F) = \frac{1}{4}\int d^4x\, F^a_{\mu\nu} F^a_{\mu\nu} \tag{11}$$

Given two lattice actions $\Gamma(F)$ should be the same in the physical continuum limit and thus

$$0 = \frac{1}{g^2} - \frac{1}{g'^2} + (l_1 - l_1') + (l_2 - l_2')g^2 + \ldots \tag{12}$$

Comparing with Eq. (2) we can relate the l_i coefficients to the $\Lambda_L/\Lambda_{L'}$ ratio and the quantity δ, viz

$$\Lambda_L/\Lambda_{L'} = e^{\left(\frac{l_1 - l_1'}{2\beta_0}\right)} \tag{13}$$

and

$$\delta_{LL'} = \frac{\ell_2 - \ell_2'}{2\beta_0} - \frac{\beta_1}{\beta_0}\left(\frac{\ell_1 - \ell_1'}{2\beta_0}\right) \qquad (14)$$

The easiest way to calculate the effective action is to use the background field method[8]. The plaquette variable in the action is then replaced by

$$U_{\mu\nu} = e^{ia^2 F_{\mu\nu}} e^{ia^2 g f_{\mu\nu}} = e^{ia^2 E_{\mu\nu}}$$

with $F_{\mu\nu}$, $f_{\mu\nu}$ related to the covariant curls of the background and quantum fields respectively[8]. Expanding, we obtain densities of the form

$$\begin{aligned} \mathcal{L}_{\mu\nu} = \frac{a^4}{g^2}\Big\{ & \tfrac{1}{4} b \left(F^a_{\mu\nu} F^a_{\mu\nu} + g^2 f^a_{\mu\nu} f^a_{\mu\nu} \right) \\ & - \frac{a^4}{4!}\left(g^2 c^{(2)}_{abcd} F^a_{\mu\nu} F^b_{\mu\nu} f^c_{\mu\nu} f^d_{\mu\nu} + g^3 c^{(3)}_{abcd} F^a_{\mu\nu} f^b_{\mu\nu} f^c_{\mu\nu} f^d_{\mu\nu} \right. \\ & \left. \qquad + g^4 c^{(4)}_{abcd} f^a_{\mu\nu} f^b_{\mu\nu} f^c_{\mu\nu} f^d_{\mu\nu} \right) \\ & + \frac{a^8}{6!} g^4 d^{(4)}_{abcdef} F^a_{\mu\nu} F^b_{\mu\nu} f^c_{\mu\nu} f^d_{\mu\nu} f^e_{\mu\nu} f^f_{\mu\nu} + \ldots \Big\} \end{aligned} \qquad (15)$$

+ terms linear in $f_{\mu\nu}$

+ terms more than quadratic in $F_{\mu\nu}$

where b, $c^{(a)}$, $d^{(4)}$ ... are functions of g^2, regular at $g^2 = 0$ and $b(0) = 1$. These coefficients depend on the lattice action used and those we need are tabulated in Table 2 for the Mixed Wilson, the Manton and the Villain actions for which the action densities are,

$$\mathcal{L}^{\text{Wilson}}_{\mu\nu} = \frac{1}{a^4 g^2}\left[\mathrm{Re}\,\mathrm{tr}(1-U_{\mu\nu}) + \gamma\, |\mathrm{tr}(1-U_{\mu\nu})|^2 \right]$$

$$\mathcal{L}^{\text{Manton}}_{\mu\nu} = \frac{1}{g^2}\,\mathrm{tr}\, E^2_{\mu\nu}$$

$$\mathcal{L}^{\text{Villain SU(2)}}_{\mu\nu} = \left(\frac{1}{g^2} - \frac{1}{12}\right)\mathrm{tr}\, E^2_{\mu\nu} - \frac{a^4}{360}\,\mathrm{tr}\, E^4_{\mu\nu} + \qquad (16)$$

The effective action has a diagrammatic loop expansion

$$\Gamma(F) = \sum_{\ell=0}^{\infty} \Gamma_\ell\, Q(F) + \ldots \qquad (17)$$

with

$$\Gamma_0 = b/g^2 \qquad (18)$$

Γ_1 was effectively first calculated by Hasenfratz and Hasenfratz[9] and checked by various authors[8,10]. Here we reproduce only the contribution from Fig. 1 arising from vertices in the lattice action (15) not present in the classical continuum action. It is the only diagram which contributes to the $\Lambda_L/\Lambda_{L'}$ ratio (when $b = 1$) and moreover gives a significant contribution to the ratio $\Lambda_L/\Lambda_{\text{cont.}}$ in the case of standard Wilson action. One finds

$$\Gamma_{1\,(\text{fig } 1)} = -\frac{1}{12}\, b^{-1} c^{(2)} \qquad (19)$$

where

$$c^{(2)} \delta_{ab} = c^{(2)}_{abcc} \qquad (20)$$

For

$$\begin{aligned} c^{(2)}_{abcd} = & -3\gamma\left[\delta_{ab}\delta_{cd} + 2\delta_{ac}\delta_{bd}\right] \\ & + c_1\, \mathrm{tr}\, T^a T^b T^c T^d + c_2\, \mathrm{tr}\, T^a T^c T^b T^d \end{aligned} \qquad (21)$$

we have

$$c^{(2)} = -3\gamma(N^2+1) + \frac{N^2-1}{4N} c_1 - \frac{1}{4N} c_2 \qquad (22)$$

The matrices T^a in (21) (generators in the fundamental representation) are normalized by $\mathrm{tr}\, T^a T^b = \frac{1}{2}\delta_{ab}$.

It turns out that the contribution to δ in Eq. 14 from the 1-loop diagram Fig. 1 is of the opposite sign from that required to improve the discrepances of Table 1. Thus 2-loop contributions must be included.

A full 2-loop calculation requires much labor and we are not yet in a position to present the complete result. Here we present partial results, easily calculable, to obtain an order of magnitude estimate.

The contributions to Γ_2 that we calculated are given in Fig. 2. That is to say we consider diagrams arising from vertices not contained in the classical continuum action, and neglected non-abelian parts of $f_{\mu\nu}$. Ghost and measure terms are also not considered. Experience from the 1-loop calculation considered above suggests that we get the right order of magnitude. Moreover most of the neglected diagrams cancel in the evaluation of $\delta_{LL'}$. The contributions from Figs. 2a, b, c are given by:

$$\Gamma_{2a} = \frac{g^2}{720}\, d^{(4)}\, b^{-2}$$

$$\Gamma_{2b} = -\frac{g^2}{288}\, c^{(4)} c^{(4)}\, b^{-3} \qquad (23)$$

$$\Gamma_{2c} = -\frac{g^2}{144}\, S\, c^{(3,3)}\, b^{-3}$$

where

$$d^{(4)}\delta_{ab} = d^{(4)}_{abcdef}\left(\delta_{cd}\delta_{ef} + \delta_{ce}\delta_{df} + \delta_{cf}\delta_{de}\right)$$

$$c^{(4)}\delta_{cd} = c^{(4)}_{a'b'c'd'}\left(\delta_{ca'}\delta_{db'}\delta_{c'd'} + 11 \text{ perms of primed indices}\right) \qquad (24)$$

$$c^{(3,3)}\delta_{aa'} = c^{(3)}_{abcd}\, c^{(3)}_{a'b'c'd'}\left(\delta_{bb'}\delta_{cc'}\delta_{dd'} + 5 \text{ perms of primed indices}\right)$$

and S, the only nontrivial integral appearing in our expressions, is given by,

$$S = \int_{-\pi}^{\pi}\frac{d^4p}{(2\pi)^4}\int_{-\pi}^{\pi}\frac{d^4q}{(2\pi)^4}\, y(p)\,y(q)\,y(p+q) \qquad (25)$$

where

$$y(p) = \frac{\sin^2 p_1/2 + \sin^2 p_2/2}{\sum_{\mu=1}^{4}\sin^2 p_\mu/2} \qquad (26)$$

Stehr has estimated S using a non-sophisticated MC programme and obtained

$$S \approx .125 \qquad (27)$$

For

$$\begin{aligned} d^{(4)}_{abcdef} = {} & d_1 \operatorname{tr} T^aT^bT^cT^dT^eT^f + d_2 \operatorname{tr} T^aT^cT^bT^dT^eT^f \\ & + d_3 \operatorname{tr} T^aT^cT^dT^bT^eT^f \\ & + 30\gamma\left\{4 \operatorname{tr} T^aT^bT^c \operatorname{tr} T^dT^eT^f + 6 \operatorname{tr} T^aT^cT^d \operatorname{tr} T^bT^eT^f\right. \\ & \left. -3\,\delta_{cd} \operatorname{tr} T^aT^bT^eT^f - 4\,\delta_{ac} \operatorname{tr} T^bT^dT^eT^f - \tfrac{1}{2}\delta_{ab} \operatorname{tr} T^cT^dT^eT^f\right\} \end{aligned} \qquad (28)$$

$$c^{(4)}_{abcd} = e \operatorname{tr} T^aT^bT^cT^d - \frac{3\gamma}{2}\delta_{ab}\delta_{cd}$$

$$c^{(3)}_{abcd} = h \operatorname{tr} T^aT^bT^cT^d - 6\gamma\,\delta_{ab}\delta_{cd}$$

one obtains

$$\begin{aligned} d^{(4)} = {} & \frac{1}{8N^2}\left\{(2N^2-3)\left([N^2-1]d_1 - d_2\right) + (N^4-N^2+3)d_3\right\} \\ & - \frac{15}{4N}(8N^2-3)(N^2+1)\gamma \end{aligned} \qquad (29)$$

$$c^{(4)} = \frac{1}{N}(2N^2-3)e - 6(N^2+1)\gamma$$

$$c^{(3,3)} = \frac{1}{16N^2}(N^4-6N^2+18)h^2 - \frac{6}{N}(2N^2-3)h\gamma + 72(N^2+1)\gamma$$

Collecting the pieces together (Table 3) and using the fact that b = 1 for Wilson and Manton actions we have

$$\begin{aligned} \delta_{\text{Manton, Wilson}}\Big|_{N=2} & \approx (-.037 + .128) - (-.035 + .079) \\ & \approx +.05 \end{aligned} \qquad (30)$$

The sign in (30) means that "experimental value" of Λ_W/Λ_M is expected to be greater than the theoretical value, which is in fact the case (see Table 1). Taking $g_\gamma^2 \approx 2$ our correction factor would be $\approx .9$. Although this is not quite big enough to account for the results of Table 1, it is reassuring that it is of the correct order of magnitude.

For the Villain action, $b = 1 - \frac{1}{12} g^2$. One then obtains

$$l_{1\,\text{Manton}} - l_{1\,\text{Villain}} = \frac{1}{12} \tag{31}$$

There are no two-loop contributions to the difference in our crude approximation. The difference arises solely from the 1-loop contribution and the fact that coefficients b, c_1, c_2 are nontrivial functions of g (Table 2). We obtain

$$\begin{aligned} l_{2\,\text{Manton}} - l_{2\,\text{Villain}}\Big|_{N=2} &\approx \frac{1}{12}\left\{-b'(0)\,c_2(0) + c_2'(0)\right\}_{\text{Villain}\ N=2} \\ &= \frac{1}{96} \end{aligned} \tag{32}$$

From (31), (32) we estimate

$$\delta_{\text{Manton, Villain}} \approx .112 - .035 = .077 \tag{33}$$

This gives a satisfactory account for the discrepancy between Manton and Villain results in Table 1.

The Wilson action with $\gamma \neq 0$ whose phase structure has been widely studied[11], has been proposed by Lüscher[12] to be used in studies of the topological charge on the lattice. The danger is that for γ too large the correction factors may swamp the expected asymptotic freedom behaviour for reasonable values of g.

In conclusion our results for the actions used so far in Monte Carlo analyses of SU(2) Y.M. theory indicate that perturbative effects are indeed of the expected order of magnitude to explain observed discrepances. A serious MC determination of m/Λ values should take these effects into account[13] as well as intermingled finite size effects.

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References

1. M. Creutz, Phys. Rev. D21 (1980) 2308
2. D. Weingarten, Monte Carlo Evaluation of Hadron Masses in Lattice Gauge Theories with Fermions, Indiana preprint IMHET-69,
   H. Hamber and G. Parisi, Numerical estimates of hadronic masses in a pure SU(3) gauge theory, BNL preprint (1981)
3. see eg. M. Nauenberg, T. Schalk and R. Brower, Finite Size Scaling and Asymptotic Freedom of the SU(2) Lattice Gauge Model
4. K. Symanzik, Talk presented at VI International Conference on Mathematical Physics, Berlin, August 1981
5. H.D. Politzer, Phys. Rev. Lett. 30 (1973) 1346,
   D.J. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343,
   W. Caswell, Phys. Rev. Lett. 33 (1974) 344,
   D.R. Jones, Nucl. Phys. B75 (1974) 531
6. G. Martinelli, G. Parisi and R. Petronzio, Phys. Lett. 100B (1981) 485,
   B. Berg and M. Lüscher, Nucl. Phys. B190 FS3 (1981) 412,
   M. Lüscher, Bern preprint, BUTP-4/1981
7. C.B. Lang, C. Rebbi, P. Salomonson and B.S. Skagerstam, On the Definitions of the Gauge Theory Coupling in Lattice and Continuum QCD-Implications of Change in Lattice Action, Göteborg preprint 81/22
8. R. Dashen and D.J. Gross, Phys. Rev. D23 (1981) 2340,
   P. Weisz, Phys. Lett. 100B (1981) 331,
   A. and P. Hasenfratz, The Scales of Euclidean and Hamilton Lattice QCD, Budapest preprint KFKI-1981-15,
   A. Gonzalez-Arroyo and C.P. Korthals Altes, Asymptotic Freedom Scales for any Lattice Action, Marseille preprint CPT-81/P. 1303
9. A. and P. Hasenfratz, Phys. Lett. 93B (1980) 165
10. H. Kawai, R. Nakayawa and K. Seo, Nucl. Phys. B189 (1981) 40

11. I.G. Halliday and A. Schwimmer, Phys. Lett. 101 (1981) 327,
    G. Bhanot and M. Creutz, Variant Actions and Phase Structure in Lattice Gauge Theory, Brookhaven preprint, BNL 29640
12. M. Lüscher, Topology of Lattice Gauge Fields, Bern preprint, BUTP 10/81
13. C.P. Korthals Altes, has informed us that he is also calculating β_{1L}

Figure Captions

Fig. 1: 1-loop contribution to the effective action (quadratic part).

Fig. 2: 2-loop contributions to the quadratic part of the effective action coming from vertices in the lattice action not present in the continuum action.

Table 1

	Expt	Thy
$\Lambda_M/\Lambda_W \big\vert_{N=2}$	4.40	3.07
$\Lambda_M/\Lambda_V \big\vert_{N=2}$	2.99	2.45

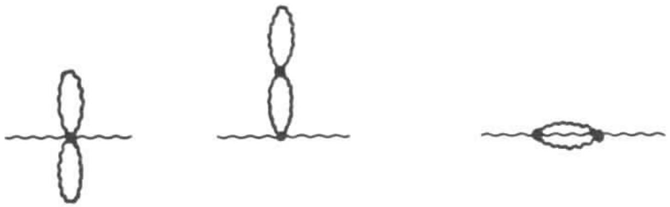
Table 2

Action	b	c_1	c_2	d_1	d_2	d_3	e	h
Wilson	1	6	0	15	0	0	1	4
Manton	1	2	-2	-1	4	-3	0	0
Villain, SU(2)	$1 - \frac{1}{12}g^2$	$2 + \frac{2}{15}g^2$	$-2 + \frac{1}{15}g^2$	$-1 + O(g^2)$	$4 + O(g^2)$	$-3 + O(g^2)$	$O(g^2)$	$O(g^2)$

Table 3

		Wilson	Manton
	$\Gamma_{1(\text{Fig.1})}$	$-\frac{1}{4}\left[\frac{N^2-1}{2N}-\gamma(N^2+1)\right]$ $\underset{N=2}{=} -\frac{3}{16}\left(1-\frac{20}{3}\gamma\right)$	$-\frac{N}{24}$ $\underset{N=2}{=} -\frac{1}{12}$
	$\Gamma_{2(a)}$	$\frac{1}{192N}\left[(2N^2-3)\left(\frac{N^2-1}{2N}\right)-\gamma(8N^2-3)(N^2+1)\right]$ $\underset{N=2}{=} \frac{5}{512}\left(1-\frac{116}{3}\gamma\right)$	$-\frac{N^2}{1152}$ $\underset{N=2}{=} -\frac{1}{288}$
	$\Gamma_{2(b)}$	$-\frac{1}{48}\left[\frac{2N^2-3}{2N}-3\gamma(N^2+1)\right]\left[\frac{N^2-1}{2N}-\gamma(N^2+1)\right]$ $\underset{N=2}{=} -\frac{5}{256}(1-12\gamma)\left(1-\frac{20}{3}\gamma\right)$	0
	$\Gamma_{2(c)}$	$-\frac{5}{144}\left[\left(N^2-6+\frac{18}{N^2}\right)-48\gamma\left(\frac{2N^2-3}{2N}\right)+72\gamma^2(N^2+1)\right]$ $\underset{N=2}{=} -\frac{5}{288}(1-12\gamma)^2$	0
N=2	$\ell_{1(\text{Fig.1})}\frac{\beta_1}{2\beta_0^2}$	$-.079 + .527\gamma$	$-.035$
N=2	$\ell_{2(a)}\frac{1}{2\beta_0}$	$.105 - 4.066\gamma$	$-.037$
N=2	$\ell_{2(b)}\frac{1}{2\beta_0}$	$-.210 + 3.925\gamma - 16.823\gamma^2$	0
N=2	$\ell_{2(c)}\,(S=\frac{1}{3})\;\frac{1}{2\beta_0}$	$-.023 + .561\gamma - 3.565\gamma^2$	0

Fig. 1



(a) (b) (c)

Fig. 2